Computer algebra independent integration tests

`1_Algebraic_functions/1.2_Trinomial_products/1.2.3General/1.2.3.2(dx)^m(a+bx^n+cx^(2`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

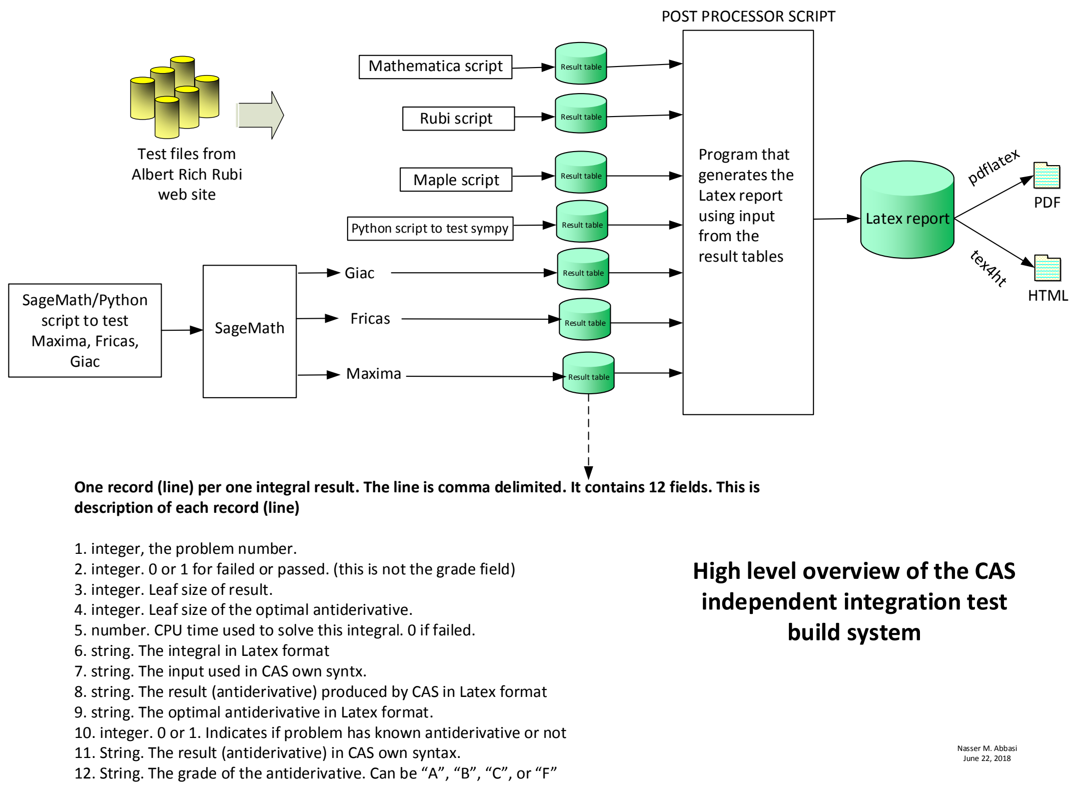
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (664)	% 0. (0)
Rubi in Sympy	% 88.86 (590)	% 11.14 (74)
Mathematica	% 99.7 (662)	% 0.3 (2)
Maple	% 74.55 (495)	% 25.45 (169)
Maxima	% 36.6 (243)	% 63.4 (421)
Fricas	% 82.23 (546)	% 17.77 (118)
Sympy	% 42.47 (282)	% 57.53 (382)
Giac	% 54.52 (362)	% 45.48 (302)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

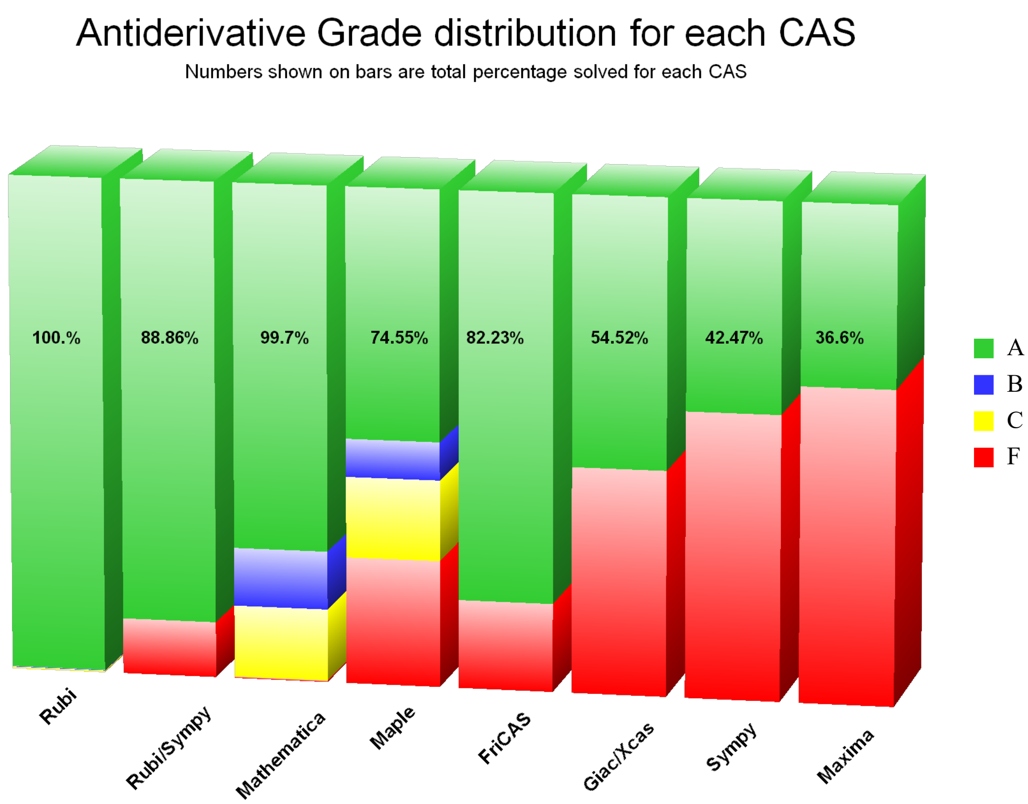
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

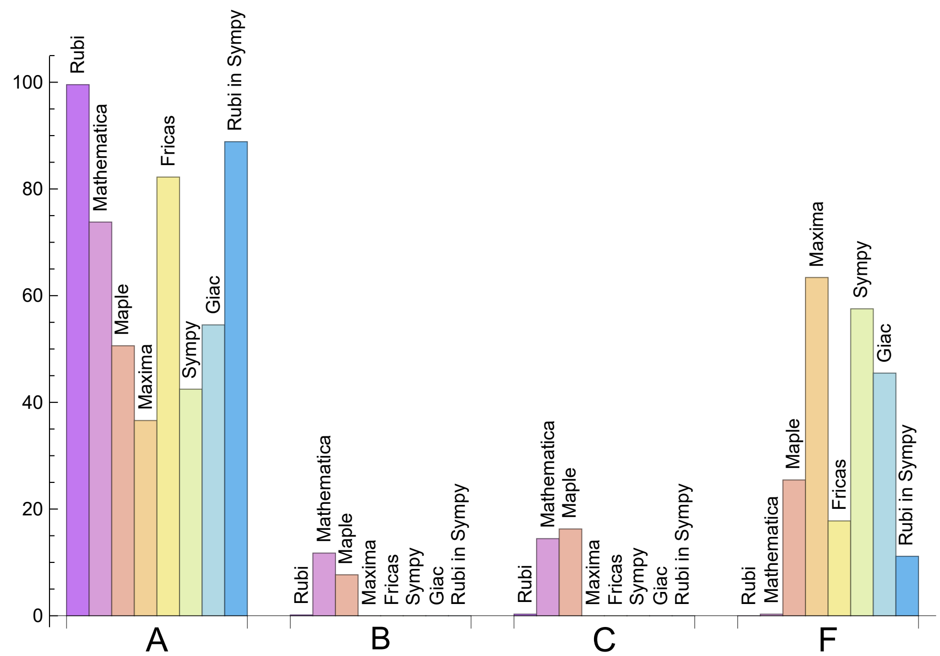
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.55	0.15	0.3	0.
Rubi in Sympy	88.86	0.	0.	11.14
Mathematica	73.8	11.75	14.46	0.3
Maple	50.6	7.68	16.27	25.45
Maxima	36.6	0.	0.	63.4
Fricas	82.23	0.	0.	17.77
Sympy	42.47	0.	0.	57.53
Giac	54.52	0.	0.	45.48

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.38	157.98	1.	137.	1.
Rubi in Sympy	34.63	139.54	0.91	121.	0.89
Mathematica	0.67	236.6	1.63	83.	0.94
Maple	0.03	289.31	1.71	78.	0.79
Maxima	0.8	85.64	0.98	57.	1.01
Fricas	0.32	641.24	2.21	74.	1.
Sympy	10.33	241.1	2.03	65.	0.94
Giac	0.32	129.06	1.17	90.	1.09

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 104, 105, 107, 109, 110, 112, 113, 115, 116, 143, 144, 149, 150, 151, 412, 413, 414, 422, 423, 424, 431, 432, 440, 441, 457, 493, 494, 524, 549, 560, 600, 601, 607, 608, 609, 610, 611, 612, 629, 634, 636, 653, 659, 663, 664}

Not solved by Mathematica {661, 662}

Not solved by Maple {30, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 472, 473, 474, 475, 476, 477, 478, 506, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 545, 546, 547, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662}

Not solved by Maxima {6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 52, 55, 58, 61, 64, 67, 70, 73, 76, 79, 82, 85, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 353, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 396, 397, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 462, 463, 464, 465, 472, 476, 477, 500, 501, 502, 503, 504, 505, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662}

Not solved by Fricas {121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 459, 461, 472, 476, 477, 481, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662}

Not solved by Sympy {1, 2, 3, 4, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 142, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 318, 327, 347, 367, 385, 449, 450, 451, 452, 453, 454, 455, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 621, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 646, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662}

Not solved by Giac {3, 4, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 182, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 340, 341, 343, 344, 346, 347, 353, 367, 378, 379, 380, 381, 382, 383, 384, 385, 449, 450, 451, 452, 453, 454, 457, 458, 459, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 613, 614, 615, 616, 617, 620, 621, 622, 623, 624, 625, 626, 627, 629, 630, 631, 632, 633, 636, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 659, 661, 662}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {378, 379, 382, 383, 384, 662}

Mathematica {132, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 309, 327, 329, 331, 333, 338, 340, 341, 343, 344, 346, 347, 351, 353, 367, 385, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 602, 603, 604, 605, 606}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	39	62	72	0	107	44
normalized size	1	1.	0.81	0.75	1.19	1.38	0.	2.06	0.85
time (sec)	N/A	0.078	0.041	0.008	0.783	0.267	0.	0.279	7.881

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	19	38	0	19	19
normalized size	1	1.	1.	1.16	0.76	1.52	0.	0.76	0.76
time (sec)	N/A	0.013	0.016	0.006	0.772	0.264	0.	0.286	1.38

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	23	28	0	0	19
normalized size	1	1.	1.	1.17	1.	1.22	0.	0.	0.83
time (sec)	N/A	0.013	0.022	0.006	0.768	0.274	0.	0.	1.385

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	46	46	51	73	0	0	70
normalized size	1	1.	0.6	0.6	0.66	0.95	0.	0.	0.91
time (sec)	N/A	0.097	0.036	0.007	0.771	0.274	0.	0.	9.217

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	50	76	48	51	44
normalized size	1	1.	1.	0.79	1.04	1.58	1.	1.06	0.92
time (sec)	N/A	0.052	0.026	0.01	0.847	0.27	0.357	0.292	7.486

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	18	12	31	0
normalized size	1	1.	0.49	0.46	0.	0.23	0.15	0.39	0.
time (sec)	N/A	0.069	0.023	0.006	0.	0.268	0.209	0.281	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	18	12	39	0
normalized size	1	1.	0.49	0.46	0.23	0.23	0.15	0.49	0.
time (sec)	N/A	0.067	0.013	0.005	0.782	0.264	0.215	0.273	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	18	12	39	0
normalized size	1	1.	0.49	0.46	0.23	0.23	0.15	0.49	0.
time (sec)	N/A	0.066	0.012	0.004	0.785	0.266	0.22	0.279	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	18	12	30	0
normalized size	1	1.	1.06	0.97	0.	0.5	0.33	0.83	0.
time (sec)	N/A	0.068	0.012	0.004	0.	0.266	0.204	0.308	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	18	12	39	0
normalized size	1	1.	0.49	0.46	0.23	0.23	0.15	0.49	0.
time (sec)	N/A	0.054	0.013	0.004	0.817	0.266	0.207	0.277	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	14	14	8	27	0
normalized size	1	1.	0.49	0.45	0.19	0.19	0.11	0.36	0.
time (sec)	N/A	0.034	0.015	0.003	0.815	0.265	0.195	0.268	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	0	15	10	38	0
normalized size	1	1.	0.49	0.45	0.	0.2	0.13	0.51	0.
time (sec)	N/A	0.061	0.016	0.014	0.	0.274	0.277	0.269	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	38	36	19	19	8	39	0
normalized size	1	1.	0.49	0.47	0.25	0.25	0.1	0.51	0.
time (sec)	N/A	0.062	0.019	0.004	0.766	0.264	1.108	0.281	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	37	34	20	20	8	35	0
normalized size	1	1.	0.5	0.46	0.27	0.27	0.11	0.47	0.
time (sec)	N/A	0.062	0.013	0.004	0.756	0.247	1.213	0.257	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	0	23	10	58	0
normalized size	1	1.	0.52	0.51	0.	0.31	0.13	0.77	0.
time (sec)	N/A	0.064	0.016	0.015	0.	0.282	1.197	0.297	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	18	18	14	41	0
normalized size	1	1.	0.48	0.44	0.23	0.23	0.18	0.53	0.
time (sec)	N/A	0.065	0.013	0.004	0.765	0.25	1.323	0.271	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	20	15	42	0
normalized size	1	1.	0.49	0.46	0.25	0.25	0.19	0.53	0.
time (sec)	N/A	0.064	0.014	0.004	0.785	0.253	1.255	0.265	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	37	34	0	18	14	41	0
normalized size	1	1.	0.47	0.43	0.	0.23	0.18	0.52	0.
time (sec)	N/A	0.063	0.017	0.004	0.	0.249	1.272	0.285	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	20	15	42	0
normalized size	1	1.	0.49	0.46	0.25	0.25	0.19	0.53	0.
time (sec)	N/A	0.063	0.016	0.004	0.784	0.257	1.319	0.274	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	20	15	42	0
normalized size	1	1.	0.49	0.46	0.25	0.25	0.19	0.53	0.
time (sec)	N/A	0.063	0.013	0.005	0.791	0.272	1.373	0.287	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	20	15	42	0
normalized size	1	1.	0.49	0.46	0.	0.25	0.19	0.53	0.
time (sec)	N/A	0.063	0.014	0.005	0.	0.259	1.404	0.269	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	20	15	42	0
normalized size	1	1.	0.49	0.46	0.25	0.25	0.19	0.53	0.
time (sec)	N/A	0.064	0.014	0.006	0.776	0.275	1.413	0.287	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.124	0.033	0.011	0.781	0.255	0.	0.266	17.258

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	167	61	58	0	47	0	90	107
normalized size	1	1.4	0.51	0.49	0.	0.39	0.	0.76	0.9
time (sec)	N/A	0.153	0.029	0.011	0.	0.254	0.	0.303	21.886

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.12	0.03	0.01	0.76	0.251	0.	0.269	17.323

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.121	0.032	0.01	0.762	0.256	0.	0.272	17.968

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	58	0	47	0	61	65
normalized size	1	1.	0.78	0.74	0.	0.6	0.	0.78	0.83
time (sec)	N/A	0.128	0.031	0.01	0.	0.249	0.	0.299	14.732

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.122	0.027	0.01	0.77	0.252	0.	0.269	17.326

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.12	0.031	0.009	0.807	0.255	0.	0.302	17.912

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	60	0	0	47	0	59	34
normalized size	1	1.	1.67	0.	0.	1.31	0.	1.64	0.94
time (sec)	N/A	0.069	0.028	180.	0.	0.254	0.	0.309	9.141

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.105	0.029	0.007	0.798	0.25	0.	0.297	15.581

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	59	56	43	43	0	86	129
normalized size	1	1.	0.36	0.35	0.27	0.27	0.	0.53	0.8
time (sec)	N/A	0.08	0.031	0.005	0.785	0.251	0.	0.26	5.605

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	60	57	0	43	0	88	117
normalized size	1	1.	0.38	0.36	0.	0.27	0.	0.55	0.73
time (sec)	N/A	0.121	0.037	0.012	0.	0.259	0.	0.291	16.882

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	50	0	90	128
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.55	0.78
time (sec)	N/A	0.114	0.029	0.01	0.8	0.27	0.	0.278	17.269

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	50	50	0	88	134
normalized size	1	1.	0.37	0.36	0.31	0.31	0.	0.54	0.82
time (sec)	N/A	0.113	0.032	0.009	0.784	0.255	0.	0.295	16.866

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	62	59	0	51	0	115	126
normalized size	1	1.	0.39	0.37	0.	0.32	0.	0.71	0.78
time (sec)	N/A	0.129	0.039	0.019	0.	0.26	0.	0.274	16.755

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	50	0	93	139
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.84
time (sec)	N/A	0.114	0.034	0.009	0.803	0.266	0.	0.275	16.337

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	50	50	0	92	136
normalized size	1	1.	0.37	0.36	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.112	0.033	0.009	0.791	0.264	0.	0.292	11.524

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	61	60	0	53	0	116	128
normalized size	1	1.	0.38	0.37	0.	0.33	0.	0.72	0.79
time (sec)	N/A	0.125	0.028	0.018	0.	0.291	0.	0.273	17.026

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	50	0	95	134
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.58	0.81
time (sec)	N/A	0.116	0.023	0.009	0.775	0.257	0.	0.277	17.031

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	61	58	50	50	0	90	138
normalized size	1	1.	0.38	0.36	0.31	0.31	0.	0.56	0.85
time (sec)	N/A	0.11	0.024	0.009	0.778	0.255	0.	0.296	16.924

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	63	60	0	53	0	115	138
normalized size	1	1.	0.39	0.37	0.	0.33	0.	0.71	0.86
time (sec)	N/A	0.125	0.043	0.018	0.	0.257	0.	0.27	22.474

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.84
time (sec)	N/A	0.114	0.027	0.01	0.758	0.28	0.	0.277	16.998

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.114	0.031	0.01	0.758	0.262	0.	0.26	17.05

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	0	47	0	92	39
normalized size	1	1.	1.44	1.37	0.	1.15	0.	2.24	0.95
time (sec)	N/A	0.056	0.026	0.011	0.	0.265	0.	0.319	8.428

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.115	0.024	0.01	0.81	0.267	0.	0.282	16.971

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.115	0.034	0.011	0.834	0.264	0.	0.275	17.099

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	58	0	50	0	93	68
normalized size	1	1.	0.73	0.69	0.	0.6	0.	1.11	0.81
time (sec)	N/A	0.108	0.026	0.011	0.	0.264	0.	0.266	8.707

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.115	0.025	0.011	0.771	0.266	0.	0.287	16.992

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.172	0.044	0.012	0.777	0.262	0.	0.268	27.247

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.166	0.037	0.012	0.781	0.252	0.	0.286	27.142

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	0	77	0	142	151
normalized size	1	1.	0.52	0.5	0.	0.48	0.	0.89	0.94
time (sec)	N/A	0.264	0.04	0.01	0.	0.257	0.	0.27	29.952

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.171	0.037	0.012	0.776	0.267	0.	0.267	27.112

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.169	0.041	0.011	0.789	0.274	0.	0.287	27.242

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	0	77	0	142	107
normalized size	1	1.	0.7	0.67	0.	0.65	0.	1.19	0.9
time (sec)	N/A	0.208	0.037	0.009	0.	0.25	0.	0.289	21.982

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.166	0.039	0.011	0.816	0.254	0.	0.27	27.213

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.166	0.036	0.011	0.802	0.251	0.	0.3	28.113

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	80	0	77	0	90	65
normalized size	1	1.	1.06	1.03	0.	0.99	0.	1.15	0.83
time (sec)	N/A	0.14	0.036	0.009	0.	0.25	0.	0.267	14.741

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.164	0.037	0.01	0.822	0.254	0.	0.265	27.09

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	76	76	0	140	207
normalized size	1	1.	0.33	0.32	0.3	0.3	0.	0.56	0.82
time (sec)	N/A	0.163	0.041	0.01	0.782	0.256	0.	0.289	28.149

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	79	0	77	0	89	34
normalized size	1	1.	2.28	2.19	0.	2.14	0.	2.47	0.94
time (sec)	N/A	0.069	0.036	0.01	0.	0.268	0.	0.27	9.212

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	76	76	0	140	207
normalized size	1	1.	0.33	0.32	0.3	0.3	0.	0.56	0.82
time (sec)	N/A	0.148	0.035	0.008	0.807	0.264	0.	0.277	24.454

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	81	78	72	72	0	136	197
normalized size	1	1.	0.33	0.32	0.29	0.29	0.	0.55	0.8
time (sec)	N/A	0.122	0.037	0.006	0.811	0.259	0.	0.272	9.86

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	0	74	0	140	178
normalized size	1	1.	0.33	0.31	0.	0.29	0.	0.56	0.71
time (sec)	N/A	0.172	0.051	0.012	0.	0.272	0.	0.297	26.481

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	80	0	142	196
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.78
time (sec)	N/A	0.158	0.038	0.01	0.788	0.263	0.	0.28	26.71

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	80	0	139	206
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.55	0.82
time (sec)	N/A	0.157	0.036	0.01	0.82	0.271	0.	0.287	26.698

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	82	0	167	199
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.66	0.79
time (sec)	N/A	0.186	0.052	0.019	0.	0.262	0.	0.283	26.165

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	80	80	0	144	216
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.58	0.87
time (sec)	N/A	0.159	0.05	0.009	0.764	0.27	0.	0.3	25.973

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	80	0	143	211
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.84
time (sec)	N/A	0.158	0.038	0.011	0.762	0.268	0.	0.27	16.208

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	82	0	170	201
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.67	0.8
time (sec)	N/A	0.181	0.049	0.019	0.	0.259	0.	0.289	27.134

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	80	80	0	144	209
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.58	0.84
time (sec)	N/A	0.16	0.041	0.01	0.772	0.274	0.	0.263	27.25

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	80	0	142	214
normalized size	1	1.	0.34	0.32	0.32	0.32	0.	0.57	0.87
time (sec)	N/A	0.158	0.037	0.01	0.794	0.262	0.	0.258	27.166

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	82	0	171	209
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.68	0.83
time (sec)	N/A	0.181	0.046	0.019	0.	0.269	0.	0.3	27.082

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	80	0	146	212
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.58	0.84
time (sec)	N/A	0.161	0.033	0.01	0.817	0.253	0.	0.259	26.364

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	80	0	143	209
normalized size	1	1.	0.34	0.32	0.32	0.32	0.	0.58	0.85
time (sec)	N/A	0.157	0.03	0.009	0.796	0.287	0.	0.297	21.39

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	0	82	0	169	206
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.67	0.82
time (sec)	N/A	0.179	0.035	0.02	0.	0.258	0.	0.305	27.041

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	80	0	146	207
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.58	0.82
time (sec)	N/A	0.162	0.031	0.01	0.789	0.268	0.	0.279	26.948

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	80	80	0	142	211
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.85
time (sec)	N/A	0.157	0.033	0.009	0.803	0.256	0.	0.294	26.695

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	0	82	0	166	209
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.66	0.83
time (sec)	N/A	0.175	0.048	0.019	0.	0.263	0.	0.283	32.273

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.84
time (sec)	N/A	0.158	0.038	0.011	0.803	0.269	0.	0.275	26.837

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.83
time (sec)	N/A	0.159	0.035	0.012	0.795	0.255	0.	0.275	26.81

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	0	77	0	143	39
normalized size	1	1.	1.98	1.9	0.	1.88	0.	3.49	0.95
time (sec)	N/A	0.059	0.031	0.011	0.	0.259	0.	0.286	8.404

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.83
time (sec)	N/A	0.163	0.035	0.011	0.79	0.27	0.	0.29	27.987

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.163	0.034	0.012	0.803	0.253	0.	0.279	26.891

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	80	0	80	0	144	68
normalized size	1	1.	0.99	0.95	0.	0.95	0.	1.71	0.81
time (sec)	N/A	0.11	0.032	0.012	0.	0.254	0.	0.28	8.64

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.161	0.031	0.012	0.785	0.252	0.	0.28	26.814

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.161	0.037	0.012	0.783	0.254	0.	0.272	26.811

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	0	80	0	144	112
normalized size	1	1.	0.65	0.62	0.	0.62	0.	1.12	0.88
time (sec)	N/A	0.145	0.035	0.015	0.	0.252	0.	0.261	15.649

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	131	113	0	190	32	197	0
normalized size	1	1.	0.55	0.47	0.	0.79	0.13	0.82	0.
time (sec)	N/A	0.278	0.106	0.016	0.	0.266	1.352	0.304	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	128	110	0	157	22	193	0
normalized size	1	1.	0.54	0.47	0.	0.67	0.09	0.82	0.
time (sec)	N/A	0.264	0.058	0.01	0.	0.275	1.325	0.317	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	23	18	10	30	29
normalized size	1	1.	0.8	0.73	0.52	0.41	0.23	0.68	0.66
time (sec)	N/A	0.083	0.015	0.009	0.782	0.258	0.421	0.296	2.175

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	109	97	0	134	24	184	0
normalized size	1	1.	0.54	0.48	0.	0.66	0.12	0.91	0.
time (sec)	N/A	0.191	0.048	0.008	0.	0.282	0.432	0.283	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	109	97	0	120	20	165	0
normalized size	1	1.	0.54	0.48	0.	0.59	0.1	0.82	0.
time (sec)	N/A	0.254	0.04	0.006	0.	0.26	0.467	0.294	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	37	0	24	15	43	0
normalized size	1	1.	0.52	0.46	0.	0.3	0.19	0.54	0.
time (sec)	N/A	0.096	0.024	0.013	0.	0.274	0.721	0.278	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	133	111	0	171	29	177	0
normalized size	1	1.	0.56	0.47	0.	0.72	0.12	0.74	0.
time (sec)	N/A	0.247	0.065	0.013	0.	0.271	1.451	0.28	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	140	118	0	217	32	169	0
normalized size	1	1.	0.58	0.49	0.	0.9	0.13	0.7	0.
time (sec)	N/A	0.242	0.068	0.013	0.	0.264	1.571	0.28	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	54	52	0	45	31	68	0
normalized size	1	1.	0.44	0.43	0.	0.37	0.25	0.56	0.
time (sec)	N/A	0.125	0.028	0.018	0.	0.26	1.949	0.28	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	235	299	0	278	0	4	0
normalized size	1	1.	0.84	1.07	0.	0.99	0.	0.01	0.
time (sec)	N/A	0.303	0.144	0.024	0.	0.278	0.	0.643	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	235	299	0	259	0	4	0
normalized size	1	1.	0.85	1.08	0.	0.94	0.	0.01	0.
time (sec)	N/A	0.294	0.136	0.023	0.	0.271	0.	0.708	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	35	0	4	36
normalized size	1	1.	0.71	0.63	0.63	0.92	0.	0.11	0.95
time (sec)	N/A	0.072	0.02	0.01	0.807	0.261	0.	0.741	9.281

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	237	301	0	277	0	4	0
normalized size	1	1.	0.86	1.09	0.	1.	0.	0.01	0.
time (sec)	N/A	0.294	0.135	0.014	0.	0.272	0.	0.734	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	235	299	0	259	0	4	0
normalized size	1	1.	0.82	1.05	0.	0.91	0.	0.01	0.
time (sec)	N/A	0.306	0.148	0.012	0.	0.273	0.	0.708	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	0	122	0	4	144
normalized size	1	1.	0.5	0.73	0.	0.83	0.	0.03	0.98
time (sec)	N/A	0.184	0.051	0.02	0.	0.265	0.	0.721	28.117

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	260	316	0	306	0	4	0
normalized size	1	1.	0.82	1.	0.	0.97	0.	0.01	0.
time (sec)	N/A	0.353	0.177	0.027	0.	0.266	0.	0.707	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	266	322	0	355	0	4	0
normalized size	1	1.	0.84	1.02	0.	1.12	0.	0.01	0.
time (sec)	N/A	0.357	0.197	0.027	0.	0.274	0.	0.732	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	97	133	0	161	0	4	180
normalized size	1	1.	0.52	0.71	0.	0.86	0.	0.02	0.96
time (sec)	N/A	0.215	0.061	0.023	0.	0.275	0.	0.697	35.764

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	218	519	0	416	0	4	0
normalized size	1	1.	0.61	1.45	0.	1.16	0.	0.01	0.
time (sec)	N/A	0.411	0.266	0.028	0.	0.274	0.	1.108	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	39	32	65	78	0	4	66
normalized size	1	1.	0.5	0.41	0.83	1.	0.	0.05	0.85
time (sec)	N/A	0.124	0.034	0.012	0.785	0.262	0.	0.734	14.879

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	229	521	0	431	0	4	0
normalized size	1	1.	0.62	1.42	0.	1.17	0.	0.01	0.
time (sec)	N/A	0.425	0.27	0.028	0.	0.28	0.	0.71	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	221	519	0	413	0	4	0
normalized size	1	1.	0.61	1.44	0.	1.15	0.	0.01	0.
time (sec)	N/A	0.406	0.279	0.027	0.	0.275	0.	0.724	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	65	0	4	36
normalized size	1	1.	0.71	0.63	0.63	1.71	0.	0.11	0.95
time (sec)	N/A	0.072	0.022	0.011	0.791	0.265	0.	0.708	9.266

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	219	521	0	425	0	4	0
normalized size	1	1.	0.61	1.45	0.	1.18	0.	0.01	0.
time (sec)	N/A	0.398	0.236	0.015	0.	0.278	0.	0.722	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	211	519	0	408	0	4	0
normalized size	1	1.	0.58	1.43	0.	1.12	0.	0.01	0.
time (sec)	N/A	0.406	0.231	0.013	0.	0.271	0.	0.66	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	0	240	0	4	211
normalized size	1	1.	0.43	0.87	0.	1.08	0.	0.02	0.95
time (sec)	N/A	0.267	0.083	0.022	0.	0.28	0.	0.676	38.002

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	242	536	0	455	0	4	0
normalized size	1	1.	0.61	1.35	0.	1.14	0.	0.01	0.
time (sec)	N/A	0.477	0.276	0.033	0.	0.268	0.	0.657	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	234	542	0	504	0	4	0
normalized size	1	1.	0.59	1.36	0.	1.27	0.	0.01	0.
time (sec)	N/A	0.487	0.279	0.033	0.	0.277	0.	0.699	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	119	219	0	279	0	4	264
normalized size	1	1.	0.44	0.81	0.	1.04	0.	0.01	0.98
time (sec)	N/A	0.311	0.096	0.026	0.	0.269	0.	0.659	46.066

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	328	498	0	1312	298
normalized size	1	1.	0.35	1.45	1.05	1.59	0.	4.19	0.95
time (sec)	N/A	0.32	0.121	0.011	0.802	0.28	0.	0.318	66.403

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	79	199	161	215	0	562	182
normalized size	1	1.	0.39	0.97	0.79	1.05	0.	2.74	0.89
time (sec)	N/A	0.199	0.07	0.009	0.8	0.267	0.	0.302	30.631

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	47	56	47	47	0	123	80
normalized size	1	1.	0.48	0.58	0.48	0.48	0.	1.27	0.82
time (sec)	N/A	0.093	0.044	0.005	0.795	0.276	0.	0.315	10.822

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	61
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.088	0.049	0.04	0.	0.	0.	0.	16.506

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	63
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.087	0.056	0.034	0.	0.	0.	0.	16.231

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	63
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.085	0.064	0.051	0.	0.	0.	0.	16.174

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	66
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.063	0.064	0.134	0.	0.	0.	0.	19.466

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	110	150	155	220	0	536	184
normalized size	1	1.	0.64	0.87	0.9	1.28	0.	3.12	1.07
time (sec)	N/A	0.226	0.073	0.012	0.794	0.268	0.	0.296	45.869

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	107	146	0	336	128
normalized size	1	1.	0.59	0.74	0.82	1.12	0.	2.58	0.98
time (sec)	N/A	0.169	0.048	0.01	0.796	0.285	0.	0.27	29.11

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	60	73	95	0	189	71
normalized size	1	1.	0.61	0.71	0.87	1.13	0.	2.25	0.85
time (sec)	N/A	0.124	0.029	0.009	0.778	0.267	0.	0.281	17.161

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	53
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.055	0.029	0.07	0.	0.	0.	0.	17.47

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	53
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.055	0.021	0.056	0.	0.	0.	0.	18.287

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	40	41	50	0	84	37
normalized size	1	1.	0.73	0.98	1.	1.22	0.	2.05	0.9
time (sec)	N/A	0.061	0.018	0.008	0.772	0.268	0.	0.283	10.49

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	51	0	0	0	0	0	53
normalized size	1	1.03	0.88	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.043	0.018	0.041	0.	0.	0.	0.	15.677

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	204	0	0	0	0	0	49
normalized size	1	1.04	3.85	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.038	0.315	0.02	0.	0.	0.	0.	7.031

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	0	0	0	0	0	76
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	1.21
time (sec)	N/A	0.08	0.036	0.03	0.	0.	0.	0.	18.152

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	53
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.05	0.029	0.05	0.	0.	0.	0.	17.219

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	56
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.049	0.024	0.057	0.	0.	0.	0.	17.187

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	75
normalized size	1	1.	0.97	0.	0.	0.	0.	0.	1.17
time (sec)	N/A	0.085	0.028	0.059	0.	0.	0.	0.	18.104

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	58
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.049	0.031	0.07	0.	0.	0.	0.	17.218

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	1	316	101	73
normalized size	1	1.	0.96	1.37	0.	0.01	3.9	1.25	0.9
time (sec)	N/A	0.165	0.094	0.006	0.	0.275	8.017	0.279	29.638

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	1	223	80	54
normalized size	1	1.	0.98	0.95	0.	0.02	3.54	1.27	0.86
time (sec)	N/A	0.113	0.042	0.004	0.	0.275	4.177	0.31	19.092

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	1	131	49	37
normalized size	1	1.	1.11	0.97	0.	0.03	3.45	1.29	0.97
time (sec)	N/A	0.07	0.016	0.002	0.	0.271	1.934	0.297	10.927

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	1	253	89	63
normalized size	1	1.	0.96	0.96	0.	0.01	3.67	1.29	0.91
time (sec)	N/A	0.134	0.04	0.007	0.	0.282	12.134	0.263	26.878

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	119	0	1	0	126	87
normalized size	1	1.	1.03	1.34	0.	0.01	0.	1.42	0.98
time (sec)	N/A	0.259	0.053	0.007	0.	0.323	0.	0.263	42.901

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	70	61	0	7657	279	0	0
normalized size	1	1.	0.11	0.1	0.	12.04	0.44	0.	0.
time (sec)	N/A	2.612	0.053	0.119	0.	0.609	13.096	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	6885	196	0	0
normalized size	1	1.	0.11	0.09	0.	10.91	0.31	0.	0.
time (sec)	N/A	2.48	0.055	0.007	0.	0.419	8.912	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	44	43	0	5165	175	0	529
normalized size	1	1.	0.08	0.08	0.	9.26	0.31	0.	0.95
time (sec)	N/A	1.275	0.03	0.005	0.	0.329	5.784	0.	149.312

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	42	43	0	3260	122	0	529
normalized size	1	1.	0.08	0.08	0.	5.84	0.22	0.	0.95
time (sec)	N/A	1.294	0.027	0.004	0.	0.293	6.328	0.	139.29

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	43	41	0	4020	158	0	529
normalized size	1	1.	0.08	0.07	0.	7.2	0.28	0.	0.95
time (sec)	N/A	1.234	0.03	0.004	0.	0.304	4.724	0.	147.85

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	45	40	0	5284	155	0	529
normalized size	1	1.	0.08	0.07	0.	9.47	0.28	0.	0.95
time (sec)	N/A	1.262	0.03	0.004	0.	0.359	9.293	0.	133.387

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	71	61	0	7272	252	0	0
normalized size	1	1.	0.12	0.1	0.	11.92	0.41	0.	0.
time (sec)	N/A	1.703	0.054	0.008	0.	0.55	8.33	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	75	62	0	7645	241	0	0
normalized size	1	1.	0.12	0.1	0.	12.49	0.39	0.	0.
time (sec)	N/A	1.95	0.056	0.009	0.	0.494	18.548	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	36	29	39	0
normalized size	1	1.	1.	0.8	1.03	1.03	0.83	1.11	0.
time (sec)	N/A	0.056	0.011	0.009	0.781	0.261	0.256	0.285	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	30	30	22	32	22
normalized size	1	1.	1.	0.82	1.07	1.07	0.79	1.14	0.79
time (sec)	N/A	0.047	0.009	0.007	0.78	0.263	0.266	0.277	11.133

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	15	26	15
normalized size	1	1.	1.	0.86	1.1	1.1	0.71	1.24	0.71
time (sec)	N/A	0.036	0.007	0.007	0.795	0.258	0.236	0.301	7.919

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	21	21	18	23	23	15	26	15
normalized size	1	2.1	2.1	1.8	2.3	2.3	1.5	2.6	1.5
time (sec)	N/A	0.03	0.006	0.007	0.782	0.254	0.216	0.262	5.733

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	31	31	28	20	32	22
normalized size	1	1.	1.	1.15	1.15	1.04	0.74	1.19	0.81
time (sec)	N/A	0.045	0.01	0.013	0.789	0.253	0.3	0.288	10.989

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	38	47	29	49	31
normalized size	1	1.	1.	1.06	1.12	1.38	0.85	1.44	0.91
time (sec)	N/A	0.076	0.01	0.013	0.762	0.253	0.422	0.284	16.495

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	41	47	54	34	55	37
normalized size	1	1.	1.	1.	1.15	1.32	0.83	1.34	0.9
time (sec)	N/A	0.085	0.009	0.013	0.773	0.249	0.532	0.303	15.142

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	118	94	127	163	144	0	119
normalized size	1	1.	0.95	0.76	1.02	1.31	1.16	0.	0.96
time (sec)	N/A	0.243	0.095	0.014	0.858	0.258	2.222	0.	34.979

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	92	124	140	129	0	117
normalized size	1	1.	0.93	0.75	1.02	1.15	1.06	0.	0.96
time (sec)	N/A	0.202	0.052	0.01	0.841	0.258	2.206	0.	36.238

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	89	120	154	134	0	110
normalized size	1	1.	0.93	0.75	1.01	1.29	1.13	0.	0.92
time (sec)	N/A	0.167	0.048	0.009	0.868	0.279	2.245	0.	27.703

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	85	115	142	126	0	105
normalized size	1	1.	0.98	0.75	1.02	1.26	1.12	0.	0.93
time (sec)	N/A	0.146	0.051	0.009	0.852	0.266	2.218	0.	28.85

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	113	163	134	0	104
normalized size	1	1.	0.96	0.75	1.01	1.46	1.2	0.	0.93
time (sec)	N/A	0.136	0.046	0.009	0.846	0.267	2.19	0.	21.621

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	106	84	113	139	110	0	104
normalized size	1	1.	0.95	0.75	1.01	1.24	0.98	0.	0.93
time (sec)	N/A	0.133	0.046	0.009	0.844	0.267	2.193	0.	22.816

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	84	113	126	119	0	104
normalized size	1	1.	0.96	0.75	1.01	1.12	1.06	0.	0.93
time (sec)	N/A	0.129	0.045	0.008	0.876	0.274	4.497	0.	20.752

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	113	178	124	0	104
normalized size	1	1.	0.96	0.75	1.01	1.59	1.11	0.	0.93
time (sec)	N/A	0.118	0.05	0.009	0.867	0.263	4.676	0.	17.927

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	89	120	181	139	0	109
normalized size	1	1.	0.99	0.75	1.01	1.52	1.17	0.	0.92
time (sec)	N/A	0.166	0.086	0.011	0.871	0.259	4.462	0.	27.92

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	89	120	182	128	0	110
normalized size	1	1.	0.95	0.75	1.01	1.53	1.08	0.	0.92
time (sec)	N/A	0.146	0.105	0.011	0.856	0.279	4.458	0.	27.479

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	130	171	141	0	116
normalized size	1	1.	0.94	0.75	1.03	1.36	1.12	0.	0.92
time (sec)	N/A	0.211	0.1	0.014	0.867	0.264	4.618	0.	34.484

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	130	230	136	0	117
normalized size	1	1.	0.94	0.75	1.03	1.83	1.08	0.	0.93
time (sec)	N/A	0.203	0.11	0.014	0.883	0.27	4.365	0.	34.234

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	59	44	0	884	26	861	354
normalized size	1	1.	0.14	0.11	0.	2.15	0.06	2.09	0.86
time (sec)	N/A	0.934	0.023	0.009	0.	0.281	0.467	0.288	103.886

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	49	37	43	34
normalized size	1	1.	1.	0.85	1.1	1.26	0.95	1.1	0.87
time (sec)	N/A	0.074	0.019	0.004	0.866	0.254	0.27	0.278	9.676

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	41	40	0	1436	26	1	335
normalized size	1	1.	0.1	0.1	0.	3.49	0.06	0.	0.82
time (sec)	N/A	0.715	0.016	0.006	0.	0.272	0.474	0.276	99.801

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	39	40	0	873	24	860	335
normalized size	1	1.	0.09	0.1	0.	2.12	0.06	2.09	0.82
time (sec)	N/A	0.742	0.014	0.007	0.	0.272	0.479	0.286	98.813

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	24	27	24	24
normalized size	1	1.	1.	0.83	1.04	1.04	1.17	1.04	1.04
time (sec)	N/A	0.049	0.01	0.004	0.843	0.25	0.244	0.261	6.25

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	40	38	0	1436	26	1	335
normalized size	1	1.	0.11	0.1	0.	3.83	0.07	0.	0.89
time (sec)	N/A	0.564	0.016	0.007	0.	0.269	0.464	0.326	95.351

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	375	42	37	0	876	20	849	335
normalized size	1	2.02	0.23	0.2	0.	4.71	0.11	4.56	1.8
time (sec)	N/A	0.573	0.014	0.006	0.	0.28	0.467	0.291	88.56

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	51	58	41	47	41
normalized size	1	1.	1.34	0.85	1.24	1.41	1.	1.15	1.
time (sec)	N/A	0.078	0.02	0.007	0.851	0.259	0.319	0.277	13.694

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	61	50	0	1442	24	1	355
normalized size	1	1.	0.15	0.12	0.	3.47	0.06	0.	0.85
time (sec)	N/A	0.662	0.023	0.011	0.	0.287	0.54	0.306	105.556

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	65	50	0	927	31	867	359
normalized size	1	1.	0.16	0.12	0.	2.22	0.07	2.07	0.86
time (sec)	N/A	0.755	0.021	0.012	0.	0.278	0.568	0.324	115.871

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	58	81	48	61	48
normalized size	1	1.	1.06	0.83	1.21	1.69	1.	1.27	1.
time (sec)	N/A	0.116	0.023	0.009	0.872	0.26	0.434	0.281	15.876

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	54	51	0	1501	37	1	345
normalized size	1	1.	0.13	0.12	0.	3.55	0.09	0.	0.82
time (sec)	N/A	0.827	0.025	0.012	0.	0.29	0.628	0.311	134.623

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	38	33	0	1477	24	0	345
normalized size	1	1.	0.1	0.09	0.	3.88	0.06	0.	0.91
time (sec)	N/A	0.888	0.015	0.008	0.	0.278	0.365	0.	96.776

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	24	27	24	24
normalized size	1	1.	1.	0.83	1.04	1.04	1.17	1.04	1.04
time (sec)	N/A	0.049	0.014	0.004	0.847	0.255	0.238	0.283	5.272

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	37	36	0	1454	24	0	345
normalized size	1	1.	0.09	0.09	0.	3.64	0.06	0.	0.86
time (sec)	N/A	0.743	0.015	0.007	0.	0.277	0.36	0.	108.427

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	203	0	0	1	0	0	221
normalized size	1	1.	0.88	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.661	0.203	0.053	0.	0.294	0.	0.	51.703

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	159	0	0	1	0	0	163
normalized size	1	1.	0.93	0.	0.	0.01	0.	0.	0.95
time (sec)	N/A	0.341	0.12	0.031	0.	0.289	0.	0.	32.031

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	0	0	1	0	0	141
normalized size	1	1.	0.88	0.	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.279	0.143	0.031	0.	0.283	0.	0.	29.916

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	0	0	1	0	132	97
normalized size	1	1.	0.9	0.	0.	0.01	0.	1.22	0.9
time (sec)	N/A	0.17	0.141	0.029	0.	0.282	0.	0.325	17.265

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	1	0	103	73
normalized size	1	1.	1.	0.	0.	0.01	0.	1.24	0.88
time (sec)	N/A	0.119	0.056	0.027	0.	0.27	0.	0.286	10.178

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	1	0	0	95
normalized size	1	1.	1.	0.	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.252	0.273	0.022	0.	0.3	0.	0.	30.025

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	0	0	1	0	0	99
normalized size	1	1.	1.	0.	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.252	0.211	0.053	0.	0.296	0.	0.	28.797

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	0	0	1	0	0	76
normalized size	1	1.	1.06	0.	0.	0.01	0.	0.	0.86
time (sec)	N/A	0.154	0.225	0.041	0.	0.279	0.	0.	18.823

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	0	0	1	0	0	104
normalized size	1	1.	0.97	0.	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.21	0.377	0.047	0.	0.292	0.	0.	24.851

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	141	0	0	1	0	0	148
normalized size	1	1.	0.88	0.	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.349	0.191	0.058	0.	0.288	0.	0.	37.702

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	175	0	0	1	0	0	185
normalized size	1	1.	0.88	0.	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.528	0.301	0.069	0.	0.296	0.	0.	54.99

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	1043	0	0	0	0	0	124
normalized size	1	1.	7.45	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.53	5.149	0.046	0.	0.	0.	0.	37.196

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	701	0	0	0	0	0	124
normalized size	1	1.	5.01	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.355	3.827	0.041	0.	0.	0.	0.	28.739

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	702	0	0	0	0	0	121
normalized size	1	1.	5.2	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.205	1.075	0.036	0.	0.	0.	0.	37.389

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	702	0	0	0	0	0	124
normalized size	1	1.	5.09	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.422	1.082	0.046	0.	0.	0.	0.	35.066

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	702	0	0	0	0	0	128
normalized size	1	1.	5.01	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.419	1.117	0.046	0.	0.	0.	0.	32.72

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	289	0	0	1	0	0	284
normalized size	1	1.	0.99	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.8	0.313	0.036	0.	0.334	0.	0.	67.24

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	222	0	0	1	0	0	214
normalized size	1	1.	1.	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.437	0.239	0.034	0.	0.311	0.	0.	41.359

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	194	0	0	1	0	0	190
normalized size	1	1.	0.95	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.364	0.195	0.031	0.	0.311	0.	0.	39.281

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	0	0	1	0	232	138
normalized size	1	1.	0.95	0.	0.	0.01	0.	1.55	0.92
time (sec)	N/A	0.231	0.156	0.03	0.	0.306	0.	0.295	23.651

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	0	0	1	0	182	112
normalized size	1	1.	0.9	0.	0.	0.01	0.	1.47	0.9
time (sec)	N/A	0.175	0.107	0.028	0.	0.295	0.	0.29	15.166

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	181	0	0	1	0	0	139
normalized size	1	1.	1.17	0.	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.428	0.474	0.022	0.	0.395	0.	0.	46.631

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	173	0	0	1	0	0	134
normalized size	1	1.	1.15	0.	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.414	0.349	0.055	0.	0.349	0.	0.	44.57

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	149	0	0	1	0	0	136
normalized size	1	1.	0.99	0.	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.402	0.696	0.056	0.	0.334	0.	0.	44.113

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	174	0	0	1	0	0	146
normalized size	1	1.	1.07	0.	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.428	0.472	0.066	0.	0.361	0.	0.	45.878

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	125	0	0	1	0	0	119
normalized size	1	1.	0.94	0.	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.235	0.234	0.057	0.	0.305	0.	0.	28.032

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	163	0	0	1	0	0	148
normalized size	1	1.	1.01	0.	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.296	0.271	0.072	0.	0.304	0.	0.	34.394

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	206	0	0	1	0	0	201
normalized size	1	1.	0.95	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.453	0.353	0.089	0.	0.338	0.	0.	48.431

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	247	0	0	1	0	0	238
normalized size	1	1.	0.97	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.671	0.401	0.106	0.	0.372	0.	0.	70.726

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1746	0	0	0	0	0	126
normalized size	1	1.	12.38	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.504	5.159	0.048	0.	0.	0.	0.	39.423

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1391	0	0	0	0	0	126
normalized size	1	1.	9.87	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.355	3.567	0.045	0.	0.	0.	0.	27.506

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	1389	0	0	0	0	0	122
normalized size	1	1.	10.21	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.2	3.412	0.041	0.	0.	0.	0.	35.564

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	1058	0	0	0	0	0	126
normalized size	1	1.	7.61	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.421	1.853	0.049	0.	0.	0.	0.	36.849

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1054	0	0	0	0	0	129
normalized size	1	1.	7.48	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.417	1.867	0.045	0.	0.	0.	0.	36.471

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	135	0	0	1	0	0	162
normalized size	1	1.	0.79	0.	0.	0.01	0.	0.	0.95
time (sec)	N/A	0.497	0.185	0.036	0.	0.291	0.	0.	42.043

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	0	0	1	0	0	114
normalized size	1	1.	0.84	0.	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.253	0.1	0.034	0.	0.28	0.	0.	26.01

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	0	0	1	0	0	92
normalized size	1	1.	0.83	0.	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.195	0.069	0.033	0.	0.277	0.	0.	24.318

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	1	0	82	58
normalized size	1	1.	0.97	0.	0.	0.01	0.	1.21	0.85
time (sec)	N/A	0.114	0.038	0.032	0.	0.291	0.	0.291	12.975

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	0	0	1	0	54	37
normalized size	1	1.	0.95	0.	0.	0.02	0.	1.26	0.86
time (sec)	N/A	0.073	0.025	0.015	0.	0.266	0.	0.293	7.523

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	50	0	0	1	0	0	39
normalized size	1	1.	1.14	0.	0.	0.02	0.	0.	0.89
time (sec)	N/A	0.089	0.122	0.015	0.	0.282	0.	0.	12.249

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	78	0	0	1	0	0	61
normalized size	1	1.	1.08	0.	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.135	0.133	0.035	0.	0.286	0.	0.	16.838

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	0	0	1	0	0	95
normalized size	1	1.	0.9	0.	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.243	0.306	0.041	0.	0.281	0.	0.	27.351

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	117	0	0	1	0	0	133
normalized size	1	1.	0.81	0.	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.394	0.489	0.043	0.	0.292	0.	0.	44.584

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	146	0	0	1	0	0	180
normalized size	1	1.	0.76	0.	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.563	0.241	0.047	0.	0.299	0.	0.	63.064

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	380	0	0	0	0	0	122
normalized size	1	1.	2.71	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.476	0.276	0.018	0.	0.	0.	0.	39.959

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	380	0	0	0	0	0	122
normalized size	1	1.	2.71	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.356	0.278	0.016	0.	0.	0.	0.	27.572

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	378	0	0	0	0	0	119
normalized size	1	1.	2.8	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.203	0.266	0.016	0.	0.	0.	0.	40.235

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	705	0	0	0	0	0	122
normalized size	1	1.	5.11	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.432	1.163	0.044	0.	0.	0.	0.	32.749

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	705	0	0	0	0	0	126
normalized size	1	1.	5.04	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.431	1.159	0.046	0.	0.	0.	0.	32.776

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	147	0	0	1	0	0	182
normalized size	1	1.	0.75	0.	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.546	0.269	0.06	0.	0.352	0.	0.	47.499

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	127	0	0	1	0	0	126
normalized size	1	1.	0.93	0.	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.257	0.199	0.053	0.	0.303	0.	0.	29.109

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	95	0	0	1	0	0	107
normalized size	1	1.	0.79	0.	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.197	0.18	0.023	0.	0.292	0.	0.	27.602

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	38	0	92	0	61	34
normalized size	1	1.	1.05	0.97	0.	2.36	0.	1.56	0.87
time (sec)	N/A	0.072	0.037	0.009	0.	0.277	0.	0.274	10.32

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	0	90	0	61	36
normalized size	1	1.	1.	0.97	0.	2.37	0.	1.61	0.95
time (sec)	N/A	0.059	0.027	0.009	0.	0.277	0.	0.301	7.025

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	96	0	0	1	0	0	83
normalized size	1	1.	1.04	0.	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.172	0.347	0.028	0.	0.294	0.	0.	22.423

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	132	0	0	1	0	0	131
normalized size	1	1.	0.93	0.	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.306	0.22	0.062	0.	0.302	0.	0.	37.534

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	159	0	0	1	0	0	184
normalized size	1	1.	0.8	0.	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.496	0.35	0.074	0.	0.332	0.	0.	57.032

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	191	0	0	1	0	0	241
normalized size	1	1.	0.75	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.683	0.522	0.091	0.	0.388	0.	0.	94.452

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	711	0	0	0	0	0	124
normalized size	1	1.	4.97	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.499	1.71	0.023	0.	0.	0.	0.	44.792

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	1054	0	0	0	0	0	124
normalized size	1	1.	7.37	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.352	1.468	0.022	0.	0.	0.	0.	28.239

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	1056	0	0	0	0	0	121
normalized size	1	1.	7.65	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.209	1.81	0.021	0.	0.	0.	0.	36.777

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1599	0	0	0	0	0	124
normalized size	1	1.	11.34	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.426	3.265	0.056	0.	0.	0.	0.	44.201

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	1593	0	0	0	0	0	128
normalized size	1	1.	11.14	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.423	3.527	0.055	0.	0.	0.	0.	44.863

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	0	325	1510	687	90
normalized size	1	1.	0.69	2.98	0.	3.22	14.95	6.8	0.89
time (sec)	N/A	0.128	0.084	0.009	0.	0.302	18.522	0.292	26.39

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	78	0	96	314	185	42
normalized size	1	1.	0.67	1.5	0.	1.85	6.04	3.56	0.81
time (sec)	N/A	0.047	0.032	0.005	0.	0.27	4.401	0.266	11.127

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	84	0	0	0	0	0	148
normalized size	1	1.	0.49	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.483	0.094	0.044	0.	0.	0.	0.	31.779

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	376	0	0	0	0	0	264
normalized size	1	1.	1.19	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	1.511	1.652	0.035	0.	0.	0.	0.	114.466

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	1083	0	0	0	0	0	139
normalized size	1	1.	6.85	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.464	10.557	0.014	0.	0.	0.	0.	41.412

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	424	0	0	0	0	0	138
normalized size	1	1.	2.7	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.449	0.289	0.015	0.	0.	0.	0.	35.343

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	426	0	0	0	0	0	136
normalized size	1	1.	2.71	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.446	2.357	0.017	0.	0.	0.	0.	36.162

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	426	0	0	0	0	0	138
normalized size	1	1.	2.66	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.446	2.888	0.015	0.	0.	0.	0.	45.774

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	501	0	0	0	0	0	129
normalized size	1	1.	3.23	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.278	4.333	0.085	0.	0.	0.	0.	33.88

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	395	0	0	0	0	0	192
normalized size	1	1.	1.76	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.478	0.613	0.035	0.	0.	0.	0.	48.286

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	439	0	0	0	0	0	136
normalized size	1	1.	2.73	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.253	0.903	0.047	0.	0.	0.	0.	22.111

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0	112
normalized size	1	1.	1.06	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.152	0.176	0.033	0.	0.	0.	0.	11.906

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	411	0	0	0	0	0	116
normalized size	1	1.	2.98	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.302	3.206	0.041	0.	0.	0.	0.	30.877

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	456	0	0	0	0	0	116
normalized size	1	1.	3.3	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.282	3.452	0.038	0.	0.	0.	0.	33.567

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	454	0	0	0	0	0	116
normalized size	1	1.	3.29	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.231	3.198	0.033	0.	0.	0.	0.	26.724

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	487	0	0	0	0	0	112
normalized size	1	1.	3.66	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.151	3.671	0.021	0.	0.	0.	0.	37.502

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	500	0	0	0	0	0	128
normalized size	1	1.	3.18	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.329	2.94	0.022	0.	0.	0.	0.	27.779

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	408	0	0	0	0	0	116
normalized size	1	1.	3.	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.25	3.195	0.036	0.	0.	0.	0.	30.203

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	474	0	0	0	0	0	119
normalized size	1	1.	3.43	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.245	3.643	0.038	0.	0.	0.	0.	30.495

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	510	0	0	0	0	0	151
normalized size	1	1.	3.11	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.323	3.317	0.041	0.	0.	0.	0.	28.935

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	455	0	0	0	0	0	121
normalized size	1	1.	3.3	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.262	3.426	0.049	0.	0.	0.	0.	30.623

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	411	0	0	0	0	0	121
normalized size	1	1.	2.98	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.253	2.942	0.052	0.	0.	0.	0.	31.625

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	507	0	0	0	0	0	150
normalized size	1	1.	3.02	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.324	3.153	0.053	0.	0.	0.	0.	29.543

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	0	0	0	24
normalized size	1	1.	1.06	0.	0.	0.	0.	0.	0.75
time (sec)	N/A	0.022	0.02	0.03	0.	0.	0.	0.	4.524

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	24	25	32	42	22	32	24
normalized size	1	1.	0.8	0.83	1.07	1.4	0.73	1.07	0.8
time (sec)	N/A	0.036	0.022	0.009	0.841	0.25	0.296	0.285	8.048

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	24	31	15	24	15
normalized size	1	1.	0.82	0.86	1.09	1.41	0.68	1.09	0.68
time (sec)	N/A	0.026	0.008	0.009	0.792	0.245	0.242	0.276	4.806

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	32	15	26	15
normalized size	1	1.	1.	0.87	1.13	1.39	0.65	1.13	0.65
time (sec)	N/A	0.027	0.014	0.009	0.844	0.245	0.323	0.289	5.846

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	12	8	12	8
normalized size	1	1.	1.	0.91	1.09	1.09	0.73	1.09	0.73
time (sec)	N/A	0.007	0.003	0.004	0.766	0.249	0.228	0.262	2.854

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	31	15	26	15
normalized size	1	1.	0.87	0.87	1.13	1.35	0.65	1.13	0.65
time (sec)	N/A	0.019	0.008	0.005	0.856	0.246	0.282	0.261	3.635

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	32	43	19	39	22
normalized size	1	1.	1.	0.88	1.33	1.79	0.79	1.62	0.92
time (sec)	N/A	0.028	0.013	0.019	0.764	0.254	0.302	0.303	5.37

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	34	42	26	34	26
normalized size	1	1.	1.	0.83	1.13	1.4	0.87	1.13	0.87
time (sec)	N/A	0.033	0.018	0.015	0.851	0.251	0.394	0.299	7.587

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	45	59	29	45	29
normalized size	1	1.	1.	0.85	1.36	1.79	0.88	1.36	0.88
time (sec)	N/A	0.036	0.019	0.021	0.76	0.252	0.428	0.297	5.746

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	41	49	29	42	32
normalized size	1	1.	0.89	0.76	1.11	1.32	0.78	1.14	0.86
time (sec)	N/A	0.042	0.016	0.017	0.861	0.246	0.515	0.271	9.521

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	69	112	181	90	112	95
normalized size	1	1.	0.9	0.66	1.08	1.74	0.87	1.08	0.91
time (sec)	N/A	0.111	0.14	0.012	0.849	0.266	0.514	0.301	18.151

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	70	113	177	90	113	90
normalized size	1	1.	0.94	0.71	1.14	1.79	0.91	1.14	0.91
time (sec)	N/A	0.106	0.099	0.01	0.848	0.279	0.525	0.267	17.599

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	68	111	173	82	111	82
normalized size	1	1.	0.93	0.7	1.14	1.78	0.85	1.14	0.85
time (sec)	N/A	0.099	0.159	0.009	0.854	0.271	0.51	0.296	16.548

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	70	113	176	83	113	83
normalized size	1	1.	0.93	0.71	1.14	1.78	0.84	1.14	0.84
time (sec)	N/A	0.105	0.103	0.006	0.845	0.261	0.533	0.278	17.399

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	68	111	174	88	111	88
normalized size	1	1.	0.94	0.7	1.14	1.79	0.91	1.14	0.91
time (sec)	N/A	0.093	0.085	0.005	0.862	0.281	0.511	0.277	14.342

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	98	75	119	178	95	119	95
normalized size	1	1.	0.92	0.71	1.12	1.68	0.9	1.12	0.9
time (sec)	N/A	0.109	0.125	0.012	0.847	0.272	0.607	0.293	18.7

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	96	73	122	192	97	117	99
normalized size	1	1.	0.91	0.69	1.15	1.81	0.92	1.1	0.93
time (sec)	N/A	0.104	0.133	0.012	0.852	0.274	0.642	0.274	17.609

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	103	80	128	198	102	130	104
normalized size	1	1.	0.91	0.71	1.13	1.75	0.9	1.15	0.92
time (sec)	N/A	0.115	0.133	0.015	0.874	0.276	0.738	0.263	20.403

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	78	128	198	102	127	105
normalized size	1	1.	0.89	0.69	1.13	1.75	0.9	1.12	0.93
time (sec)	N/A	0.113	0.14	0.015	0.854	0.26	0.778	0.31	18.973

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	0	0	0	0	0	22
normalized size	1	1.	1.07	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.018	0.021	0.028	0.	0.	0.	0.	4.474

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	39	41	46	62	34	47	24
normalized size	1	1.	1.22	1.28	1.44	1.94	1.06	1.47	0.75
time (sec)	N/A	0.038	0.044	0.015	0.77	0.249	0.28	0.28	8.159

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	24	31	15	26	15
normalized size	1	1.	0.85	0.73	0.92	1.19	0.58	1.	0.58
time (sec)	N/A	0.032	0.01	0.009	0.766	0.249	0.243	0.262	4.825

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	39	54	26	41	15
normalized size	1	1.	1.32	1.44	1.56	2.16	1.04	1.64	0.6
time (sec)	N/A	0.029	0.018	0.013	0.779	0.251	0.274	0.283	5.94

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	12	12	8	12	7
normalized size	1	1.	0.85	0.77	0.92	0.92	0.62	0.92	0.54
time (sec)	N/A	0.007	0.004	0.005	0.771	0.241	0.219	0.275	2.852

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	39	54	26	41	15
normalized size	1	1.	1.32	1.44	1.56	2.16	1.04	1.64	0.6
time (sec)	N/A	0.021	0.013	0.013	0.779	0.253	0.277	0.263	3.72

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	47	32	43	19	41	22
normalized size	1	1.	0.93	1.68	1.14	1.54	0.68	1.46	0.79
time (sec)	N/A	0.033	0.014	0.021	0.781	0.249	0.298	0.289	5.351

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	50	50	73	36	51	26
normalized size	1	1.	1.28	1.56	1.56	2.28	1.12	1.59	0.81
time (sec)	N/A	0.035	0.033	0.024	0.77	0.284	0.4	0.287	7.867

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	54	47	68	29	49	29
normalized size	1	1.	0.95	1.46	1.27	1.84	0.78	1.32	0.78
time (sec)	N/A	0.039	0.021	0.022	0.765	0.251	0.426	0.269	6.139

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	49	55	57	80	41	57	32
normalized size	1	1.	1.26	1.41	1.46	2.05	1.05	1.46	0.82
time (sec)	N/A	0.043	0.026	0.022	0.759	0.275	0.543	0.305	9.804

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	43	38	66	32	41	27
normalized size	1	1.	1.12	1.26	1.12	1.94	0.94	1.21	0.79
time (sec)	N/A	0.026	0.025	0.02	0.846	0.264	0.455	0.305	6.271

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	42	39	62	32	42	22
normalized size	1	1.	1.21	1.45	1.34	2.14	1.1	1.45	0.76
time (sec)	N/A	0.025	0.023	0.02	0.839	0.256	0.448	0.299	5.698

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	42	36	58	26	39	17
normalized size	1	1.	1.15	1.56	1.33	2.15	0.96	1.44	0.63
time (sec)	N/A	0.02	0.021	0.018	0.86	0.283	0.436	0.282	4.343

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	33	42	39	61	27	42	19
normalized size	1	1.	1.14	1.45	1.34	2.1	0.93	1.45	0.66
time (sec)	N/A	0.025	0.019	0.016	0.851	0.258	0.438	0.274	5.776

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	42	36	59	31	39	20
normalized size	1	1.	1.22	1.56	1.33	2.19	1.15	1.44	0.74
time (sec)	N/A	0.014	0.015	0.019	0.856	0.258	0.456	0.284	2.002

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	40	47	47	74	37	50	27
normalized size	1	1.	1.11	1.31	1.31	2.06	1.03	1.39	0.75
time (sec)	N/A	0.031	0.026	0.022	0.847	0.258	0.545	0.279	6.954

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	47	50	85	39	46	31
normalized size	1	1.	1.06	1.31	1.39	2.36	1.08	1.28	0.86
time (sec)	N/A	0.026	0.031	0.021	0.861	0.269	0.584	0.288	6.304

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	52	57	92	44	58	36
normalized size	1	1.	1.19	1.21	1.33	2.14	1.02	1.35	0.84
time (sec)	N/A	0.038	0.034	0.024	0.85	0.257	0.713	0.274	8.7

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	57	92	44	55	37
normalized size	1	1.	1.	1.21	1.33	2.14	1.02	1.28	0.86
time (sec)	N/A	0.033	0.034	0.024	0.855	0.281	0.73	0.281	7.282

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	82	0	0	0	0	0	141
normalized size	1	1.	0.5	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.325	0.084	0.023	0.	0.	0.	0.	32.074

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	1	316	101	73
normalized size	1	1.	0.96	1.37	0.	0.01	3.9	1.25	0.9
time (sec)	N/A	0.174	0.085	0.006	0.	0.307	11.156	0.286	31.197

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	210	360	0	1446	134	1	196
normalized size	1	1.	1.09	1.88	0.	7.53	0.7	0.01	1.02
time (sec)	N/A	0.709	0.222	0.029	0.	0.295	12.38	0.44	72.569

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	1	223	80	54
normalized size	1	1.	0.98	0.95	0.	0.02	3.54	1.27	0.86
time (sec)	N/A	0.119	0.039	0.003	0.	0.277	6.109	0.277	19.568

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	171	216	0	765	76	1	144
normalized size	1	1.	1.08	1.36	0.	4.81	0.48	0.01	0.91
time (sec)	N/A	0.274	0.149	0.021	0.	0.275	7.94	0.395	38.766

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	1	131	49	36
normalized size	1	1.	1.11	0.97	0.	0.03	3.45	1.29	0.95
time (sec)	N/A	0.072	0.015	0.002	0.	0.263	2.34	0.26	11.471

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	133	120	0	836	88	1	144
normalized size	1	1.	0.86	0.78	0.	5.43	0.57	0.01	0.94
time (sec)	N/A	0.228	0.137	0.016	0.	0.28	8.742	0.41	29.942

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	1	253	92	63
normalized size	1	1.	0.96	0.96	0.	0.01	3.67	1.33	0.91
time (sec)	N/A	0.141	0.038	0.008	0.	0.342	18.493	0.308	27.741

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	75	240	0	1531	153	1	184
normalized size	1	1.	0.41	1.3	0.	8.32	0.83	0.01	1.
time (sec)	N/A	0.472	0.049	0.024	0.	0.283	13.555	0.457	62.193

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	119	0	1	0	127	87
normalized size	1	1.	1.03	1.34	0.	0.01	0.	1.43	0.98
time (sec)	N/A	0.265	0.048	0.011	0.	0.419	0.	0.291	44.035

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	70	63	0	8504	360	0	386
normalized size	1	1.	0.18	0.17	0.	22.32	0.94	0.	1.01
time (sec)	N/A	1.363	0.058	0.003	0.	1.19	139.482	0.	149.894

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	5154	218	0	382
normalized size	1	1.	0.19	0.16	0.	13.71	0.58	0.	1.02
time (sec)	N/A	1.288	0.053	0.002	0.	0.45	52.681	0.	142.909

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	44	43	0	5292	230	0	291
normalized size	1	1.	0.14	0.13	0.	16.28	0.71	0.	0.9
time (sec)	N/A	0.7	0.035	0.002	0.	0.365	23.921	0.	92.289

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	42	43	0	2464	126	0	291
normalized size	1	1.	0.13	0.13	0.	7.58	0.39	0.	0.9
time (sec)	N/A	0.596	0.03	0.002	0.	0.303	16.031	0.	89.758

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	43	43	0	3767	172	0	291
normalized size	1	1.	0.14	0.14	0.	11.96	0.55	0.	0.92
time (sec)	N/A	0.6	0.032	0.001	0.	0.336	12.088	0.	90.859

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	45	40	0	4000	177	0	291
normalized size	1	1.	0.14	0.13	0.	12.7	0.56	0.	0.92
time (sec)	N/A	0.626	0.035	0.002	0.	0.375	22.42	0.	83.996

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	71	63	0	6971	304	0	357
normalized size	1	1.	0.2	0.17	0.	19.2	0.84	0.	0.98
time (sec)	N/A	0.896	0.053	0.002	0.	0.754	56.783	0.	129.01

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	75	62	0	6533	277	0	360
normalized size	1	1.	0.21	0.17	0.	17.9	0.76	0.	0.99
time (sec)	N/A	0.841	0.058	0.002	0.	0.728	123.503	0.	123.403

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	488	0	0	0	0	0	102
normalized size	1	1.	3.84	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.152	2.427	0.028	0.	0.	0.	0.	17.576

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	47	58	42	47	39
normalized size	1	1.	1.	0.82	1.07	1.32	0.95	1.07	0.89
time (sec)	N/A	0.078	0.018	0.004	0.823	0.253	0.333	0.265	11.498

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	98	43	57	55	51	57	51
normalized size	1	1.	1.81	0.8	1.06	1.02	0.94	1.06	0.94
time (sec)	N/A	0.109	0.293	0.008	0.82	0.249	0.343	0.292	15.721

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	46	37	41	34
normalized size	1	1.	1.	0.84	1.11	1.24	1.	1.11	0.92
time (sec)	N/A	0.062	0.017	0.004	0.822	0.25	0.313	0.27	8.278

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	94	62	82	88	76	82	70
normalized size	1	1.	1.25	0.83	1.09	1.17	1.01	1.09	0.93
time (sec)	N/A	0.142	0.21	0.005	0.818	0.262	0.598	0.316	25.247

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	24	26	24	22
normalized size	1	1.	1.	0.83	1.04	1.04	1.13	1.04	0.96
time (sec)	N/A	0.044	0.01	0.002	0.826	0.25	0.287	0.283	5.264

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	62	82	88	76	82	70
normalized size	1	1.	1.05	0.83	1.09	1.17	1.01	1.09	0.93
time (sec)	N/A	0.114	0.093	0.005	0.821	0.259	0.605	0.291	14.964

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	138	87	49	55	41	49	41
normalized size	1	1.	3.54	2.23	1.26	1.41	1.05	1.26	1.05
time (sec)	N/A	0.066	0.158	0.013	0.823	0.25	0.37	0.289	11.555

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	100	57	57	59	53	57	53
normalized size	1	1.	1.85	1.06	1.06	1.09	0.98	1.06	0.98
time (sec)	N/A	0.1	0.085	0.009	0.82	0.247	0.441	0.311	15.751

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	141	94	55	78	48	62	48
normalized size	1	1.	2.94	1.96	1.15	1.62	1.	1.29	1.
time (sec)	N/A	0.1	0.253	0.012	0.822	0.254	0.522	0.309	14.362

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	142	95	99	126	88	99	83
normalized size	1	1.	1.6	1.07	1.11	1.42	0.99	1.11	0.93
time (sec)	N/A	0.189	0.234	0.012	0.822	0.258	0.801	0.299	37.357

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	139	110	0	220	192	0	129
normalized size	1	1.	0.99	0.78	0.	1.56	1.36	0.	0.91
time (sec)	N/A	0.183	0.447	0.03	0.	0.278	2.888	0.	33.481

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	93	82	97	83
normalized size	1	1.	0.77	0.76	0.	1.06	0.93	1.1	0.94
time (sec)	N/A	0.112	0.038	0.018	0.	0.259	0.505	0.285	24.398

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	193	197	0	128
normalized size	1	1.	0.96	0.78	0.	1.38	1.41	0.	0.91
time (sec)	N/A	0.205	0.305	0.017	0.	0.288	2.886	0.	48.708

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	212	214	0	128
normalized size	1	1.	0.96	0.78	0.	1.51	1.53	0.	0.91
time (sec)	N/A	0.165	0.297	0.012	0.	0.271	2.89	0.	29.847

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	90	82	97	83
normalized size	1	1.	0.77	0.76	0.	1.02	0.93	1.1	0.94
time (sec)	N/A	0.099	0.035	0.015	0.	0.269	0.515	0.301	21.16

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	140	114	0	235	218	0	131
normalized size	1	1.	0.97	0.79	0.	1.62	1.5	0.	0.9
time (sec)	N/A	0.226	0.387	0.016	0.	0.281	2.961	0.	55.735

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	148	114	0	259	197	0	134
normalized size	1	1.	1.01	0.78	0.	1.76	1.34	0.	0.91
time (sec)	N/A	0.188	0.555	0.009	0.	0.275	3.002	0.	34.914

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	75	0	128	94	113	94
normalized size	1	1.	0.97	0.77	0.	1.31	0.96	1.15	0.96
time (sec)	N/A	0.166	0.064	0.011	0.	0.265	0.696	0.278	34.866

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	171	119	0	250	209	0	141
normalized size	1	1.	1.11	0.77	0.	1.62	1.36	0.	0.92
time (sec)	N/A	0.285	0.693	0.012	0.	0.274	3.126	0.	62.757

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	0	0	0	0	0	100
normalized size	1	1.	0.62	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.115	0.077	0.027	0.	0.	0.	0.	18.287

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	50	59	42	50	39
normalized size	1	1.	1.	0.83	1.09	1.28	0.91	1.09	0.85
time (sec)	N/A	0.078	0.019	0.006	0.823	0.266	0.326	0.279	12.499

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	74	48	348	48
normalized size	1	1.	0.96	0.77	0.	1.3	0.84	6.11	0.84
time (sec)	N/A	0.087	0.025	0.01	0.	0.261	0.289	0.343	22.418

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	49	37	43	34
normalized size	1	1.	1.	0.85	1.1	1.26	0.95	1.1	0.87
time (sec)	N/A	0.067	0.012	0.002	0.826	0.27	0.314	0.292	9.43

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	98	65	0	159	70	342	70
normalized size	1	1.	1.2	0.79	0.	1.94	0.85	4.17	0.85
time (sec)	N/A	0.133	0.228	0.01	0.	0.276	0.593	0.335	29.147

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	24	26	24	22
normalized size	1	1.	1.	0.83	1.04	1.04	1.13	1.04	0.96
time (sec)	N/A	0.046	0.011	0.001	0.825	0.256	0.292	0.275	5.982

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	65	0	159	70	0	70
normalized size	1	1.	1.01	0.79	0.	1.94	0.85	0.	0.85
time (sec)	N/A	0.108	0.105	0.009	0.	0.265	0.618	0.	18.366

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	51	58	41	51	41
normalized size	1	1.	1.34	0.85	1.24	1.41	1.	1.24	1.
time (sec)	N/A	0.072	0.021	0.011	0.828	0.257	0.383	0.275	13.469

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	77	49	348	49
normalized size	1	1.	0.96	0.77	0.	1.35	0.86	6.11	0.86
time (sec)	N/A	0.08	0.029	0.008	0.	0.275	0.405	0.337	22.123

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	58	81	48	65	48
normalized size	1	1.	1.06	0.83	1.21	1.69	1.	1.35	1.
time (sec)	N/A	0.107	0.022	0.01	0.826	0.252	0.54	0.276	15.745

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	56	75	0	198	82	358	83
normalized size	1	1.	0.58	0.78	0.	2.06	0.85	3.73	0.86
time (sec)	N/A	0.185	0.027	0.009	0.	0.269	0.861	0.344	40.912

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	59	44	0	1284	26	343	496
normalized size	1	1.	0.17	0.12	0.	3.61	0.07	0.96	1.39
time (sec)	N/A	0.693	0.021	0.012	0.	0.296	4.971	0.3	78.869

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	41	32	0	271	165	277	508
normalized size	1	1.	0.15	0.12	0.	0.99	0.6	1.01	1.85
time (sec)	N/A	0.516	0.018	0.028	0.	0.267	0.633	0.303	73.759

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	39	40	0	1067	24	342	311
normalized size	1	1.	0.11	0.12	0.	3.07	0.07	0.99	0.9
time (sec)	N/A	0.457	0.016	0.01	0.	0.286	4.859	0.288	64.081

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	40	40	0	1107	26	342	311
normalized size	1	1.	0.11	0.11	0.	3.12	0.07	0.96	0.88
time (sec)	N/A	0.436	0.016	0.01	0.	0.285	4.89	0.288	43.02

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	257	165	277	529
normalized size	1	1.	0.15	0.11	0.	0.93	0.6	1.01	1.92
time (sec)	N/A	0.596	0.016	0.007	0.	0.265	0.643	0.295	65.942

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	61	52	0	1311	29	348	484
normalized size	1	1.	0.17	0.14	0.	3.64	0.08	0.97	1.34
time (sec)	N/A	0.694	0.022	0.015	0.	0.282	4.919	0.292	99.62

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	65	50	0	1354	31	348	549
normalized size	1	1.	0.18	0.14	0.	3.66	0.08	0.94	1.48
time (sec)	N/A	0.543	0.019	0.014	0.	0.286	5.136	0.298	81.148

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	54	43	0	302	180	293	518
normalized size	1	1.	0.19	0.15	0.	1.05	0.63	1.02	1.8
time (sec)	N/A	0.572	0.024	0.014	0.	0.271	0.866	0.286	86.354

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	54	51	0	1218	36	358	325
normalized size	1	1.	0.14	0.14	0.	3.23	0.1	0.95	0.86
time (sec)	N/A	0.659	0.023	0.015	0.	0.292	5.168	0.306	76.811

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	0	0	0	0	0	104
normalized size	1	1.	0.68	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.153	0.073	0.026	0.	0.	0.	0.	13.605

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	38	68	96	60	68	68
normalized size	1	1.	0.92	0.61	1.1	1.55	0.97	1.1	1.1
time (sec)	N/A	0.106	0.055	0.005	0.827	0.249	0.33	0.271	11.643

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	117	0	203	54	89	102
normalized size	1	1.	1.08	1.3	0.	2.26	0.6	0.99	1.13
time (sec)	N/A	0.265	0.264	0.044	0.	0.266	0.618	0.282	20.182

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	61	81	53	61	60
normalized size	1	1.	0.96	0.6	1.11	1.47	0.96	1.11	1.09
time (sec)	N/A	0.068	0.04	0.004	0.826	0.262	0.3	0.284	8.486

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	110	0	196	49	63	97
normalized size	1	1.	0.93	1.36	0.	2.42	0.6	0.78	1.2
time (sec)	N/A	0.155	0.093	0.029	0.	0.271	0.609	0.268	13.717

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	42	59	42	42	24
normalized size	1	1.	1.65	0.83	1.83	2.57	1.83	1.83	1.04
time (sec)	N/A	0.05	0.016	0.003	0.824	0.258	0.261	0.28	5.657

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	60	0	212	49	55	73
normalized size	1	1.	0.99	0.8	0.	2.83	0.65	0.73	0.97
time (sec)	N/A	0.104	0.062	0.018	0.	0.281	0.593	0.284	7.372

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	35	69	90	58	69	66
normalized size	1	1.	0.96	0.61	1.21	1.58	1.02	1.21	1.16
time (sec)	N/A	0.079	0.059	0.01	0.825	0.257	0.377	0.276	11.232

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	65	117	0	217	56	92	105
normalized size	1	1.	0.73	1.31	0.	2.44	0.63	1.03	1.18
time (sec)	N/A	0.167	0.027	0.006	0.	0.288	0.724	0.28	18.056

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	42	76	117	65	85	78
normalized size	1	1.	0.91	0.64	1.15	1.77	0.98	1.29	1.18
time (sec)	N/A	0.144	0.059	0.011	0.824	0.251	0.53	0.282	16.592

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	122	0	215	65	104	110
normalized size	1	1.	0.75	1.26	0.	2.22	0.67	1.07	1.13
time (sec)	N/A	0.281	0.029	0.026	0.	0.283	0.87	0.279	25.167

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	440	58	46	0	1837	29	0	605
normalized size	1	0.96	0.13	0.1	0.	4.01	0.06	0.	1.32
time (sec)	N/A	0.873	0.022	0.011	0.	0.308	3.862	0.	91.068

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	431	41	40	0	1864	26	0	542
normalized size	1	0.96	0.09	0.09	0.	4.15	0.06	0.	1.21
time (sec)	N/A	0.705	0.017	0.009	0.	0.31	3.796	0.	86.547

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	451	39	40	0	1378	24	0	604
normalized size	1	0.96	0.08	0.09	0.	2.94	0.05	0.	1.29
time (sec)	N/A	0.662	0.016	0.01	0.	0.31	3.821	0.	84.724

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	431	40	40	0	1640	26	0	488
normalized size	1	0.96	0.09	0.09	0.	3.65	0.06	0.	1.09
time (sec)	N/A	0.518	0.015	0.009	0.	0.311	3.753	0.	86.542

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	414	42	37	0	1805	26	0	549
normalized size	1	1.33	0.14	0.12	0.	5.8	0.08	0.	1.77
time (sec)	N/A	0.593	0.014	0.009	0.	0.34	3.765	0.	80.713

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	416	61	52	0	1902	32	0	546
normalized size	1	0.96	0.14	0.12	0.	4.38	0.07	0.	1.26
time (sec)	N/A	0.74	0.023	0.014	0.	0.321	3.952	0.	95.785

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	466	65	50	0	1897	34	0	610
normalized size	1	0.96	0.13	0.1	0.	3.92	0.07	0.	1.26
time (sec)	N/A	0.808	0.024	0.014	0.	0.329	4.08	0.	93.59

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	575	0	0	0	0	0	105
normalized size	1	1.	4.91	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.128	1.022	0.027	0.	0.	0.	0.	14.954

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	38	68	96	58	72	68
normalized size	1	1.	0.9	0.61	1.1	1.55	0.94	1.16	1.1
time (sec)	N/A	0.106	0.058	0.005	0.824	0.295	0.348	0.299	12.228

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	103	67	124	166	170	131	102
normalized size	1	1.	1.14	0.74	1.38	1.84	1.89	1.46	1.13
time (sec)	N/A	0.246	0.088	0.008	0.82	0.271	1.754	0.315	21.808

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	61	84	53	65	60
normalized size	1	1.	0.96	0.6	1.11	1.53	0.96	1.18	1.09
time (sec)	N/A	0.076	0.033	0.003	0.82	0.282	0.314	0.291	8.804

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	91	62	117	155	165	124	99
normalized size	1	1.	1.12	0.77	1.44	1.91	2.04	1.53	1.22
time (sec)	N/A	0.147	0.05	0.005	0.826	0.313	1.74	0.295	15.415

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	42	59	42	45	24
normalized size	1	1.	1.65	0.83	1.83	2.57	1.83	1.96	1.04
time (sec)	N/A	0.056	0.018	0.002	0.834	0.276	0.265	0.291	5.984

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	91	62	117	150	165	124	73
normalized size	1	1.	1.21	0.83	1.56	2.	2.2	1.65	0.97
time (sec)	N/A	0.085	0.054	0.005	0.822	0.293	1.907	0.294	8.711

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	64	69	93	58	73	66
normalized size	1	1.	0.96	1.12	1.21	1.63	1.02	1.28	1.16
time (sec)	N/A	0.07	0.048	0.015	0.821	0.275	0.37	0.293	11.605

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	103	67	124	177	172	131	104
normalized size	1	1.	1.16	0.75	1.39	1.99	1.93	1.47	1.17
time (sec)	N/A	0.157	0.106	0.013	0.826	0.311	1.851	0.297	20.037

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	71	76	117	66	89	78
normalized size	1	1.	0.92	1.08	1.15	1.77	1.	1.35	1.18
time (sec)	N/A	0.144	0.063	0.016	0.829	0.306	0.533	0.303	16.768

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	72	134	188	197	140	110
normalized size	1	1.	1.14	0.74	1.38	1.94	2.03	1.44	1.13
time (sec)	N/A	0.261	0.121	0.016	0.821	0.275	2.028	0.303	26.771

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	160	205	0	383	58	200	238
normalized size	1	1.	0.94	1.21	0.	2.25	0.34	1.18	1.4
time (sec)	N/A	0.326	0.515	0.082	0.	0.315	3.249	0.343	26.351

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	160	206	0	425	53	198	189
normalized size	1	1.	0.96	1.23	0.	2.54	0.32	1.19	1.13
time (sec)	N/A	0.225	0.258	0.036	0.	0.303	3.208	0.35	23.368

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	132	206	0	374	49	198	238
normalized size	1	1.	0.76	1.19	0.	2.16	0.28	1.14	1.38
time (sec)	N/A	0.231	0.361	0.046	0.	0.287	3.153	0.35	20.263

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	166	131	110	0	441	53	198	162
normalized size	1	1.14	0.9	0.76	0.	3.04	0.37	1.37	1.12
time (sec)	N/A	0.141	0.069	0.037	0.	0.298	3.148	0.348	23.154

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	160	206	0	401	53	198	209
normalized size	1	1.	0.95	1.22	0.	2.37	0.31	1.17	1.24
time (sec)	N/A	0.138	0.279	0.035	0.	0.334	3.22	0.337	15.41

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	174	211	0	386	63	205	192
normalized size	1	1.	1.01	1.23	0.	2.24	0.37	1.19	1.12
time (sec)	N/A	0.234	0.59	0.037	0.	0.317	3.393	0.347	31.054

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	166	209	0	482	63	205	243
normalized size	1	1.	0.91	1.15	0.	2.65	0.35	1.13	1.34
time (sec)	N/A	0.297	0.483	0.041	0.	0.29	3.444	0.344	26.524

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	189	216	0	478	71	215	199
normalized size	1	1.	1.09	1.25	0.	2.76	0.41	1.24	1.15
time (sec)	N/A	0.372	0.519	0.051	0.	0.297	3.482	0.345	37.436

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	189	216	0	489	68	215	248
normalized size	1	1.	1.	1.14	0.	2.59	0.36	1.14	1.31
time (sec)	N/A	0.403	0.528	0.057	0.	0.282	3.563	0.354	31.417

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	15	23	15
normalized size	1	1.	1.	0.86	1.1	1.1	0.71	1.1	0.71
time (sec)	N/A	0.029	0.006	0.009	0.746	0.254	0.255	0.281	5.654

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	30	30	19	30	19
normalized size	1	1.	1.	0.88	1.15	1.15	0.73	1.15	0.73
time (sec)	N/A	0.045	0.007	0.008	0.744	0.252	0.299	0.281	11.105

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	46	37	41	34
normalized size	1	1.	1.	0.84	1.11	1.24	1.	1.11	0.92
time (sec)	N/A	0.069	0.019	0.004	0.822	0.298	0.351	0.271	8.267

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	24	27	24	24
normalized size	1	1.	1.	0.83	1.04	1.04	1.17	1.04	1.04
time (sec)	N/A	0.045	0.009	0.003	0.821	0.26	0.324	0.295	5.2

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	197	66	49	55	41	45	41
normalized size	1	1.	5.05	1.69	1.26	1.41	1.05	1.15	1.05
time (sec)	N/A	0.072	0.057	0.07	0.82	0.265	0.406	0.295	12.415

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	208	73	55	78	48	61	48
normalized size	1	1.	4.33	1.52	1.15	1.62	1.	1.27	1.
time (sec)	N/A	0.103	0.066	0.034	0.826	0.267	0.61	0.282	15.337

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	197	66	0	55	41	45	41
normalized size	1	1.	5.05	1.69	0.	1.41	1.05	1.15	1.05
time (sec)	N/A	0.067	0.027	0.017	0.	0.255	0.428	0.266	13.402

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	140	236	0	1	600	196	0
normalized size	1	1.	0.95	1.61	0.	0.01	4.08	1.33	0.
time (sec)	N/A	0.276	0.215	0.009	0.	0.262	3.934	0.278	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	190	0	1	496	153	0
normalized size	1	1.	0.95	1.61	0.	0.01	4.2	1.3	0.
time (sec)	N/A	0.221	0.143	0.005	0.	0.28	3.489	0.297	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	132	0	1	381	116	0
normalized size	1	1.	0.94	1.48	0.	0.01	4.28	1.3	0.
time (sec)	N/A	0.171	0.184	0.004	0.	0.26	2.884	0.274	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	101	0	1	306	90	65
normalized size	1	1.	1.04	1.44	0.	0.01	4.37	1.29	0.93
time (sec)	N/A	0.105	0.103	0.004	0.	0.281	2.371	0.291	21.157

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	1	216	74	49
normalized size	1	1.	1.02	1.	0.	0.02	3.86	1.32	0.88
time (sec)	N/A	0.078	0.055	0.003	0.	0.268	0.909	0.267	17.741

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	1	124	46	32
normalized size	1	1.	1.06	0.97	0.	0.03	3.44	1.28	0.89
time (sec)	N/A	0.064	0.011	0.001	0.	0.269	0.548	0.287	11.2

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	1	564	84	54
normalized size	1	1.	0.98	1.	0.	0.02	9.1	1.35	0.87
time (sec)	N/A	0.106	0.112	0.007	0.	0.283	6.623	0.306	24.9

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	112	0	1	862	107	75
normalized size	1	1.	0.95	1.38	0.	0.01	10.64	1.32	0.93
time (sec)	N/A	0.222	0.142	0.01	0.	0.278	12.363	0.316	40.392

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	150	0	1	1525	142	97
normalized size	1	1.	0.98	1.44	0.	0.01	14.66	1.37	0.93
time (sec)	N/A	0.326	0.251	0.012	0.	0.308	17.802	0.295	49.421

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	214	0	1	2105	184	129
normalized size	1	1.	0.96	1.56	0.	0.01	15.36	1.34	0.94
time (sec)	N/A	0.398	0.173	0.013	0.	0.332	28.619	0.269	62.512

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	163	662	0	1	1012	254	0
normalized size	1	1.	0.83	3.38	0.	0.01	5.16	1.3	0.
time (sec)	N/A	0.439	0.35	0.019	0.	0.27	7.779	0.292	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	132	569	0	1	842	217	0
normalized size	1	1.	0.88	3.79	0.	0.01	5.61	1.45	0.
time (sec)	N/A	0.307	0.299	0.012	0.	0.272	6.13	0.262	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	109	330	0	1	729	169	0
normalized size	1	1.	0.96	2.89	0.	0.01	6.39	1.48	0.
time (sec)	N/A	0.215	0.226	0.017	0.	0.261	4.532	0.269	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	81	97	0	1	280	119	60
normalized size	1	1.	1.14	1.37	0.	0.01	3.94	1.68	0.85
time (sec)	N/A	0.093	0.15	0.008	0.	0.274	2.832	0.284	11.796

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	1	252	103	60
normalized size	1	1.	1.05	1.06	0.	0.02	3.82	1.56	0.91
time (sec)	N/A	0.076	0.105	0.004	0.	0.266	2.657	0.271	13.737

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	1	265	103	60
normalized size	1	1.	1.06	1.03	0.	0.02	4.02	1.56	0.91
time (sec)	N/A	0.069	0.123	0.003	0.	0.277	2.711	0.277	11.336

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	389	0	1	2236	170	102
normalized size	1	1.	0.99	3.6	0.	0.01	20.7	1.57	0.94
time (sec)	N/A	0.309	0.315	0.016	0.	0.342	25.511	0.275	45.787

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	131	545	0	1	2672	231	143
normalized size	1	1.	0.89	3.68	0.	0.01	18.05	1.56	0.97
time (sec)	N/A	0.4	0.463	0.021	0.	0.421	38.659	0.287	74.922

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	175	646	0	1	4083	309	192
normalized size	1	1.	0.87	3.2	0.	0.	20.21	1.53	0.95
time (sec)	N/A	0.507	0.772	0.023	0.	0.522	59.168	0.276	85.639

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	260	1524	0	1	1714	381	0
normalized size	1	1.	1.09	6.4	0.	0.	7.2	1.6	0.
time (sec)	N/A	0.638	0.649	0.027	0.	0.27	16.245	0.268	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	221	806	0	1	1510	331	0
normalized size	1	1.	1.16	4.24	0.	0.01	7.95	1.74	0.
time (sec)	N/A	0.575	0.565	0.027	0.	0.274	10.652	0.288	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	174	260	0	1	547	273	95
normalized size	1	1.	1.57	2.34	0.	0.01	4.93	2.46	0.86
time (sec)	N/A	0.139	0.275	0.015	0.	0.27	6.665	0.295	17.395

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	126	223	0	1	510	220	102
normalized size	1	1.	1.18	2.08	0.	0.01	4.77	2.06	0.95
time (sec)	N/A	0.113	0.348	0.013	0.	0.272	5.712	0.264	24.14

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	131	262	0	1	570	208	110
normalized size	1	1.	1.14	2.28	0.	0.01	4.96	1.81	0.96
time (sec)	N/A	0.148	0.259	0.013	0.	0.273	5.515	0.274	29.026

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	130	0	1	479	182	97
normalized size	1	1.	0.99	1.26	0.	0.01	4.65	1.77	0.94
time (sec)	N/A	0.099	0.17	0.005	0.	0.267	5.113	0.283	18.18

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	129	0	1	474	184	95
normalized size	1	1.	0.96	1.28	0.	0.01	4.69	1.82	0.94
time (sec)	N/A	0.091	0.159	0.005	0.	0.262	5.194	0.314	16.696

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	178	1159	0	1	4862	323	180
normalized size	1	1.	0.96	6.26	0.	0.01	26.28	1.75	0.97
time (sec)	N/A	0.482	0.595	0.024	0.	0.645	66.515	0.305	82.002

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	1438	0	1	5722	417	238
normalized size	1	1.	0.92	6.02	0.	0.	23.94	1.74	1.
time (sec)	N/A	0.634	0.742	0.03	0.	0.86	153.597	0.299	141.342

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	41	41	34	43	0
normalized size	1	1.	1.	0.78	1.02	1.02	0.85	1.08	0.
time (sec)	N/A	0.057	0.009	0.009	0.739	0.263	0.247	0.269	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	34	27	36	0
normalized size	1	1.	1.	0.79	1.03	1.03	0.82	1.09	0.
time (sec)	N/A	0.048	0.007	0.008	0.752	0.264	0.257	0.288	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	27	20	30	20
normalized size	1	1.	1.	0.81	1.04	1.04	0.77	1.15	0.77
time (sec)	N/A	0.035	0.006	0.008	0.743	0.259	0.24	0.26	7.456

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	17	26	17
normalized size	1	1.	1.	0.86	1.1	1.1	0.81	1.24	0.81
time (sec)	N/A	0.031	0.005	0.008	0.743	0.286	0.218	0.297	9.102

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	18	23	23	15	26	15
normalized size	1	1.	0.91	0.78	1.	1.	0.65	1.13	0.65
time (sec)	N/A	0.03	0.004	0.007	0.75	0.247	0.217	0.301	6.829

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	28	24	32	24
normalized size	1	1.	1.	0.81	1.04	1.04	0.89	1.19	0.89
time (sec)	N/A	0.044	0.007	0.009	0.745	0.254	0.314	0.265	12.126

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	41	31	39	31
normalized size	1	1.	1.	0.79	1.03	1.21	0.91	1.15	0.91
time (sec)	N/A	0.075	0.006	0.012	0.743	0.258	0.364	0.285	17.374

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	42	53	36	46	37
normalized size	1	1.	1.	0.78	1.02	1.29	0.88	1.12	0.9
time (sec)	N/A	0.082	0.008	0.012	0.74	0.253	0.397	0.28	17.628

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	49	59	41	53	44
normalized size	1	1.	1.	0.77	1.02	1.23	0.85	1.1	0.92
time (sec)	N/A	0.094	0.008	0.012	0.751	0.254	0.443	0.281	17.477

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	233	701	0	1	0	0	178
normalized size	1	1.	1.14	3.44	0.	0.	0.	0.	0.87
time (sec)	N/A	0.607	1.021	0.02	0.	0.424	0.	0.	61.534

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	236	334	0	1	0	0	124
normalized size	1	1.	1.63	2.3	0.	0.01	0.	0.	0.86
time (sec)	N/A	0.37	0.287	0.011	0.	0.345	0.	0.	37.652

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	138	121	0	1	0	0	85
normalized size	1	1.	1.31	1.15	0.	0.01	0.	0.	0.81
time (sec)	N/A	0.216	0.154	0.01	0.	0.296	0.	0.	22.431

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	87	88	0	1	0	0	53
normalized size	1	1.	1.3	1.31	0.	0.01	0.	0.	0.79
time (sec)	N/A	0.096	0.097	0.01	0.	0.276	0.	0.	11.689

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	136	197	0	1	0	0	117
normalized size	1	1.	1.02	1.48	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.25	0.193	0.013	0.	0.313	0.	0.	30.297

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	235	376	0	1	0	0	206
normalized size	1	1.	1.07	1.71	0.	0.	0.	0.	0.94
time (sec)	N/A	0.473	0.527	0.016	0.	0.423	0.	0.	67.417

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	32	40	11	11	0	39	58
normalized size	1	1.	0.44	0.55	0.15	0.15	0.	0.53	0.79
time (sec)	N/A	0.086	0.03	0.014	0.751	0.267	0.	0.294	5.41

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	1430	129	1	189
normalized size	1	1.	1.13	1.92	0.	7.99	0.72	0.01	1.06
time (sec)	N/A	0.601	0.222	0.013	0.	0.274	5.948	0.83	53.151

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	6885	196	0	0
normalized size	1	1.	0.11	0.09	0.	10.91	0.31	0.	0.
time (sec)	N/A	2.6	0.059	0.023	0.	0.418	9.067	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	5154	218	0	382
normalized size	1	1.	0.19	0.16	0.	13.71	0.58	0.	1.02
time (sec)	N/A	1.364	0.062	0.037	0.	0.44	52.03	0.	137.529

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	110	84	0	0	0	0	97
normalized size	1	1.	1.04	0.79	0.	0.	0.	0.	0.92
time (sec)	N/A	0.235	0.124	0.034	0.	0.	0.	0.	28.403

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	60	52	73	72	58	66	34
normalized size	1	1.	1.5	1.3	1.82	1.8	1.45	1.65	0.85
time (sec)	N/A	0.046	0.017	0.003	0.747	0.275	1.632	0.295	13.352

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-1)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	50	31	0	0	65	73
normalized size	1	1.	0.67	0.67	0.41	0.	0.	0.87	0.97
time (sec)	N/A	0.085	0.038	0.033	0.761	0.	0.	0.299	9.53

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	115	109	0	113	0	189	126
normalized size	1	1.	0.84	0.8	0.	0.82	0.	1.38	0.92
time (sec)	N/A	0.171	0.063	0.014	0.	0.274	0.	0.304	15.006

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	91	87	0	82	0	138	126
normalized size	1	1.	0.66	0.64	0.	0.6	0.	1.01	0.92
time (sec)	N/A	0.157	0.042	0.004	0.	0.275	0.	0.284	14.949

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	65	65	0	43	0	86	122
normalized size	1	1.	0.47	0.47	0.	0.31	0.	0.63	0.89
time (sec)	N/A	0.123	0.033	0.004	0.	0.273	0.	0.283	15.021

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	43	43	0	14	0	35	70
normalized size	1	1.	0.49	0.49	0.	0.16	0.	0.4	0.8
time (sec)	N/A	0.091	0.016	0.004	0.	0.277	0.	0.271	9.328

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	65	103	62	45	0	82	131
normalized size	1	1.	0.44	0.7	0.42	0.31	0.	0.56	0.89
time (sec)	N/A	0.153	0.041	0.044	0.744	0.277	0.	0.283	16.162

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	72	92	88	93	0	86	131
normalized size	1	1.	0.55	0.71	0.68	0.72	0.	0.66	1.01
time (sec)	N/A	0.154	0.047	0.012	0.746	0.272	0.	0.285	11.943

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	58	54	85	90	0	0	75
normalized size	1	1.	0.43	0.4	0.63	0.67	0.	0.	0.56
time (sec)	N/A	0.157	0.031	0.01	0.75	0.274	0.	0.	8.865

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	85	123	0	0	124
normalized size	1	1.	0.42	0.39	0.62	0.9	0.	0.	0.91
time (sec)	N/A	0.16	0.041	0.01	0.745	0.275	0.	0.	10.941

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	85	151	0	0	128
normalized size	1	1.	0.42	0.39	0.62	1.1	0.	0.	0.93
time (sec)	N/A	0.159	0.032	0.011	0.751	0.275	0.	0.	11.128

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	85	181	0	0	126
normalized size	1	1.	0.42	0.39	0.62	1.32	0.	0.	0.92
time (sec)	N/A	0.156	0.039	0.011	0.768	0.277	0.	0.	11.374

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	68	0	0	0	0	0	75
normalized size	1	1.	0.88	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.084	0.096	0.121	0.	0.	0.	0.	26.542

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	367	0	489	782	0	1	614
normalized size	1	1.	0.78	0.	1.04	1.67	0.	0.	1.31
time (sec)	N/A	0.45	0.337	0.012	0.795	0.447	0.	0.29	97.904

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	199	0	267	401	0	1065	410
normalized size	1	1.	0.63	0.	0.85	1.27	0.	3.38	1.3
time (sec)	N/A	0.279	0.268	0.012	0.791	0.364	0.	0.285	65.384

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	83	0	104	149	0	328	144
normalized size	1	1.	0.58	0.	0.73	1.05	0.	2.31	1.01
time (sec)	N/A	0.139	0.056	0.008	0.78	0.314	0.	0.278	24.417

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	0	0	0	0	0	85
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	1.23
time (sec)	N/A	0.068	0.038	0.012	0.	0.	0.	0.	17.81

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	0	0	0	0	0	87
normalized size	1	1.	0.88	0.	0.	0.	0.	0.	1.16
time (sec)	N/A	0.079	0.038	0.011	0.	0.	0.	0.	18.905

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	162	64	0	85	111	0	0	185
normalized size	1	1.11	0.44	0.	0.58	0.76	0.	0.	1.27
time (sec)	N/A	0.2	0.084	0.016	1.213	0.376	0.	0.	50.017

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	93	114	200	123	0	0	160
normalized size	1	1.	0.53	0.65	1.14	0.7	0.	0.	0.91
time (sec)	N/A	0.198	0.073	0.021	0.78	0.266	0.	0.	17.88

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	121	174	161	212	0	0	250
normalized size	1	1.	0.45	0.65	0.6	0.79	0.	0.	0.93
time (sec)	N/A	0.295	0.091	0.022	0.794	0.277	0.	0.	27.63

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	66	68	42	0	0	108	134
normalized size	1	1.	0.37	0.38	0.23	0.	0.	0.6	0.75
time (sec)	N/A	0.201	0.052	0.036	0.786	0.	0.	0.292	22.706

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	125	115	107	116	0	234	313
normalized size	1	1.	0.32	0.29	0.27	0.3	0.	0.6	0.8
time (sec)	N/A	0.398	0.087	0.031	0.755	0.274	0.	0.305	45.79

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	99	91	77	82	0	173	228
normalized size	1	1.	0.34	0.31	0.26	0.28	0.	0.59	0.78
time (sec)	N/A	0.304	0.08	0.016	0.755	0.273	0.	0.298	32.739

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	77	69	41	45	0	107	163
normalized size	1	1.	0.41	0.37	0.22	0.24	0.	0.57	0.86
time (sec)	N/A	0.209	0.049	0.008	0.75	0.277	0.	0.293	27.374

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	49	50	14	14	0	46	70
normalized size	1	1.	0.56	0.57	0.16	0.16	0.	0.52	0.8
time (sec)	N/A	0.127	0.019	0.008	0.75	0.269	0.	0.275	9.808

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	86	78	59	58	0	104	235
normalized size	1	1.	0.45	0.41	0.31	0.31	0.	0.55	1.24
time (sec)	N/A	0.261	0.039	0.009	0.753	0.272	0.	0.308	43.351

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	126	141	131	154	0	163	314
normalized size	1	1.	0.42	0.47	0.44	0.51	0.	0.54	1.05
time (sec)	N/A	0.384	0.086	0.017	0.752	0.274	0.	0.317	53.815

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	152	199	188	238	0	190	396
normalized size	1	1.	0.37	0.49	0.46	0.58	0.	0.46	0.97
time (sec)	N/A	0.521	0.129	0.018	0.751	0.276	0.	0.319	68.309

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	98	94	77	82	0	170	231
normalized size	1	1.	0.34	0.33	0.27	0.28	0.	0.59	0.8
time (sec)	N/A	0.302	0.07	0.044	0.755	0.275	0.	0.307	32.521

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	103	91	70	74	0	169	246
normalized size	1	1.	0.35	0.31	0.24	0.25	0.	0.58	0.85
time (sec)	N/A	0.289	0.074	0.025	0.76	0.276	0.	0.302	37.987

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	98	152	134	182	0	113	218
normalized size	1	1.	0.44	0.68	0.6	0.82	0.	0.51	0.98
time (sec)	N/A	0.253	0.09	0.018	0.76	0.276	0.	0.288	20.884

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	124	116	107	112	0	232	313
normalized size	1	1.	0.32	0.3	0.27	0.29	0.	0.59	0.8
time (sec)	N/A	0.392	0.086	0.035	0.757	0.274	0.	0.39	44.408

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	62	61	51	0	0	0
normalized size	1	1.	0.83	1.35	1.33	1.11	0.	0.	0.
time (sec)	N/A	0.072	0.033	0.04	0.755	0.303	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	33	43	32	0	0	0
normalized size	1	1.	0.86	1.18	1.54	1.14	0.	0.	0.
time (sec)	N/A	0.048	0.018	0.032	0.745	0.298	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	26	20	37	0	10
normalized size	1	1.	1.	1.2	1.73	1.33	2.47	0.	0.67
time (sec)	N/A	0.023	0.004	0.029	0.747	0.287	58.471	0.	4.485

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	26	36	30	66	34	19
normalized size	1	1.	0.96	1.13	1.57	1.3	2.87	1.48	0.83
time (sec)	N/A	0.036	0.014	0.029	0.749	0.296	95.702	0.271	7.815

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	69	76	80	0	0	51
normalized size	1	1.	0.81	1.21	1.33	1.4	0.	0.	0.89
time (sec)	N/A	0.08	0.043	0.034	0.749	0.29	0.	0.	14.712

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	88	93	97	0	0	66
normalized size	1	1.	0.8	1.16	1.22	1.28	0.	0.	0.87
time (sec)	N/A	0.093	0.039	0.041	0.755	0.291	0.	0.	17.295

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	74	105	111	115	0	0	82
normalized size	1	1.	0.8	1.13	1.19	1.24	0.	0.	0.88
time (sec)	N/A	0.109	0.059	0.045	0.75	0.292	0.	0.	20.232

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	60	54	0	335	0	0	206
normalized size	1	1.	0.25	0.23	0.	1.42	0.	0.	0.87
time (sec)	N/A	0.386	0.044	0.322	0.	0.307	0.	0.	64.237

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	60	54	0	293	0	0	138
normalized size	1	1.	0.38	0.34	0.	1.83	0.	0.	0.86
time (sec)	N/A	0.245	0.044	0.091	0.	0.301	0.	0.	40.681

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	79	0	1	0	0	39
normalized size	1	1.	1.	1.58	0.	0.02	0.	0.	0.78
time (sec)	N/A	0.068	0.041	0.093	0.	0.298	0.	0.	13.454

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	97	0	1	0	0	56
normalized size	1	1.	0.91	1.43	0.	0.01	0.	0.	0.82
time (sec)	N/A	0.09	0.089	0.122	0.	0.301	0.	0.	17.867

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	70	73	0	228	0	0	151
normalized size	1	1.	0.4	0.41	0.	1.3	0.	0.	0.86
time (sec)	N/A	0.272	0.077	0.106	0.	0.302	0.	0.	48.843

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	70	73	0	317	0	0	219
normalized size	1	1.	0.28	0.29	0.	1.26	0.	0.	0.87
time (sec)	N/A	0.427	0.081	0.118	0.	0.312	0.	0.	73.783

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	0	58	80	0	0	27
normalized size	1	1.	1.03	0.	1.57	2.16	0.	0.	0.73
time (sec)	N/A	0.067	0.049	0.109	0.813	0.291	0.	0.	9.398

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	43	0	0	80	0	0	31
normalized size	1	1.	1.13	0.	0.	2.11	0.	0.	0.82
time (sec)	N/A	0.067	0.086	0.112	0.	0.292	0.	0.	9.562

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	96	208	100	100	0	0	71
normalized size	1	1.	0.86	1.86	0.89	0.89	0.	0.	0.63
time (sec)	N/A	0.115	0.081	0.102	0.771	0.297	0.	0.	15.434

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	70	135	65	65	0	0	71
normalized size	1	1.	0.62	1.21	0.58	0.58	0.	0.	0.63
time (sec)	N/A	0.109	0.051	0.038	0.765	0.29	0.	0.	15.129

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	44	64	30	30	0	0	80
normalized size	1	1.	0.44	0.65	0.3	0.3	0.	0.	0.81
time (sec)	N/A	0.081	0.027	0.034	0.761	0.269	0.	0.	9.195

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	44	71	43	32	0	0	73
normalized size	1	1.	0.49	0.79	0.48	0.36	0.	0.	0.81
time (sec)	N/A	0.114	0.048	0.048	0.76	0.272	0.	0.	16.774

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	37	55	55	0	0	42
normalized size	1	1.	0.83	0.77	1.15	1.15	0.	0.	0.88
time (sec)	N/A	0.071	0.041	0.038	0.765	0.267	0.	0.	8.92

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	93	93	0	0	73
normalized size	1	1.	0.45	0.42	1.06	1.06	0.	0.	0.83
time (sec)	N/A	0.124	0.042	0.044	0.758	0.273	0.	0.	15.365

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	131	131	0	0	73
normalized size	1	1.	0.45	0.42	1.49	1.49	0.	0.	0.83
time (sec)	N/A	0.122	0.049	0.048	0.768	0.271	0.	0.	15.202

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	132	63	77	0	236	97
normalized size	1	1.	0.51	1.22	0.58	0.71	0.	2.19	0.9
time (sec)	N/A	0.104	0.05	0.064	0.765	0.274	0.	0.286	11.412

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	34	38	0	74	80
normalized size	1	1.	0.49	0.66	0.37	0.41	0.	0.8	0.86
time (sec)	N/A	0.079	0.028	0.021	0.754	0.283	0.	0.279	9.443

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	34	38	0	74	80
normalized size	1	1.	0.49	0.66	0.37	0.41	0.	0.8	0.86
time (sec)	N/A	0.066	0.027	0.022	0.755	0.272	0.	0.279	8.611

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	39	56	26	27	0	34	76
normalized size	1	1.	0.44	0.64	0.3	0.31	0.	0.39	0.86
time (sec)	N/A	0.047	0.02	0.02	0.75	0.279	0.	0.279	3.288

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	37	54	18	20	0	0	70
normalized size	1	1.	0.44	0.64	0.21	0.24	0.	0.	0.82
time (sec)	N/A	0.074	0.021	0.026	0.759	0.275	0.	0.	8.768

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	42	61	30	31	0	0	76
normalized size	1	1.	0.45	0.65	0.32	0.33	0.	0.	0.81
time (sec)	N/A	0.079	0.032	0.029	0.754	0.277	0.	0.	9.318

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	47	61	30	31	0	0	80
normalized size	1	1.	0.49	0.64	0.31	0.32	0.	0.	0.83
time (sec)	N/A	0.078	0.035	0.028	0.759	0.273	0.	0.	9.378

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	90	532	373	527	0	1	248
normalized size	1	1.	0.38	2.24	1.57	2.21	0.	0.	1.04
time (sec)	N/A	0.228	0.154	0.086	0.767	0.286	0.	0.38	42.771

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	123	146	146	194	0	410	197
normalized size	1	1.	0.58	0.69	0.69	0.92	0.	1.93	0.93
time (sec)	N/A	0.16	0.107	0.025	0.755	0.271	0.	0.291	29.548

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	124	145	147	196	0	410	0
normalized size	1	1.	0.59	0.69	0.7	0.93	0.	1.94	0.
time (sec)	N/A	0.139	0.107	0.025	0.755	0.284	0.	0.291	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	122	138	136	176	0	371	194
normalized size	1	1.	0.59	0.67	0.66	0.85	0.	1.8	0.94
time (sec)	N/A	0.12	0.098	0.024	0.756	0.276	0.	0.289	18.179

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	66	127	58	59	0	0	143
normalized size	1	1.	0.34	0.65	0.3	0.3	0.	0.	0.73
time (sec)	N/A	0.139	0.064	0.031	0.753	0.272	0.	0.	18.013

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	124	147	136	177	0	0	194
normalized size	1	1.	0.58	0.69	0.64	0.83	0.	0.	0.92
time (sec)	N/A	0.17	0.147	0.038	0.756	0.289	0.	0.	30.245

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	124	145	136	181	0	0	197
normalized size	1	1.	0.57	0.67	0.62	0.83	0.	0.	0.9
time (sec)	N/A	0.169	0.143	0.039	0.752	0.273	0.	0.	29.873

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	0	0	0	0	0	66
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.099	0.048	0.109	0.	0.	0.	0.	16.828

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	56
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.078	0.036	0.03	0.	0.	0.	0.	15.392

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	56
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.06	0.032	0.03	0.	0.	0.	0.	13.887

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	0	0	0	0	0	53
normalized size	1	1.	0.8	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.038	0.02	0.03	0.	0.	0.	0.	4.947

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	42	66	36	30	0	0	90
normalized size	1	1.	0.49	0.78	0.42	0.35	0.	0.	1.06
time (sec)	N/A	0.102	0.035	0.029	0.763	0.282	0.	0.	20.457

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	56
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.074	0.029	0.03	0.	0.	0.	0.	14.881

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	0	0	0	0	0	60
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.073	0.029	0.03	0.	0.	0.	0.	15.062

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	119	0	0	0	0	0	68
normalized size	1	1.	1.57	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.095	0.2	0.057	0.	0.	0.	0.	16.356

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	98	0	0	0	0	0	58
normalized size	1	1.	1.53	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.077	0.103	0.088	0.	0.	0.	0.	14.669

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	0	0	0	0	0	58
normalized size	1	1.	1.45	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.06	0.094	0.087	0.	0.	0.	0.	13.195

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	93	0	0	0	0	0	54
normalized size	1	1.	1.63	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.037	0.125	0.09	0.	0.	0.	0.	5.088

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	78	104	95	143	0	0	163
normalized size	1	1.	0.49	0.65	0.6	0.9	0.	0.	1.03
time (sec)	N/A	0.185	0.104	0.031	0.756	0.289	0.	0.	31.241

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	98	0	0	0	0	0	58
normalized size	1	1.	1.51	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.074	0.127	0.091	0.	0.	0.	0.	13.967

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	94	0	0	0	0	0	61
normalized size	1	1.	1.4	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.074	0.11	0.084	0.	0.	0.	0.	14.204

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	121	0	0	107	0	0	49
normalized size	1	1.	2.33	0.	0.	2.06	0.	0.	0.94
time (sec)	N/A	0.039	0.165	0.354	0.	0.294	0.	0.	2.598

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	51	0	61	0	0	41
normalized size	1	1.	0.74	1.19	0.	1.42	0.	0.	0.95
time (sec)	N/A	0.029	0.095	0.053	0.	0.281	0.	0.	2.734

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	165	0	0	139	0	0	95
normalized size	1	1.	1.27	0.	0.	1.07	0.	0.	0.73
time (sec)	N/A	0.11	0.308	0.391	0.	0.288	0.	0.	4.684

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	59	0	0	111	0	0	80
normalized size	1	1.	0.58	0.	0.	1.09	0.	0.	0.78
time (sec)	N/A	0.087	0.073	0.145	0.	0.286	0.	0.	4.315

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	124	75	0	0	223	0	0	104
normalized size	1	1.06	0.64	0.	0.	1.91	0.	0.	0.89
time (sec)	N/A	0.146	0.103	0.084	0.	0.277	0.	0.	13.515

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	54	148	80	105	0	0	78
normalized size	1	1.	0.52	1.44	0.78	1.02	0.	0.	0.76
time (sec)	N/A	0.139	0.054	0.109	0.775	0.271	0.	0.	18.887

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	97	973	0	1	0	0	0
normalized size	1	1.	0.87	8.77	0.	0.01	0.	0.	0.
time (sec)	N/A	0.239	0.26	0.269	0.	0.296	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	664	0	1	0	0	76
normalized size	1	1.	0.94	7.63	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.152	0.152	0.148	0.	0.285	0.	0.	30.808

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	402	0	1	0	0	58
normalized size	1	1.	0.97	5.91	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.104	0.086	0.125	0.	0.276	0.	0.	19.815

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	113	0	1	0	53	37
normalized size	1	1.	1.1	2.9	0.	0.03	0.	1.36	0.95
time (sec)	N/A	0.066	0.037	0.074	0.	0.285	0.	0.271	11.042

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	658	0	1	0	0	90
normalized size	1	1.	0.89	6.71	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.27	0.233	0.173	0.	0.284	0.	0.	44.347

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	105	958	0	1	0	0	116
normalized size	1	1.	0.83	7.6	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.379	0.339	0.204	0.	0.286	0.	0.	53.684

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	136	1300	0	1	0	0	151
normalized size	1	1.	0.83	7.93	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.474	0.527	0.256	0.	0.296	0.	0.	70.459

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	62	280	0	4443	0	0	311
normalized size	1	1.	0.18	0.79	0.	12.59	0.	0.	0.88
time (sec)	N/A	1.127	0.071	0.391	0.	0.472	0.	0.	111.414

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	62	260	0	6064	0	0	559
normalized size	1	1.	0.1	0.43	0.	9.94	0.	0.	0.92
time (sec)	N/A	2.166	0.068	0.348	0.	0.43	0.	0.	170.553

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	60	114	0	1081	0	0	151
normalized size	1	1.	0.36	0.67	0.	6.4	0.	0.	0.89
time (sec)	N/A	0.368	0.056	0.179	0.	0.295	0.	0.	34.218

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	105	268	0	1659	0	0	202
normalized size	1	1.	0.51	1.31	0.	8.09	0.	0.	0.99
time (sec)	N/A	0.795	0.098	0.307	0.	0.311	0.	0.	77.293

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	699	699	107	534	0	7922	0	0	0
normalized size	1	1.	0.15	0.76	0.	11.33	0.	0.	0.
time (sec)	N/A	3.151	0.115	0.585	0.	0.56	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	105	630	0	5652	0	0	403
normalized size	1	1.	0.25	1.52	0.	13.65	0.	0.	0.97
time (sec)	N/A	1.563	0.109	0.841	0.	0.626	0.	0.	179.699

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	265	0	0	0	0	0	122
normalized size	1	1.	1.89	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.28	1.168	0.074	0.	0.	0.	0.	24.611

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	263	0	0	0	0	0	112
normalized size	1	1.	1.93	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.116	1.437	0.04	0.	0.	0.	0.	22.162

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	261	0	0	0	0	0	112
normalized size	1	1.	2.1	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.125	1.086	0.033	0.	0.	0.	0.	11.773

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	397	0	1	0	0	66
normalized size	1	1.	1.	5.36	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.152	0.177	0.112	0.	0.298	0.	0.	28.356

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	240	0	0	0	0	0	114
normalized size	1	1.	1.69	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.14	0.727	0.044	0.	0.	0.	0.	24.342

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	258	0	0	0	0	0	114
normalized size	1	1.	1.84	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.125	0.75	0.049	0.	0.	0.	0.	24.295

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	820	0	0	0	0	0	128
normalized size	1	1.	5.54	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.507	7.548	0.142	0.	0.	0.	0.	34.124

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	825	0	0	0	0	0	128
normalized size	1	1.	5.57	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.47	8.243	0.079	0.	0.	0.	0.	33.535

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	816	0	0	0	0	0	128
normalized size	1	1.	5.51	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.351	8.443	0.078	0.	0.	0.	0.	28.421

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	786	0	0	0	0	0	124
normalized size	1	1.	5.65	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.222	8.739	0.077	0.	0.	0.	0.	37.13

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	125	0	1	0	0	102
normalized size	1	1.	0.99	1.05	0.	0.01	0.	0.	0.86
time (sec)	N/A	0.237	0.284	0.05	0.	0.326	0.	0.	31.04

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	821	0	0	0	0	0	128
normalized size	1	1.	5.51	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.469	8.588	0.084	0.	0.	0.	0.	33.633

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	816	0	0	0	0	0	131
normalized size	1	1.	5.4	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.452	8.921	0.084	0.	0.	0.	0.	33.812

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	3165	0	0	0	0	0	129
normalized size	1	1.	21.24	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.469	6.333	0.074	0.	0.	0.	0.	38.329

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	3165	0	0	0	0	0	129
normalized size	1	1.	21.24	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.475	6.416	0.072	0.	0.	0.	0.	38.642

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	3165	0	0	0	0	0	129
normalized size	1	1.	21.24	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.352	6.384	0.074	0.	0.	0.	0.	30.011

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	3058	0	0	0	0	0	126
normalized size	1	1.	21.84	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.225	6.327	0.071	0.	0.	0.	0.	37.09

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	161	209	0	1	0	0	151
normalized size	1	1.	0.93	1.21	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.398	0.721	0.051	0.	0.418	0.	0.	48.871

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	3181	0	0	0	0	0	129
normalized size	1	1.	21.21	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.466	6.349	0.084	0.	0.	0.	0.	37.896

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	3165	0	0	0	0	0	133
normalized size	1	1.	20.82	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.448	6.362	0.085	0.	0.	0.	0.	36.7

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	415	0	0	0	0	0	126
normalized size	1	1.	2.8	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.479	0.497	0.029	0.	0.	0.	0.	33.23

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	417	0	0	0	0	0	126
normalized size	1	1.	2.82	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.476	0.483	0.029	0.	0.	0.	0.	33.173

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	415	0	0	0	0	0	126
normalized size	1	1.	2.8	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.35	0.49	0.03	0.	0.	0.	0.	28.203

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	400	0	0	0	0	0	122
normalized size	1	1.	2.88	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.219	0.35	0.029	0.	0.	0.	0.	40.05

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	0	0	1	0	0	41
normalized size	1	1.	1.06	0.	0.	0.02	0.	0.	0.87
time (sec)	N/A	0.086	0.113	0.036	0.	0.285	0.	0.	12.446

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	415	0	0	0	0	0	126
normalized size	1	1.	2.79	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.463	0.424	0.031	0.	0.	0.	0.	33.261

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	415	0	0	0	0	0	129
normalized size	1	1.	2.75	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.461	0.518	0.03	0.	0.	0.	0.	33.177

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	1947	0	0	0	0	0	128
normalized size	1	1.	12.89	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.481	5.765	0.019	0.	0.	0.	0.	41.371

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	2229	0	0	0	0	0	128
normalized size	1	1.	14.76	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.483	6.221	0.019	0.	0.	0.	0.	43.309

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	1947	0	0	0	0	0	128
normalized size	1	1.	12.89	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.356	5.44	0.019	0.	0.	0.	0.	28.734

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	2144	0	0	0	0	0	124
normalized size	1	1.	15.1	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.221	6.184	0.019	0.	0.	0.	0.	36.35

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	1	0	0	87
normalized size	1	1.	1.02	0.	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.166	0.884	0.02	0.	0.33	0.	0.	23.072

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	2225	0	0	0	0	0	128
normalized size	1	1.	14.64	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.464	6.188	0.02	0.	0.	0.	0.	43.036

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	2221	0	0	0	0	0	131
normalized size	1	1.	14.42	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.46	6.207	0.02	0.	0.	0.	0.	41.401

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	137	3798	0	3109	0	1	221
normalized size	1	1.	0.75	20.87	0.	17.08	0.	0.01	1.21
time (sec)	N/A	0.301	0.391	0.154	0.	0.305	0.	0.333	49.422

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	86	1065	0	953	0	1	141
normalized size	1	1.	0.74	9.1	0.	8.15	0.	0.01	1.21
time (sec)	N/A	0.147	0.139	0.101	0.	0.295	0.	0.283	27.872

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	205	0	192	0	765	63
normalized size	1	1.	0.71	3.53	0.	3.31	0.	13.19	1.09
time (sec)	N/A	0.053	0.101	0.07	0.	0.277	0.	0.288	10.646

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	307	0	0	0	0	0	144
normalized size	1	1.	1.75	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.406	1.915	0.045	0.	0.	0.	0.	29.536

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	3515	0	0	0	0	0	0
normalized size	1	1.	10.72	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.882	6.452	0.058	0.	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	637	12289	0	0	0	0	0	0
normalized size	1	1.04	19.98	0.	0.	0.	0.	0.	0.
time (sec)	N/A	19.027	7.104	0.074	0.	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	5259	0	0	0	0	0	139
normalized size	1	1.	32.66	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.485	11.538	0.29	0.	0.	0.	0.	39.935

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	930	0	0	0	0	0	138
normalized size	1	1.	5.81	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.471	6.098	0.187	0.	0.	0.	0.	35.759

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	440	0	0	0	0	0	136
normalized size	1	1.	2.75	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.469	0.503	0.047	0.	0.	0.	0.	36.631

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	3743	0	0	0	0	0	138
normalized size	1	1.	22.96	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.488	6.269	0.032	0.	0.	0.	0.	45.251

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	534	0	0	0	0	0	129
normalized size	1	1.	3.38	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.321	5.63	0.081	0.	0.	0.	0.	35.015

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	150	298	192	1	178	224	0
normalized size	1	1.	3.26	6.48	4.17	0.02	3.87	4.87	0.
time (sec)	N/A	0.11	0.065	0.002	0.753	0.257	0.202	0.266	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	401	1314	544	1	559	730	0
normalized size	1	1.	4.51	14.76	6.11	0.01	6.28	8.2	0.
time (sec)	N/A	0.396	0.215	0.002	0.753	0.267	0.47	0.269	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	797	7550	1177	1	1314	1	0
normalized size	1	1.	5.78	54.71	8.53	0.01	9.52	0.01	0.
time (sec)	N/A	0.755	0.635	0.003	0.754	0.279	0.946	0.27	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	154	349	224	1	240	297	0
normalized size	1	1.	2.8	6.35	4.07	0.02	4.36	5.4	0.
time (sec)	N/A	0.122	0.061	0.002	0.747	0.242	0.232	0.27	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	405	1413	593	1	722	925	0
normalized size	1	1.	3.89	13.59	5.7	0.01	6.94	8.89	0.
time (sec)	N/A	0.407	0.214	0.002	0.759	0.245	0.532	0.27	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	801	7697	1242	1	1654	1	0
normalized size	1	1.	5.04	48.41	7.81	0.01	10.4	0.01	0.
time (sec)	N/A	0.758	0.639	0.002	0.759	0.259	1.055	0.271	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	219	158	0	1662	178	0	204
normalized size	1	1.	1.13	0.82	0.	8.61	0.92	0.	1.06
time (sec)	N/A	0.874	0.257	0.095	0.	0.298	9.929	0.	85.418

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	151	0	1	280	0	68
normalized size	1	1.	0.95	1.86	0.	0.01	3.46	0.	0.84
time (sec)	N/A	0.265	0.069	0.007	0.	0.273	6.713	0.	40.125

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	175	140	0	949	104	0	151
normalized size	1	1.	1.07	0.85	0.	5.79	0.63	0.	0.92
time (sec)	N/A	0.391	0.163	0.006	0.	0.275	5.851	0.	50.958

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	129	0	1	168	0	39
normalized size	1	1.	1.07	3.	0.	0.02	3.91	0.	0.91
time (sec)	N/A	0.124	0.023	0.008	0.	0.277	3.577	0.	27.095

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	128	184	0	1	320	0	82
normalized size	1	1.	1.36	1.96	0.	0.01	3.4	0.	0.87
time (sec)	N/A	0.26	0.137	0.013	0.	0.278	16.228	0.	47.578

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	206	168	0	1808	211	1	192
normalized size	1	1.	1.06	0.86	0.	9.27	1.08	0.01	0.98
time (sec)	N/A	0.616	0.638	0.012	0.	0.281	13.529	1.	73.632

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	154	213	0	1	464	138	110
normalized size	1	1.	1.27	1.76	0.	0.01	3.83	1.14	0.91
time (sec)	N/A	0.402	0.25	0.018	0.	0.302	46.369	0.297	64.721

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	235	188	0	2759	347	0	228
normalized size	1	1.	1.05	0.84	0.	12.32	1.55	0.	1.02
time (sec)	N/A	1.041	0.378	0.015	0.	0.296	47.041	0.	122.508

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	263	323	0	3313	0	0	243
normalized size	1	1.	0.97	1.2	0.	12.27	0.	0.	0.9
time (sec)	N/A	1.108	0.888	0.058	0.	0.328	0.	0.	81.945

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	100	276	0	1	493	0	82
normalized size	1	1.	1.03	2.85	0.	0.01	5.08	0.	0.85
time (sec)	N/A	0.269	0.228	0.031	0.	0.319	164.745	0.	29.006

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	247	319	0	3340	578	0	226
normalized size	1	1.	0.97	1.26	0.	13.15	2.28	0.	0.89
time (sec)	N/A	0.87	1.749	0.028	0.	0.337	151.335	0.	68.046

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	98	270	0	1	495	0	83
normalized size	1	1.	1.02	2.81	0.	0.01	5.16	0.	0.86
time (sec)	N/A	0.246	0.219	0.033	0.	0.296	114.037	0.	23.889

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	271	364	0	4358	740	0	255
normalized size	1	1.	0.91	1.22	0.	14.58	2.47	0.	0.85
time (sec)	N/A	1.462	1.684	0.055	0.	0.346	95.609	0.	90.529

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	235	693	0	1	0	0	146
normalized size	1	1.	1.45	4.28	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.578	0.733	0.046	0.	0.489	0.	0.	66.26

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	339	1304	0	5846	0	0	313
normalized size	1	1.	0.97	3.75	0.	16.8	0.	0.	0.9
time (sec)	N/A	3.192	2.99	0.041	0.	0.417	0.	0.	172.702

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	284	1014	0	1	0	302	192
normalized size	1	1.	1.33	4.76	0.	0.	0.	1.42	0.9
time (sec)	N/A	0.725	0.873	0.054	0.	0.637	0.	0.288	100.686

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	384	1518	0	7741	0	0	0
normalized size	1	1.	0.94	3.72	0.	18.97	0.	0.	0.
time (sec)	N/A	6.612	5.511	0.048	0.	0.609	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	347	704	0	8955	0	0	314
normalized size	1	1.	1.02	2.06	0.	26.26	0.	0.	0.92
time (sec)	N/A	1.891	6.248	0.094	0.	0.511	0.	0.	124.697

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	146	544	0	1	0	0	134
normalized size	1	1.	0.97	3.63	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.373	0.348	0.064	0.	0.44	0.	0.	36.178

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	405	885	0	10396	0	0	332
normalized size	1	1.	1.12	2.44	0.	28.64	0.	0.	0.91
time (sec)	N/A	2.013	6.272	0.094	0.	0.585	0.	0.	117.204

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	147	541	0	1	0	0	134
normalized size	1	1.	0.98	3.61	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.357	0.306	0.068	0.	0.431	0.	0.	32.401

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	463	1010	0	11548	0	1	0
normalized size	1	1.	1.06	2.31	0.	26.43	0.	0.	0.
time (sec)	N/A	9.918	6.271	0.097	0.	0.756	0.	0.314	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	421	4477	0	1	0	1	238
normalized size	1	1.	1.65	17.56	0.	0.	0.	0.	0.93
time (sec)	N/A	0.939	6.235	0.103	0.	1.29	0.	0.348	106.21

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	560	6821	0	13851	0	0	0
normalized size	1	1.	1.16	14.09	0.	28.62	0.	0.	0.
time (sec)	N/A	2.607	6.355	0.093	0.	1.15	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	491	5575	0	1	0	509	313
normalized size	1	1.	1.51	17.15	0.	0.	0.	1.57	0.96
time (sec)	N/A	1.166	6.278	0.111	0.	2.393	0.	0.295	172.743

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	222	164	0	1817	219	0	214
normalized size	1	1.	1.1	0.81	0.	9.	1.08	0.	1.06
time (sec)	N/A	0.938	0.253	0.004	0.	0.313	12.782	0.	77.226

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	154	0	1	332	0	75
normalized size	1	1.	0.92	1.77	0.	0.01	3.82	0.	0.86
time (sec)	N/A	0.281	0.074	0.003	0.	0.283	8.324	0.	32.944

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	178	143	0	1079	124	0	158
normalized size	1	1.	1.05	0.84	0.	6.35	0.73	0.	0.93
time (sec)	N/A	0.447	0.167	0.003	0.	0.325	6.985	0.	42.694

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	1	189	0	41
normalized size	1	1.	1.07	2.95	0.	0.02	4.3	0.	0.93
time (sec)	N/A	0.135	0.027	0.002	0.	0.32	5.239	0.	18.757

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	131	190	0	1	348	0	87
normalized size	1	1.	1.27	1.84	0.	0.01	3.38	0.	0.84
time (sec)	N/A	0.283	0.126	0.007	0.	0.295	20.461	0.	39.074

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	209	174	0	1994	258	1	202
normalized size	1	1.	1.02	0.85	0.	9.77	1.26	0.	0.99
time (sec)	N/A	0.694	0.644	0.007	0.	0.289	16.99	1.019	64.948

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	157	222	0	1	532	0	124
normalized size	1	1.	1.18	1.67	0.	0.01	4.	0.	0.93
time (sec)	N/A	0.432	0.246	0.007	0.	0.399	58.493	0.	56.659

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	238	197	0	2986	411	0	241
normalized size	1	1.	1.01	0.83	0.	12.65	1.74	0.	1.02
time (sec)	N/A	1.2	0.373	0.007	0.	0.303	56.729	0.	115.569

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	266	695	0	3480	0	0	253
normalized size	1	1.	0.95	2.49	0.	12.47	0.	0.	0.91
time (sec)	N/A	1.214	0.884	0.008	0.	0.33	0.	0.	82.47

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	500	0	1	554	0	88
normalized size	1	1.	1.	4.85	0.	0.01	5.38	0.	0.85
time (sec)	N/A	0.288	0.226	0.008	0.	0.287	167.661	0.	29.355

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	250	693	0	3510	646	0	236
normalized size	1	1.	0.95	2.63	0.	13.35	2.46	0.	0.9
time (sec)	N/A	0.967	1.728	0.008	0.	0.309	157.283	0.	72.592

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	484	0	1	525	0	87
normalized size	1	1.	1.01	4.94	0.	0.01	5.36	0.	0.89
time (sec)	N/A	0.26	0.223	0.007	0.	0.288	115.044	0.	25.372

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	238	714	0	1	0	0	153
normalized size	1	1.	1.37	4.1	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.63	0.715	0.018	0.	0.64	0.	0.	73.249

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	342	1346	0	6102	0	0	326
normalized size	1	1.	0.95	3.74	0.	16.95	0.	0.	0.91
time (sec)	N/A	3.296	2.978	0.017	0.	0.394	0.	0.	178.924

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	287	1047	0	1	0	0	209
normalized size	1	1.	1.26	4.59	0.	0.	0.	0.	0.92
time (sec)	N/A	0.761	0.858	0.015	0.	1.599	0.	0.	104.26

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	387	1569	0	8038	0	0	0
normalized size	1	1.	0.91	3.71	0.	19.	0.	0.	0.
time (sec)	N/A	6.708	5.639	0.015	0.	0.533	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	351	3432	0	9140	0	0	328
normalized size	1	1.	0.99	9.72	0.	25.89	0.	0.	0.93
time (sec)	N/A	1.895	6.22	0.015	0.	0.509	0.	0.	131.81

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	149	2181	0	1	0	0	144
normalized size	1	1.	0.94	13.72	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.402	0.355	0.016	0.	0.437	0.	0.	39.026

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	409	4751	0	10581	0	1	345
normalized size	1	1.	1.09	12.67	0.	28.22	0.	0.	0.92
time (sec)	N/A	2.087	6.246	0.015	0.	0.583	0.	0.284	123.568

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	148	2132	0	1	0	0	139
normalized size	1	1.	0.97	13.93	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.383	0.318	0.014	0.	0.444	0.	0.	34.325

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	436	4606	0	1	0	1	246
normalized size	1	1.	1.61	17.06	0.	0.	0.	0.	0.91
time (sec)	N/A	1.004	6.204	0.034	0.	2.104	0.	0.338	113.605

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	575	7019	0	14199	0	0	0
normalized size	1	1.	1.15	14.07	0.	28.45	0.	0.	0.
time (sec)	N/A	2.732	6.325	0.033	0.	1.159	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	509	5737	0	1	0	1	333
normalized size	1	1.	1.48	16.73	0.	0.	0.	0.	0.97
time (sec)	N/A	1.225	6.247	0.027	0.	6.1	0.	0.335	176.663

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	0	0	0	299
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	1.51	20.617	0.037	0.	0.	0.	0.	85.53

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	0	0	0	0	0	0	354
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.61	2.211	0.123	0.	0.	0.	0.	101.353

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	105	140	1	107	38	0
normalized size	1	1.	1.	3.09	4.12	0.03	3.15	1.12	0.
time (sec)	N/A	0.08	0.014	0.003	0.783	0.245	0.177	0.273	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	188	175	235	1	187	62	0
normalized size	1	1.	3.36	3.12	4.2	0.02	3.34	1.11	0.
time (sec)	N/A	0.191	0.012	0.005	0.782	0.226	0.307	0.269	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [411] had the largest ratio of [0.8]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.	15	0.133
2	A	1	1	1.	15	0.067
3	A	1	1	1.	15	0.067
4	A	3	3	1.	15	0.2
5	A	8	8	1.	11	0.727
6	A	3	2	1.	26	0.077
7	A	3	2	1.	26	0.077
8	A	3	2	1.	26	0.077
9	A	2	2	1.	26	0.077
10	A	3	2	1.	24	0.083
11	A	2	1	1.	22	0.045
12	A	3	2	1.	26	0.077
13	A	3	2	1.	26	0.077
14	A	3	2	1.	26	0.077
15	A	3	2	1.	26	0.077
16	A	3	2	1.	26	0.077
17	A	3	2	1.	26	0.077
18	A	3	2	1.	26	0.077
19	A	3	2	1.	26	0.077
20	A	3	2	1.	26	0.077
21	A	3	2	1.	26	0.077
22	A	3	2	1.	26	0.077
23	A	3	2	1.	26	0.077
24	A	4	3	1.4	26	0.115
25	A	3	2	1.	26	0.077
26	A	3	2	1.	26	0.077

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	4	3	1.	26	0.115
28	A	3	2	1.	26	0.077
29	A	3	2	1.	26	0.077
30	A	2	2	1.	26	0.077
31	A	3	2	1.	24	0.083
32	A	3	2	1.	22	0.091
33	A	4	3	1.	26	0.115
34	A	3	2	1.	26	0.077
35	A	3	2	1.	26	0.077
36	A	4	3	1.	26	0.115
37	A	3	2	1.	26	0.077
38	A	3	2	1.	26	0.077
39	A	4	3	1.	26	0.115
40	A	3	2	1.	26	0.077
41	A	3	2	1.	26	0.077
42	A	4	3	1.	26	0.115
43	A	3	2	1.	26	0.077
44	A	3	2	1.	26	0.077
45	A	2	2	1.	26	0.077
46	A	3	2	1.	26	0.077
47	A	3	2	1.	26	0.077
48	A	4	4	1.	26	0.154
49	A	3	2	1.	26	0.077
50	A	3	2	1.	26	0.077
51	A	3	2	1.	26	0.077
52	A	4	3	1.	26	0.115
53	A	3	2	1.	26	0.077
54	A	3	2	1.	26	0.077
55	A	4	3	1.	26	0.115
56	A	3	2	1.	26	0.077
57	A	3	2	1.	26	0.077
58	A	4	3	1.	26	0.115
59	A	3	2	1.	26	0.077
60	A	3	2	1.	26	0.077

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.	26	0.077
62	A	3	2	1.	24	0.083
63	A	3	2	1.	22	0.091
64	A	4	3	1.	26	0.115
65	A	3	2	1.	26	0.077
66	A	3	2	1.	26	0.077
67	A	4	3	1.	26	0.115
68	A	3	2	1.	26	0.077
69	A	3	2	1.	26	0.077
70	A	4	3	1.	26	0.115
71	A	3	2	1.	26	0.077
72	A	3	2	1.	26	0.077
73	A	4	3	1.	26	0.115
74	A	3	2	1.	26	0.077
75	A	3	2	1.	26	0.077
76	A	4	3	1.	26	0.115
77	A	3	2	1.	26	0.077
78	A	3	2	1.	26	0.077
79	A	4	3	1.	26	0.115
80	A	3	2	1.	26	0.077
81	A	3	2	1.	26	0.077
82	A	2	2	1.	26	0.077
83	A	3	2	1.	26	0.077
84	A	3	2	1.	26	0.077
85	A	4	4	1.	26	0.154
86	A	3	2	1.	26	0.077
87	A	3	2	1.	26	0.077
88	A	5	4	1.	26	0.154
89	A	8	8	1.	26	0.308
90	A	8	8	1.	26	0.308
91	A	3	3	1.	26	0.115
92	A	7	7	1.	24	0.292
93	A	7	7	1.	22	0.318
94	A	5	5	1.	26	0.192

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	8	1.	26	0.308
96	A	8	8	1.	26	0.308
97	A	4	3	1.	26	0.115
98	A	9	9	1.	26	0.346
99	A	9	9	1.	26	0.346
100	A	2	2	1.	26	0.077
101	A	9	8	1.	24	0.333
102	A	9	8	1.	22	0.364
103	A	4	3	1.	26	0.115
104	A	10	9	1.	26	0.346
105	A	10	9	1.	26	0.346
106	A	4	3	1.	26	0.115
107	A	11	9	1.	26	0.346
108	A	4	3	1.	26	0.115
109	A	11	9	1.	26	0.346
110	A	11	9	1.	26	0.346
111	A	2	2	1.	26	0.077
112	A	11	8	1.	24	0.333
113	A	11	8	1.	22	0.364
114	A	4	3	1.	26	0.115
115	A	12	9	1.	26	0.346
116	A	12	9	1.	26	0.346
117	A	4	3	1.	26	0.115
118	A	3	2	1.	28	0.071
119	A	3	2	1.	28	0.071
120	A	3	2	1.	28	0.071
121	A	2	2	1.	28	0.071
122	A	2	2	1.	28	0.071
123	A	2	2	1.	28	0.071
124	A	2	2	1.	26	0.077
125	A	4	3	1.	24	0.125
126	A	4	3	1.	24	0.125
127	A	4	3	1.	24	0.125
128	A	2	2	1.	24	0.083

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	2	2	1.	24	0.083
130	A	2	2	1.	24	0.083
131	A	2	2	1.03	22	0.091
132	A	3	3	1.04	20	0.15
133	A	3	3	1.	24	0.125
134	A	2	2	1.	24	0.083
135	A	2	2	1.	24	0.083
136	A	3	3	1.	24	0.125
137	A	2	2	1.	24	0.083
138	A	6	6	1.	18	0.333
139	A	5	5	1.	18	0.278
140	A	3	3	1.	18	0.167
141	A	7	7	1.	18	0.389
142	A	8	7	1.	18	0.389
143	A	14	8	1.	18	0.444
144	A	14	8	1.	18	0.444
145	A	13	7	1.	18	0.389
146	A	13	7	1.	18	0.389
147	A	13	7	1.	16	0.438
148	A	13	7	1.	14	0.5
149	A	14	8	1.	18	0.444
150	A	14	8	1.	18	0.444
151	A	6	4	1.	16	0.25
152	A	5	4	1.	16	0.25
153	A	4	3	1.	16	0.188
154	B	4	3	2.1	16	0.188
155	A	6	5	1.	16	0.312
156	A	4	3	1.	16	0.188
157	A	4	3	1.	16	0.188
158	A	15	10	1.	16	0.625
159	A	15	10	1.	16	0.625
160	A	14	9	1.	16	0.562
161	A	14	9	1.	16	0.562
162	A	13	8	1.	16	0.5

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	13	8	1.	16	0.5
164	A	13	8	1.	14	0.571
165	A	13	8	1.	12	0.667
166	A	14	9	1.	16	0.562
167	A	14	9	1.	16	0.562
168	A	15	10	1.	16	0.625
169	A	15	10	1.	16	0.625
170	A	14	8	1.	16	0.5
171	A	5	5	1.	16	0.312
172	A	13	7	1.	16	0.438
173	A	13	7	1.	16	0.438
174	A	3	3	1.	16	0.188
175	A	13	7	1.	14	0.5
176	C	13	7	2.02	12	0.583
177	A	7	7	1.	16	0.438
178	A	14	8	1.	16	0.5
179	A	14	8	1.	16	0.5
180	A	8	7	1.	16	0.438
181	A	16	10	1.	16	0.625
182	A	13	7	1.	10	0.7
183	A	3	3	1.	14	0.214
184	A	13	7	1.	14	0.5
185	A	7	7	1.	20	0.35
186	A	6	6	1.	20	0.3
187	A	6	6	1.	20	0.3
188	A	5	5	1.	20	0.25
189	A	4	4	1.	20	0.2
190	A	7	6	1.	20	0.3
191	A	7	6	1.	20	0.3
192	A	4	4	1.	20	0.2
193	A	5	5	1.	20	0.25
194	A	6	6	1.	20	0.3
195	A	7	7	1.	20	0.35
196	A	2	2	1.	20	0.1

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	2	2	1.	18	0.111
198	A	2	2	1.	16	0.125
199	A	2	2	1.	20	0.1
200	A	2	2	1.	20	0.1
201	A	8	7	1.	20	0.35
202	A	7	6	1.	20	0.3
203	A	7	6	1.	20	0.3
204	A	6	5	1.	20	0.25
205	A	5	4	1.	20	0.2
206	A	8	7	1.	20	0.35
207	A	8	7	1.	20	0.35
208	A	8	7	1.	20	0.35
209	A	8	7	1.	20	0.35
210	A	5	4	1.	20	0.2
211	A	6	5	1.	20	0.25
212	A	7	6	1.	20	0.3
213	A	8	7	1.	20	0.35
214	A	2	2	1.	20	0.1
215	A	2	2	1.	18	0.111
216	A	2	2	1.	16	0.125
217	A	2	2	1.	20	0.1
218	A	2	2	1.	20	0.1
219	A	6	6	1.	20	0.3
220	A	5	5	1.	20	0.25
221	A	5	5	1.	20	0.25
222	A	4	4	1.	20	0.2
223	A	3	3	1.	20	0.15
224	A	3	3	1.	20	0.15
225	A	4	4	1.	20	0.2
226	A	5	5	1.	20	0.25
227	A	6	6	1.	20	0.3
228	A	7	6	1.	20	0.3
229	A	2	2	1.	20	0.1
230	A	2	2	1.	18	0.111

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	2	2	1.	16	0.125
232	A	2	2	1.	20	0.1
233	A	2	2	1.	20	0.1
234	A	6	6	1.	20	0.3
235	A	5	5	1.	20	0.25
236	A	5	5	1.	20	0.25
237	A	2	2	1.	20	0.1
238	A	2	2	1.	20	0.1
239	A	5	5	1.	20	0.25
240	A	5	5	1.	20	0.25
241	A	6	6	1.	20	0.3
242	A	7	6	1.	20	0.3
243	A	2	2	1.	20	0.1
244	A	2	2	1.	18	0.111
245	A	2	2	1.	16	0.125
246	A	2	2	1.	20	0.1
247	A	2	2	1.	20	0.1
248	A	2	1	1.	20	0.05
249	A	2	1	1.	18	0.056
250	A	3	2	1.	20	0.1
251	A	4	3	1.	20	0.15
252	A	2	2	1.	22	0.091
253	A	2	2	1.	22	0.091
254	A	2	2	1.	22	0.091
255	A	2	2	1.	22	0.091
256	A	2	2	1.	20	0.1
257	A	4	4	1.	18	0.222
258	A	3	3	1.	18	0.167
259	A	2	2	1.	18	0.111
260	A	2	2	1.	18	0.111
261	A	2	2	1.	18	0.111
262	A	2	2	1.	16	0.125
263	A	2	2	1.	14	0.143
264	A	3	3	1.	18	0.167

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	2	2	1.	18	0.111
266	A	2	2	1.	18	0.111
267	A	3	3	1.	18	0.167
268	A	2	2	1.	18	0.111
269	A	2	2	1.	18	0.111
270	A	3	3	1.	18	0.167
271	A	2	2	1.	16	0.125
272	A	5	5	1.	16	0.312
273	A	4	3	1.	16	0.188
274	A	4	4	1.	16	0.25
275	A	2	2	1.	16	0.125
276	A	4	4	1.	14	0.286
277	A	4	3	1.	16	0.188
278	A	5	5	1.	16	0.312
279	A	4	3	1.	16	0.188
280	A	6	5	1.	16	0.312
281	A	12	9	1.	16	0.562
282	A	11	8	1.	16	0.5
283	A	11	8	1.	16	0.5
284	A	11	8	1.	16	0.5
285	A	11	8	1.	12	0.667
286	A	12	9	1.	16	0.562
287	A	12	9	1.	16	0.562
288	A	13	9	1.	16	0.562
289	A	13	9	1.	16	0.562
290	A	2	2	1.	16	0.125
291	A	5	5	1.	16	0.312
292	A	4	3	1.	16	0.188
293	A	4	4	1.	16	0.25
294	A	2	2	1.	16	0.125
295	A	4	4	1.	14	0.286
296	A	4	3	1.	16	0.188
297	A	5	5	1.	16	0.312
298	A	4	3	1.	16	0.188

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	6	5	1.	16	0.312
300	A	6	6	1.	16	0.375
301	A	5	5	1.	16	0.312
302	A	5	5	1.	16	0.312
303	A	5	5	1.	16	0.312
304	A	5	5	1.	12	0.417
305	A	6	6	1.	16	0.375
306	A	6	6	1.	16	0.375
307	A	7	6	1.	16	0.375
308	A	7	6	1.	16	0.375
309	A	3	2	1.	18	0.111
310	A	6	6	1.	18	0.333
311	A	5	4	1.	18	0.222
312	A	5	5	1.	18	0.278
313	A	4	3	1.	18	0.167
314	A	3	3	1.	18	0.167
315	A	4	3	1.	16	0.188
316	A	7	7	1.	18	0.389
317	A	5	4	1.	18	0.222
318	A	8	7	1.	18	0.389
319	A	8	5	1.	18	0.278
320	A	8	5	1.	18	0.278
321	A	7	4	1.	18	0.222
322	A	7	4	1.	18	0.222
323	A	7	4	1.	18	0.222
324	A	7	4	1.	14	0.286
325	A	8	5	1.	18	0.278
326	A	8	5	1.	18	0.278
327	A	3	2	1.	14	0.143
328	A	6	6	1.	14	0.429
329	A	7	5	1.	14	0.357
330	A	5	5	1.	14	0.357
331	A	10	7	1.	14	0.5
332	A	3	3	1.	14	0.214

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	10	6	1.	12	0.5
334	A	7	7	1.	14	0.5
335	A	7	5	1.	14	0.357
336	A	8	7	1.	14	0.5
337	A	13	10	1.	14	0.714
338	A	20	7	1.	14	0.5
339	A	9	6	1.	14	0.429
340	A	19	7	1.	14	0.5
341	A	19	6	1.	14	0.429
342	A	9	6	1.	10	0.6
343	A	20	8	1.	14	0.571
344	A	20	7	1.	14	0.5
345	A	12	9	1.	14	0.643
346	A	22	10	1.	14	0.714
347	A	3	2	1.	16	0.125
348	A	6	6	1.	16	0.375
349	A	5	4	1.	16	0.25
350	A	5	5	1.	16	0.312
351	A	10	7	1.	16	0.438
352	A	3	3	1.	16	0.188
353	A	10	6	1.	14	0.429
354	A	7	7	1.	16	0.438
355	A	5	4	1.	16	0.25
356	A	8	7	1.	16	0.438
357	A	13	10	1.	16	0.625
358	A	20	7	1.	16	0.438
359	A	19	6	1.	16	0.375
360	A	19	7	1.	16	0.438
361	A	19	6	1.	16	0.375
362	A	19	6	1.	12	0.5
363	A	22	8	1.	16	0.5
364	A	20	7	1.	16	0.438
365	A	22	9	1.	16	0.562
366	A	22	10	1.	16	0.625

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	3	2	1.	16	0.125
368	A	5	4	1.	16	0.25
369	A	5	4	1.	16	0.25
370	A	4	3	1.	16	0.188
371	A	4	3	1.	16	0.188
372	A	3	3	1.	16	0.188
373	A	4	3	1.	14	0.214
374	A	6	5	1.	16	0.312
375	A	5	4	1.	16	0.25
376	A	7	5	1.	16	0.312
377	A	6	5	1.	16	0.312
378	A	20	8	0.96	16	0.5
379	A	19	7	0.96	16	0.438
380	A	19	7	0.96	16	0.438
381	A	19	7	0.96	16	0.438
382	A	19	7	1.33	12	0.583
383	A	20	8	0.96	16	0.5
384	A	20	8	0.96	16	0.5
385	A	3	2	1.	16	0.125
386	A	5	4	1.	16	0.25
387	A	5	4	1.	16	0.25
388	A	4	3	1.	16	0.188
389	A	4	3	1.	16	0.188
390	A	3	3	1.	16	0.188
391	A	4	3	1.	14	0.214
392	A	6	5	1.	16	0.312
393	A	5	4	1.	16	0.25
394	A	7	5	1.	16	0.312
395	A	6	5	1.	16	0.312
396	A	8	5	1.	16	0.312
397	A	7	4	1.	16	0.25
398	A	7	4	1.	16	0.25
399	A	7	4	1.14	16	0.25
400	A	7	4	1.	12	0.333

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	8	5	1.	16	0.312
402	A	8	5	1.	16	0.312
403	A	9	6	1.	16	0.375
404	A	9	6	1.	16	0.375
405	A	4	3	1.	16	0.188
406	A	5	4	1.	16	0.25
407	A	5	5	1.	14	0.357
408	A	3	3	1.	14	0.214
409	A	7	7	1.	14	0.5
410	A	8	7	1.	14	0.5
411	A	8	8	1.	10	0.8
412	A	7	6	1.	18	0.333
413	A	7	6	1.	18	0.333
414	A	7	6	1.	16	0.375
415	A	6	6	1.	14	0.429
416	A	5	5	1.	18	0.278
417	A	3	3	1.	18	0.167
418	A	7	7	1.	18	0.389
419	A	8	7	1.	18	0.389
420	A	8	7	1.	18	0.389
421	A	8	7	1.	18	0.389
422	A	8	7	1.	16	0.438
423	A	8	7	1.	14	0.5
424	A	7	7	1.	18	0.389
425	A	4	4	1.	18	0.222
426	A	4	4	1.	18	0.222
427	A	4	4	1.	18	0.222
428	A	8	7	1.	18	0.389
429	A	8	7	1.	18	0.389
430	A	8	7	1.	18	0.389
431	A	9	8	1.	14	0.571
432	A	8	8	1.	18	0.444
433	A	5	4	1.	18	0.222
434	A	5	5	1.	18	0.278

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
435	A	5	5	1.	18	0.278
436	A	5	5	1.	18	0.278
437	A	5	4	1.	18	0.222
438	A	9	8	1.	18	0.444
439	A	9	8	1.	18	0.444
440	A	6	4	1.	18	0.222
441	A	6	4	1.	16	0.25
442	A	5	4	1.	14	0.286
443	A	4	3	1.	18	0.167
444	A	4	3	1.	18	0.167
445	A	6	5	1.	18	0.278
446	A	4	3	1.	18	0.167
447	A	4	3	1.	18	0.167
448	A	4	3	1.	18	0.167
449	A	9	7	1.	16	0.438
450	A	8	7	1.	16	0.438
451	A	7	6	1.	16	0.375
452	A	4	4	1.	16	0.25
453	A	5	5	1.	16	0.312
454	A	6	6	1.	16	0.375
455	A	4	3	1.	22	0.136
456	A	5	4	1.	14	0.286
457	A	15	9	1.	14	0.643
458	A	9	6	1.	14	0.429
459	A	7	6	1.	20	0.3
460	A	4	3	1.	23	0.13
461	A	4	4	1.	22	0.182
462	A	4	3	1.	26	0.115
463	A	4	3	1.	26	0.115
464	A	3	2	1.	26	0.077
465	A	4	3	1.	26	0.115
466	A	4	3	1.	26	0.115
467	A	4	3	1.	26	0.115
468	A	4	3	1.	26	0.115

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	4	3	1.	26	0.115
470	A	4	3	1.	26	0.115
471	A	4	3	1.	26	0.115
472	A	4	4	1.	30	0.133
473	A	4	3	1.	28	0.107
474	A	4	3	1.	26	0.115
475	A	4	3	1.	24	0.125
476	A	3	3	1.	28	0.107
477	A	3	3	1.	28	0.107
478	C	7	4	1.11	77	0.052
479	A	4	3	1.	26	0.115
480	A	4	3	1.	26	0.115
481	A	5	4	1.	24	0.167
482	A	5	4	1.	26	0.154
483	A	5	4	1.	26	0.154
484	A	5	4	1.	26	0.154
485	A	4	3	1.	26	0.115
486	A	5	4	1.	26	0.154
487	A	5	4	1.	26	0.154
488	A	5	4	1.	26	0.154
489	A	5	4	1.	26	0.154
490	A	5	4	1.	26	0.154
491	A	4	3	1.	26	0.115
492	A	5	4	1.	26	0.154
493	A	4	3	1.	23	0.13
494	A	4	3	1.	23	0.13
495	A	2	2	1.	23	0.087
496	A	5	5	1.	21	0.238
497	A	4	3	1.	23	0.13
498	A	4	3	1.	23	0.13
499	A	4	3	1.	23	0.13
500	A	12	9	1.	25	0.36
501	A	9	9	1.	25	0.36
502	A	5	5	1.	25	0.2

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
503	A	6	6	1.	25	0.24
504	A	11	11	1.	25	0.44
505	A	14	11	1.	25	0.44
506	A	1	1	1.	26	0.038
507	A	1	1	1.	28	0.036
508	A	4	3	1.	32	0.094
509	A	4	3	1.	32	0.094
510	A	3	2	1.	32	0.062
511	A	4	3	1.	32	0.094
512	A	2	2	1.	32	0.062
513	A	4	3	1.	32	0.094
514	A	4	3	1.	32	0.094
515	A	5	4	1.	30	0.133
516	A	3	2	1.	28	0.071
517	A	3	2	1.	26	0.077
518	A	2	1	1.	24	0.042
519	A	3	2	1.	28	0.071
520	A	3	2	1.	28	0.071
521	A	3	2	1.	28	0.071
522	A	9	4	1.	30	0.133
523	A	3	2	1.	28	0.071
524	A	3	2	1.	26	0.077
525	A	3	2	1.	24	0.083
526	A	4	3	1.	28	0.107
527	A	3	2	1.	28	0.071
528	A	3	2	1.	28	0.071
529	A	2	2	1.	30	0.067
530	A	2	2	1.	28	0.071
531	A	2	2	1.	26	0.077
532	A	2	2	1.	24	0.083
533	A	5	5	1.	28	0.179
534	A	2	2	1.	28	0.071
535	A	2	2	1.	28	0.071
536	A	2	2	1.	30	0.067

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
537	A	2	2	1.	28	0.071
538	A	2	2	1.	26	0.077
539	A	2	2	1.	24	0.083
540	A	4	3	1.	28	0.107
541	A	2	2	1.	28	0.071
542	A	2	2	1.	28	0.071
543	A	2	2	1.	36	0.056
544	A	2	2	1.	31	0.065
545	A	3	3	1.	34	0.088
546	A	3	3	1.	33	0.091
547	A	3	3	1.06	35	0.086
548	A	4	3	1.	30	0.1
549	A	7	6	1.	24	0.25
550	A	6	6	1.	24	0.25
551	A	5	5	1.	24	0.208
552	A	3	3	1.	22	0.136
553	A	8	7	1.	24	0.292
554	A	8	7	1.	24	0.292
555	A	8	7	1.	24	0.292
556	A	8	5	1.	26	0.192
557	A	14	8	1.	26	0.308
558	A	4	3	1.	26	0.115
559	A	6	5	1.	26	0.192
560	A	16	10	1.	26	0.385
561	A	10	7	1.	26	0.269
562	A	3	2	1.	20	0.1
563	A	3	2	1.	18	0.111
564	A	3	2	1.	16	0.125
565	A	7	7	1.	20	0.35
566	A	3	2	1.	20	0.1
567	A	3	2	1.	20	0.1
568	A	2	2	1.	22	0.091
569	A	2	2	1.	22	0.091
570	A	2	2	1.	20	0.1

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
571	A	2	2	1.	18	0.111
572	A	7	6	1.	22	0.273
573	A	2	2	1.	22	0.091
574	A	2	2	1.	22	0.091
575	A	2	2	1.	22	0.091
576	A	2	2	1.	22	0.091
577	A	2	2	1.	20	0.1
578	A	2	2	1.	18	0.111
579	A	8	7	1.	22	0.318
580	A	2	2	1.	22	0.091
581	A	2	2	1.	22	0.091
582	A	2	2	1.	22	0.091
583	A	2	2	1.	22	0.091
584	A	2	2	1.	20	0.1
585	A	2	2	1.	18	0.111
586	A	3	3	1.	22	0.136
587	A	2	2	1.	22	0.091
588	A	2	2	1.	22	0.091
589	A	2	2	1.	22	0.091
590	A	2	2	1.	22	0.091
591	A	2	2	1.	20	0.1
592	A	2	2	1.	18	0.111
593	A	5	5	1.	22	0.227
594	A	2	2	1.	22	0.091
595	A	2	2	1.	22	0.091
596	A	14	3	1.	22	0.136
597	A	10	3	1.	22	0.136
598	A	6	3	1.	20	0.15
599	A	3	2	1.	22	0.091
600	A	5	3	1.	22	0.136
601	A	6	4	1.04	22	0.182
602	A	2	2	1.	24	0.083
603	A	2	2	1.	24	0.083
604	A	2	2	1.	24	0.083

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
605	A	2	2	1.	24	0.083
606	A	2	2	1.	22	0.091
607	A	3	2	1.	28	0.071
608	A	4	3	1.	30	0.1
609	A	4	3	1.	30	0.1
610	A	3	2	1.	31	0.065
611	A	4	3	1.	33	0.091
612	A	4	3	1.	33	0.091
613	A	5	4	1.	30	0.133
614	A	6	6	1.	30	0.2
615	A	4	3	1.	30	0.1
616	A	4	4	1.	28	0.143
617	A	8	8	1.	30	0.267
618	A	5	4	1.	30	0.133
619	A	9	8	1.	30	0.267
620	A	6	5	1.	30	0.167
621	A	5	4	1.	30	0.133
622	A	5	5	1.	30	0.167
623	A	5	4	1.	30	0.133
624	A	5	5	1.	28	0.179
625	A	5	4	1.	22	0.182
626	A	9	8	1.	30	0.267
627	A	6	5	1.	30	0.167
628	A	9	8	1.	30	0.267
629	A	7	5	1.	30	0.167
630	A	6	5	1.	30	0.167
631	A	6	6	1.	30	0.2
632	A	6	5	1.	30	0.167
633	A	6	5	1.	28	0.179
634	A	6	5	1.	22	0.227
635	A	10	9	1.	30	0.3
636	A	7	6	1.	30	0.2
637	A	10	9	1.	30	0.3
638	A	5	4	1.	33	0.121

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
639	A	6	6	1.	33	0.182
640	A	4	3	1.	33	0.091
641	A	4	4	1.	31	0.129
642	A	8	8	1.	33	0.242
643	A	5	4	1.	33	0.121
644	A	9	8	1.	33	0.242
645	A	6	5	1.	33	0.152
646	A	5	4	1.	33	0.121
647	A	5	5	1.	33	0.152
648	A	5	4	1.	33	0.121
649	A	5	5	1.	31	0.161
650	A	9	8	1.	33	0.242
651	A	6	5	1.	33	0.152
652	A	9	8	1.	33	0.242
653	A	7	5	1.	33	0.152
654	A	6	5	1.	33	0.152
655	A	6	6	1.	33	0.182
656	A	6	5	1.	33	0.152
657	A	6	5	1.	31	0.161
658	A	10	9	1.	33	0.273
659	A	7	6	1.	33	0.182
660	A	10	9	1.	33	0.273
661	A	7	6	1.	26	0.231
662	A	10	9	1.	28	0.321
663	A	3	2	1.	24	0.083
664	A	4	3	1.	26	0.115

3 Listing of integrals

3.1 $\int \left(ax^3+bx^6\right)^{5/3}\,dx$

Optimal. Leaf size=52

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

```
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)
/(11*b*x^5)
```

Rubi [A] time = 0.0778471, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)
/(11*b*x^5)
```

Rubi in Sympy [A] time = 7.88126, size = 44, normalized size = 0.85

$$-\frac{3a\left(ax^3+bx^6\right)^{\frac{8}{3}}}{88b^2x^8}+\frac{\left(ax^3+bx^6\right)^{\frac{8}{3}}}{11bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**6+a*x**3)**(5/3),x)
```

```
[Out] -3*a*(a*x**3 + b*x**6)**(8/3)/(88*b**2*x**8) + (a*x**3 + b*x**6)*
*(8/3)/(11*b*x**5)
```

Mathematica [A] time = 0.0405374, size = 42, normalized size = 0.81

$$\frac{x\left(a+bx^3\right)^3\left(8bx^3-3a\right)}{88b^2\sqrt[3]{x^3\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (x*(a + b*x^3)^3*(-3*a + 8*b*x^3))/(88*b^2*(x^3*(a + b*x^3))^(1/3
))
```

Maple [A] time = 0.008, size = 39, normalized size = 0.8

$$-\frac{\left(bx^3+a\right)\left(-8\,bx^3+3\,a\right)}{88\,b^2x^5}\left(bx^6+ax^3\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^6+a*x^3)^(5/3),x)
```

```
[Out] -1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

Maxima [A] time = 0.782731, size = 62, normalized size = 1.19

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{88\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)
^(2/3)/b^2
```

Fricas [A] time = 0.267117, size = 72, normalized size = 1.38

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^6+ax^3\right)^{\frac{2}{3}}}{88\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*
x^3)^(2/3)/(b^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax^3 + bx^6\right)^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(5/3), x)
```

GIAC/XCAS [A] time = 0.278751, size = 107, normalized size = 2.06

$$\frac{\frac{11\left(5\,(bx^3+a)^{\frac{8}{3}}-8\,(bx^3+a)^{\frac{5}{3}}a\right)a}{b}+\frac{2\left(20\,(bx^3+a)^{\frac{11}{3}}-55\,(bx^3+a)^{\frac{8}{3}}a+44\,(bx^3+a)^{\frac{5}{3}}a^2\right)}{b}}{440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(5/3),x, algorithm="giac")
```

```
[Out] 1/440*(11*(5*(b*x^3 + a)^(8/3) - 8*(b*x^3 + a)^(5/3)*a)*a/b + 2*(
20*(b*x^3 + a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 44*(b*x^3 + a)^(
5/3)*a^2)/b)/b
```

3.2 $\int \left(ax^3+bx^6\right)^{2/3}\,dx$

Optimal. Leaf size=25

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rubi [A] time = 0.0126025, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^3 + b*x^6)^(2/3),x]

[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rubi in Sympy [A] time = 1.37952, size = 19, normalized size = 0.76

$$\frac{\left(ax^3+bx^6\right)^{\frac{5}{3}}}{5bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**6+a*x**3)**(2/3),x)

[Out] (a*x**3 + b*x**6)**(5/3)/(5*b*x**5)
```

Mathematica [A] time = 0.0160823, size = 25, normalized size = 1.

$$\frac{\left(x^3\left(a+bx^3\right)\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

Maple [A] time = 0.006, size = 29, normalized size = 1.2

$$\frac{bx^3+a}{5\,bx^2}\left(bx^6+ax^3\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^6+a*x^3)^(2/3),x)
```

```
[Out] 1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

Maxima [A] time = 0.771503, size = 19, normalized size = 0.76

$$\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

Fricas [A] time = 0.263995, size = 38, normalized size = 1.52

$$\frac{\left(bx^6+ax^3\right)^{\frac{2}{3}}\left(bx^3+a\right)}{5\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax^3 + bx^6\right)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(2/3), x)
```

GIAC/XCAS [A] time = 0.285731, size = 19, normalized size = 0.76

$$\frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(2/3),x, algorithm="giac")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

3.3 $\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

```
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rubi [A] time = 0.0130483, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^3 + b*x^6)^(-2/3),x]

[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rubi in Sympy [A] time = 1.38468, size = 19, normalized size = 0.83

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**6+a*x**3)**(2/3),x)

[Out] -(a*x**3 + b*x**6)**(1/3)/(a*x**2)
```

Mathematica [A] time = 0.0216424, size = 23, normalized size = 1.

$$-\frac{\sqrt[3]{x^3\left(a+bx^3\right)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^3 + b*x^6)^(-2/3),x]

[Out] -((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

Maple [A] time = 0.006, size = 27, normalized size = 1.2

$$-\frac{x\left(bx^{3}+a\right)}{a}\left(bx^{6}+ax^{3}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^6+a*x^3)^(2/3),x)

[Out] -x*(b*x^3+a)/a/(b*x^6+a*x^3)^(2/3)
```

Maxima [A] time = 0.767714, size = 23, normalized size = 1.

$$-\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(-2/3),x, algorithm="maxima")

[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

Fricas [A] time = 0.274283, size = 28, normalized size = 1.22

$$-\frac{\left(bx^{6}+ax^{3}\right)^{\frac{1}{3}}}{ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(-2/3),x, algorithm="fricas")

[Out] -(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^3 + bx^6)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-2/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^6 + ax^3)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(-2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^6 + a*x^3)^(-2/3), x)
```

3.4 $\int \frac{1}{(ax^3+bx^6)^{5/3}} \, dx$

Optimal. Leaf size=77

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*
a^2*x^5) + (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rubi [A] time = 0.0969952, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*
a^2*x^5) + (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rubi in Sympy [A] time = 9.21689, size = 70, normalized size = 0.91

$$\frac{1}{2ax^2\left(ax^3+bx^6\right)^{\frac{2}{3}}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

```
[Out] 1/(2*a*x**2*(a*x**3 + b*x**6)**(2/3)) - 3*(a*x**3 + b*x**6)**(1/3
)/(4*a**2*x**5) + 9*b*(a*x**3 + b*x**6)**(1/3)/(4*a**3*x**2)
```

Mathematica [A] time = 0.0359869, size = 46, normalized size = 0.6

$$\frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2\left(x^3\left(a+bx^3\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3)
)
```

Maple [A] time = 0.007, size = 46, normalized size = 0.6

$$-\frac{x\left(bx^{3}+a\right)\left(-9\,b^{2}x^{6}-6\,bx^{3}a+a^{2}\right)}{4\,a^{3}}\left(bx^{6}+ax^{3}\right)^{-\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^6+a*x^3)^(5/3),x)
```

```
[Out] -1/4*x*(b*x^3+a)*(-9*b^2*x^6-6*a*b*x^3+a^2)/a^3/(b*x^6+a*x^3)^(5/
3)
```

Maxima [A] time = 0.771116, size = 51, normalized size = 0.66

$$\frac{9\,b^{2}x^{6}+6\,abx^{3}-a^{2}}{4\,(bx^{3}+a)^{\frac{2}{3}}a^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(-5/3),x, algorithm="maxima")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

Fricas [A] time = 0.27397, size = 73, normalized size = 0.95

$$\frac{\left(9\,b^{2}x^{6}+6\,abx^{3}-a^{2}\right)\left(bx^{6}+ax^{3}\right)^{\frac{1}{3}}}{4\,(a^{3}bx^{8}+a^{4}x^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(-5/3),x, algorithm="fricas")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^
8 + a^4*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^3 + bx^6)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-5/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^6 + ax^3)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^6 + a*x^3)^(-5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^6 + a*x^3)^(-5/3), x)
```

3.5 $\int \frac{1}{-x^3+x^6}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Lo
g[1 + x + x^2]/6
```

Rubi [A] time = 0.0516987, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Lo
g[1 + x + x^2]/6
```

Rubi in Sympy [A] time = 7.48566, size = 44, normalized size = 0.92

$$\frac{\log\left(-x+1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-x**3),x)
```

```
[Out] log(-x + 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(sqrt(3)*(2*x/3
 + 1/3))/3 + 1/(2*x**2)
```

Mathematica [A] time = 0.0261167, size = 48, normalized size = 1.

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Lo
g[1 + x + x^2]/6
```

Maple [A] time = 0.01, size = 38, normalized size = 0.8

$$-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln(-1+x)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-x^3),x)
```

```
[Out] -1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(-
1+x)+1/2/x^2
```

Maxima [A] time = 0.846924, size = 50, normalized size = 1.04

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^
2 + x + 1) + 1/3*log(x - 1)
```

Fricas [A] time = 0.270274, size = 76, normalized size = 1.58

$$-\frac{\sqrt{3}\left(\sqrt{3}x^2\log\left(x^2+x+1\right)-2\sqrt{3}x^2\log(x-1)+6x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-3\sqrt{3}\right)}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*x^2*log(x^2 + x + 1) - 2*sqrt(3)*x^2*log(x
 - 1) + 6*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*sqrt(3))/x^2
```

Sympy [A] time = 0.356674, size = 48, normalized size = 1.

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3 + 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.291603, size = 51, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\ln\left(x^2+x+1\right)+\frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*ln(x^2
 + x + 1) + 1/3*ln(abs(x - 1))
```

3.6 $\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}$$

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rubi [A] time = 0.0688549, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.0234656, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^3)^2}\left(3ax^6+2bx^9\right)}{18(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

Maple [A] time = 0.006, size = 36, normalized size = 0.5

$$\frac{x^6\left(2\,bx^3+3\,a\right)}{18\,bx^3+18\,a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*((b*x^3+a)^2)^(1/2),x)

[Out] 1/18*x^6*(2*b*x^3+3*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26779, size = 18, normalized size = 0.23

$$\frac{1}{9}\,bx^9+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^5,x, algorithm="fricas")

[Out] 1/9*b*x^9 + 1/6*a*x^6
```

Sympy [A] time = 0.209381, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**9/9
```

GIAC/XCAS [A] time = 0.281125, size = 31, normalized size = 0.39

$$\frac{1}{18}\left(2\,bx^9 + 3\,ax^6\right)\mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^5,x, algorithm="giac")
```

```
[Out] 1/18*(2*b*x^9 + 3*a*x^6)*sign(b*x^3 + a)
```

3.7 $\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi [A] time = 0.0668339, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.0127001, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(8ax^5+5bx^8\right)}{40\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$\frac{x^5\left(5\,bx^3+8\,a\right)}{40\,bx^3+40\,a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 0.781704, size = 18, normalized size = 0.23

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Fricas [A] time = 0.264422, size = 18, normalized size = 0.23

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Sympy [A] time = 0.214934, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**8/8
```

GIAC/XCAS [A] time = 0.27303, size = 39, normalized size = 0.49

$$\frac{1}{8}\,bx^8 \mathrm{sign}\left(bx^3 + a\right) + \frac{1}{5}\,ax^5 \mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^4,x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*sign(b*x^3 + a) + 1/5*a*x^5*sign(b*x^3 + a)
```

3.8 $\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi [A] time = 0.0659783, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.012017, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^3)^2}\left(7ax^4+4bx^7\right)}{28(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$\frac{x^4\left(4\,bx^3+7\,a\right)}{28\,bx^3+28\,a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 0.784532, size = 18, normalized size = 0.23

$$\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Fricas [A] time = 0.265921, size = 18, normalized size = 0.23

$$\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Sympy [A] time = 0.220402, size = 12, normalized size = 0.15

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**7/7
```

GIAC/XCAS [A] time = 0.27898, size = 39, normalized size = 0.49

$$\frac{1}{7} bx^7 \mathrm{sign}\left(bx^3 + a\right) + \frac{1}{4} ax^4 \mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^3,x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*sign(b*x^3 + a) + 1/4*a*x^4*sign(b*x^3 + a)
```

3.9 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

```
[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rubi [A] time = 0.0681423, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{(a+bx^3)^2}\int^{a+bx^3} x\,dx}{3b\,(a+bx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*((b*x**3+a)**2)**(1/2),x)

[Out] sqrt((a + b*x**3)**2)*Integral(x, (x, a + b*x**3))/(3*b*(a + b*x*
*3))
```

Mathematica [A] time = 0.0120992, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^3)^2}\,(2ax^3+bx^6)}{6\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))
```

Maple [A] time = 0.004, size = 35, normalized size = 1.

$$\frac{x^3 \left(bx^3 + 2a\right)}{6bx^3 + 6a}\sqrt{\left(bx^3 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265928, size = 18, normalized size = 0.5

$$\frac{1}{6}bx^6 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/3*a*x^3
```

Sympy [A] time = 0.20392, size = 12, normalized size = 0.33

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**6/6
```

GIAC/XCAS [A] time = 0.307718, size = 30, normalized size = 0.83

$$\frac{1}{6}\left(bx^6 + 2\,ax^3\right)\operatorname{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^6 + 2*a*x^3)*sign(b*x^3 + a)
```

3.10 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rubi [A] time = 0.0537178, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.0133414, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(5ax^2+2bx^5\right)}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$\frac{x^2\left(2\,bx^3+5\,a\right)}{10\,bx^3+10\,a}\sqrt{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((b*x^3+a)^2)^(1/2),x)

[Out] 1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 0.816568, size = 18, normalized size = 0.23

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x,x, algorithm="maxima")

[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Fricas [A] time = 0.26646, size = 18, normalized size = 0.23

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x,x, algorithm="fricas")

[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Sympy [A] time = 0.207347, size = 12, normalized size = 0.15

$$\frac{ax^2}{2} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**5/5
```

GIAC/XCAS [A] time = 0.277304, size = 39, normalized size = 0.49

$$\frac{1}{5}\,bx^5\mathrm{sign}\left(bx^3 + a\right) + \frac{1}{2}\,ax^2\mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)*x,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*sign(b*x^3 + a) + 1/2*a*x^2*sign(b*x^3 + a)
```

3.11 $\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{bx^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4\,(a + bx^3)}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi [A] time = 0.0344427, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{ax\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{bx^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4\,(a + bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(a + bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.0152645, size = 36, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2}\,\left(4ax + bx^4\right)}{4\,(a + bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

Maple [A] time = 0.003, size = 33, normalized size = 0.5

$$\frac{x\left(bx^{3}+4\,a\right)}{4\,bx^{3}+4\,a}\sqrt{\left(bx^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2),x)

[Out] 1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A] time = 0.815274, size = 14, normalized size = 0.19

$$\frac{1}{4}\,bx^{4}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2),x, algorithm="maxima")

[Out] 1/4*b*x^4 + a*x
```

Fricas [A] time = 0.264772, size = 14, normalized size = 0.19

$$\frac{1}{4}\,bx^{4}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2),x, algorithm="fricas")

[Out] 1/4*b*x^4 + a*x
```

Sympy [A] time = 0.195398, size = 8, normalized size = 0.11

$$ax + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**4/4
```

GIAC/XCAS [A] time = 0.2681, size = 27, normalized size = 0.36

$$\frac{1}{4}\left(bx^4 + 4\,ax\right)\operatorname{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2),x, algorithm="giac")
```

```
[Out] 1/4*(b*x^4 + 4*a*x)*sign(b*x^3 + a)
```

3.12 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$

Optimal. Leaf size=75

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.0611676, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^3\right)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x, x)
```

Mathematica [A] time = 0.0155816, size = 37, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3a\log(x)+bx^3\right)}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3 + 3*a*Log[x]))/(3*(a + b*x^3))
```

Maple [A] time = 0.014, size = 34, normalized size = 0.5

$$\frac{bx^3+3\,a\ln(x)}{3\,bx^3+3\,a}\sqrt{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x,x)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(b*x^3+3*a*ln(x))/(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273561, size = 15, normalized size = 0.2

$$\frac{1}{3}\,bx^3+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*log(x)
```

Sympy [A] time = 0.277418, size = 10, normalized size = 0.13

$$a \log(x) + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x,x)

[Out] a*log(x) + b*x**3/3
```

GIAC/XCAS [A] time = 0.269077, size = 38, normalized size = 0.51

$$\frac{1}{3} bx^3 \operatorname{sign}\left(bx^3 + a\right) + a\ln\left(|x|\right) \operatorname{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x,x, algorithm="giac")

[Out] 1/3*b*x^3*sign(b*x^3 + a) + a*ln(abs(x))*sign(b*x^3 + a)
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx$

Optimal. Leaf size=77

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}$$

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rubi [A] time = 0.0624719, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^3)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**2, x)
```

Mathematica [A] time = 0.0185209, size = 38, normalized size = 0.49

$$\frac{(bx^3-2a)\sqrt{(a+bx^3)^2}}{2x\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$-\frac{-bx^3+2a}{2x(bx^3+a)}\sqrt{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Maxima [A] time = 0.765795, size = 19, normalized size = 0.25

$$\frac{bx^3-2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 - 2*a)/x
```

Fricas [A] time = 0.264117, size = 19, normalized size = 0.25

$$\frac{bx^3-2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3 - 2*a)/x
```

Sympy [A] time = 1.10838, size = 8, normalized size = 0.1

$$-\frac{a}{x} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**2,x)

[Out] -a/x + b*x**2/2
```

GIAC/XCAS [A] time = 0.281235, size = 39, normalized size = 0.51

$$\frac{1}{2}\,bx^2\text{sign}\left(bx^3+a\right) - \frac{a\text{sign}\left(bx^3+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^2,x, algorithm="giac")

[Out] 1/2*b*x^2*sign(b*x^3 + a) - a*sign(b*x^3 + a)/x
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi [A] time = 0.0618748, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^3\right)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**3, x)
```

Mathematica [A] time = 0.0127244, size = 37, normalized size = 0.5

$$-\frac{\left(a-2bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]

[Out] -((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x^2*(a + b*x^3))
```

Maple [A] time = 0.004, size = 34, normalized size = 0.5

$$-\frac{-2\,bx^{3}+a}{2\,x^{2}\left(bx^{3}+a\right)}\sqrt{\left(bx^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^3,x)

[Out] -1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

Maxima [A] time = 0.756498, size = 20, normalized size = 0.27

$$\frac{2\,bx^{3}-a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^3,x, algorithm="maxima")

[Out] 1/2*(2*b*x^3 - a)/x^2
```

Fricas [A] time = 0.247426, size = 20, normalized size = 0.27

$$\frac{2\,bx^{3}-a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^3,x, algorithm="fricas")

[Out] 1/2*(2*b*x^3 - a)/x^2
```

Sympy [A] time = 1.21255, size = 8, normalized size = 0.11

$$-\frac{a}{2x^2} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*x
```

GIAC/XCAS [A] time = 0.257151, size = 35, normalized size = 0.47

$$bx\text{sign}\left(bx^3 + a\right) - \frac{a\text{sign}\left(bx^3 + a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^3,x, algorithm="giac")
```

```
[Out] b*x*sign(b*x^3 + a) - 1/2*a*sign(b*x^3 + a)/x^2
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\,(a+bx^3)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.0637153, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^3)^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**4, x)
```

Mathematica [A] time = 0.016082, size = 39, normalized size = 0.52

$$-\frac{\sqrt{(a+bx^3)^2}\,\left(a-3bx^3\log(x)\right)}{3x^3\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a - 3*b*x^3*Log[x]))/(3*x^3*(a + b*x^3))
```

Maple [A] time = 0.015, size = 38, normalized size = 0.5

$$\frac{3\,b\ln(x)\,x^3 - a}{(3\,bx^3 + 3\,a)\,x^3}\sqrt{(bx^3 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^4,x)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(3*b*ln(x)*x^3-a)/(b*x^3+a)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282301, size = 23, normalized size = 0.31

$$\frac{3\,bx^3\log(x) - a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*log(x) - a)/x^3
```

Sympy [A] time = 1.19694, size = 10, normalized size = 0.13

$$-\frac{a}{3x^3} + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

```
[Out] -a/(3*x**3) + b*log(x)
```

GIAC/XCAS [A] time = 0.29695, size = 58, normalized size = 0.77

$$b\ln(|x|)\operatorname{sign}\left(bx^3 + a\right) - \frac{bx^3\operatorname{sign}\left(bx^3 + a\right) + a\operatorname{sign}\left(bx^3 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^4,x, algorithm="giac")
```

```
[Out] b*ln(abs(x))*sign(b*x^3 + a) - 1/3*(b*x^3*sign(b*x^3 + a) + a*sig
n(b*x^3 + a))/x^3
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$

Optimal. Leaf size=77

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rubi [A] time = 0.0648164, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^3\right)^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**5, x)
```

Mathematica [A] time = 0.0131551, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+4bx^3\right)}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(4*x^4*(a + b*x^3))
```

Maple [A] time = 0.004, size = 34, normalized size = 0.4

$$-\frac{4\,bx^3+a}{4\,x^4\,(bx^3+a)}\sqrt{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^5,x)
```

```
[Out] -1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Maxima [A] time = 0.764604, size = 18, normalized size = 0.23

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Fricas [A] time = 0.250252, size = 18, normalized size = 0.23

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Sympy [A] time = 1.32295, size = 14, normalized size = 0.18

$$-\frac{a+4bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

```
[Out] -(a + 4*b*x**3)/(4*x**4)
```

GIAC/XCAS [A] time = 0.271324, size = 41, normalized size = 0.53

$$-\frac{4\,bx^3\text{sign}\left(bx^3+a\right)+a\text{sign}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b*x^3*sign(b*x^3 + a) + a*sign(b*x^3 + a))/x^4
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rubi [A] time = 0.0642564, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^3\right)^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**6, x)
```

Mathematica [A] time = 0.0136681, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(10*x^5*(a + b*x^3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$-\frac{5\,bx^{3}+2\,a}{10\,x^{5}\left(bx^{3}+a\right)}\sqrt{\left(bx^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^6,x)
```

```
[Out] -1/10*(5*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Maxima [A] time = 0.785436, size = 20, normalized size = 0.25

$$-\frac{5\,bx^{3}+2\,a}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Fricas [A] time = 0.253413, size = 20, normalized size = 0.25

$$-\frac{5\,bx^{3}+2\,a}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Sympy [A] time = 1.25528, size = 15, normalized size = 0.19

$$-\frac{2a+5bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**6,x)

[Out] -(2*a + 5*b*x**3)/(10*x**5)
```

GIAC/XCAS [A] time = 0.264726, size = 42, normalized size = 0.53

$$-\frac{5\,bx^3\mathrm{sign}\left(bx^3+a\right)+2\,a\mathrm{sign}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^6,x, algorithm="giac")

[Out] -1/10*(5*b*x^3*sign(b*x^3 + a) + 2*a*sign(b*x^3 + a))/x^5
```

3.18 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rubi [A] time = 0.0633717, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^3)^2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**7, x)
```

Mathematica [A] time = 0.0167524, size = 37, normalized size = 0.47

$$-\frac{\sqrt{(a+bx^3)^2}\left(a+2bx^3\right)}{6x^6(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(6*x^6*(a + b*x^3))
```

Maple [A] time = 0.004, size = 34, normalized size = 0.4

$$-\frac{2\,bx^{3}+a}{6\,x^{6}\left(bx^{3}+a\right)}\sqrt{\left(bx^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^7,x)
```

```
[Out] -1/6*(2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24892, size = 18, normalized size = 0.23

$$-\frac{2\,bx^{3}+a}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/x^6
```

Sympy [A] time = 1.27151, size = 14, normalized size = 0.18

$$-\frac{a+2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**7,x)

[Out] -(a + 2*b*x**3)/(6*x**6)
```

GIAC/XCAS [A] time = 0.285397, size = 41, normalized size = 0.52

$$-\frac{2\,bx^3\mathrm{sign}\left(bx^3+a\right)+a\mathrm{sign}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^7,x, algorithm="giac")

[Out] -1/6*(2*b*x^3*sign(b*x^3 + a) + a*sign(b*x^3 + a))/x^6
```

3.19 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rubi [A] time = 0.0634568, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^3\right)^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**8, x)
```

Mathematica [A] time = 0.0157928, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a+7bx^3\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(28*x^7*(a + b*x^3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$-\frac{7\,bx^{3}+4\,a}{28\,x^{7}\,(bx^{3}+a)}\sqrt{(bx^{3}+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^8,x)
```

```
[Out] -1/28*(7*b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Maxima [A] time = 0.783568, size = 20, normalized size = 0.25

$$-\frac{7\,bx^{3}+4\,a}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Fricas [A] time = 0.256891, size = 20, normalized size = 0.25

$$-\frac{7\,bx^{3}+4\,a}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Sympy [A] time = 1.31883, size = 15, normalized size = 0.19

$$-\frac{4a+7bx^3}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

```
[Out] -(4*a + 7*b*x**3)/(28*x**7)
```

GIAC/XCAS [A] time = 0.273598, size = 42, normalized size = 0.53

$$-\frac{7\,bx^3\text{sign}\left(bx^3+a\right)+4\,a\text{sign}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/28*(7*b*x^3*sign(b*x^3 + a) + 4*a*sign(b*x^3 + a))/x^7
```

3.20 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9} dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rubi [A] time = 0.0629029, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^3)^2}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**9, x)
```

Mathematica [A] time = 0.0134127, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^3)^2}\left(5a+8bx^3\right)}{40x^8(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]

[Out] -(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(40*x^8*(a + b*x^3))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$-\frac{8\,bx^3+5\,a}{40\,x^8\,(bx^3+a)}\sqrt{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^9,x)

[Out] -1/40*(8*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

Maxima [A] time = 0.791242, size = 20, normalized size = 0.25

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^9,x, algorithm="maxima")

[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Fricas [A] time = 0.27184, size = 20, normalized size = 0.25

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^9,x, algorithm="fricas")

[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Sympy [A] time = 1.37279, size = 15, normalized size = 0.19

$$-\frac{5a+8bx^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
```

```
[Out] -(5*a + 8*b*x**3)/(40*x**8)
```

GIAC/XCAS [A] time = 0.286681, size = 42, normalized size = 0.53

$$-\frac{8\,bx^3\mathrm{sign}\left(bx^3+a\right)+5\,a\mathrm{sign}\left(bx^3+a\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^9,x, algorithm="giac")
```

```
[Out] -1/40*(8*b*x^3*sign(b*x^3 + a) + 5*a*sign(b*x^3 + a))/x^8
```

3.21 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}} dx$

Optimal. Leaf size=79

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rubi [A] time = 0.0629637, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^3)^2}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**10, x)
```

Mathematica [A] time = 0.0138159, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^3)^2}\left(2a+3bx^3\right)}{18x^9(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(18*x^9*(a + b*x^3))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$-\frac{3\,bx^{3}+2\,a}{18\,x^{9}\,(bx^{3}+a)}\sqrt{(bx^{3}+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^10,x)
```

```
[Out] -1/18*(3*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258882, size = 20, normalized size = 0.25

$$-\frac{3\,bx^{3}+2\,a}{18\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/18*(3*b*x^3 + 2*a)/x^9
```

Sympy [A] time = 1.404, size = 15, normalized size = 0.19

$$-\frac{2a+3bx^3}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

```
[Out] -(2*a + 3*b*x**3)/(18*x**9)
```

GIAC/XCAS [A] time = 0.268966, size = 42, normalized size = 0.53

$$-\frac{3\,bx^3\mathrm{sign}\left(bx^3+a\right)+2\,a\mathrm{sign}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^10,x, algorithm="giac")
```

```
[Out] -1/18*(3*b*x^3*sign(b*x^3 + a) + 2*a*sign(b*x^3 + a))/x^9
```

3.22 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi [A] time = 0.0642167, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^3\right)^2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt((a + b*x**3)**2)/x**11, x)
```

Mathematica [A] time = 0.0137263, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a+10bx^3\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]

[Out] -(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(70*x^10*(a + b*x^3))
```

Maple [A] time = 0.006, size = 36, normalized size = 0.5

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}\,(bx^3+a)}\sqrt{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^3+a)^2)^(1/2)/x^11,x)

[Out] -1/70*(10*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)
```

Maxima [A] time = 0.775506, size = 20, normalized size = 0.25

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^11,x, algorithm="maxima")

[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Fricas [A] time = 0.274876, size = 20, normalized size = 0.25

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^11,x, algorithm="fricas")

[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Sympy [A] time = 1.413, size = 15, normalized size = 0.19

$$-\frac{7a + 10bx^3}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
```

```
[Out] -(7*a + 10*b*x**3)/(70*x**10)
```

GIAC/XCAS [A] time = 0.286571, size = 42, normalized size = 0.53

$$-\frac{10\,bx^3 \operatorname{sign}\left(bx^3 + a\right) + 7\,a \operatorname{sign}\left(bx^3 + a\right)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^3 + a)^2)/x^11,x, algorithm="giac")
```

```
[Out] -1/70*(10*b*x^3*sign(b*x^3 + a) + 7*a*sign(b*x^3 + a))/x^10
```

3.23 $\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*
a^2*b*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3
*a*b^2*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (
b^3*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))
```

Rubi [A] time = 0.123926, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*
a^2*b*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3
*a*b^2*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (
b^3*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.2577, size = 136, normalized size = 0.81

$$\frac{81a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{19760\left(a+bx^3\right)}+\frac{27a^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{1976}+\frac{9ax^{10}\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{304}+\frac{x^{10}\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 81*a**3*x**10*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(19760*(a + b*x
**3)) + 27*a**2*x**10*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/1976 +
9*a*x**10*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/304 +
x**10*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/19
```

Mathematica [A] time = 0.0328731, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{(a+bx^3)^2}\left(1976a^3+4560a^2bx^3+3705ab^2x^6+1040b^3x^9\right)}{19760\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2
*x^6 + 1040*b^3*x^9))/(19760*(a + b*x^3))
```

Maple [A] time = 0.011, size = 58, normalized size = 0.4

$$\frac{x^{10}\left(1040\,b^3x^9+3705\,ab^2x^6+4560\,a^2bx^3+1976\,a^3\right)}{19760\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/19760*x^10*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3
)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 0.780585, size = 47, normalized size = 0.28

$$\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^9,x, algorithm="maxima")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Fricas [A] time = 0.255237, size = 47, normalized size = 0.28

$$\frac{1}{19}\,b^{3}x^{19}+\frac{3}{16}\,ab^{2}x^{16}+\frac{3}{13}\,a^{2}bx^{13}+\frac{1}{10}\,a^{3}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^9,x, algorithm="fricas")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{9}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.26618, size = 90, normalized size = 0.54

$$\frac{1}{19}\,b^{3}x^{19}\mathrm{sign}\left(bx^{3}+a\right)+\frac{3}{16}\,ab^{2}x^{16}\mathrm{sign}\left(bx^{3}+a\right)+\frac{3}{13}\,a^{2}bx^{13}\mathrm{sign}\left(bx^{3}+a\right)+\frac{1}{10}\,a^{3}x^{10}\mathrm{sign}\left(bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^9,x, algorithm="giac")
```

```
[Out] 1/19*b^3*x^19*sign(b*x^3 + a) + 3/16*a*b^2*x^16*sign(b*x^3 + a) +
 3/13*a^2*b*x^13*sign(b*x^3 + a) + 1/10*a^3*x^10*sign(b*x^3 + a)
```

3.24 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^4}{15b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^3}{12b^3}$$

```
[Out] (a^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^3) - (2
*a*(a + b*x^3)^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^3) + ((a
+ b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3)
```

Rubi [A] time = 0.152593, antiderivative size = 167, normalized size of antiderivative = 1.4, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^3x^{18}\sqrt{a^2+2abx^3+b^2x^6}}{18\left(a+bx^3\right)}+\frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^2*
b*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (a*b^2*
x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^18
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*(a + b*x^3))
```

Rubi in Sympy [A] time = 21.886, size = 107, normalized size = 0.9

$$\frac{a^2\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{72b^3}-\frac{a\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{45b^3}+\frac{x^6\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] a**2*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(72*
b**3) - a*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(45*b**3) + x**6
*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(36*b)
```

Mathematica [A] time = 0.0292304, size = 61, normalized size = 0.51

$$\frac{x^9\sqrt{(a+bx^3)^2}\,\left(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9\right)}{180\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 +
10*b^3*x^9))/(180*(a + b*x^3))
```

Maple [A] time = 0.011, size = 58, normalized size = 0.5

$$\frac{x^9\,\left(10\,b^3x^9+36\,ab^2x^6+45\,a^2bx^3+20\,a^3\right)}{180\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/180*x^9*(10*b^3*x^9+36*a*b^2*x^6+45*a^2*b*x^3+20*a^3)*((b*x^3+a
)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254318, size = 47, normalized size = 0.39

$$\frac{1}{18}\,b^3x^{18}+\frac{1}{5}\,ab^2x^{15}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.303493, size = 90, normalized size = 0.76

$$\frac{1}{18}\,b^3x^{18}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{5}\,ab^2x^{15}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{4}\,a^2bx^{12}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{9}\,a^3x^9\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] 1/18*b^3*x^18*sign(b*x^3 + a) + 1/5*a*b^2*x^15*sign(b*x^3 + a) +
1/4*a^2*b*x^12*sign(b*x^3 + a) + 1/9*a^3*x^9*sign(b*x^3 + a)
```

3.25 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}$$

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^
2*b*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a
*b^2*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^
3*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
```

Rubi [A] time = 0.120454, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```


```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^
2*b*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a
*b^2*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^
3*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.3234, size = 136, normalized size = 0.81

$$\frac{81a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{10472(a+bx^3)}+\frac{27a^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{1309}+\frac{9ax^8\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{238}+\frac{x^8\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 81*a**3*x**8*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(10472*(a + b*x*
*3)) + 27*a**2*x**8*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/1309 + 9*
a*x**8*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/238 + x**
8*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/17
```

Mathematica [A] time = 0.0295808, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{(a+bx^3)^2}\left(1309a^3+2856a^2bx^3+2244ab^2x^6+616b^3x^9\right)}{10472\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*
x^6 + 616*b^3*x^9))/(10472*(a + b*x^3))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$\frac{x^8\left(616\,b^3x^9+2244\,ab^2x^6+2856\,a^2bx^3+1309\,a^3\right)}{10472\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*
((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 0.759951, size = 47, normalized size = 0.28

$$\frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^7,x, algorithm="maxima")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Fricas [A] time = 0.250942, size = 47, normalized size = 0.28

$$\frac{1}{17}\,b^{3}x^{17}+\frac{3}{14}\,ab^{2}x^{14}+\frac{3}{11}\,a^{2}bx^{11}+\frac{1}{8}\,a^{3}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^7,x, algorithm="fricas")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{7}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.26885, size = 90, normalized size = 0.54

$$\frac{1}{17}\,b^{3}x^{17}\mathrm{sign}\left(bx^{3}+a\right)+\frac{3}{14}\,ab^{2}x^{14}\mathrm{sign}\left(bx^{3}+a\right)+\frac{3}{11}\,a^{2}bx^{11}\mathrm{sign}\left(bx^{3}+a\right)+\frac{1}{8}\,a^{3}x^{8}\mathrm{sign}\left(bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^7,x, algorithm="giac")
```

```
[Out] 1/17*b^3*x^17*sign(b*x^3 + a) + 3/14*a*b^2*x^14*sign(b*x^3 + a) +
 3/11*a^2*b*x^11*sign(b*x^3 + a) + 1/8*a^3*x^8*sign(b*x^3 + a)
```

3.26 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^
2*b*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a
*b^2*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^
3*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3))
```

Rubi [A] time = 0.1208, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^
2*b*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a
*b^2*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^
3*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.9684, size = 136, normalized size = 0.81

$$\frac{81a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7280\left(a+bx^3\right)}+\frac{27a^2x^7\sqrt{a^2+2abx^3+b^2x^6}}{1040}+\frac{9ax^7\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{208}+\frac{x^7\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 81*a**3*x**7*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(7280*(a + b*x**
3)) + 27*a**2*x**7*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/1040 + 9*a
*x**7*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/208 + x**7
*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/16
```

Mathematica [A] time = 0.0324748, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{(a+bx^3)^2}\left(1040a^3+2184a^2bx^3+1680ab^2x^6+455b^3x^9\right)}{7280\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*
x^6 + 455*b^3*x^9))/(7280*(a + b*x^3))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$\frac{x^7\left(455\,b^3x^9+1680\,ab^2x^6+2184\,a^2bx^3+1040\,a^3\right)}{7280\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*(
(b*x^3+a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 0.762293, size = 47, normalized size = 0.28

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Fricas [A] time = 0.256258, size = 47, normalized size = 0.28

$$\frac{1}{16}\,b^3x^{16} + \frac{3}{13}\,ab^2x^{13} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.27162, size = 90, normalized size = 0.54

$$\frac{1}{16}\,b^3x^{16}\mathrm{sign}\left(bx^3 + a\right) + \frac{3}{13}\,ab^2x^{13}\mathrm{sign}\left(bx^3 + a\right) + \frac{3}{10}\,a^2bx^{10}\mathrm{sign}\left(bx^3 + a\right) + \frac{1}{7}\,a^3x^7\mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sign(b*x^3 + a) + 3/13*a*b^2*x^13*sign(b*x^3 + a) +
 3/10*a^2*b*x^10*sign(b*x^3 + a) + 1/7*a^3*x^7*sign(b*x^3 + a)
```

3.27 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

```
[Out] -(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^2) + ((a
 + b*x^3)^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rubi [A] time = 0.127671, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^2) + ((a
 + b*x^3)^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rubi in Sympy [A] time = 14.7323, size = 65, normalized size = 0.83

$$-\frac{a\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{24b^2}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] -a*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(24*b*
*2) + (a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(15*b**2)
```

Mathematica [A] time = 0.0311487, size = 61, normalized size = 0.78

$$\frac{x^6\sqrt{\left(a+bx^3\right)^2}\left(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9\right)}{60\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 +
4*b^3*x^9))/(60*(a + b*x^3))
```

Maple [A]   time = 0.01, size = 58, normalized size = 0.7

$$\frac{x^6\left(4\,b^3x^9+15\,ab^2x^6+20\,a^2bx^3+10\,a^3\right)}{60\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/60*x^6*(4*b^3*x^9+15*a*b^2*x^6+20*a^2*b*x^3+10*a^3)*((b*x^3+a)^
2)^(3/2)/(b*x^3+a)^3
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.249463, size = 47, normalized size = 0.6

$$\frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.298897, size = 61, normalized size = 0.78

$$\frac{1}{60}\left(4\, b^3 x^{15} + 15\, ab^2 x^{12} + 20\, a^2 b x^9 + 10\, a^3 x^6\right) \operatorname{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sig
n(b*x^3 + a)
```

3.28 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^
2*b*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b
^2*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*
x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3))
```

Rubi [A] time = 0.122196, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^
2*b*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b
^2*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*
x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.3258, size = 136, normalized size = 0.81

$$\frac{81a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{3080\left(a+bx^3\right)}+\frac{27a^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{616}+\frac{9ax^5\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{154}+\frac{x^5\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 81*a**3*x**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(3080*(a + b*x**
3)) + 27*a**2*x**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/616 + 9*a*
x**5*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/154 + x**5*
(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/14
```

Mathematica [A] time = 0.0268024, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{(a+bx^3)^2}\left(616a^3+1155a^2bx^3+840ab^2x^6+220b^3x^9\right)}{3080\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^
6 + 220*b^3*x^9))/(3080*(a + b*x^3))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$\frac{x^5\left(220\,b^3x^9+840\,ab^2x^6+1155\,a^2bx^3+616\,a^3\right)}{3080\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b
*x^3+a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 0.76952, size = 47, normalized size = 0.28

$$\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Fricas [A] time = 0.252331, size = 47, normalized size = 0.28

$$\frac{1}{14}\,b^3x^{14} + \frac{3}{11}\,ab^2x^{11} + \frac{3}{8}\,a^2bx^8 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.269335, size = 90, normalized size = 0.54

$$\frac{1}{14}\,b^3x^{14}\mathrm{sign}\left(bx^3+a\right) + \frac{3}{11}\,ab^2x^{11}\mathrm{sign}\left(bx^3+a\right) + \frac{3}{8}\,a^2bx^8\mathrm{sign}\left(bx^3+a\right) + \frac{1}{5}\,a^3x^5\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sign(b*x^3 + a) + 3/11*a*b^2*x^11*sign(b*x^3 + a) +
 3/8*a^2*b*x^8*sign(b*x^3 + a) + 1/5*a^3*x^5*sign(b*x^3 + a)
```

3.29 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^
2*b*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b
^2*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*
x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rubi [A] time = 0.120476, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^
2*b*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b
^2*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*
x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.9119, size = 136, normalized size = 0.81

$$\frac{81a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{1820\left(a+bx^3\right)}+\frac{27a^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{455}+\frac{9ax^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{130}+\frac{x^4\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 81*a**3*x**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(1820*(a + b*x**
3)) + 27*a**2*x**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/455 + 9*a*
x**4*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/130 + x**4*
(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/13
```

Mathematica [A] time = 0.0309094, size = 61, normalized size = 0.37

$$\frac{x^4\sqrt{(a+bx^3)^2}\left(455a^3+780a^2bx^3+546ab^2x^6+140b^3x^9\right)}{1820\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6
 + 140*b^3*x^9))/(1820*(a + b*x^3))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$\frac{x^4\left(140\,b^3x^9+546\,ab^2x^6+780\,a^2bx^3+455\,a^3\right)}{1820\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*
x^3+a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 0.806784, size = 47, normalized size = 0.28

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Fricas [A] time = 0.254919, size = 47, normalized size = 0.28

$$\frac{1}{13}\,b^3x^{13} + \frac{3}{10}\,ab^2x^{10} + \frac{3}{7}\,a^2bx^7 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.30196, size = 90, normalized size = 0.54

$$\frac{1}{13}\,b^3x^{13}\mathrm{sign}\left(bx^3+a\right) + \frac{3}{10}\,ab^2x^{10}\mathrm{sign}\left(bx^3+a\right) + \frac{3}{7}\,a^2bx^7\,\mathrm{sign}\left(bx^3+a\right) + \frac{1}{4}\,a^3x^4\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sign(b*x^3 + a) + 3/10*a*b^2*x^10*sign(b*x^3 + a) +
 3/7*a^2*b*x^7*sign(b*x^3 + a) + 1/4*a^3*x^4*sign(b*x^3 + a)
```

3.30 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rubi [A] time = 0.0692568, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rubi in Sympy [A] time = 9.14128, size = 34, normalized size = 0.94

$$\frac{\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] (2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(24*b)
```

Mathematica [A] time = 0.0284362, size = 60, normalized size = 1.67

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9\right)}{12\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3
*x^9))/(12*(a + b*x^3))
```

Maple [F] time = 180., size = 0, normalized size = 0.

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Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254083, size = 47, normalized size = 1.31

$$\frac{1}{12}\,b^3x^{12} + \frac{1}{3}\,ab^2x^9 + \frac{1}{2}\,a^2bx^6 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\left(a + bx^3 \right)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.308757, size = 59, normalized size = 1.64

$$\frac{1}{12} \left(b^3 x^{12} + 4\, ab^2 x^9 + 6\, a^2 b x^6 + 4\, a^3 x^3 \right) \operatorname{sign}\left(bx^3 + a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/12*(b^3*x^12 + 4*a*b^2*x^9 + 6*a^2*b*x^6 + 4*a^3*x^3)*sign(b*x^
3 + a)
```

3.31 $\int x \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^
2*b*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b
^2*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^
11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rubi [A] time = 0.105138, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^
2*b*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b
^2*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^
11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rubi in Sympy [A] time = 15.5806, size = 136, normalized size = 0.81

$$\frac{81a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{440(a+bx^3)}+\frac{27a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{220}+\frac{9ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{88}+\frac{x^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 81*a**3*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(440*(a + b*x**3
)) + 27*a**2*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/220 + 9*a*x
**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/88 + x**2*(a
**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/11
```

Mathematica [A] time = 0.0288461, size = 61, normalized size = 0.37

$$\frac{x^2\sqrt{(a+bx^3)^2}\left(220a^3+264a^2bx^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6
 + 40*b^3*x^9))/(440*(a + b*x^3))
```

Maple [A] time = 0.007, size = 58, normalized size = 0.4

$$\frac{x^2\left(40\,b^3x^9+165\,ab^2x^6+264\,a^2bx^3+220\,a^3\right)}{440\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^
3+a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 0.797611, size = 47, normalized size = 0.28

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Fricas [A] time = 0.249562, size = 47, normalized size = 0.28

$$\frac{1}{11}\,b^3x^{11} + \frac{3}{8}\,ab^2x^8 + \frac{3}{5}\,a^2bx^5 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.296884, size = 90, normalized size = 0.54

$$\frac{1}{11}\,b^3x^{11}\mathrm{sign}\left(bx^3+a\right) + \frac{3}{8}\,ab^2x^8\mathrm{sign}\left(bx^3+a\right) + \frac{3}{5}\,a^2bx^5\mathrm{sign}\left(bx^3+a\right) + \frac{1}{2}\,a^3x^2\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sign(b*x^3 + a) + 3/8*a*b^2*x^8*sign(b*x^3 + a) + 3
/5*a^2*b*x^5*sign(b*x^3 + a) + 1/2*a^3*x^2*sign(b*x^3 + a)
```

3.32 $\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3}$$

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*
b*x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a
*b^2*x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (
b^3*x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rubi [A] time = 0.0801765, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*
b*x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a
*b^2*x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (
b^3*x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rubi in Sympy [A] time = 5.60486, size = 129, normalized size = 0.8

$$\frac{81a^3x\sqrt{a^2+2abx^3+b^2x^6}}{140\left(a+bx^3\right)}+\frac{27a^2x\sqrt{a^2+2abx^3+b^2x^6}}{140}+\frac{9ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{70}+\frac{x\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 81*a**3*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(140*(a + b*x**3))
+ 27*a**2*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/140 + 9*a*x*(a +
b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/70 + x*(a**2 + 2*a*b*
x**3 + b**2*x**6)**(3/2)/10
```

Mathematica [A] time = 0.0311651, size = 59, normalized size = 0.36

$$\frac{x\sqrt{(a+bx^3)^2}\left(140a^3+105a^2bx^3+60ab^2x^6+14b^3x^9\right)}{140\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 +
14*b^3*x^9))/(140*(a + b*x^3))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.4

$$\frac{x\left(14\,b^3x^9+60\,ab^2x^6+105\,a^2bx^3+140\,a^3\right)}{140\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a
)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A] time = 0.78541, size = 43, normalized size = 0.27

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{7}\,ab^2x^7+\frac{3}{4}\,a^2bx^4+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Fricas [A] time = 0.250921, size = 43, normalized size = 0.27

$$\frac{1}{10}b^3x^{10} + \frac{3}{7}ab^2x^7 + \frac{3}{4}a^2bx^4 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

GIAC/XCAS [A] time = 0.259844, size = 86, normalized size = 0.53

$$\frac{1}{10}b^3x^{10}\text{sign}\left(bx^3+a\right) + \frac{3}{7}ab^2x^7\text{sign}\left(bx^3+a\right) + \frac{3}{4}a^2bx^4\text{sign}\left(bx^3+a\right) + a^3x\text{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*sign(b*x^3 + a) + 3/7*a*b^2*x^7*sign(b*x^3 + a) + 3
/4*a^2*b*x^4*sign(b*x^3 + a) + a^3*x*sign(b*x^3 + a)
```

3.33 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx$

Optimal. Leaf size=160

$$\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.120659, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 16.8819, size = 117, normalized size = 0.73

$$\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}}{3}+\frac{a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

```
[Out] a**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**3) + a*
*2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/3 + a*(a + b*x**3)*sqrt(a*
*2 + 2*a*b*x**3 + b**2*x**6)/6 + (a**2 + 2*a*b*x**3 + b**2*x**6)*
*(3/2)/9
```

Mathematica [A] time = 0.0365753, size = 60, normalized size = 0.38

$$\frac{\sqrt{(a+bx^3)^2}\left(18a^3\log(x)+bx^3\left(18a^2+9abx^3+2b^2x^6\right)\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18
*a^3*Log[x]))/(18*(a + b*x^3))
```

Maple [A] time = 0.012, size = 57, normalized size = 0.4

$$\frac{2\,b^3x^9+9\,ax^6b^2+18\,x^3a^2b+18\,a^3\ln(x)}{18\,\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+9*a*x^6*b^2+18*x^3*a^2*b+18*a
^3*ln(x))/(b*x^3+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259378, size = 43, normalized size = 0.27

$$\frac{1}{9}b^3x^9 + \frac{1}{2}ab^2x^6 + a^2bx^3 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

GIAC/XCAS [A] time = 0.290963, size = 88, normalized size = 0.55

$$\frac{1}{9}b^3x^9\operatorname{sign}(bx^3+a) + \frac{1}{2}ab^2x^6\operatorname{sign}(bx^3+a) + a^2bx^3\operatorname{sign}(bx^3+a) + a^3\ln(|x|)\operatorname{sign}(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sign(b*x^3 + a) + 1/2*a*b^2*x^6*sign(b*x^3 + a) + a^2
*b*x^3*sign(b*x^3 + a) + a^3*ln(abs(x))*sign(b*x^3 + a)
```

3.34 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=165

$$\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2
*b*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^
2*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi [A] time = 0.11365, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2
*b*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^
2*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.2694, size = 128, normalized size = 0.78

$$-\frac{81a^3\sqrt{a^2+2abx^3+b^2x^6}}{40x\left(a+bx^3\right)}+\frac{27a^2\sqrt{a^2+2abx^3+b^2x^6}}{40x}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{40x}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
```

```
[Out] -81*a**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(40*x*(a + b*x**3))
+ 27*a**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(40*x) + 9*a*(a + b
```

```
*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(40*x) + (a**2 + 2*a*b
*x**3 + b**2*x**6)**(3/2)/(8*x)
```

Mathematica [A] time = 0.0288916, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\left(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9\right)}{40x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b
^3*x^9))/(40*x*(a + b*x^3))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{-5\,b^3x^9-24\,ax^6b^2-60\,x^3a^2b+40\,a^3}{40\,x\,(bx^3+a)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)
^(3/2)/x/(b*x^3+a)^3
```

Maxima [A] time = 0.799569, size = 50, normalized size = 0.3

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Fricas [A] time = 0.270011, size = 50, normalized size = 0.3

$$\frac{5\,b^{3}x^{9}+24\,ab^{2}x^{6}+60\,a^{2}bx^{3}-40\,a^{3}}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**2, x)
```

GIAC/XCAS [A] time = 0.278078, size = 90, normalized size = 0.55

$$\frac{1}{8}\,b^{3}x^{8}\mathrm{sign}\left(bx^{3}+a\right)+\frac{3}{5}\,ab^{2}x^{5}\mathrm{sign}\left(bx^{3}+a\right)+\frac{3}{2}\,a^{2}bx^{2}\mathrm{sign}\left(bx^{3}+a\right)-\frac{a^{3}\mathrm{sign}\left(bx^{3}+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8*sign(b*x^3 + a) + 3/5*a*b^2*x^5*sign(b*x^3 + a) + 3/2
*a^2*b*x^2*sign(b*x^3 + a) - a^3*sign(b*x^3 + a)/x
```

3.35 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx$

Optimal. Leaf size=163

$$\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a
^2*b*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi [A] time = 0.112662, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a
^2*b*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi in Sympy [A] time = 16.8656, size = 134, normalized size = 0.82

$$-\frac{81a^3\sqrt{a^2+2abx^3+b^2x^6}}{28x^2(a+bx^3)}+\frac{27a^2\sqrt{a^2+2abx^3+b^2x^6}}{14x^2}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{28x^2}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
```

```
[Out] -81*a**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(28*x**2*(a + b*x**3
)) + 27*a**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(14*x**2) + 9*a*
```

```
(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(28*x**2) + (a**
2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(7*x**2)
```

Mathematica [A] time = 0.0321532, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\left(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9\right)}{28x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b
^3*x^9))/(28*x^2*(a + b*x^3))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$-\frac{-4\,b^3x^9-21\,ax^6b^2-84\,x^3a^2b+14\,a^3}{28\,x^2\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x)
```

```
[Out] -1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)
^(3/2)/x^2/(b*x^3+a)^3
```

Maxima [A] time = 0.784053, size = 50, normalized size = 0.31

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Fricas [A] time = 0.254784, size = 50, normalized size = 0.31

$$\frac{4\,b^3x^9 + 21\,ab^2x^6 + 84\,a^2bx^3 - 14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

GIAC/XCAS [A] time = 0.294515, size = 88, normalized size = 0.54

$$\frac{1}{7}\,b^3x^7\mathrm{sign}\left(bx^3 + a\right) + \frac{3}{4}\,ab^2x^4\mathrm{sign}\left(bx^3 + a\right) + 3\,a^2bx\mathrm{sign}\left(bx^3 + a\right) - \frac{a^3\mathrm{sign}\left(bx^3 + a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sign(b*x^3 + a) + 3/4*a*b^2*x^4*sign(b*x^3 + a) + 3*a
^2*b*x*sign(b*x^3 + a) - 1/2*a^3*sign(b*x^3 + a)/x^2
```

3.36 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (a*b
^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.128956, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (a*b
^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 16.7554, size = 126, normalized size = 0.78

$$\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\log\left(x\right)}{a+bx^3}+ab\sqrt{a^2+2abx^3+b^2x^6}-\frac{a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{2x^3}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

```
[Out] 3*a**2*b*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**3)
+ a*b*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6) - a*(a + b*x**3)*sqrt(a
**2 + 2*a*b*x**3 + b**2*x**6)/(2*x**3) + (a**2 + 2*a*b*x**3 + b**
2*x**6)**(3/2)/(6*x**3)
```

Mathematica [A] time = 0.038804, size = 62, normalized size = 0.39

$$\frac{\sqrt{(a+bx^3)^2}\left(-2a^3+18a^2bx^3\log(x)+6ab^2x^6+b^3x^9\right)}{6x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x
^3*Log[x]))/(6*x^3*(a + b*x^3))
```

Maple [A] time = 0.019, size = 59, normalized size = 0.4

$$\frac{b^3x^9+6\,ax^6b^2+18\,a^2b\ln(x)\,x^3-2\,a^3}{6\,(bx^3+a)^3\,x^3}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*a*x^6*b^2+18*a^2*b*ln(x)*x^3-2
*a^3)/(b*x^3+a)^3/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259681, size = 51, normalized size = 0.32

$$\frac{b^3x^9 + 6\,ab^2x^6 + 18\,a^2bx^3\log\left(x\right) - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

GIAC/XCAS [A] time = 0.27397, size = 115, normalized size = 0.71

$$\frac{1}{6}\,b^3x^6\mathrm{sign}\left(bx^3 + a\right) + ab^2x^3\mathrm{sign}\left(bx^3 + a\right) + 3\,a^2b\ln\left(|x|\right)\mathrm{sign}\left(bx^3 + a\right) - \frac{3\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + a^3\mathrm{sign}\left(bx^3 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sign(b*x^3 + a) + a*b^2*x^3*sign(b*x^3 + a) + 3*a^2*b
*ln(abs(x))*sign(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sign(b*x^3 + a) +
a^3*sign(b*x^3 + a))/x^3
```

3.37 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5} dx$

Optimal. Leaf size=165

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*
x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rubi [A] time = 0.113774, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*
x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rubi in Sympy [A] time = 16.3367, size = 139, normalized size = 0.84

$$\frac{81ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{4x^4}+\frac{27b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{10}-\frac{5\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
```

```
[Out] 81*a*b**2*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(20*(a + b*x**
3)) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(4*x**
```

```
4) + 27*b**2*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/10 - 5*(a**
2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(2*x**4)
```

Mathematica [A] time = 0.0339256, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\left(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9\right)}{20x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^
3*x^9))/(20*x^4*(a + b*x^3))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$-\frac{-4\,b^3x^9-30\,ax^6b^2+60\,x^3a^2b+5\,a^3}{20\,x^4\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x)
```

```
[Out] -1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^
(3/2)/x^4/(b*x^3+a)^3
```

Maxima [A] time = 0.803365, size = 50, normalized size = 0.3

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Fricas [A] time = 0.26626, size = 50, normalized size = 0.3

$$\frac{4\,b^3x^9 + 30\,ab^2x^6 - 60\,a^2bx^3 - 5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

GIAC/XCAS [A] time = 0.275357, size = 93, normalized size = 0.56

$$\frac{1}{5}\,b^3x^5\mathrm{sign}\left(bx^3 + a\right) + \frac{3}{2}\,ab^2x^2\mathrm{sign}\left(bx^3 + a\right) - \frac{12\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + a^3\mathrm{sign}\left(bx^3 + a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sign(b*x^3 + a) + 3/2*a*b^2*x^2*sign(b*x^3 + a) - 1/4
*(12*a^2*b*x^3*sign(b*x^3 + a) + a^3*sign(b*x^3 + a))/x^4
```

3.38 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*
b^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi [A] time = 0.112273, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*
b^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi in Sympy [A] time = 11.524, size = 136, normalized size = 0.83

$$\frac{81ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{20(a+bx^3)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{10x^5}+\frac{27b^2x\sqrt{a^2+2abx^3+b^2x^6}}{20}-\frac{11\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
```

```
[Out] 81*a*b**2*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(20*(a + b*x**3))
 + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(10*x**5)
```

```
 + 27*b**2*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/20 - 11*(a**2 +
2*a*b*x**3 + b**2*x**6)**(3/2)/(10*x**5)
```

Mathematica [A] time = 0.0334181, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\left(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9\right)}{20x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^
3*x^9))/(20*x^5*(a + b*x^3))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$-\frac{-5\,b^3x^9-60\,ax^6b^2+30\,x^3a^2b+4\,a^3}{20\,x^5\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^
(3/2)/x^5/(b*x^3+a)^3
```

Maxima [A] time = 0.790617, size = 50, normalized size = 0.31

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Fricas [A] time = 0.263934, size = 50, normalized size = 0.31

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

GIAC/XCAS [A] time = 0.29151, size = 92, normalized size = 0.56

$$\frac{1}{4}\,b^3x^4\mathrm{sign}\left(bx^3+a\right)+3\,ab^2x\mathrm{sign}\left(bx^3+a\right)-\frac{15\,a^2bx^3\mathrm{sign}\left(bx^3+a\right)+2\,a^3\mathrm{sign}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sign(b*x^3 + a) + 3*a*b^2*x*sign(b*x^3 + a) - 1/10*(1
5*a^2*b*x^3*sign(b*x^3 + a) + 2*a^3*sign(b*x^3 + a))/x^5
```

3.39 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=162

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (a^2
*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.125499, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (a^2
*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 17.0262, size = 128, normalized size = 0.79

$$\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\log{(x)}}{a+bx^3}+\frac{a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{2x^6}+b^2\sqrt{a^2+2abx^3+b^2x^6}-\frac{2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
```

```
[Out] 3*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**3)
+ a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(2*x**6) + b
**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6) - 2*(a**2 + 2*a*b*x**3 +
b**2*x**6)**(3/2)/(3*x**6)
```

Mathematica [A] time = 0.0278203, size = 61, normalized size = 0.38

$$-\frac{\sqrt{(a+bx^3)^2}\,\left(a^3+6a^2bx^3-18ab^2x^6\log(x)-2b^3x^9\right)}{6x^6\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a^3 + 6*a^2*b*x^3 - 2*b^3*x^9 - 18*a*b^2*x
^6*Log[x]))/(6*x^6*(a + b*x^3))
```

Maple [A] time = 0.018, size = 60, normalized size = 0.4

$$\frac{2\,b^3x^9+18\,ab^2\ln(x)\,x^6-6\,x^3a^2b-a^3}{6\,(bx^3+a)^3\,x^6}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+18*a*b^2*ln(x)*x^6-6*x^3*a^2*b
-a^3)/(b*x^3+a)^3/x^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290712, size = 53, normalized size = 0.33

$$\frac{2\,b^{3}x^{9} + 18\,ab^{2}x^{6}\log\left(x\right) - 6\,a^{2}bx^{3} - a^{3}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

GIAC/XCAS [A] time = 0.272545, size = 116, normalized size = 0.72

$$\frac{1}{3}\,b^{3}x^{3}\mathrm{sign}\left(bx^{3} + a\right) + 3\,ab^{2}\ln\left(|x|\right)\mathrm{sign}\left(bx^{3} + a\right) - \frac{9\,ab^{2}x^{6}\mathrm{sign}\left(bx^{3} + a\right) + 6\,a^{2}bx^{3}\mathrm{sign}\left(bx^{3} + a\right) + a^{3}\mathrm{sign}\left(bx^{3} + a\right)}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sign(b*x^3 + a) + 3*a*b^2*ln(abs(x))*sign(b*x^3 + a)
- 1/6*(9*a*b^2*x^6*sign(b*x^3 + a) + 6*a^2*b*x^3*sign(b*x^3 + a)
+ a^3*sign(b*x^3 + a))/x^6
```

3.40 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=165

$$-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rubi [A] time = 0.115792, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.0312, size = 134, normalized size = 0.81

$$-\frac{81ab^2\sqrt{a^2+2abx^3+b^2x^6}}{28x(a+bx^3)}+\frac{9a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{28x^7}+\frac{27b^2\sqrt{a^2+2abx^3+b^2x^6}}{28x}-\frac{13(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
```

```
[Out] -81*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(28*x*(a + b*x**3)
) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(28*x**7
```

```
) + 27*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(28*x) - 13*(a**2
 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(28*x**7)
```

Mathematica [A] time = 0.0228199, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\left(4a^3+21a^2bx^3+84ab^2x^6-14b^3x^9\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b
^3*x^9))/(28*x^7*(a + b*x^3))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$-\frac{-14\,b^3x^9+84\,ax^6b^2+21\,x^3a^2b+4\,a^3}{28\,x^7\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)
^(3/2)/x^7/(b*x^3+a)^3
```

Maxima [A] time = 0.775011, size = 50, normalized size = 0.3

$$\frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Fricas [A] time = 0.257037, size = 50, normalized size = 0.3

$$\frac{14\,b^3x^9 - 84\,ab^2x^6 - 21\,a^2bx^3 - 4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

GIAC/XCAS [A] time = 0.27689, size = 95, normalized size = 0.58

$$\frac{1}{2}\,b^3x^2\mathrm{sign}\left(bx^3 + a\right) - \frac{84\,ab^2x^6\mathrm{sign}\left(bx^3 + a\right) + 21\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + 4\,a^3\mathrm{sign}\left(bx^3 + a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sign(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sign(b*x^3 + a)
+ 21*a^2*b*x^3*sign(b*x^3 + a) + 4*a^3*sign(b*x^3 + a))/x^7
```

3.41 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=162

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi [A] time = 0.110234, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi in Sympy [A] time = 16.9236, size = 138, normalized size = 0.85

$$-\frac{81ab^2\sqrt{a^2+2abx^3+b^2x^6}}{40x^2(a+bx^3)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{40x^8}+\frac{27b^2\sqrt{a^2+2abx^3+b^2x^6}}{20x^2}-\frac{7\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{20x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
```

```
[Out] -81*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(40*x**2*(a + b*x*
*3)) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(40*x
```

```
**8) + 27*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(20*x**2) - 7*
(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(20*x**8)
```

Mathematica [A] time = 0.0243241, size = 61, normalized size = 0.38

$$-\frac{\sqrt{(a+bx^3)^2}\left(5a^3+24a^2bx^3+60ab^2x^6-40b^3x^9\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b
^3*x^9))/(40*x^8*(a + b*x^3))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$-\frac{-40\,b^3x^9+60\,ax^6b^2+24\,x^3a^2b+5\,a^3}{40\,x^8\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)
^(3/2)/x^8/(b*x^3+a)^3
```

Maxima [A] time = 0.777922, size = 50, normalized size = 0.31

$$\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Fricas [A] time = 0.255093, size = 50, normalized size = 0.31

$$\frac{40\,b^3x^9 - 60\,ab^2x^6 - 24\,a^2bx^3 - 5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**9, x)
```

GIAC/XCAS [A] time = 0.295689, size = 90, normalized size = 0.56

$$b^3x\mathrm{sign}\left(bx^3 + a\right) - \frac{60\,ab^2x^6\mathrm{sign}\left(bx^3 + a\right) + 24\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + 5\,a^3\mathrm{sign}\left(bx^3 + a\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] b^3*x*sign(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sign(b*x^3 + a) + 24*a
^2*b*x^3*sign(b*x^3 + a) + 5*a^3*sign(b*x^3 + a))/x^8
```

3.42 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=161

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (a^2
*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.124858, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (a^2
*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 22.4744, size = 138, normalized size = 0.86

$$-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6x^9}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\log\left(x\right)}{a+bx^3}-\frac{5\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

```
[Out] -a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(3*x**3*(a + b*x**3))
 + a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(6*x**9) +
```

```
b**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**3) - 5*
(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(18*x**9)
```

Mathematica [A] time = 0.0425568, size = 63, normalized size = 0.39

$$-\frac{\sqrt{(a+bx^3)^2}\left(a\left(2a^2+9abx^3+18b^2x^6\right)-18b^3x^9\log(x)\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a*(2*a^2 + 9*a*b*x^3 + 18*b^2*x^6) - 18*b^
3*x^9*Log[x]))/(18*x^9*(a + b*x^3))
```

Maple [A] time = 0.018, size = 60, normalized size = 0.4

$$\frac{18\,b^3\ln(x)\,x^9-18\,ax^6b^2-9\,x^3a^2b-2\,a^3}{18\,\left(bx^3+a\right)^3x^9}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(18*b^3*ln(x)*x^9-18*a*x^6*b^2-9*x^3*a^2
*b-2*a^3)/(b*x^3+a)^3/x^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256578, size = 53, normalized size = 0.33

$$\frac{18\,b^3x^9\log\left(x\right) - 18\,ab^2x^6 - 9\,a^2bx^3 - 2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

GIAC/XCAS [A] time = 0.269732, size = 115, normalized size = 0.71

$$b^3\ln\left(|x|\right)\mathrm{sign}\left(bx^3 + a\right) - \frac{11\,b^3x^9\mathrm{sign}\left(bx^3 + a\right) + 18\,ab^2x^6\mathrm{sign}\left(bx^3 + a\right) + 9\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + 2\,a^3\mathrm{sign}\left(bx^3 + a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] b^3*ln(abs(x))*sign(b*x^3 + a) - 1/18*(11*b^3*x^9*sign(b*x^3 + a)
 + 18*a*b^2*x^6*sign(b*x^3 + a) + 9*a^2*b*x^3*sign(b*x^3 + a) + 2
*a^3*sign(b*x^3 + a))/x^9
```

3.43 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=165

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*
a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rubi [A] time = 0.114287, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*
a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rubi in Sympy [A] time = 16.9982, size = 138, normalized size = 0.84

$$\frac{81ab^2\sqrt{a^2+2abx^3+b^2x^6}}{140x^4(a+bx^3)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{70x^{10}}-\frac{27b^2\sqrt{a^2+2abx^3+b^2x^6}}{35x^4}-\frac{8\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{35x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

```
[Out] 81*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(140*x**4*(a + b*x*
*3)) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(70*x
```

```
**10) - 27*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(35*x**4) - 8
*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(35*x**10)
```

Mathematica [A] time = 0.0271454, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(14a^3+60a^2bx^3+105ab^2x^6+140b^3x^9)}{140x^{10}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 14
0*b^3*x^9))/(140*x^10*(a + b*x^3))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{140\,b^3x^9+105\,ax^6b^2+60\,x^3a^2b+14\,a^3}{140\,x^{10}\,(bx^3+a)^3}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/140*(140*b^3*x^9+105*a*b^2*x^6+60*a^2*b*x^3+14*a^3)*((b*x^3+a)
^2)^(3/2)/x^10/(b*x^3+a)^3
```

Maxima [A] time = 0.757781, size = 50, normalized size = 0.3

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Fricas [A] time = 0.280252, size = 50, normalized size = 0.3

$$-\frac{140\,b^3x^9 + 105\,ab^2x^6 + 60\,a^2bx^3 + 14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

GIAC/XCAS [A] time = 0.277043, size = 93, normalized size = 0.56

$$-\frac{140\,b^3x^9\mathrm{sign}\left(bx^3 + a\right) + 105\,ab^2x^6\mathrm{sign}\left(bx^3 + a\right) + 60\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + 14\,a^3\mathrm{sign}\left(bx^3 + a\right)}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/140*(140*b^3*x^9*sign(b*x^3 + a) + 105*a*b^2*x^6*sign(b*x^3 +
a) + 60*a^2*b*x^3*sign(b*x^3 + a) + 14*a^3*sign(b*x^3 + a))/x^10
```

3.44 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*
a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rubi [A] time = 0.114206, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*
a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.0504, size = 138, normalized size = 0.83

$$\frac{81ab^2\sqrt{a^2+2abx^3+b^2x^6}}{440x^5\left(a+bx^3\right)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{88x^{11}}-\frac{27b^2\sqrt{a^2+2abx^3+b^2x^6}}{88x^5}-\frac{17\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
```

```
[Out] 81*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(440*x**5*(a + b*x*
*3)) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(88*x
```

```
**11) - 27*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(88*x**5) - 1
7*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(88*x**11)
```

Mathematica [A] time = 0.0313977, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\left(40a^3+165a^2bx^3+264ab^2x^6+220b^3x^9\right)}{440x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 2
20*b^3*x^9))/(440*x^11*(a + b*x^3))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{220\,b^3x^9+264\,ax^6b^2+165\,x^3a^2b+40\,a^3}{440\,x^{11}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x)
```

```
[Out] -1/440*(220*b^3*x^9+264*a*b^2*x^6+165*a^2*b*x^3+40*a^3)*((b*x^3+a
)^2)^(3/2)/x^11/(b*x^3+a)^3
```

Maxima [A] time = 0.758175, size = 50, normalized size = 0.3

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^1
1
```

Fricas [A] time = 0.261998, size = 50, normalized size = 0.3

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^1
1
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```

GIAC/XCAS [A] time = 0.259786, size = 93, normalized size = 0.56

$$-\frac{220\,b^3x^9\mathrm{sign}\left(bx^3+a\right)+264\,ab^2x^6\mathrm{sign}\left(bx^3+a\right)+165\,a^2bx^3\mathrm{sign}\left(bx^3+a\right)+40\,a^3\mathrm{sign}\left(bx^3+a\right)}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/440*(220*b^3*x^9*sign(b*x^3 + a) + 264*a*b^2*x^6*sign(b*x^3 +
a) + 165*a^2*b*x^3*sign(b*x^3 + a) + 40*a^3*sign(b*x^3 + a))/x^11
```

3.45 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*a*x^12)
```

Rubi [A] time = 0.0561068, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*a*x^12)
```

Rubi in Sympy [A] time = 8.42827, size = 39, normalized size = 0.95

$$-\frac{\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{24ax^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

```
[Out] -(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(24*a*x*
*12)
```

Mathematica [A] time = 0.0263874, size = 59, normalized size = 1.44

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^3+4a^2bx^3+6ab^2x^6+4b^3x^9\right)}{12x^{12}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^
9))/(12*x^12*(a + b*x^3))
```

Maple [A] time = 0.011, size = 56, normalized size = 1.4

$$-\frac{4\,b^3x^9+6\,ax^6b^2+4\,x^3a^2b+a^3}{12\,x^{12}\,(bx^3+a)^3}\left((bx^3+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/12*(4*b^3*x^9+6*a*b^2*x^6+4*a^2*b*x^3+a^3)*((b*x^3+a)^2)^(3/2)
/x^12/(b*x^3+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264754, size = 47, normalized size = 1.15

$$-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

GIAC/XCAS [A] time = 0.318631, size = 92, normalized size = 2.24

$$-\frac{4\, b^{3} x^{9} \operatorname{sign}\left(bx^{3} + a\right) + 6\, ab^{2} x^{6} \operatorname{sign}\left(bx^{3} + a\right) + 4\, a^{2} bx^{3} \operatorname{sign}\left(bx^{3} + a\right) + a^{3} \operatorname{sign}\left(bx^{3} + a\right)}{12\, x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^3*x^9*sign(b*x^3 + a) + 6*a*b^2*x^6*sign(b*x^3 + a) +
4*a^2*b*x^3*sign(b*x^3 + a) + a^3*sign(b*x^3 + a))/x^12
```

3.46 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rubi [A] time = 0.115019, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rubi in Sympy [A] time = 16.9713, size = 138, normalized size = 0.83

$$\frac{81ab^2\sqrt{a^2+2abx^3+b^2x^6}}{1820x^7\left(a+bx^3\right)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{130x^{13}}-\frac{27b^2\sqrt{a^2+2abx^3+b^2x^6}}{260x^7}-\frac{19\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{130x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
```

```
[Out] 81*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(1820*x**7*(a + b*x
**3)) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(130
```

```
*x**13) - 27*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(260*x**7)
- 19*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(130*x**13)
```

Mathematica [A] time = 0.0239923, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\left(140a^3+546a^2bx^3+780ab^2x^6+455b^3x^9\right)}{1820x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 +
455*b^3*x^9))/(1820*x^13*(a + b*x^3))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{455\,b^3x^9+780\,ax^6b^2+546\,x^3a^2b+140\,a^3}{1820\,x^{13}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/1820*(455*b^3*x^9+780*a*b^2*x^6+546*a^2*b*x^3+140*a^3)*((b*x^3
+a)^2)^(3/2)/x^13/(b*x^3+a)^3
```

Maxima [A] time = 0.810479, size = 50, normalized size = 0.3

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x
^13
```

Fricas [A] time = 0.266836, size = 50, normalized size = 0.3

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x
^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```

GIAC/XCAS [A] time = 0.282329, size = 93, normalized size = 0.56

$$-\frac{455\,b^3x^9\mathrm{sign}\left(bx^3+a\right)+780\,ab^2x^6\mathrm{sign}\left(bx^3+a\right)+546\,a^2bx^3\mathrm{sign}\left(bx^3+a\right)+140\,a^3\mathrm{sign}\left(bx^3+a\right)}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1820*(455*b^3*x^9*sign(b*x^3 + a) + 780*a*b^2*x^6*sign(b*x^3 +
 a) + 546*a^2*b*x^3*sign(b*x^3 + a) + 140*a^3*sign(b*x^3 + a))/x^
13
```

3.47 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rubi [A] time = 0.11513, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rubi in Sympy [A] time = 17.0988, size = 138, normalized size = 0.83

$$\frac{81ab^2\sqrt{a^2+2abx^3+b^2x^6}}{3080x^8(a+bx^3)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{154x^{14}}-\frac{27b^2\sqrt{a^2+2abx^3+b^2x^6}}{385x^8}-\frac{10\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{77x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
```

```
[Out] 81*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(3080*x**8*(a + b*x
**3)) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(154
```

```
*x**14) - 27*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(385*x**8)
- 10*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(77*x**14)
```

Mathematica [A] time = 0.0335502, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\left(220a^3+840a^2bx^3+1155ab^2x^6+616b^3x^9\right)}{3080x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 +
 616*b^3*x^9))/(3080*x^14*(a + b*x^3))
```

Maple [A] time = 0.011, size = 58, normalized size = 0.4

$$-\frac{616\,b^3x^9+1155\,ax^6b^2+840\,x^3a^2b+220\,a^3}{3080\,x^{14}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/3080*(616*b^3*x^9+1155*a*b^2*x^6+840*a^2*b*x^3+220*a^3)*((b*x^
3+a)^2)^(3/2)/x^14/(b*x^3+a)^3
```

Maxima [A] time = 0.833543, size = 50, normalized size = 0.3

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/
x^14
```

Fricas [A] time = 0.263617, size = 50, normalized size = 0.3

$$-\frac{616\,b^3x^9 + 1155\,ab^2x^6 + 840\,a^2bx^3 + 220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/
x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

GIAC/XCAS [A] time = 0.275275, size = 93, normalized size = 0.56

$$-\frac{616\,b^3x^9\mathrm{sign}\left(bx^3 + a\right) + 1155\,ab^2x^6\mathrm{sign}\left(bx^3 + a\right) + 840\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + 220\,a^3\mathrm{sign}\left(bx^3 + a\right)}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/3080*(616*b^3*x^9*sign(b*x^3 + a) + 1155*a*b^2*x^6*sign(b*x^3
+ a) + 840*a^2*b*x^3*sign(b*x^3 + a) + 220*a^3*sign(b*x^3 + a))/x
^14
```

3.48 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*a*x^15) + (b
*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rubi [A] time = 0.107785, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*a*x^15) + (b
*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rubi in Sympy [A] time = 8.70683, size = 68, normalized size = 0.81

$$-\frac{\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{24ax^{15}}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{60a^2x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

```
[Out] -(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(24*a*x*
*15) + (a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(60*a**2*x**15)
```

Mathematica [A] time = 0.025709, size = 61, normalized size = 0.73

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a^3+15a^2bx^3+20ab^2x^6+10b^3x^9\right)}{60x^{15}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b
^3*x^9))/(60*x^15*(a + b*x^3))
```

Maple [A] time = 0.011, size = 58, normalized size = 0.7

$$-\frac{10\,b^{3}x^{9}+20\,ax^{6}b^{2}+15\,x^{3}a^{2}b+4\,a^{3}}{60\,x^{15}\left(bx^{3}+a\right)^{3}}\left(\left(bx^{3}+a\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/60*(10*b^3*x^9+20*a*b^2*x^6+15*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^
(3/2)/x^15/(b*x^3+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264416, size = 50, normalized size = 0.6

$$-\frac{10\,b^{3}x^{9}+20\,ab^{2}x^{6}+15\,a^{2}bx^{3}+4\,a^{3}}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

GIAC/XCAS [A] time = 0.266375, size = 93, normalized size = 1.11

$$-\frac{10\,b^3x^9\text{sign}\left(bx^3+a\right)+20\,ab^2x^6\text{sign}\left(bx^3+a\right)+15\,a^2bx^3\text{sign}\left(bx^3+a\right)+4\,a^3\text{sign}\left(bx^3+a\right)}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*x^9*sign(b*x^3 + a) + 20*a*b^2*x^6*sign(b*x^3 + a)
+ 15*a^2*b*x^3*sign(b*x^3 + a) + 4*a^3*sign(b*x^3 + a))/x^15
```

3.49 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) -
(b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi [A] time = 0.114646, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) -
(b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi in Sympy [A] time = 16.9918, size = 138, normalized size = 0.83

$$\frac{81ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7280x^{10}\left(a+bx^3\right)}+\frac{9a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{208x^{16}}-\frac{27b^2\sqrt{a^2+2abx^3+b^2x^6}}{728x^{10}}-\frac{11\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{104x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

```
[Out] 81*a*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(7280*x**10*(a + b*
x**3)) + 9*a*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(20
```

```
8*x**16) - 27*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(728*x**10
) - 11*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(104*x**16)
```

Mathematica [A] time = 0.0252863, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\left(455a^3+1680a^2bx^3+2184ab^2x^6+1040b^3x^9\right)}{7280x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6
+ 1040*b^3*x^9))/(7280*x^16*(a + b*x^3))
```

Maple [A] time = 0.011, size = 58, normalized size = 0.4

$$-\frac{1040\,b^3x^9+2184\,ax^6b^2+1680\,x^3a^2b+455\,a^3}{7280\,x^{16}\left(bx^3+a\right)^3}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x)
```

```
[Out] -1/7280*(1040*b^3*x^9+2184*a*b^2*x^6+1680*a^2*b*x^3+455*a^3)*((b*
x^3+a)^2)^(3/2)/x^16/(b*x^3+a)^3
```

Maxima [A] time = 0.770807, size = 50, normalized size = 0.3

$$-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3
)/x^16
```

Fricas [A] time = 0.265585, size = 50, normalized size = 0.3

$$-\frac{1040\,b^3x^9 + 2184\,ab^2x^6 + 1680\,a^2bx^3 + 455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3
)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

GIAC/XCAS [A] time = 0.287192, size = 93, normalized size = 0.56

$$-\frac{1040\,b^3x^9\mathrm{sign}\left(bx^3 + a\right) + 2184\,ab^2x^6\mathrm{sign}\left(bx^3 + a\right) + 1680\,a^2bx^3\mathrm{sign}\left(bx^3 + a\right) + 455\,a^3\mathrm{sign}\left(bx^3 + a\right)}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/7280*(1040*b^3*x^9*sign(b*x^3 + a) + 2184*a*b^2*x^6*sign(b*x^3
 + a) + 1680*a^2*b*x^3*sign(b*x^3 + a) + 455*a^3*sign(b*x^3 + a))
/x^16
```

3.50 $\int x^{13} \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29(a+bx^3)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26(a+bx^3)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23(a+bx^3)}+\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*
a^4*b*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a
^3*b^2*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (1
0*a^2*b^3*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3))
+ (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*(a + b*x^3))
 + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*(a + b*x^3))
```

Rubi [A] time = 0.17194, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29(a+bx^3)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26(a+bx^3)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23(a+bx^3)}+\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*
a^4*b*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a
^3*b^2*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (1
0*a^2*b^3*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3))
+ (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*(a + b*x^3))
 + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.2469, size = 207, normalized size = 0.81

$$\frac{729a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{2063698(a+bx^3)}+\frac{243a^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{147407}+\frac{81a^3x^{14}\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{17342}+\frac{90a^2x^{14}\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{8671}+\frac{15ax^{14}\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{754}+\frac{x^{14}\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**14*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(2063698*(a +
b*x**3)) + 243*a**4*x**14*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/147
407 + 81*a**3*x**14*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x*
*6)/17342 + 90*a**2*x**14*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/
8671 + 15*a*x**14*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(
3/2)/754 + x**14*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/29
```

Mathematica [A] time = 0.043758, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{(a+bx^3)^2}\left(147407a^5+606970a^4bx^3+1031849a^3b^2x^6+897260a^2b^3x^9+396865ab^4x^{12}+71162b^5x^{15}\right)}{2063698\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 103184
9*a^3*b^2*x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^
5*x^15))/(2063698*(a + b*x^3))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$\frac{x^{14}\left(71162\,b^5x^{15}+396865\,ab^4x^{12}+897260\,a^2b^3x^9+1031849\,a^3b^2x^6+606970\,a^4bx^3+147407\,a^5\right)}{2063698\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2063698*x^14*(71162*b^5*x^15+396865*a*b^4*x^12+897260*a^2*b^3*x
^9+1031849*a^3*b^2*x^6+606970*a^4*b*x^3+147407*a^5)*((b*x^3+a)^2)
^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 0.776602, size = 77, normalized size = 0.3

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^13,x, algorithm="maxima")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^
2*x^20 + 5/17*a^4*b*x^17 + 1/14*a^5*x^14
```

Fricas [A] time = 0.262242, size = 77, normalized size = 0.3

$$\frac{1}{29}\,b^5x^{29} + \frac{5}{26}\,ab^4x^{26} + \frac{10}{23}\,a^2b^3x^{23} + \frac{1}{2}\,a^3b^2x^{20} + \frac{5}{17}\,a^4bx^{17} + \frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^13,x, algorithm="fricas")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^
2*x^20 + 5/17*a^4*b*x^17 + 1/14*a^5*x^14
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267532, size = 142, normalized size = 0.56

$$\frac{1}{29}\,b^5x^{29}\mathrm{sign}\left(bx^3+a\right) + \frac{5}{26}\,ab^4x^{26}\mathrm{sign}\left(bx^3+a\right) + \frac{10}{23}\,a^2b^3x^{23}\mathrm{sign}\left(bx^3+a\right) + \frac{1}{2}\,a^3b^2x^{20}\mathrm{sign}\left(bx^3+a\right) + \frac{5}{17}\,a^4bx^{17}\mathrm{sign}\left(bx^3+a\right) + \frac{1}{14}\,a^5x^{14}\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^13,x, algorithm="giac")
```

```
[Out] 1/29*b^5*x^29*sign(b*x^3 + a) + 5/26*a*b^4*x^26*sign(b*x^3 + a) +
 10/23*a^2*b^3*x^23*sign(b*x^3 + a) + 1/2*a^3*b^2*x^20*sign(b*x^3
 + a) + 5/17*a^4*b*x^17*sign(b*x^3 + a) + 1/14*a^5*x^14*sign(b*x^
3 + a)
```

3.51 $\int x^{12} \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*
a^4*b*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (1
0*a^3*b^2*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))
+ (5*a^2*b^3*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3
)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
 + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*(a + b*x^3))
```

Rubi [A] time = 0.166108, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*
a^4*b*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (1
0*a^3*b^2*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))
+ (5*a^2*b^3*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3
)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
 + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.1422, size = 207, normalized size = 0.81

$$\frac{729a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{1521520\left(a+bx^3\right)}+\frac{243a^4x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{117040}+\frac{81a^3x^{13}\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{14630}+\frac{9a^2x^{13}\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{770}+\frac{3ax^{13}\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{140}+\frac{x^{13}\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**13*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(1521520*(a +
b*x**3)) + 243*a**4*x**13*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/117
040 + 81*a**3*x**13*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x*
*6)/14630 + 9*a**2*x**13*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/7
70 + 3*a*x**13*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2
)/140 + x**13*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/28
```

Mathematica [A] time = 0.0365222, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{(a+bx^3)^2}\left(117040a^5+475475a^4bx^3+800800a^3b^2x^6+691600a^2b^3x^9+304304ab^4x^{12}+54340b^5x^{15}\right)}{1521520\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800
*a^3*b^2*x^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5
*x^15))/(1521520*(a + b*x^3))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$\frac{x^{13}\left(54340\,b^5x^{15}+304304\,ab^4x^{12}+691600\,a^2b^3x^9+800800\,a^3b^2x^6+475475\,a^4bx^3+117040\,a^5\right)}{1521520\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1521520*x^13*(54340*b^5*x^15+304304*a*b^4*x^12+691600*a^2*b^3*x
^9+800800*a^3*b^2*x^6+475475*a^4*b*x^3+117040*a^5)*((b*x^3+a)^2)^
(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 0.780853, size = 77, normalized size = 0.3

$$\frac{1}{28}\,b^5x^{28}+\frac{1}{5}\,ab^4x^{25}+\frac{5}{11}\,a^2b^3x^{22}+\frac{10}{19}\,a^3b^2x^{19}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^12,x, algorithm="maxima")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^
2*x^19 + 5/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Fricas [A] time = 0.251931, size = 77, normalized size = 0.3

$$\frac{1}{28}\,b^5x^{28}+\frac{1}{5}\,ab^4x^{25}+\frac{5}{11}\,a^2b^3x^{22}+\frac{10}{19}\,a^3b^2x^{19}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^12,x, algorithm="fricas")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^
2*x^19 + 5/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{12}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.28566, size = 142, normalized size = 0.56

$$\frac{1}{28}\,b^5x^{28}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{5}\,ab^4x^{25}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{11}\,a^2b^3x^{22}\mathrm{sign}\left(bx^3+a\right)+\frac{10}{19}\,a^3b^2x^{19}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{16}\,a^4bx^{16}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{13}\,a^5x^{13}\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^12,x, algorithm="giac")
```

```
[Out] 1/28*b^5*x^28*sign(b*x^3 + a) + 1/5*a*b^4*x^25*sign(b*x^3 + a) +
5/11*a^2*b^3*x^22*sign(b*x^3 + a) + 10/19*a^3*b^2*x^19*sign(b*x^3
 + a) + 5/16*a^4*b*x^16*sign(b*x^3 + a) + 1/13*a^5*x^13*sign(b*x^
3 + a)
```

3.52 $\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^4}$$

```
[Out] -(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^4) + (
a^2*(a + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(
a + b*x^3)^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x
^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(27*b^4)
```

Rubi [A] time = 0.263847, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^4) + (
a^2*(a + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(
a + b*x^3)^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x
^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(27*b^4)
```

Rubi in Sympy [A] time = 29.9518, size = 151, normalized size = 0.94

$$-\frac{a^3\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{432b^4}+\frac{a^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{7}{2}}}{252b^4}-\frac{ax^6\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{144b^2}+\frac{x^9\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{54b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] -a**3*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(43
2*b**4) + a**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(7/2)/(252*b**4)
- a*x**6*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/
(144*b**2) + x**9*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6
)**(5/2)/(54*b)
```

Mathematica [A] time = 0.0403947, size = 83, normalized size = 0.52

$$\frac{x^{12}\sqrt{(a+bx^3)^2}\left(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15}\right)}{1512\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*
x^6 + 720*a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a +
 b*x^3))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.5

$$\frac{x^{12}\left(56\,b^5x^{15}+315\,ab^4x^{12}+720\,a^2b^3x^9+840\,a^3b^2x^6+504\,a^4bx^3+126\,a^5\right)}{1512\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1512*x^12*(56*b^5*x^15+315*a*b^4*x^12+720*a^2*b^3*x^9+840*a^3*b
^2*x^6+504*a^4*b*x^3+126*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256543, size = 77, normalized size = 0.48

$$\frac{1}{27}\,b^5x^{27}+\frac{5}{24}\,ab^4x^{24}+\frac{10}{21}\,a^2b^3x^{21}+\frac{5}{9}\,a^3b^2x^{18}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^11,x, algorithm="fricas")
```

```
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^
2*x^18 + 1/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.269772, size = 142, normalized size = 0.89

$$\frac{1}{27}\,b^5x^{27}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{24}\,ab^4x^{24}\mathrm{sign}\left(bx^3+a\right)+\frac{10}{21}\,a^2b^3x^{21}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^11,x, algorithm="giac")
```

```
[Out] 1/27*b^5*x^27*sign(b*x^3 + a) + 5/24*a*b^4*x^24*sign(b*x^3 + a) +
 10/21*a^2*b^3*x^21*sign(b*x^3 + a) + 5/9*a^3*b^2*x^18*sign(b*x^3
 + a) + 1/3*a^4*b*x^15*sign(b*x^3 + a) + 1/12*a^5*x^12*sign(b*x^3
 + a)
```

3.53 $\int x^{10} \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26(a+bx^3)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23(a+bx^3)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}$$

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*
a^4*b*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (1
0*a^3*b^2*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
+ (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
+ (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3))
 + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*(a + b*x^3))
```

Rubi [A] time = 0.170755, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26(a+bx^3)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23(a+bx^3)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*
a^4*b*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (1
0*a^3*b^2*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
+ (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
+ (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3))
 + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.1124, size = 207, normalized size = 0.81

$$\frac{729a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{782782(a+bx^3)}+\frac{243a^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{71162}+\frac{81a^3x^{11}\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{10166}+\frac{9a^2x^{11}\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{598}+\frac{15ax^{11}\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{598}+\frac{x^{11}\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**11*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(782782*(a + b
*x**3)) + 243*a**4*x**11*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/7116
2 + 81*a**3*x**11*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6
)/10166 + 9*a**2*x**11*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/598
 + 15*a*x**11*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)
/598 + x**11*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/26
```

Mathematica [A] time = 0.0370438, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{(a+bx^3)^2}\left(71162a^5+279565a^4bx^3+460460a^3b^2x^6+391391a^2b^3x^9+170170ab^4x^{12}+30107b^5x^{15}\right)}{782782\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*
a^3*b^2*x^6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*
x^15))/(782782*(a + b*x^3))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$\frac{x^{11}\left(30107\,b^5x^{15}+170170\,ab^4x^{12}+391391\,a^2b^3x^9+460460\,a^3b^2x^6+279565\,a^4bx^3+71162\,a^5\right)}{782782\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/782782*x^11*(30107*b^5*x^15+170170*a*b^4*x^12+391391*a^2*b^3*x^
9+460460*a^3*b^2*x^6+279565*a^4*b*x^3+71162*a^5)*((b*x^3+a)^2)^(5
/2)/(b*x^3+a)^5
```

Maxima [A] time = 0.775507, size = 77, normalized size = 0.3

$$\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^10,x, algorithm="maxima")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^
2*x^17 + 5/14*a^4*b*x^14 + 1/11*a^5*x^11
```

Fricas [A] time = 0.266822, size = 77, normalized size = 0.3

$$\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^10,x, algorithm="fricas")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^
2*x^17 + 5/14*a^4*b*x^14 + 1/11*a^5*x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{10}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.266949, size = 142, normalized size = 0.56

$$\frac{1}{26}\,b^5x^{26}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{23}\,ab^4x^{23}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sign}\left(bx^3+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^10,x, algorithm="giac")
```

```
[Out] 1/26*b^5*x^26*sign(b*x^3 + a) + 5/23*a*b^4*x^23*sign(b*x^3 + a) +
 1/2*a^2*b^3*x^20*sign(b*x^3 + a) + 10/17*a^3*b^2*x^17*sign(b*x^3
 + a) + 5/14*a^4*b*x^14*sign(b*x^3 + a) + 1/11*a^5*x^11*sign(b*x^
3 + a)
```

3.54 $\int x^9 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*
a^4*b*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5
*a^3*b^2*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) +
(10*a^2*b^3*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)
) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a + b*x^3
)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(25*(a + b*x^3))
```

Rubi [A] time = 0.169372, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*
a^4*b*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5
*a^3*b^2*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) +
(10*a^2*b^3*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)
) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a + b*x^3
)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(25*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.2422, size = 207, normalized size = 0.81

$$\frac{729a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{543400\left(a+bx^3\right)}+\frac{243a^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{54340}+\frac{81a^3x^{10}\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{8360}+\frac{18a^2x^{10}\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{1045}+\frac{3ax^{10}\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{110}+\frac{x^{10}\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**10*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(543400*(a + b
*x**3)) + 243*a**4*x**10*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/5434
0 + 81*a**3*x**10*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6
)/8360 + 18*a**2*x**10*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/104
5 + 3*a*x**10*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)
/110 + x**10*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/25
```

Mathematica [A] time = 0.0408829, size = 83, normalized size = 0.33

$$\frac{x^{10}\sqrt{(a+bx^3)^2}\left(54340a^5+209000a^4bx^3+339625a^3b^2x^6+286000a^2b^3x^9+123500ab^4x^{12}+21736b^5x^{15}\right)}{543400\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*
a^3*b^2*x^6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*
x^15))/(543400*(a + b*x^3))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$\frac{x^{10}\left(21736\,b^5x^{15}+123500\,ab^4x^{12}+286000\,a^2b^3x^9+339625\,a^3b^2x^6+209000\,a^4bx^3+54340\,a^5\right)}{543400\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^
9+339625*a^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5
/2)/(b*x^3+a)^5
```

Maxima [A] time = 0.788878, size = 77, normalized size = 0.3

$$\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^9,x, algorithm="maxima")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^
2*x^16 + 5/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Fricas [A] time = 0.274036, size = 77, normalized size = 0.3

$$\frac{1}{25}\,b^5x^{25} + \frac{5}{22}\,ab^4x^{22} + \frac{10}{19}\,a^2b^3x^{19} + \frac{5}{8}\,a^3b^2x^{16} + \frac{5}{13}\,a^4bx^{13} + \frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^9,x, algorithm="fricas")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^
2*x^16 + 5/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.286785, size = 142, normalized size = 0.56

$$\frac{1}{25}\,b^5x^{25}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{22}\,ab^4x^{22}\mathrm{sign}\left(bx^3 + a\right) + \frac{10}{19}\,a^2b^3x^{19}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{8}\,a^3b^2x^{16}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{13}\,a^4bx^{13}\mathrm{sign}\left(bx^3 + a\right) + \frac{1}{10}\,a^5x^{10}\mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^9,x, algorithm="giac")
```

```
[Out] 1/25*b^5*x^25*sign(b*x^3 + a) + 5/22*a*b^4*x^22*sign(b*x^3 + a) +
 10/19*a^2*b^3*x^19*sign(b*x^3 + a) + 5/8*a^3*b^2*x^16*sign(b*x^3
 + a) + 5/13*a^4*b*x^13*sign(b*x^3 + a) + 1/10*a^5*x^10*sign(b*x^
3 + a)
```

3.55 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}$$

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2
*a*(a + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a
+ b*x^3)^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rubi [A] time = 0.207968, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2
*a*(a + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a
+ b*x^3)^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rubi in Sympy [A] time = 21.9823, size = 107, normalized size = 0.9

$$\frac{a^2\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{144b^3}-\frac{a\left(a^2+2abx^3+b^2x^6\right)^{\frac{7}{2}}}{84b^3}+\frac{x^6\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] a**2*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(144
*b**3) - a*(a**2 + 2*a*b*x**3 + b**2*x**6)**(7/2)/(84*b**3) + x**
6*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(48*b)
```

Mathematica [A] time = 0.0367037, size = 83, normalized size = 0.7

$$\frac{x^9\sqrt{(a+bx^3)^2}\left(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15}\right)}{504\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^
6 + 280*a^2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*
x^3))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.7

$$\frac{x^9\left(21\,b^5x^{15}+120\,ab^4x^{12}+280\,a^2b^3x^9+336\,a^3b^2x^6+210\,a^4bx^3+56\,a^5\right)}{504\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/504*x^9*(21*b^5*x^15+120*a*b^4*x^12+280*a^2*b^3*x^9+336*a^3*b^2
*x^6+210*a^4*b*x^3+56*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250057, size = 77, normalized size = 0.65

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{21}\,ab^4x^{21}+\frac{5}{9}\,a^2b^3x^{18}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*
x^15 + 5/12*a^4*b*x^12 + 1/9*a^5*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.288693, size = 142, normalized size = 1.19

$$\frac{1}{24}\,b^5x^{24}\operatorname{sign}\left(bx^3+a\right)+\frac{5}{21}\,ab^4x^{21}\operatorname{sign}\left(bx^3+a\right)+\frac{5}{9}\,a^2b^3x^{18}\operatorname{sign}\left(bx^3+a\right)+\frac{2}{3}\,a^3b^2x^{15}\operatorname{sign}\left(bx^3+a\right)+\frac{5}{12}\,a^4bx^{12}\operatorname{sign}\left(bx^3+a\right)+\frac{1}{9}\,a^5x^9\operatorname{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^8,x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sign(b*x^3 + a) + 5/21*a*b^4*x^21*sign(b*x^3 + a) +
 5/9*a^2*b^3*x^18*sign(b*x^3 + a) + 2/3*a^3*b^2*x^15*sign(b*x^3 +
 a) + 5/12*a^4*b*x^12*sign(b*x^3 + a) + 1/9*a^5*x^9*sign(b*x^3 +
a)
```

3.56 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23(a+bx^3)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^
4*b*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a
^3*b^2*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (1
0*a^2*b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
+ (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) +
(b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3))
```

Rubi [A] time = 0.165533, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23(a+bx^3)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^
4*b*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a
^3*b^2*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (1
0*a^2*b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
+ (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) +
(b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.2127, size = 207, normalized size = 0.81

$$\frac{729a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{240856(a+bx^3)}+\frac{243a^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{30107}+\frac{81a^3x^8\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{5474}+\frac{9a^2x^8\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{391}+\frac{3ax^8\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{92}+\frac{x^8\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**8*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(240856*(a + b*
x**3)) + 243*a**4*x**8*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/30107
+ 81*a**3*x**8*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/5
474 + 9*a**2*x**8*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/391 + 3*
a*x**8*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/92 + x
**8*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/23
```

Mathematica [A] time = 0.038653, size = 83, normalized size = 0.33

$$\frac{x^8\sqrt{(a+bx^3)^2}\left(30107a^5+109480a^4bx^3+172040a^3b^2x^6+141680a^2b^3x^9+60214ab^4x^{12}+10472b^5x^{15}\right)}{240856\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a
^3*b^2*x^6 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^
15))/(240856*(a + b*x^3))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$\frac{x^8\left(10472\,b^5x^{15}+60214\,ab^4x^{12}+141680\,a^2b^3x^9+172040\,a^3b^2x^6+109480\,a^4bx^3+30107\,a^5\right)}{240856\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+
172040*a^3*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2
)/(b*x^3+a)^5
```

Maxima [A] time = 0.815834, size = 77, normalized size = 0.3

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^7,x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2
*x^14 + 5/11*a^4*b*x^11 + 1/8*a^5*x^8
```

Fricas [A] time = 0.254456, size = 77, normalized size = 0.3

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^7,x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2
*x^14 + 5/11*a^4*b*x^11 + 1/8*a^5*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.269834, size = 142, normalized size = 0.56

$$\frac{1}{23}\,b^5x^{23}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sign}\left(bx^3+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{8}\,a^5x^8\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^7,x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sign(b*x^3 + a) + 1/4*a*b^4*x^20*sign(b*x^3 + a) +
10/17*a^2*b^3*x^17*sign(b*x^3 + a) + 5/7*a^3*b^2*x^14*sign(b*x^3
+ a) + 5/11*a^4*b*x^11*sign(b*x^3 + a) + 1/8*a^5*x^8*sign(b*x^3 +
 a)
```

3.57 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22(a+bx^3)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19(a+bx^3)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}$$

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*
b*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3
*b^2*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*
a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (
5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) +
(b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a + b*x^3))
```

Rubi [A] time = 0.165741, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22(a+bx^3)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19(a+bx^3)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*
b*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3
*b^2*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*
a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (
5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) +
(b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a + b*x^3))
```

Rubi in Sympy [A] time = 28.1131, size = 207, normalized size = 0.81

$$\frac{729a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{152152(a+bx^3)}+\frac{243a^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{21736}+\frac{405a^3x^7\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{21736}+\frac{45a^2x^7\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{1672}+\frac{15ax^7\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{418}+\frac{x^7\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**7*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(152152*(a + b*
x**3)) + 243*a**4*x**7*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/21736
+ 405*a**3*x**7*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/
21736 + 45*a**2*x**7*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/1672
+ 15*a*x**7*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/4
18 + x**7*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/22
```

Mathematica [A] time = 0.0360826, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{(a+bx^3)^2}\left(21736a^5+76076a^4bx^3+117040a^3b^2x^6+95095a^2b^3x^9+40040ab^4x^{12}+6916b^5x^{15}\right)}{152152\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^
3*b^2*x^6 + 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15)
)/(152152*(a + b*x^3))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$\frac{x^7\left(6916\,b^5x^{15}+40040\,ab^4x^{12}+95095\,a^2b^3x^9+117040\,a^3b^2x^6+76076\,a^4bx^3+21736\,a^5\right)}{152152\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+11
7040*a^3*b^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(
b*x^3+a)^5
```

Maxima [A] time = 0.802327, size = 77, normalized size = 0.3

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^
2*x^13 + 1/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Fricas [A] time = 0.250694, size = 77, normalized size = 0.3

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^
2*x^13 + 1/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.300202, size = 142, normalized size = 0.56

$$\frac{1}{22}\,b^5x^{22}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sign}\left(bx^3+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{sign}\left(bx^3+a\right)+\frac{1}{7}\,a^5x^7\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^6,x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sign(b*x^3 + a) + 5/19*a*b^4*x^19*sign(b*x^3 + a) +
 5/8*a^2*b^3*x^16*sign(b*x^3 + a) + 10/13*a^3*b^2*x^13*sign(b*x^3
 + a) + 1/2*a^4*b*x^10*sign(b*x^3 + a) + 1/7*a^5*x^7*sign(b*x^3 +
 a)
```

3.58 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

```
[Out] -(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^2) + ((a
 + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rubi [A] time = 0.139693, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^2) + ((a
 + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rubi in Sympy [A] time = 14.7412, size = 65, normalized size = 0.83

$$-\frac{a\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{36b^2}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{7}{2}}}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] -a*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(36*b*
*2) + (a**2 + 2*a*b*x**3 + b**2*x**6)**(7/2)/(21*b**2)
```

Mathematica [A] time = 0.0361341, size = 83, normalized size = 1.06

$$\frac{x^6\sqrt{\left(a+bx^3\right)^2}\left(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15}\right)}{126\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6
 + 84*a^2*b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3)
)
```

Maple [A] time = 0.009, size = 80, normalized size = 1.

$$\frac{x^6\left(6\,b^5x^{15}+35\,ab^4x^{12}+84\,a^2b^3x^9+105\,a^3b^2x^6+70\,a^4bx^3+21\,a^5\right)}{126\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/126*x^6*(6*b^5*x^15+35*a*b^4*x^12+84*a^2*b^3*x^9+105*a^3*b^2*x^
6+70*a^4*b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249862, size = 77, normalized size = 0.99

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{18}\,ab^4x^{18}+\frac{2}{3}\,a^2b^3x^{15}+\frac{5}{6}\,a^3b^2x^{12}+\frac{5}{9}\,a^4bx^9+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*
x^12 + 5/9*a^4*b*x^9 + 1/6*a^5*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(\left(a + bx^3\right)^2 \right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.267288, size = 90, normalized size = 1.15

$$\frac{1}{126} \left(6\, b^5 x^{21} + 35\, ab^4 x^{18} + 84\, a^2 b^3 x^{15} + 105\, a^3 b^2 x^{12} + 70\, a^4 b x^9 + 21\, a^5 x^6\right) \operatorname{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2
*x^12 + 70*a^4*b*x^9 + 21*a^5*x^6)*sign(b*x^3 + a)
```

3.59 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20(a+bx^3)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}$$

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^
4*b*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^
3*b^2*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5
*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) +
(5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) +
 (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a + b*x^3))
```

Rubi [A] time = 0.163892, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20(a+bx^3)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^
4*b*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^
3*b^2*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5
*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) +
(5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) +
 (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.0897, size = 207, normalized size = 0.81

$$\frac{729a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{52360(a+bx^3)}+\frac{243a^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{10472}+\frac{81a^3x^5\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{2618}+\frac{9a^2x^5\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{238}+\frac{3ax^5\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{68}+\frac{x^5\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(52360*(a + b*x
**3)) + 243*a**4*x**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/10472 +
 81*a**3*x**5*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/26
18 + 9*a**2*x**5*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/238 + 3*a
*x**5*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/68 + x*
*5*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/20
```

Mathematica [A] time = 0.0369222, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{(a+bx^3)^2}\left(10472a^5+32725a^4bx^3+47600a^3b^2x^6+37400a^2b^3x^9+15400ab^4x^{12}+2618b^5x^{15}\right)}{52360\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3
*b^2*x^6 + 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))
/(52360*(a + b*x^3))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$\frac{x^5\left(2618\,b^5x^{15}+15400\,ab^4x^{12}+37400\,a^2b^3x^9+47600\,a^3b^2x^6+32725\,a^4bx^3+10472\,a^5\right)}{52360\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+476
00*a^3*b^2*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*
x^3+a)^5
```

Maxima [A] time = 0.822102, size = 77, normalized size = 0.3

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^
2*x^11 + 5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Fricas [A] time = 0.25404, size = 77, normalized size = 0.3

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{17}\,ab^4x^{17} + \frac{5}{7}\,a^2b^3x^{14} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{8}\,a^4bx^8 + \frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^
2*x^11 + 5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.265001, size = 142, normalized size = 0.56

$$\frac{1}{20}\,b^5x^{20}\text{sign}\left(bx^3+a\right) + \frac{5}{17}\,ab^4x^{17}\text{sign}\left(bx^3+a\right) + \frac{5}{7}\,a^2b^3x^{14}\text{sign}\left(bx^3+a\right) + \frac{10}{11}\,a^3b^2x^{11}\text{sign}\left(bx^3+a\right) + \frac{5}{8}\,a^4bx^8\text{sign}\left(bx^3+a\right) + \frac{1}{5}\,a^5x^5\text{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sign(b*x^3 + a) + 5/17*a*b^4*x^17*sign(b*x^3 + a) +
 5/7*a^2*b^3*x^14*sign(b*x^3 + a) + 10/11*a^3*b^2*x^11*sign(b*x^3
 + a) + 5/8*a^4*b*x^8*sign(b*x^3 + a) + 1/5*a^5*x^5*sign(b*x^3 +
a)
```

3.60 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=252

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^
4*b*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b
^2*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^
3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a*b
^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^5*
x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))
```

Rubi [A] time = 0.163034, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^
4*b*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b
^2*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^
3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a*b
^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^5*
x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))
```

Rubi in Sympy [A] time = 28.149, size = 207, normalized size = 0.82

$$\frac{729a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{27664\left(a+bx^3\right)}+\frac{243a^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{6916}+\frac{81a^3x^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{1976}+\frac{45a^2x^4\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{988}+\frac{15ax^4\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{304}+\frac{x^4\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(27664*(a + b*x
**3)) + 243*a**4*x**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/6916 +
81*a**3*x**4*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/197
6 + 45*a**2*x**4*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/988 + 15*
a*x**4*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/304 +
x**4*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/19
```

Mathematica [A] time = 0.0412161, size = 83, normalized size = 0.33

$$\frac{x^4\sqrt{(a+bx^3)^2}\left(6916a^5+19760a^4bx^3+27664a^3b^2x^6+21280a^2b^3x^9+8645ab^4x^{12}+1456b^5x^{15}\right)}{27664\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*
b^2*x^6 + 21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(
27664*(a + b*x^3))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$\frac{x^4\left(1456\,b^5x^{15}+8645\,ab^4x^{12}+21280\,a^2b^3x^9+27664\,a^3b^2x^6+19760\,a^4bx^3+6916\,a^5\right)}{27664\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+2766
4*a^3*b^2*x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^
3+a)^5
```

Maxima [A] time = 0.782088, size = 76, normalized size = 0.3

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^
10 + 5/7*a^4*b*x^7 + 1/4*a^5*x^4
```

Fricas [A] time = 0.256267, size = 76, normalized size = 0.3

$$\frac{1}{19}\,b^5x^{19} + \frac{5}{16}\,ab^4x^{16} + \frac{10}{13}\,a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7}\,a^4bx^7 + \frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^
10 + 5/7*a^4*b*x^7 + 1/4*a^5*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.288848, size = 140, normalized size = 0.56

$$\frac{1}{19}\,b^5x^{19}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{16}\,ab^4x^{16}\mathrm{sign}\left(bx^3 + a\right) + \frac{10}{13}\,a^2b^3x^{13}\mathrm{sign}\left(bx^3 + a\right) + a^3b^2x^{10}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{7}\,a^4bx^7\mathrm{sign}\left(bx^3 + a\right) + \frac{1}{4}\,a^5x^4\mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sign(b*x^3 + a) + 5/16*a*b^4*x^16*sign(b*x^3 + a) +
 10/13*a^2*b^3*x^13*sign(b*x^3 + a) + a^3*b^2*x^10*sign(b*x^3 + a
) + 5/7*a^4*b*x^7*sign(b*x^3 + a) + 1/4*a^5*x^4*sign(b*x^3 + a)
```

3.61 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rubi [A] time = 0.0688459, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rubi in Sympy [A] time = 9.21185, size = 34, normalized size = 0.94

$$\frac{\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] (2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(36*b)
```

Mathematica [B] time = 0.0356727, size = 82, normalized size = 2.28

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15}\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 +
 15*a^2*b^3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

Maple [B] time = 0.01, size = 79, normalized size = 2.2

$$\frac{x^{3}\left(b^{5}x^{15}+6\,ab^{4}x^{12}+15\,a^{2}b^{3}x^{9}+20\,a^{3}b^{2}x^{6}+15\,a^{4}bx^{3}+6\,a^{5}\right)}{18\,\left(bx^{3}+a\right)^{5}}\left(\left(bx^{3}+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/18*x^3*(b^5*x^15+6*a*b^4*x^12+15*a^2*b^3*x^9+20*a^3*b^2*x^6+15*
a^4*b*x^3+6*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267522, size = 77, normalized size = 2.14

$$\frac{1}{18}\,b^{5}x^{18}+\frac{1}{3}\,ab^{4}x^{15}+\frac{5}{6}\,a^{2}b^{3}x^{12}+\frac{10}{9}\,a^{3}b^{2}x^{9}+\frac{5}{6}\,a^{4}bx^{6}+\frac{1}{3}\,a^{5}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*
x^9 + 5/6*a^4*b*x^6 + 1/3*a^5*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.269938, size = 89, normalized size = 2.47

$$\frac{1}{18}\left(b^5x^{18}+6\,ab^4x^{15}+15\,a^2b^3x^{12}+20\,a^3b^2x^9+15\,a^4bx^6+6\,a^5x^3\right)\operatorname{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/18*(b^5*x^18 + 6*a*b^4*x^15 + 15*a^2*b^3*x^12 + 20*a^3*b^2*x^9
+ 15*a^4*b*x^6 + 6*a^5*x^3)*sign(b*x^3 + a)
```

3.62 $\int x \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=252

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*
b*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x
^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3
*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a*b^
4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^5*x
^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
```

Rubi [A] time = 0.148092, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*
b*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x
^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3
*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a*b^
4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^5*x
^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))
```

Rubi in Sympy [A] time = 24.4538, size = 207, normalized size = 0.82

$$\frac{729a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{5236\,(a+bx^3)}+\frac{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2618}+\frac{405a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{5236}+\frac{90a^2x^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{1309}+\frac{15ax^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{238}+\frac{x^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(5236*(a + b*x*
*3)) + 243*a**4*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/2618 + 4
05*a**3*x**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/523
6 + 90*a**2*x**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/1309 + 15
*a*x**2*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/238 +
 x**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/17
```

Mathematica [A] time = 0.0352532, size = 83, normalized size = 0.33

$$\frac{x^2\sqrt{(a+bx^3)^2}\left(2618a^5+5236a^4bx^3+6545a^3b^2x^6+4760a^2b^3x^9+1870ab^4x^{12}+308b^5x^{15}\right)}{5236\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^
2*x^6 + 4760*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236
*(a + b*x^3))
```

Maple [A] time = 0.008, size = 80, normalized size = 0.3

$$\frac{x^2\left(308\,b^5x^{15}+1870\,ab^4x^{12}+4760\,a^2b^3x^9+6545\,a^3b^2x^6+5236\,a^4bx^3+2618\,a^5\right)}{5236\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^
3*b^2*x^6+5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^
5
```

Maxima [A] time = 0.807452, size = 76, normalized size = 0.3

$$\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x,x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^
2*x^8 + a^4*b*x^5 + 1/2*a^5*x^2
```

Fricas [A] time = 0.263643, size = 76, normalized size = 0.3

$$\frac{1}{17}\,b^5x^{17} + \frac{5}{14}\,ab^4x^{14} + \frac{10}{11}\,a^2b^3x^{11} + \frac{5}{4}\,a^3b^2x^8 + a^4bx^5 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^
2*x^8 + a^4*b*x^5 + 1/2*a^5*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.277485, size = 140, normalized size = 0.56

$$\frac{1}{17}\,b^5x^{17}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{14}\,ab^4x^{14}\mathrm{sign}\left(bx^3 + a\right) + \frac{10}{11}\,a^2b^3x^{11}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{4}\,a^3b^2x^8\mathrm{sign}\left(bx^3 + a\right) + a^4bx^5\mathrm{sign}\left(bx^3 + a\right) + \frac{1}{2}\,a^5x^2\mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x,x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sign(b*x^3 + a) + 5/14*a*b^4*x^14*sign(b*x^3 + a) +
 10/11*a^2*b^3*x^11*sign(b*x^3 + a) + 5/4*a^3*b^2*x^8*sign(b*x^3
+ a) + a^4*b*x^5*sign(b*x^3 + a) + 1/2*a^5*x^2*sign(b*x^3 + a)
```

3.63 $\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{7\left(a+bx^3\right)^5}$$

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*
b*x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*
a^3*b^2*x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5)
+ (a^2*b^3*x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5
+ (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(13*(a + b*x^3
)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(16*(a + b*x^
3)^5)
```

Rubi [A] time = 0.12218, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{7\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*
b*x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*
a^3*b^2*x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5)
+ (a^2*b^3*x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5
+ (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(13*(a + b*x^3
)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(16*(a + b*x^
3)^5)
```

Rubi in Sympy [A] time = 9.86023, size = 197, normalized size = 0.8

$$\frac{729a^5x\sqrt{a^2+2abx^3+b^2x^6}}{1456\,(a+bx^3)}+\frac{243a^4x\sqrt{a^2+2abx^3+b^2x^6}}{1456}+\frac{81a^3x\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{728}+\frac{9a^2x\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{104}+\frac{15ax\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{208}+\frac{x\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 729*a**5*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(1456*(a + b*x**3)
) + 243*a**4*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/1456 + 81*a**3
*x*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/728 + 9*a**2*
x*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/104 + 15*a*x*(a + b*x**3
)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/208 + x*(a**2 + 2*a*b*x*
*3 + b**2*x**6)**(5/2)/16
```

Mathematica [A] time = 0.0367961, size = 81, normalized size = 0.33

$$\frac{x\sqrt{(a+bx^3)^2}\left(1456a^5+1820a^4bx^3+2080a^3b^2x^6+1456a^2b^3x^9+560ab^4x^{12}+91b^5x^{15}\right)}{1456\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*
x^6 + 1456*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a
+ b*x^3))
```

Maple [A] time = 0.006, size = 78, normalized size = 0.3

$$\frac{x\left(91\,b^5x^{15}+560\,ab^4x^{12}+1456\,a^2b^3x^9+2080\,a^3b^2x^6+1820\,a^4bx^3+1456\,a^5\right)}{1456\,\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^
2*x^6+1820*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A] time = 0.811495, size = 72, normalized size = 0.29

$$\frac{1}{16}\,b^5x^{16} + \frac{5}{13}\,ab^4x^{13} + a^2b^3x^{10} + \frac{10}{7}\,a^3b^2x^7 + \frac{5}{4}\,a^4bx^4 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7
 + 5/4*a^4*b*x^4 + a^5*x
```

Fricas [A] time = 0.258865, size = 72, normalized size = 0.29

$$\frac{1}{16}\,b^5x^{16} + \frac{5}{13}\,ab^4x^{13} + a^2b^3x^{10} + \frac{10}{7}\,a^3b^2x^7 + \frac{5}{4}\,a^4bx^4 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7
 + 5/4*a^4*b*x^4 + a^5*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

GIAC/XCAS [A] time = 0.271573, size = 136, normalized size = 0.55

$$\frac{1}{16}\,b^5x^{16}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{13}\,ab^4x^{13}\mathrm{sign}\left(bx^3+a\right)+a^2b^3x^{10}\mathrm{sign}\left(bx^3+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sign}\left(bx^3+a\right)+\frac{5}{4}\,a^4bx^4\mathrm{sign}\left(bx^3+a\right)+a^5x\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sign(b*x^3 + a) + 5/13*a*b^4*x^13*sign(b*x^3 + a) +
 a^2*b^3*x^10*sign(b*x^3 + a) + 10/7*a^3*b^2*x^7*sign(b*x^3 + a)
+ 5/4*a^4*b*x^4*sign(b*x^3 + a) + a^5*x*sign(b*x^3 + a)
```

3.64 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\,(a+bx^3)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\,(a+bx^3)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}+\frac{a^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}$$

```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (
5*a^3*b^2*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) +
(10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
+ (5*a*b^4*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*(a + b*x^3))
 + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*(a + b*x^3)) +
(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.171591, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\,(a+bx^3)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\,(a+bx^3)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}+\frac{a^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (
5*a^3*b^2*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) +
(10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
+ (5*a*b^4*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*(a + b*x^3))
 + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*(a + b*x^3)) +
(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 26.4812, size = 178, normalized size = 0.71

$$\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}+\frac{a^4\sqrt{a^2+2abx^3+b^2x^6}}{3}+\frac{a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6}+\frac{a^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{9}+\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{12}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
```

```
[Out] a**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**3) + a*
*4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/3 + a**3*(a + b*x**3)*sqrt
(a**2 + 2*a*b*x**3 + b**2*x**6)/6 + a**2*(a**2 + 2*a*b*x**3 + b**
2*x**6)**(3/2)/9 + a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)
**(3/2)/12 + (a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/15
```

Mathematica [A] time = 0.0505826, size = 82, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\left(180a^5\log(x)+bx^3\left(300a^4+300a^3bx^3+200a^2b^2x^6+75ab^3x^9+12b^4x^{12}\right)\right)}{180\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^
2*x^6 + 75*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a +
b*x^3))
```

Maple [A] time = 0.012, size = 79, normalized size = 0.3

$$\frac{12\,b^5x^{15}+75\,ab^4x^{12}+200\,a^2b^3x^9+300\,a^3b^2x^6+300\,a^4bx^3+180\,a^5\ln(x)}{180\,(bx^3+a)^5}\left((bx^3+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+75*a*b^4*x^12+200*a^2*b^3*
x^9+300*a^3*b^2*x^6+300*a^4*b*x^3+180*a^5*ln(x))/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272122, size = 74, normalized size = 0.29

$$\frac{1}{15}\,b^5x^{15} + \frac{5}{12}\,ab^4x^{12} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{3}\,a^3b^2x^6 + \frac{5}{3}\,a^4bx^3 + a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*
x^6 + 5/3*a^4*b*x^3 + a^5*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x, x)
```

GIAC/XCAS [A] time = 0.296822, size = 140, normalized size = 0.56

$$\frac{1}{15}\,b^5x^{15}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{12}\,ab^4x^{12}\mathrm{sign}\left(bx^3 + a\right) + \frac{10}{9}\,a^2b^3x^9\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{3}\,a^3b^2x^6\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{3}\,a^4bx^3\mathrm{sign}\left(bx^3 + a\right) + a^5\ln\left(|x|\right)\mathrm{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sign(b*x^3 + a) + 5/12*a*b^4*x^12*sign(b*x^3 + a) +
 10/9*a^2*b^3*x^9*sign(b*x^3 + a) + 5/3*a^3*b^2*x^6*sign(b*x^3 +
a) + 5/3*a^4*b*x^3*sign(b*x^3 + a) + a^5*ln(abs(x))*sign(b*x^3 +
a)
```

3.65 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4
*b*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*
b^2*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3
*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*
x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^5*x^1
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3))
```

Rubi [A] time = 0.157973, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4
*b*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*
b^2*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3
*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*
x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^5*x^1
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.7104, size = 196, normalized size = 0.78

$$-\frac{729a^5\sqrt{a^2+2abx^3+b^2x^6}}{308x\left(a+bx^3\right)}+\frac{243a^4\sqrt{a^2+2abx^3+b^2x^6}}{308x}+\frac{81a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{308x}+\frac{45a^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{308x}+\frac{15a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{154x}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{14x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

```
[Out] -729*a**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(308*x*(a + b*x**3)
) + 243*a**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(308*x) + 81*a**
3*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(308*x) + 45*a
**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(308*x) + 15*a*(a + b*
x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(154*x) + (a**2 + 2*
a*b*x**3 + b**2*x**6)**(5/2)/(14*x)
```

Mathematica [A] time = 0.0381417, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\left(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+22b^5x^{15}\right)}{308x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6
+ 385*a^2*b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*
x^3))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{-22\,b^5x^{15}-140\,ab^4x^{12}-385\,a^2b^3x^9-616\,a^3b^2x^6-770\,a^4bx^3+308\,a^5}{308\,x\,(bx^3+a)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x)
```

```
[Out] -1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x
^6-770*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5
```

Maxima [A] time = 0.788433, size = 80, normalized size = 0.32

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b
^2*x^6 + 770*a^4*b*x^3 - 308*a^5)/x
```

Fricas [A] time = 0.263446, size = 80, normalized size = 0.32

$$\frac{22\,b^5x^{15} + 140\,ab^4x^{12} + 385\,a^2b^3x^9 + 616\,a^3b^2x^6 + 770\,a^4bx^3 - 308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b
^2*x^6 + 770*a^4*b*x^3 - 308*a^5)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**2, x)
```

GIAC/XCAS [A] time = 0.27993, size = 142, normalized size = 0.57

$$\frac{1}{14}\,b^5x^{14}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{11}\,ab^4x^{11}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{4}\,a^2b^3x^8\mathrm{sign}\left(bx^3 + a\right) + 2\,a^3b^2x^5\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{2}\,a^4bx^2\mathrm{sign}\left(bx^3 + a\right) - \frac{a^5\mathrm{sign}\left(bx^3 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14*sign(b*x^3 + a) + 5/11*a*b^4*x^11*sign(b*x^3 + a) +
 5/4*a^2*b^3*x^8*sign(b*x^3 + a) + 2*a^3*b^2*x^5*sign(b*x^3 + a)
+ 5/2*a^4*b*x^2*sign(b*x^3 + a) - a^5*sign(b*x^3 + a)/x
```

3.66 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (5*a
^4*b*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*
x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^
3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x
^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^5*x^13*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rubi [A] time = 0.156713, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (5*a
^4*b*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*
x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^
3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x
^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^5*x^13*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.6983, size = 206, normalized size = 0.82

$$-\frac{729a^5\sqrt{a^2+2abx^3+b^2x^6}}{182x^2(a+bx^3)}+\frac{243a^4\sqrt{a^2+2abx^3+b^2x^6}}{91x^2}+\frac{81a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{182x^2}+\frac{18a^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{91x^2}+\frac{3a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{26x^2}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{13x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

```
[Out] -729*a**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(182*x**2*(a + b*x*
*3)) + 243*a**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(91*x**2) + 8
1*a**3*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(182*x**2
) + 18*a**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(91*x**2) + 3*
a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(26*x**2) +
 (a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(13*x**2)
```

Mathematica [A]   time = 0.0361411, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\left(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14b^5x^{15}\right)}{182x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 +
 260*a^2*b^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*
x^3))
```

Maple [A]   time = 0.01, size = 80, normalized size = 0.3

$$-\frac{-14\,b^5x^{15}-91\,ab^4x^{12}-260\,a^2b^3x^9-455\,a^3b^2x^6-910\,a^4bx^3+91\,a^5}{182\,x^2\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x)
```

```
[Out] -1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^
6-910*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

Maxima [A]   time = 0.820378, size = 80, normalized size = 0.32

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^
2*x^6 + 910*a^4*b*x^3 - 91*a^5)/x^2
```

Fricas [A] time = 0.271278, size = 80, normalized size = 0.32

$$\frac{14\,b^5x^{15} + 91\,ab^4x^{12} + 260\,a^2b^3x^9 + 455\,a^3b^2x^6 + 910\,a^4bx^3 - 91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^
2*x^6 + 910*a^4*b*x^3 - 91*a^5)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

GIAC/XCAS [A] time = 0.287128, size = 139, normalized size = 0.55

$$\frac{1}{13}\,b^5x^{13}\mathrm{sign}\left(bx^3 + a\right) + \frac{1}{2}\,ab^4x^{10}\mathrm{sign}\left(bx^3 + a\right) + \frac{10}{7}\,a^2b^3x^7\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{2}\,a^3b^2x^4\mathrm{sign}\left(bx^3 + a\right) + 5\,a^4bx\mathrm{sign}\left(bx^3 + a\right) - \frac{a^5\mathrm{sign}\left(bx^3 + a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3,x, algorithm="giac")

[Out] 1/13*b^5*x^13*sign(b*x^3 + a) + 1/2*a*b^4*x^10*sign(b*x^3 + a) +
10/7*a^2*b^3*x^7*sign(b*x^3 + a) + 5/2*a^3*b^2*x^4*sign(b*x^3 + a
) + 5*a^4*b*x*sign(b*x^3 + a) - 1/2*a^5*sign(b*x^3 + a)/x^2
```

3.67 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4} dx$

Optimal. Leaf size=252

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*
a^3*b^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5
*a^2*b^3*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (
5*a*b^4*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (b
^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (5*a^
4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.185517, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*
a^3*b^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5
*a^2*b^3*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (
5*a*b^4*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (b
^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (5*a^
4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 26.1648, size = 199, normalized size = 0.79

$$\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}+\frac{5a^3b\sqrt{a^2+2abx^3+b^2x^6}}{3}+\frac{5a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6}+\frac{5ab\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{9}-\frac{5a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{12x^3}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

```
[Out] 5*a**4*b*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**3)
+ 5*a**3*b*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/3 + 5*a**2*b*(a +
b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/6 + 5*a*b*(a**2 + 2*a
*b*x**3 + b**2*x**6)**(3/2)/9 - 5*a*(a + b*x**3)*(a**2 + 2*a*b*x*
*3 + b**2*x**6)**(3/2)/(12*x**3) + (a**2 + 2*a*b*x**3 + b**2*x**6
)**(5/2)/(12*x**3)
```

Mathematica [A] time = 0.0517115, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\left(-12a^5+180a^4bx^3\log(x)+120a^3b^2x^6+60a^2b^3x^9+20ab^4x^{12}+3b^5x^{15}\right)}{36x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9
+ 20*a*b^4*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a
+ b*x^3))
```

Maple [A] time = 0.019, size = 82, normalized size = 0.3

$$\frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4b\ln(x)\,x^3-12\,a^5}{36\,\left(bx^3+a\right)^5x^3}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+20*a*b^4*x^12+60*a^2*b^3*x^9
+120*a^3*b^2*x^6+180*a^4*b*ln(x)*x^3-12*a^5)/(b*x^3+a)^5/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261885, size = 82, normalized size = 0.33

$$\frac{3\,b^5x^{15} + 20\,ab^4x^{12} + 60\,a^2b^3x^9 + 120\,a^3b^2x^6 + 180\,a^4bx^3\log\left(x\right) - 12\,a^5}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x
^6 + 180*a^4*b*x^3*log(x) - 12*a^5)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

GIAC/XCAS [A] time = 0.282666, size = 167, normalized size = 0.66

$$\frac{1}{12}\,b^5x^{12}\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{9}\,ab^4x^9\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{3}\,a^2b^3x^6\mathrm{sign}\left(bx^3 + a\right) + \frac{10}{3}\,a^3b^2x^3\mathrm{sign}\left(bx^3 + a\right) + 5\,a^4b\ln\left(|x|\right)\mathrm{sign}\left(bx^3 + a\right) - \frac{5\,a^4bx^3\mathrm{sign}\left(bx^3 + a\right) + a^5\mathrm{sign}\left(bx^3 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12*sign(b*x^3 + a) + 5/9*a*b^4*x^9*sign(b*x^3 + a) + 5
/3*a^2*b^3*x^6*sign(b*x^3 + a) + 10/3*a^3*b^2*x^3*sign(b*x^3 + a)
 + 5*a^4*b*ln(abs(x))*sign(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sign(b*x
^3 + a) + a^5*sign(b*x^3 + a))/x^3
```

3.68 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=249

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^
2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x
^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^5*x^11*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rubi [A] time = 0.158662, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^
2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x
^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^5*x^11*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rubi in Sympy [A] time = 25.9729, size = 216, normalized size = 0.87

$$\frac{729a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{88\left(a+bx^3\right)}+\frac{243a^2b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{44}+\frac{405ab^2x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{88}+\frac{15a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{4x^4}+\frac{45b^2x^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{11}-\frac{4\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

```
[Out] 729*a**3*b**2*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(88*(a + b
*x**3)) + 243*a**2*b**2*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/
44 + 405*a*b**2*x**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x
**6)/88 + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2
)/(4*x**4) + 45*b**2*x**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/
11 - 4*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/x**4
```

Mathematica [A] time = 0.0497039, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-22a^5-440a^4bx^3+440a^3b^2x^6+176a^2b^3x^9+55ab^4x^{12}+8b^5x^{15}\right)}{88x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 +
 176*a^2*b^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^
3))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$-\frac{-8\,b^5x^{15}-55\,ab^4x^{12}-176\,a^2b^3x^9-440\,a^3b^2x^6+440\,a^4bx^3+22\,a^5}{88\,x^4\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x)
```

```
[Out] -1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+
440*a^4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

Maxima [A] time = 0.764393, size = 80, normalized size = 0.32

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*
x^6 - 440*a^4*b*x^3 - 22*a^5)/x^4
```

Fricas [A] time = 0.270441, size = 80, normalized size = 0.32

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*
x^6 - 440*a^4*b*x^3 - 22*a^5)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

GIAC/XCAS [A] time = 0.299673, size = 144, normalized size = 0.58

$$\frac{1}{11}\,b^5x^{11}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sign}\left(bx^3+a\right)+2\,a^2b^3x^5\mathrm{sign}\left(bx^3+a\right)+5\,a^3b^2x^2\mathrm{sign}\left(bx^3+a\right)-\frac{20\,a^4bx^3\mathrm{sign}\left(bx^3+a\right)+a^5\mathrm{sign}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sign(b*x^3 + a) + 5/8*a*b^4*x^8*sign(b*x^3 + a) + 2
*a^2*b^3*x^5*sign(b*x^3 + a) + 5*a^3*b^2*x^2*sign(b*x^3 + a) - 1/
4*(20*a^4*b*x^3*sign(b*x^3 + a) + a^5*sign(b*x^3 + a))/x^4
```

3.69 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a
^3*b^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^
3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4
*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (b^5*x^10
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3))
```

Rubi [A] time = 0.157908, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a
^3*b^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^
3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4
*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (b^5*x^10
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3))
```

Rubi in Sympy [A] time = 16.2078, size = 211, normalized size = 0.84

$$\frac{729a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{70\left(a+bx^3\right)}+\frac{243a^2b^2x\sqrt{a^2+2abx^3+b^2x^6}}{70}+\frac{81ab^2x\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{35}+\frac{3a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{2x^5}+\frac{9b^2x\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{5}-\frac{17\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

```
[Out] 729*a**3*b**2*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(70*(a + b*x*
*3)) + 243*a**2*b**2*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/70 + 8
1*a*b**2*x*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/35 +
3*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(2*x**5)
+ 9*b**2*x*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/5 - 17*(a**2 +
2*a*b*x**3 + b**2*x**6)**(5/2)/(10*x**5)
```

Mathematica [A] time = 0.0381785, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\left(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b^5x^{15}\right)}{70x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 +
 175*a^2*b^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^
3))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$-\frac{-7\,b^5x^{15}-50\,ab^4x^{12}-175\,a^2b^3x^9-700\,a^3b^2x^6+175\,a^4bx^3+14\,a^5}{70\,x^5\,(bx^3+a)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+
175*a^4*b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5
```

Maxima [A] time = 0.761959, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*
x^6 - 175*a^4*b*x^3 - 14*a^5)/x^5
```

Fricas [A] time = 0.267945, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*
x^6 - 175*a^4*b*x^3 - 14*a^5)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

GIAC/XCAS [A] time = 0.27047, size = 143, normalized size = 0.57

$$\frac{1}{10}\,b^5x^{10}\mathrm{sign}\left(bx^3+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sign}\left(bx^3+a\right)+\frac{5}{2}\,a^2b^3x^4\mathrm{sign}\left(bx^3+a\right)+10\,a^3b^2x\mathrm{sign}\left(bx^3+a\right)-\frac{25\,a^4bx^3\mathrm{sign}\left(bx^3+a\right)+2\,a^5\mathrm{sign}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sign(b*x^3 + a) + 5/7*a*b^4*x^7*sign(b*x^3 + a) + 5
/2*a^2*b^3*x^4*sign(b*x^3 + a) + 10*a^3*b^2*x*sign(b*x^3 + a) - 1
/10*(25*a^4*b*x^3*sign(b*x^3 + a) + 2*a^5*sign(b*x^3 + a))/x^5
```

3.70 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a
^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*
a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b^5
*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (10*a^3*b
^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.180803, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a
^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*
a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b^5
*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (10*a^3*b
^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 27.1344, size = 201, normalized size = 0.8

$$\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\log\left(x\right)}{a+bx^3}+\frac{10a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}{3}+\frac{5ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{3}+\frac{5a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{6x^6}+\frac{10b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{9}-\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
```

```
[Out] 10*a**3*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x*
*3) + 10*a**2*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/3 + 5*a*b*
*2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/3 + 5*a*(a +
b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(6*x**6) + 10*b**2
*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/9 - (a**2 + 2*a*b*x**3 +
b**2*x**6)**(5/2)/x**6
```

Mathematica [A] time = 0.0492585, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\left(-3a^5-30a^4bx^3+180a^3b^2x^6\log(x)+60a^2b^3x^9+15ab^4x^{12}+2b^5x^{15}\right)}{18x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-3*a^5 - 30*a^4*b*x^3 + 60*a^2*b^3*x^9 + 15
*a*b^4*x^12 + 2*b^5*x^15 + 180*a^3*b^2*x^6*Log[x]))/(18*x^6*(a +
b*x^3))
```

Maple [A] time = 0.019, size = 82, normalized size = 0.3

$$\frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2\ln(x)\,x^6-30\,a^4bx^3-3\,a^5}{18\,\left(bx^3+a\right)^5x^6}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9
+180*a^3*b^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258562, size = 82, normalized size = 0.33

$$\frac{2\,b^5x^{15} + 15\,ab^4x^{12} + 60\,a^2b^3x^9 + 180\,a^3b^2x^6\log\left(x\right) - 30\,a^4bx^3 - 3\,a^5}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x
^6*log(x) - 30*a^4*b*x^3 - 3*a^5)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**7, x)
```

GIAC/XCAS [A] time = 0.28916, size = 170, normalized size = 0.67

$$\frac{1}{9}\,b^5x^9\mathrm{sign}\left(bx^3+a\right) + \frac{5}{6}\,ab^4x^6\mathrm{sign}\left(bx^3+a\right) + \frac{10}{3}\,a^2b^3x^3\mathrm{sign}\left(bx^3+a\right) + 10\,a^3b^2\ln\left(|x|\right)\mathrm{sign}\left(bx^3+a\right) - \frac{30\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right) + 10\,a^4bx^3\mathrm{sign}\left(bx^3+a\right) + a^5\mathrm{sign}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sign(b*x^3 + a) + 5/6*a*b^4*x^6*sign(b*x^3 + a) + 10/
3*a^2*b^3*x^3*sign(b*x^3 + a) + 10*a^3*b^2*ln(abs(x))*sign(b*x^3
+ a) - 1/6*(30*a^3*b^2*x^6*sign(b*x^3 + a) + 10*a^4*b*x^3*sign(b*
x^3 + a) + a^5*sign(b*x^3 + a))/x^6
```

3.71 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*
b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^4*x^5
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^5*x^8*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi [A] time = 0.160022, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*
b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^4*x^5
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^5*x^8*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.2498, size = 209, normalized size = 0.84

$$-\frac{729a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{56x(a+bx^3)}+\frac{243a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}{56x}+\frac{81ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{56x}+\frac{15a(a+bx^3)(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{28x^7}+\frac{45b^2(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{56x}-\frac{19(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

```
[Out] -729*a**3*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(56*x*(a + b*x
**3)) + 243*a**2*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(56*x)
+ 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(56*
x) + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(28
*x**7) + 45*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(56*x) -
19*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(28*x**7)
```

Mathematica [A] time = 0.0405226, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\left(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x^{15}\right)}{56x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 2
80*a^2*b^3*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3)
)
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{-7\,b^5x^{15}-56\,ab^4x^{12}-280\,a^2b^3x^9+560\,a^3b^2x^6+70\,a^4bx^3+8\,a^5}{56\,x^7\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+
70*a^4*b*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^7/(b*x^3+a)^5
```

Maxima [A] time = 0.771765, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*
x^6 - 70*a^4*b*x^3 - 8*a^5)/x^7
```

Fricas [A] time = 0.274498, size = 80, normalized size = 0.32

$$\frac{7\,b^{5}x^{15} + 56\,ab^{4}x^{12} + 280\,a^{2}b^{3}x^{9} - 560\,a^{3}b^{2}x^{6} - 70\,a^{4}bx^{3} - 8\,a^{5}}{56\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*
x^6 - 70*a^4*b*x^3 - 8*a^5)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

GIAC/XCAS [A] time = 0.26282, size = 144, normalized size = 0.58

$$\frac{1}{8}\,b^{5}x^{8}\mathrm{sign}\left(bx^{3} + a\right) + ab^{4}x^{5}\mathrm{sign}\left(bx^{3} + a\right) + 5\,a^{2}b^{3}x^{2}\mathrm{sign}\left(bx^{3} + a\right) - \frac{280\,a^{3}b^{2}x^{6}\mathrm{sign}\left(bx^{3} + a\right) + 35\,a^{4}bx^{3}\mathrm{sign}\left(bx^{3} + a\right) + 4\,a^{5}\mathrm{sign}\left(bx^{3} + a\right)}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sign(b*x^3 + a) + a*b^4*x^5*sign(b*x^3 + a) + 5*a^2*b
^3*x^2*sign(b*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sign(b*x^3 + a) +
35*a^4*b*x^3*sign(b*x^3 + a) + 4*a^5*sign(b*x^3 + a))/x^7
```

3.72 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (a^4
*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^
2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^
3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a*b^4*x^4*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^5*x^7*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi [A] time = 0.157919, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (a^4
*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^
2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^
3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a*b^4*x^4*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^5*x^7*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.1662, size = 214, normalized size = 0.87

$$-\frac{729a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{56x^2\left(a+bx^3\right)}+\frac{243a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}{28x^2}+\frac{81ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{56x^2}+\frac{3a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{8x^8}+\frac{9b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{14x^2}-\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
```

```
[Out] -729*a**3*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(56*x**2*(a +
b*x**3)) + 243*a**2*b**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(28*
x**2) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6
)/(56*x**2) + 3*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(
3/2)/(8*x**8) + 9*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(14
*x**2) - (a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(2*x**8)
```

Mathematica [A] time = 0.0366966, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\left(-7a^5-56a^4bx^3-280a^3b^2x^6+560a^2b^3x^9+70ab^4x^{12}+8b^5x^{15}\right)}{56x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 5
60*a^2*b^3*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3)
)
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{-8\,b^5x^{15}-70\,ab^4x^{12}-560\,a^2b^3x^9+280\,a^3b^2x^6+56\,a^4bx^3+7\,a^5}{56\,x^8\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x)
```

```
[Out] -1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+
56*a^4*b*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^8/(b*x^3+a)^5
```

Maxima [A] time = 0.793914, size = 80, normalized size = 0.32

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*
x^6 - 56*a^4*b*x^3 - 7*a^5)/x^8
```

Fricas [A] time = 0.261778, size = 80, normalized size = 0.32

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*
x^6 - 56*a^4*b*x^3 - 7*a^5)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**9, x)
```

GIAC/XCAS [A] time = 0.257658, size = 142, normalized size = 0.57

$$\frac{1}{7}\,b^5x^7\mathrm{sign}\left(bx^3+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sign}\left(bx^3+a\right)+10\,a^2b^3x\mathrm{sign}\left(bx^3+a\right) - \frac{40\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+8\,a^4bx^3\mathrm{sign}\left(bx^3+a\right)+a^5\mathrm{sign}\left(bx^3+a\right)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sign(b*x^3 + a) + 5/4*a*b^4*x^4*sign(b*x^3 + a) + 10*
a^2*b^3*x*sign(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sign(b*x^3 + a) +
 8*a^4*b*x^3*sign(b*x^3 + a) + a^5*sign(b*x^3 + a))/x^8
```

3.73 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*
a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (b^5
*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (10*a^2*b
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.180685, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*
a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (b^5
*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (10*a^2*b
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 27.0819, size = 209, normalized size = 0.83

$$\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}+\frac{10ab^3\sqrt{a^2+2abx^3+b^2x^6}}{3}-\frac{5ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{3x^3}+\frac{5a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{18x^9}+\frac{5b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{9x^3}-\frac{7\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

```
[Out] 10*a**2*b**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x*
*3) + 10*a*b**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/3 - 5*a*b**2*
(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(3*x**3) + 5*a*(
a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(18*x**9) + 5*
b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(9*x**3) - 7*(a**2 +
2*a*b*x**3 + b**2*x**6)**(5/2)/(18*x**9)
```

Mathematica [A] time = 0.0461057, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\left(-2a^5-15a^4bx^3-60a^3b^2x^6+180a^2b^3x^9\log(x)+30ab^4x^{12}+3b^5x^{15}\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^5 - 15*a^4*b*x^3 - 60*a^3*b^2*x^6 + 30
*a*b^4*x^12 + 3*b^5*x^15 + 180*a^2*b^3*x^9*Log[x]))/(18*x^9*(a +
b*x^3))
```

Maple [A] time = 0.019, size = 82, normalized size = 0.3

$$\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3\ln(x)\,x^9-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,\left(bx^3+a\right)^5x^9}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln
(x)*x^9-60*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269197, size = 82, normalized size = 0.33

$$\frac{3\,b^5x^{15} + 30\,ab^4x^{12} + 180\,a^2b^3x^9\log\left(x\right) - 60\,a^3b^2x^6 - 15\,a^4bx^3 - 2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^
3*b^2*x^6 - 15*a^4*b*x^3 - 2*a^5)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**10, x)
```

GIAC/XCAS [A] time = 0.299895, size = 171, normalized size = 0.68

$$\frac{1}{6}\,b^5x^6\mathrm{sign}\left(bx^3 + a\right) + \frac{5}{3}\,ab^4x^3\mathrm{sign}\left(bx^3 + a\right) + 10\,a^2b^3\ln\left(|x|\right)\mathrm{sign}\left(bx^3 + a\right) - \frac{110\,a^2b^3x^9\mathrm{sign}\left(bx^3 + a\right) + 60\,a^3b^2x^6\mathrm{sign}\left(bx^3 + a\right) + 15\,a^4bx^3\mathrm{sign}\left(bx^3 + a\right) + 2\,a^5\mathrm{sign}\left(bx^3 + a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sign(b*x^3 + a) + 5/3*a*b^4*x^3*sign(b*x^3 + a) + 10*
a^2*b^3*ln(abs(x))*sign(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sign(b
*x^3 + a) + 60*a^3*b^2*x^6*sign(b*x^3 + a) + 15*a^4*b*x^3*sign(b*
x^3 + a) + 2*a^5*sign(b*x^3 + a))/x^9
```

3.74 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=253

$$\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (1
0*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a
*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^5*
x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rubi [A] time = 0.161025, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (1
0*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a
*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^5*
x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.3638, size = 212, normalized size = 0.84

$$\frac{729ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{70\,(a+bx^3)}+\frac{81ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{14x^4}+\frac{3a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{14x^{10}}+\frac{243b^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{35}-\frac{45b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{7x^4}-\frac{11\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{35x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
```

```
[Out] 729*a*b**4*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(70*(a + b*x*
*3)) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)
/(14*x**4) + 3*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3
/2)/(14*x**10) + 243*b**4*x**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6
)/35 - 45*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(7*x**4) -
11*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(35*x**10)
```

Mathematica [A] time = 0.0332968, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(7a^5+50a^4bx^3+175a^3b^2x^6+700a^2b^3x^9-175ab^4x^{12}-14b^5x^{15}\right)}{70x^{10}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 7
00*a^2*b^3*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(70*x^10*(a + b*x
^3))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{-14\,b^5x^{15}-175\,ab^4x^{12}+700\,a^2b^3x^9+175\,a^3b^2x^6+50\,a^4bx^3+7\,a^5}{70\,x^{10}\,(bx^3+a)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x)
```

```
[Out] -1/70*(-14*b^5*x^15-175*a*b^4*x^12+700*a^2*b^3*x^9+175*a^3*b^2*x^
6+50*a^4*b*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^10/(b*x^3+a)^5
```

Maxima [A] time = 0.816737, size = 80, normalized size = 0.32

$$\frac{14\, b^5 x^{15} + 175\, a b^4 x^{12} - 700\, a^2 b^3 x^9 - 175\, a^3 b^2 x^6 - 50\, a^4 b x^3 - 7\, a^5}{70\, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^
2*x^6 - 50*a^4*b*x^3 - 7*a^5)/x^10
```

Fricas [A] time = 0.252904, size = 80, normalized size = 0.32

$$\frac{14\, b^5 x^{15} + 175\, a b^4 x^{12} - 700\, a^2 b^3 x^9 - 175\, a^3 b^2 x^6 - 50\, a^4 b x^3 - 7\, a^5}{70\, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^
2*x^6 - 50*a^4*b*x^3 - 7*a^5)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^3\right)^2\right)^{\frac{5}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**11, x)
```

GIAC/XCAS [A] time = 0.259142, size = 146, normalized size = 0.58

$$\frac{1}{5}\,b^{5}x^{5}\mathrm{sign}\left(bx^{3}+a\right)+\frac{5}{2}\,ab^{4}x^{2}\mathrm{sign}\left(bx^{3}+a\right)-\frac{700\,a^{2}b^{3}x^{9}\mathrm{sign}\left(bx^{3}+a\right)+175\,a^{3}b^{2}x^{6}\mathrm{sign}\left(bx^{3}+a\right)+50\,a^{4}bx^{3}\mathrm{sign}\left(bx^{3}+a\right)+7\,a^{5}\mathrm{sign}\left(bx^{3}+a\right)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sign(b*x^3 + a) + 5/2*a*b^4*x^2*sign(b*x^3 + a) - 1/7
0*(700*a^2*b^3*x^9*sign(b*x^3 + a) + 175*a^3*b^2*x^6*sign(b*x^3 +
 a) + 50*a^4*b*x^3*sign(b*x^3 + a) + 7*a^5*sign(b*x^3 + a))/x^10
```

3.75 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}} \, dx$

Optimal. Leaf size=247

$$\frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a
^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*
b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^5*x^4*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi [A] time = 0.157147, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a
^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*
b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^5*x^4*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rubi in Sympy [A] time = 21.3905, size = 209, normalized size = 0.85

$$\frac{729ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{88(a+bx^3)}+\frac{81ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{44x^5}+\frac{15a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{88x^{11}}+\frac{243b^4x\sqrt{a^2+2abx^3+b^2x^6}}{88}-\frac{9b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{4x^5}-\frac{23\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
```

```
[Out] 729*a*b**4*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(88*(a + b*x**3)
) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(4
4*x**5) + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2
)/(88*x**11) + 243*b**4*x*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/88
- 9*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(4*x**5) - 23*(a*
*2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(88*x**11)
```

Mathematica [A] time = 0.0304854, size = 83, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^3)^2}\left(8a^5+55a^4bx^3+176a^3b^2x^6+440a^2b^3x^9-440ab^4x^{12}-22b^5x^{15}\right)}{88x^{11}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 4
40*a^2*b^3*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(88*x^11*(a + b*x
^3))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$-\frac{-22\,b^5x^{15}-440\,ab^4x^{12}+440\,a^2b^3x^9+176\,a^3b^2x^6+55\,a^4bx^3+8\,a^5}{88\,x^{11}\,(bx^3+a)^5}\left((bx^3+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/88*(-22*b^5*x^15-440*a*b^4*x^12+440*a^2*b^3*x^9+176*a^3*b^2*x^
6+55*a^4*b*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^11/(b*x^3+a)^5
```

Maxima [A] time = 0.796162, size = 80, normalized size = 0.32

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^
2*x^6 - 55*a^4*b*x^3 - 8*a^5)/x^11
```

Fricas [A] time = 0.286694, size = 80, normalized size = 0.32

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^
2*x^6 - 55*a^4*b*x^3 - 8*a^5)/x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**12, x)
```

GIAC/XCAS [A] time = 0.296663, size = 143, normalized size = 0.58

$$\frac{1}{4}\,b^{5}x^{4}\mathrm{sign}\left(bx^{3}+a\right)+5\,ab^{4}x\mathrm{sign}\left(bx^{3}+a\right)-\frac{440\,a^{2}b^{3}x^{9}\mathrm{sign}\left(bx^{3}+a\right)+176\,a^{3}b^{2}x^{6}\mathrm{sign}\left(bx^{3}+a\right)+55\,a^{4}bx^{3}\mathrm{sign}\left(bx^{3}+a\right)+8\,a^{5}\mathrm{sign}\left(bx^{3}+a\right)}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sign(b*x^3 + a) + 5*a*b^4*x*sign(b*x^3 + a) - 1/88*(4
40*a^2*b^3*x^9*sign(b*x^3 + a) + 176*a^3*b^2*x^6*sign(b*x^3 + a)
+ 55*a^4*b*x^3*sign(b*x^3 + a) + 8*a^5*sign(b*x^3 + a))/x^11
```

3.76 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (1
0*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) +
(b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*
b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.179246, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (1
0*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) +
(b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*
b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 27.0411, size = 206, normalized size = 0.82

$$\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}+\frac{5ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6x^6}+\frac{5a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{36x^{12}}+\frac{5b^4\sqrt{a^2+2abx^3+b^2x^6}}{3}-\frac{10b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{9x^6}-\frac{2\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{9x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
```

```
[Out] 5*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**3)
+ 5*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(6*x*
*6) + 5*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(36
*x**12) + 5*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/3 - 10*b**2*
(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(9*x**6) - 2*(a**2 + 2*a*b
*x**3 + b**2*x**6)**(5/2)/(9*x**12)
```

Mathematica [A] time = 0.035137, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(3a^5+20a^4bx^3+60a^3b^2x^6+120a^2b^3x^9-180ab^4x^{12}\log(x)-12b^5x^{15}\right)}{36x^{12}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(3*a^5 + 20*a^4*b*x^3 + 60*a^3*b^2*x^6 + 12
0*a^2*b^3*x^9 - 12*b^5*x^15 - 180*a*b^4*x^12*Log[x]))/(36*x^12*(a
 + b*x^3))
```

Maple [A] time = 0.02, size = 82, normalized size = 0.3

$$\frac{12\,b^5x^{15}+180\,ab^4\ln(x)\,x^{12}-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,\left(bx^3+a\right)^5x^{12}}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+180*a*b^4*ln(x)*x^12-120*a^
2*b^3*x^9-60*a^3*b^2*x^6-20*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^12
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25801, size = 82, normalized size = 0.33

$$\frac{12\,b^5x^{15} + 180\,ab^4x^{12}\log\left(x\right) - 120\,a^2b^3x^9 - 60\,a^3b^2x^6 - 20\,a^4bx^3 - 3\,a^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*
a^3*b^2*x^6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**13, x)
```

GIAC/XCAS [A] time = 0.305178, size = 169, normalized size = 0.67

$$\frac{1}{3}\,b^{5}x^{3}\mathrm{sign}\left(bx^{3}+a\right)+5\,ab^{4}\ln\left(|x|\right)\mathrm{sign}\left(bx^{3}+a\right)-\frac{125\,ab^{4}x^{12}\mathrm{sign}\left(bx^{3}+a\right)+120\,a^{2}b^{3}x^{9}\mathrm{sign}\left(bx^{3}+a\right)+60\,a^{3}b^{2}x^{6}\mathrm{sign}\left(bx^{3}+a\right)+20\,a^{4}bx^{3}\mathrm{sign}\left(bx^{3}+a\right)+3\,a^{5}\mathrm{sign}\left(bx^{3}\right.}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sign(b*x^3 + a) + 5*a*b^4*ln(abs(x))*sign(b*x^3 + a)
- 1/36*(125*a*b^4*x^12*sign(b*x^3 + a) + 120*a^2*b^3*x^9*sign(b*x
^3 + a) + 60*a^3*b^2*x^6*sign(b*x^3 + a) + 20*a^4*b*x^3*sign(b*x^
3 + a) + 3*a^5*sign(b*x^3 + a))/x^12
```

3.77 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^5*x
^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rubi [A] time = 0.16151, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^5*x
^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.9483, size = 207, normalized size = 0.82

$$-\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{182x(a+bx^3)}+\frac{81ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{182x^7}+\frac{3a(a+bx^3)(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{26x^{13}}+\frac{243b^4\sqrt{a^2+2abx^3+b^2x^6}}{182x}-\frac{9b^2(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{14x^7}-\frac{5(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}{26x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
```

```
[Out] -729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(182*x*(a + b*x**
3)) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/
(182*x**7) + 3*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3
/2)/(26*x**13) + 243*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(18
2*x) - 9*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(14*x**7) -
5*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(26*x**13)
```

Mathematica [A] time = 0.0310825, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(14a^5+91a^4bx^3+260a^3b^2x^6+455a^2b^3x^9+910ab^4x^{12}-91b^5x^{15}\right)}{182x^{13}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 +
455*a^2*b^3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(182*x^13*(a + b
*x^3))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{-91\,b^5x^{15}+910\,ab^4x^{12}+455\,a^2b^3x^9+260\,a^3b^2x^6+91\,a^4bx^3+14\,a^5}{182\,x^{13}(bx^3+a)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x)
```

```
[Out] -1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x
^6+91*a^4*b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^13/(b*x^3+a)^5
```

Maxima [A] time = 0.78908, size = 80, normalized size = 0.32

$$\frac{91\,b^5x^{15} - 910\,ab^4x^{12} - 455\,a^2b^3x^9 - 260\,a^3b^2x^6 - 91\,a^4bx^3 - 14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b
^2*x^6 - 91*a^4*b*x^3 - 14*a^5)/x^13
```

Fricas [A] time = 0.268413, size = 80, normalized size = 0.32

$$\frac{91\,b^5x^{15} - 910\,ab^4x^{12} - 455\,a^2b^3x^9 - 260\,a^3b^2x^6 - 91\,a^4bx^3 - 14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b
^2*x^6 - 91*a^4*b*x^3 - 14*a^5)/x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

GIAC/XCAS [A] time = 0.279417, size = 146, normalized size = 0.58

$$\frac{1}{2}\,b^5x^2\mathrm{sign}\left(bx^3+a\right) - \frac{910\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right)+455\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right)+260\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+91\,a^4bx^3\mathrm{sign}\left(bx^3+a\right)+14\,a^5\mathrm{sign}\left(b\right.}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sign(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sign(b*x^3 +
a) + 455*a^2*b^3*x^9*sign(b*x^3 + a) + 260*a^3*b^2*x^6*sign(b*x^3
 + a) + 91*a^4*b*x^3*sign(b*x^3 + a) + 14*a^5*sign(b*x^3 + a))/x^
13
```

3.78 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) -
(2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b
^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi [A] time = 0.156539, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) -
(2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b
^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rubi in Sympy [A] time = 26.6954, size = 211, normalized size = 0.85

$$-\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{308x^2(a+bx^3)}+\frac{81ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{308x^8}+\frac{15a(a+bx^3)(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{154x^{14}}+\frac{243b^4\sqrt{a^2+2abx^3+b^2x^6}}{154x^2}-\frac{9b^2(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{22x^8}-\frac{13(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}{77x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
```

```
[Out] -729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(308*x**2*(a + b*
x**3)) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**
6)/(308*x**8) + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)
**(3/2)/(154*x**14) + 243*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6
)/(154*x**2) - 9*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(22*
x**8) - 13*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(77*x**14)
```

Mathematica [A] time = 0.0328613, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(22a^5+140a^4bx^3+385a^3b^2x^6+616a^2b^3x^9+770ab^4x^{12}-308b^5x^{15}\right)}{308x^{14}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 +
 616*a^2*b^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(308*x^14*(a +
 b*x^3))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$-\frac{-308\,b^5x^{15}+770\,ab^4x^{12}+616\,a^2b^3x^9+385\,a^3b^2x^6+140\,a^4bx^3+22\,a^5}{308\,x^{14}\,(bx^3+a)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*
x^6+140*a^4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5
```

Maxima [A] time = 0.803092, size = 80, normalized size = 0.32

$$\frac{308\,b^5x^{15} - 770\,ab^4x^{12} - 616\,a^2b^3x^9 - 385\,a^3b^2x^6 - 140\,a^4bx^3 - 22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*
b^2*x^6 - 140*a^4*b*x^3 - 22*a^5)/x^14
```

Fricas [A] time = 0.255989, size = 80, normalized size = 0.32

$$\frac{308\,b^5x^{15} - 770\,ab^4x^{12} - 616\,a^2b^3x^9 - 385\,a^3b^2x^6 - 140\,a^4bx^3 - 22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*
b^2*x^6 - 140*a^4*b*x^3 - 22*a^5)/x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{15}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**15, x)
```

GIAC/XCAS [A] time = 0.293629, size = 142, normalized size = 0.57

$$b^5 x\mathrm{sign}\left(bx^3+a\right) - \frac{770\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right) + 616\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right) + 385\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right) + 140\,a^4bx^3\mathrm{sign}\left(bx^3+a\right) + 22\,a^5\mathrm{sign}\left(\right.}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] b^5*x*sign(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sign(b*x^3 + a) + 6
16*a^2*b^3*x^9*sign(b*x^3 + a) + 385*a^3*b^2*x^6*sign(b*x^3 + a)
+ 140*a^4*b*x^3*sign(b*x^3 + a) + 22*a^5*sign(b*x^3 + a))/x^14
```

3.79 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=251

$$\frac{b^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*x^15*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) -
 (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3))
- (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) +
 (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi [A] time = 0.175053, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*x^15*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) -
 (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3))
- (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) +
 (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rubi in Sympy [A] time = 32.2727, size = 209, normalized size = 0.83

$$-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6x^9}+\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{12x^{15}}+\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}-\frac{5b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{18x^9}-\frac{3\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{20x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] -a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(3*x**3*(a + b*x**3))
 + a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(6*x**
9) + a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(12*x*
*15) + b**5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x)/(a + b*x**
3) - 5*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(18*x**9) - 3*
(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(20*x**15)
```

Mathematica [A] time = 0.0484905, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^3)^2}\left(a\left(12a^4+75a^3bx^3+200a^2b^2x^6+300ab^3x^9+300b^4x^{12}\right)-180b^5x^{15}\log(x)\right)}{180x^{15}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a*(12*a^4 + 75*a^3*b*x^3 + 200*a^2*b^2*x^6
 + 300*a*b^3*x^9 + 300*b^4*x^12) - 180*b^5*x^15*Log[x]))/(180*x^1
5*(a + b*x^3))
```

Maple [A] time = 0.019, size = 82, normalized size = 0.3

$$\frac{180\,b^5\ln(x)\,x^{15}-300\,ab^4x^{12}-300\,a^2b^3x^9-200\,a^3b^2x^6-75\,a^4bx^3-12\,a^5}{180\,(bx^3+a)^5\,x^{15}}\left((bx^3+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(180*b^5*ln(x)*x^15-300*a*b^4*x^12-300*
a^2*b^3*x^9-200*a^3*b^2*x^6-75*a^4*b*x^3-12*a^5)/(b*x^3+a)^5/x^15
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263079, size = 82, normalized size = 0.33

$$\frac{180\, b^5 x^{15} \log(x) - 300\, ab^4 x^{12} - 300\, a^2 b^3 x^9 - 200\, a^3 b^2 x^6 - 75\, a^4 b x^3 - 12\, a^5}{180\, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 2
00*a^3*b^2*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

GIAC/XCAS [A] time = 0.28285, size = 166, normalized size = 0.66

$$b^5 \ln(|x|)\,\mathrm{sign}\left(bx^3 + a\right) - \frac{137\, b^5 x^{15} \mathrm{sign}\left(bx^3 + a\right) + 300\, ab^4 x^{12} \mathrm{sign}\left(bx^3 + a\right) + 300\, a^2 b^3 x^9 \mathrm{sign}\left(bx^3 + a\right) + 200\, a^3 b^2 x^6 \mathrm{sign}\left(bx^3 + a\right) + 75\, a^4 b x^3 \mathrm{sig}}{180\, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] b^5*ln(abs(x))*sign(b*x^3 + a) - 1/180*(137*b^5*x^15*sign(b*x^3 +
 a) + 300*a*b^4*x^12*sign(b*x^3 + a) + 300*a^2*b^3*x^9*sign(b*x^3
 + a) + 200*a^3*b^2*x^6*sign(b*x^3 + a) + 75*a^4*b*x^3*sign(b*x^3
 + a) + 12*a^5*sign(b*x^3 + a))/x^15
```

3.80 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=251

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b
^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rubi [A] time = 0.15832, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b
^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.8369, size = 211, normalized size = 0.84

$$\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{1456x^4\,(a+bx^3)}+\frac{81ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{728x^{10}}+\frac{15a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{208x^{16}}-\frac{243b^4\sqrt{a^2+2abx^3+b^2x^6}}{364x^4}-\frac{18b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{91x^{10}}-\frac{7\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{52x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)
```

```
[Out] 729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(1456*x**4*(a + b*
x**3)) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**
6)/(728*x**10) + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6
)**(3/2)/(208*x**16) - 243*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**
6)/(364*x**4) - 18*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(9
1*x**10) - 7*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(52*x**16)
```

Mathematica [A] time = 0.0378834, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(91a^5+560a^4bx^3+1456a^3b^2x^6+2080a^2b^3x^9+1820ab^4x^{12}+1456b^5x^{15}\right)}{1456x^{16}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6
+ 2080*a^2*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(1456*x^16
*(a + b*x^3))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}\,(bx^3+a)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/1456*(1456*b^5*x^15+1820*a*b^4*x^12+2080*a^2*b^3*x^9+1456*a^3*
b^2*x^6+560*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^16/(b*x^3+a)^
5
```

Maxima [A] time = 0.803367, size = 80, normalized size = 0.32

$$-\frac{1456\, b^5x^{15} + 1820\, ab^4x^{12} + 2080\, a^2b^3x^9 + 1456\, a^3b^2x^6 + 560\, a^4bx^3 + 91\, a^5}{1456\, x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 145
6*a^3*b^2*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

Fricas [A] time = 0.269164, size = 80, normalized size = 0.32

$$-\frac{1456\, b^5x^{15} + 1820\, ab^4x^{12} + 2080\, a^2b^3x^9 + 1456\, a^3b^2x^6 + 560\, a^4bx^3 + 91\, a^5}{1456\, x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 145
6*a^3*b^2*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{17}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**17, x)
```

GIAC/XCAS [A] time = 0.274717, size = 144, normalized size = 0.57

$$-\frac{1456\,b^5x^{15}\mathrm{sign}\left(bx^3+a\right)+1820\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right)+2080\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right)+1456\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+560\,a^4b}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/1456*(1456*b^5*x^15*sign(b*x^3 + a) + 1820*a*b^4*x^12*sign(b*x
^3 + a) + 2080*a^2*b^3*x^9*sign(b*x^3 + a) + 1456*a^3*b^2*x^6*sig
n(b*x^3 + a) + 560*a^4*b*x^3*sign(b*x^3 + a) + 91*a^5*sign(b*x^3
+ a))/x^16
```

3.81 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3))
 - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)
) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (
b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rubi [A] time = 0.159328, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3))
 - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)
) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (
b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.8099, size = 211, normalized size = 0.83

$$\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{5236x^5(a+bx^3)}+\frac{405ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{5236x^{11}}+\frac{15a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{238x^{17}}-\frac{1215b^4\sqrt{a^2+2abx^3+b^2x^6}}{5236x^5}-\frac{45b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{308x^{11}}-\frac{29\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{238x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

```
[Out] 729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(5236*x**5*(a + b*
x**3)) + 405*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x*
*6)/(5236*x**11) + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x*
*6)**(3/2)/(238*x**17) - 1215*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*
x**6)/(5236*x**5) - 45*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2
)/(308*x**11) - 29*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(238*x*
*17)
```

Mathematica [A] time = 0.034577, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(308a^5+1870a^4bx^3+4760a^3b^2x^6+6545a^2b^3x^9+5236ab^4x^{12}+2618b^5x^{15}\right)}{5236x^{17}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^
6 + 6545*a^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(5236*x^
17*(a + b*x^3))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}(bx^3+a)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/5236*(2618*b^5*x^15+5236*a*b^4*x^12+6545*a^2*b^3*x^9+4760*a^3*
b^2*x^6+1870*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x^17/(b*x^3+a
)^5
```

Maxima [A] time = 0.79513, size = 80, normalized size = 0.32

$$-\frac{2618\, b^5 x^{15} + 5236\, a b^4 x^{12} + 6545\, a^2 b^3 x^9 + 4760\, a^3 b^2 x^6 + 1870\, a^4 b x^3 + 308\, a^5}{5236\, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 476
0*a^3*b^2*x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Fricas [A] time = 0.254866, size = 80, normalized size = 0.32

$$-\frac{2618\, b^5 x^{15} + 5236\, a b^4 x^{12} + 6545\, a^2 b^3 x^9 + 4760\, a^3 b^2 x^6 + 1870\, a^4 b x^3 + 308\, a^5}{5236\, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 476
0*a^3*b^2*x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^3\right)^2\right)^{\frac{5}{2}}}{x^{18}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**18, x)
```

GIAC/XCAS [A] time = 0.274794, size = 144, normalized size = 0.57

$$-\frac{2618\,b^{5}x^{15}\mathrm{sign}\left(bx^{3}+a\right)+5236\,ab^{4}x^{12}\mathrm{sign}\left(bx^{3}+a\right)+6545\,a^{2}b^{3}x^{9}\mathrm{sign}\left(bx^{3}+a\right)+4760\,a^{3}b^{2}x^{6}\mathrm{sign}\left(bx^{3}+a\right)+1870\,a^{4}}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/5236*(2618*b^5*x^15*sign(b*x^3 + a) + 5236*a*b^4*x^12*sign(b*x
^3 + a) + 6545*a^2*b^3*x^9*sign(b*x^3 + a) + 4760*a^3*b^2*x^6*sig
n(b*x^3 + a) + 1870*a^4*b*x^3*sign(b*x^3 + a) + 308*a^5*sign(b*x^
3 + a))/x^17
```

3.82 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*a*x^18)
```

Rubi [A] time = 0.0594346, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*a*x^18)
```

Rubi in Sympy [A] time = 8.40389, size = 39, normalized size = 0.95

$$-\frac{\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{36ax^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

```
[Out] -(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(36*a*x*
*18)
```

Mathematica [A] time = 0.030504, size = 81, normalized size = 1.98

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^5+6a^4bx^3+15a^3b^2x^6+20a^2b^3x^9+15ab^4x^{12}+6b^5x^{15}\right)}{18x^{18}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^
2*b^3*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(18*x^18*(a + b*x^3))
```

Maple [B] time = 0.011, size = 78, normalized size = 1.9

$$-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}\,(bx^3+a)^5}\left((bx^3+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/18*(6*b^5*x^15+15*a*b^4*x^12+20*a^2*b^3*x^9+15*a^3*b^2*x^6+6*a
^4*b*x^3+a^5)*((b*x^3+a)^2)^(5/2)/x^18/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259309, size = 77, normalized size = 1.88

$$-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x
^6 + 6*a^4*b*x^3 + a^5)/x^18
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**19, x)
```

GIAC/XCAS [A] time = 0.285515, size = 143, normalized size = 3.49

$$-\frac{6\,b^5x^{15}\mathrm{sign}\left(bx^3+a\right)+15\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right)+20\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right)+15\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+6\,a^4bx^3\mathrm{sign}\left(bx^3\right.}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(6*b^5*x^15*sign(b*x^3 + a) + 15*a*b^4*x^12*sign(b*x^3 + a)
 + 20*a^2*b^3*x^9*sign(b*x^3 + a) + 15*a^3*b^2*x^6*sign(b*x^3 + a
) + 6*a^4*b*x^3*sign(b*x^3 + a) + a^5*sign(b*x^3 + a))/x^18
```

3.83 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3))
 - (a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) -
 (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) -
(b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rubi [A] time = 0.16337, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3))
 - (a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) -
 (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) -
(b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rubi in Sympy [A] time = 27.9872, size = 211, normalized size = 0.83

$$\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{27664x^7(a+bx^3)}+\frac{81ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{1976x^{13}}+\frac{15a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{304x^{19}}-\frac{243b^4\sqrt{a^2+2abx^3+b^2x^6}}{3952x^7}-\frac{9b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{104x^{13}}-\frac{31\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{304x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
```

```
[Out] 729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(27664*x**7*(a + b
*x**3)) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x*
*6)/(1976*x**13) + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x*
*6)**(3/2)/(304*x**19) - 243*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x
**6)/(3952*x**7) - 9*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/
(104*x**13) - 31*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(304*x**1
9)
```

Mathematica [A] time = 0.0352852, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(1456a^5+8645a^4bx^3+21280a^3b^2x^6+27664a^2b^3x^9+19760ab^4x^{12}+6916b^5x^{15}\right)}{27664x^{19}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*
x^6 + 27664*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(276
64*x^19*(a + b*x^3))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/27664*(6916*b^5*x^15+19760*a*b^4*x^12+27664*a^2*b^3*x^9+21280*
a^3*b^2*x^6+8645*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/x^19/(b*
x^3+a)^5
```

Maxima [A] time = 0.790183, size = 80, normalized size = 0.32

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 +
21280*a^3*b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Fricas [A] time = 0.269533, size = 80, normalized size = 0.32

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 +
21280*a^3*b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**20, x)
```

GIAC/XCAS [A] time = 0.289573, size = 144, normalized size = 0.57

$$-\frac{6916\,b^5x^{15}\mathrm{sign}\left(bx^3+a\right)+19760\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right)+27664\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right)+21280\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+864}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(6916*b^5*x^15*sign(b*x^3 + a) + 19760*a*b^4*x^12*sign(b
*x^3 + a) + 27664*a^2*b^3*x^9*sign(b*x^3 + a) + 21280*a^3*b^2*x^6
*sign(b*x^3 + a) + 8645*a^4*b*x^3*sign(b*x^3 + a) + 1456*a^5*sign
(b*x^3 + a))/x^19
```

3.84 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*x^20*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)
) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rubi [A] time = 0.162943, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*x^20*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)
) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.8909, size = 211, normalized size = 0.83

$$\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{52360x^8(a+bx^3)}+\frac{81ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{2618x^{14}}+\frac{3a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{68x^{20}}-\frac{243b^4\sqrt{a^2+2abx^3+b^2x^6}}{6545x^8}-\frac{90b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{1309x^{14}}-\frac{8\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{85x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)
```

```
[Out] 729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(52360*x**8*(a + b
*x**3)) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x*
*6)/(2618*x**14) + 3*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**
6)**(3/2)/(68*x**20) - 243*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**
6)/(6545*x**8) - 90*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)/(
1309*x**14) - 8*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(85*x**20)
```

Mathematica [A] time = 0.0336709, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(2618a^5+15400a^4bx^3+37400a^3b^2x^6+47600a^2b^3x^9+32725ab^4x^{12}+10472b^5x^{15}\right)}{52360x^{20}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2
*x^6 + 47600*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(5
2360*x^20*(a + b*x^3))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x)
```

```
[Out] -1/52360*(10472*b^5*x^15+32725*a*b^4*x^12+47600*a^2*b^3*x^9+37400
*a^3*b^2*x^6+15400*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/x^20/(
b*x^3+a)^5
```

Maxima [A] time = 0.802587, size = 80, normalized size = 0.31

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 +
 37400*a^3*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Fricas [A] time = 0.253075, size = 80, normalized size = 0.31

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 +
 37400*a^3*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278601, size = 144, normalized size = 0.56

$$-\frac{10472\, b^5 x^{15} \operatorname{sign}\left(bx^3+a\right) + 32725\, ab^4 x^{12} \operatorname{sign}\left(bx^3+a\right) + 47600\, a^2 b^3 x^9 \operatorname{sign}\left(bx^3+a\right) + 37400\, a^3 b^2 x^6 \operatorname{sign}\left(bx^3+a\right) + 15}{52360\, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(10472*b^5*x^15*sign(b*x^3 + a) + 32725*a*b^4*x^12*sign(
b*x^3 + a) + 47600*a^2*b^3*x^9*sign(b*x^3 + a) + 37400*a^3*b^2*x^
6*sign(b*x^3 + a) + 15400*a^4*b*x^3*sign(b*x^3 + a) + 2618*a^5*si
gn(b*x^3 + a))/x^20
```

3.85 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*a*x^21) + (b
*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rubi [A] time = 0.110063, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*a*x^21) + (b
*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rubi in Sympy [A] time = 8.64034, size = 68, normalized size = 0.81

$$-\frac{\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{36ax^{21}}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{\frac{7}{2}}}{126a^2x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

```
[Out] -(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(36*a*x*
*21) + (a**2 + 2*a*b*x**3 + b**2*x**6)**(7/2)/(126*a**2*x**21)
```

Mathematica [A] time = 0.0322872, size = 83, normalized size = 0.99

$$-\frac{\sqrt{(a+bx^3)^2}\left(6a^5+35a^4bx^3+84a^3b^2x^6+105a^2b^3x^9+70ab^4x^{12}+21b^5x^{15}\right)}{126x^{21}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 10
5*a^2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(126*x^21*(a + b*x^
3))
```

Maple [A] time = 0.012, size = 80, normalized size = 1.

$$-\frac{21\,b^{5}x^{15}+70\,a b^{4}x^{12}+105\,a^{2}b^{3}x^{9}+84\,a^{3}b^{2}x^{6}+35\,a^{4}b x^{3}+6\,a^{5}}{126\,x^{21}\left(b x^{3}+a\right)^{5}}\left(\left(b x^{3}+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/126*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2*x^6+
35*a^4*b*x^3+6*a^5)*((b*x^3+a)^2)^(5/2)/x^21/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254495, size = 80, normalized size = 0.95

$$-\frac{21\,b^{5}x^{15}+70\,a b^{4}x^{12}+105\,a^{2}b^{3}x^{9}+84\,a^{3}b^{2}x^{6}+35\,a^{4}b x^{3}+6\,a^{5}}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^
2*x^6 + 35*a^4*b*x^3 + 6*a^5)/x^21
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280166, size = 144, normalized size = 1.71

$$-\frac{21\, b^5 x^{15} \mathrm{sign}\left(bx^3 + a\right) + 70\, ab^4 x^{12} \mathrm{sign}\left(bx^3 + a\right) + 105\, a^2 b^3 x^9 \mathrm{sign}\left(bx^3 + a\right) + 84\, a^3 b^2 x^6 \mathrm{sign}\left(bx^3 + a\right) + 35\, a^4 b x^3 \mathrm{sign}\left(b\right.}{126\, x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(21*b^5*x^15*sign(b*x^3 + a) + 70*a*b^4*x^12*sign(b*x^3 +
a) + 105*a^2*b^3*x^9*sign(b*x^3 + a) + 84*a^3*b^2*x^6*sign(b*x^3
+ a) + 35*a^4*b*x^3*sign(b*x^3 + a) + 6*a^5*sign(b*x^3 + a))/x^21
```

3.86 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*x^22*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3
)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3))
 - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi [A] time = 0.161161, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*x^22*(a + b*x^3)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3
)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3))
 - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.8141, size = 211, normalized size = 0.83

$$\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{152152x^{10}(a+bx^3)}+\frac{405ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{21736x^{16}}+\frac{15a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{418x^{22}}-\frac{1215b^4\sqrt{a^2+2abx^3+b^2x^6}}{76076x^{10}}-\frac{45b^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{988x^{16}}-\frac{17\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{209x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

```
[Out] 729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(152152*x**10*(a +
 b*x**3)) + 405*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2
*x**6)/(21736*x**16) + 15*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**
2*x**6)**(3/2)/(418*x**22) - 1215*b**4*sqrt(a**2 + 2*a*b*x**3 + b
**2*x**6)/(76076*x**10) - 45*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)
**(3/2)/(988*x**16) - 17*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(
209*x**22)
```

Mathematica [A] time = 0.0312652, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(6916a^5+40040a^4bx^3+95095a^3b^2x^6+117040a^2b^3x^9+76076ab^4x^{12}+21736b^5x^{15}\right)}{152152x^{22}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2
*x^6 + 117040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(
152152*x^22*(a + b*x^3))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/152152*(21736*b^5*x^15+76076*a*b^4*x^12+117040*a^2*b^3*x^9+950
95*a^3*b^2*x^6+40040*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/x^22
/(b*x^3+a)^5
```

Maxima [A] time = 0.785126, size = 80, normalized size = 0.31

$$-\frac{21736\, b^5 x^{15} + 76076\, a b^4 x^{12} + 117040\, a^2 b^3 x^9 + 95095\, a^3 b^2 x^6 + 40040\, a^4 b x^3 + 6916\, a^5}{152152\, x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9
 + 95095*a^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Fricas [A] time = 0.252037, size = 80, normalized size = 0.31

$$-\frac{21736\, b^5 x^{15} + 76076\, a b^4 x^{12} + 117040\, a^2 b^3 x^9 + 95095\, a^3 b^2 x^6 + 40040\, a^4 b x^3 + 6916\, a^5}{152152\, x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9
 + 95095*a^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280151, size = 144, normalized size = 0.56

$$-\frac{21736\,b^5x^{15}\mathrm{sign}\left(bx^3+a\right)+76076\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right)+117040\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right)+95095\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+4}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/152152*(21736*b^5*x^15*sign(b*x^3 + a) + 76076*a*b^4*x^12*sign
(b*x^3 + a) + 117040*a^2*b^3*x^9*sign(b*x^3 + a) + 95095*a^3*b^2*
x^6*sign(b*x^3 + a) + 40040*a^4*b*x^3*sign(b*x^3 + a) + 6916*a^5*
sign(b*x^3 + a))/x^22
```

3.87 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*x^23*(a + b*x^3)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) -
(5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3))
- (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3))
 - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3))
```

Rubi [A] time = 0.160998, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*x^23*(a + b*x^3)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) -
(5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3))
- (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3))
 - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3))
```

Rubi in Sympy [A] time = 26.8114, size = 211, normalized size = 0.83

$$\frac{729ab^4\sqrt{a^2+2abx^3+b^2x^6}}{240856x^{11}(a+bx^3)}+\frac{81ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{5474x^{17}}+\frac{3a(a+bx^3)(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{92x^{23}}-\frac{243b^4\sqrt{a^2+2abx^3+b^2x^6}}{21896x^{11}}-\frac{9b^2(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}{238x^{17}}-\frac{7(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}{92x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
```

```
[Out] 729*a*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(240856*x**11*(a +
 b*x**3)) + 81*a*b**2*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*
x**6)/(5474*x**17) + 3*a*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x
**6)**(3/2)/(92*x**23) - 243*b**4*sqrt(a**2 + 2*a*b*x**3 + b**2*x
**6)/(21896*x**11) - 9*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2
)/(238*x**17) - 7*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(92*x**2
3)
```

Mathematica [A] time = 0.0366637, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(10472a^5+60214a^4bx^3+141680a^3b^2x^6+172040a^2b^3x^9+109480ab^4x^{12}+30107b^5x^{15}\right)}{240856x^{23}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b
^2*x^6 + 172040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15)
)/(240856*x^23*(a + b*x^3))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}\,(bx^3+a)^5}\left((bx^3+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/240856*(30107*b^5*x^15+109480*a*b^4*x^12+172040*a^2*b^3*x^9+14
1680*a^3*b^2*x^6+60214*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/x
^23/(b*x^3+a)^5
```

Maxima [A] time = 0.783491, size = 80, normalized size = 0.31

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^
9 + 141680*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

Fricas [A] time = 0.253849, size = 80, normalized size = 0.31

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^
9 + 141680*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.272021, size = 144, normalized size = 0.56

$$-\frac{30107\,b^5x^{15}\mathrm{sign}\left(bx^3+a\right)+109480\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right)+172040\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right)+141680\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/240856*(30107*b^5*x^15*sign(b*x^3 + a) + 109480*a*b^4*x^12*sig
n(b*x^3 + a) + 172040*a^2*b^3*x^9*sign(b*x^3 + a) + 141680*a^3*b^
2*x^6*sign(b*x^3 + a) + 60214*a^4*b*x^3*sign(b*x^3 + a) + 10472*a
^5*sign(b*x^3 + a))/x^23
```

3.88 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{24ax^{24}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{84a^2x^{21}}-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{504a^3x^{18}}$$

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*a*x^24) + (b
*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (
b^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rubi [A] time = 0.145238, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{24ax^{24}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{84a^2x^{21}}-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{504a^3x^{18}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*a*x^24) + (b
*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (
b^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rubi in Sympy [A] time = 15.6485, size = 112, normalized size = 0.88

$$-\frac{(2a+2bx^3)\,(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}{48ax^{24}}+\frac{b\,(2a+2bx^3)\,(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}{144a^2x^{21}}-\frac{b\,(a^2+2abx^3+b^2x^6)^{\frac{7}{2}}}{504a^3x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

```
[Out] -(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/(48*a*x*
*24) + b*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)/
(144*a**2*x**21) - b*(a**2 + 2*a*b*x**3 + b**2*x**6)**(7/2)/(504*
a**3*x**21)
```

Mathematica [A] time = 0.0350055, size = 83, normalized size = 0.65

$$-\frac{\sqrt{(a+bx^3)^2}\left(21a^5+120a^4bx^3+280a^3b^2x^6+336a^2b^3x^9+210ab^4x^{12}+56b^5x^{15}\right)}{504x^{24}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 +
 336*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(504*x^24*(a +
b*x^3))
```

Maple [A] time = 0.015, size = 80, normalized size = 0.6

$$-\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}\left(bx^3+a\right)^5}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/504*(56*b^5*x^15+210*a*b^4*x^12+336*a^2*b^3*x^9+280*a^3*b^2*x^
6+120*a^4*b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/x^24/(b*x^3+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.251845, size = 80, normalized size = 0.62

$$-\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*
b^2*x^6 + 120*a^4*b*x^3 + 21*a^5)/x^24
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.260684, size = 144, normalized size = 1.12

$$-\frac{56\,b^5x^{15}\mathrm{sign}\left(bx^3+a\right)+210\,ab^4x^{12}\mathrm{sign}\left(bx^3+a\right)+336\,a^2b^3x^9\mathrm{sign}\left(bx^3+a\right)+280\,a^3b^2x^6\mathrm{sign}\left(bx^3+a\right)+120\,a^4bx^3\mathrm{sig}}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/504*(56*b^5*x^15*sign(b*x^3 + a) + 210*a*b^4*x^12*sign(b*x^3 +
 a) + 336*a^2*b^3*x^9*sign(b*x^3 + a) + 280*a^3*b^2*x^6*sign(b*x^
3 + a) + 120*a^4*b*x^3*sign(b*x^3 + a) + 21*a^5*sign(b*x^3 + a))/
x^24
```

3.89 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=240

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3
)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a +
 b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.278399, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3
)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a +
 b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\left(a+bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.105932, size = 131, normalized size = 0.55

$$\frac{\left(a+bx^3\right)\left(-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+3b^{2/3}x^2\right)}{6b^{5/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] -
 a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5
/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.016, size = 113, normalized size = 0.5

$$\frac{bx^3+a}{6\,b^2}\left(3\,x^2b\sqrt[3]{\frac{a}{b}}+2\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}a+2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)a-\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)\frac{1}{\sqrt{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(3*x^2*b*(a/b)^(1/3)+2*arctan(1/3*(-2*x+(a/b)^(1/3)
)*3^(1/2)/(a/b)^(1/3))*3^(1/2)*a+2*ln(x+(a/b)^(1/3))*a-ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt((b*x^3 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266416, size = 190, normalized size = 0.79

$$\frac{\sqrt{3}\left(3\sqrt{3}x^2-\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)+6\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}ax-\sqrt{3}b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}}{3b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}}\right)\right)}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt((b*x^3 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(3*sqrt(3)*x^2 - sqrt(3)*(a^2/b^2)^(1/3)*log(a*x^2 -
 b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + 2*sqrt(3)*(a^2/b^2)^(
1/3)*log(a*x + b*(a^2/b^2)^(2/3)) + 6*(a^2/b^2)^(1/3)*arctan(-1/3
*(2*sqrt(3)*a*x - sqrt(3)*b*(a^2/b^2)^(2/3))/(b*(a^2/b^2)^(2/3)))
)/b
```

Sympy [A] time = 1.35183, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^3b^5-a^2,\left(t\mapsto t\log\left(\frac{9t^2b^3}{a}+x\right)\right)\right)+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a +
x))) + x**2/(2*b)
```

GIAC/XCAS [A] time = 0.30396, size = 197, normalized size = 0.82

$$\frac{x^2 \operatorname{sign}\left(bx^3+a\right)}{2\,b}+\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sign}\left(bx^3+a\right)}{3\,b}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sign}\left(bx^3+a\right)}{3\,b^3}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sign}\left(bx^3+a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt((b*x^3 + a)^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sign(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*ln(abs(x - (-a/b)^(1
/3)))*sign(b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sign(b*x^3 + a)/b^3 - 1
/6*(-a*b^2)^(2/3)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sign(b*
x^3 + a)/b^3
```

3.90 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=235

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a
 + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x
^3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.263548, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a
 + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x
^3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\left(a+bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.0575173, size = 128, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}x\right)}{6b^{4/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1
/3)*x)/a^(1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a
^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3
)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.01, size = 110, normalized size = 0.5

$$\frac{bx^3+a}{6\,b^2}\left(6\,xb\left(\frac{a}{b}\right)^{2/3}+2\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}a-2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)a+\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)\frac{1}{\sqrt{(bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(6*x*b*(a/b)^(2/3)+2*arctan(1/3*(-2*x+(a/b)^(1/3))*
3^(1/2)/(a/b)^(1/3))*3^(1/2)*a-2*ln(x+(a/b)^(1/3))*a+ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt((b*x^3 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274634, size = 157, normalized size = 0.67

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-6\sqrt{3}x+6\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}x+\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt((b*x^3 + a)^2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3)) - 2*sqrt(3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 6*sq
rt(3)*x + 6*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*x + sqrt(3)*(-a/b)
^(1/3))/(-a/b)^(1/3)))/b
```

Sympy [A] time = 1.32502, size = 22, normalized size = 0.09

$$\operatorname{RootSum}\left(27t^3b^4+a,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

GIAC/XCAS [A] time = 0.316725, size = 193, normalized size = 0.82

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sign}\left(bx^3+a\right)}{3\,b}+\frac{x\operatorname{sign}\left(bx^3+a\right)}{b}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sign}\left(bx^3+a\right)}{3\,b^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sign}\left(bx^3+a\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt((b*x^3 + a)^2),x, algorithm="giac")
```

```
[Out] 1/3*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))*sign(b*x^3 + a)/b + x*
sign(b*x^3 + a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sign(b*x^3 + a)/b^2 - 1/6*(-a
*b^2)^(1/3)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sign(b*x^3 +
a)/b^2
```

3.91 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=44

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)
```

Rubi [A] time = 0.0825211, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)
```

Rubi in Sympy [A] time = 2.17478, size = 29, normalized size = 0.66

$$\frac{\sqrt{(a+bx^3)^2}\log(a+bx^3)}{3b(a+bx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] sqrt((a + b*x**3)**2)*log(a + b*x**3)/(3*b*(a + b*x**3))
```

Mathematica [A] time = 0.0147903, size = 35, normalized size = 0.8

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.009, size = 32, normalized size = 0.7

$$\frac{(bx^3+a)\ln(bx^3+a)}{3b}\frac{1}{\sqrt{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

Maxima [A] time = 0.782078, size = 23, normalized size = 0.52

$$\frac{1}{3}\sqrt{\frac{1}{b^2}}\log\left(x^3+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x^3 + a)^2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b^(-2))*log(x^3 + a/b)
```

Fricas [A] time = 0.258203, size = 18, normalized size = 0.41

$$\frac{\log(bx^3+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x^3 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/3*log(b*x^3 + a)/b
```

Sympy [A] time = 0.42149, size = 10, normalized size = 0.23

$$\frac{\log\left(a+bx^3\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x**3)/(3*b)
```

GIAC/XCAS [A] time = 0.295681, size = 30, normalized size = 0.68

$$\frac{\ln\left(\left|bx^3+a\right|\right)\operatorname{sign}\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x^3 + a)^2),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(b*x^3 + a))*sign(b*x^3 + a)/b
```

3.92 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=202

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a
 + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi [A] time = 0.19144, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a
 + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(a+bx^3)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.0478474, size = 109, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] - 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]))/(6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.008, size = 97, normalized size = 0.5

$$-\frac{bx^3+a}{6\,b}\left(2\,\sqrt{3}\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)+2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)\frac{1}{\sqrt{\left(bx^3+a\right)^2}}\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(
a/b)^(1/3))+2*ln(x+(a/b)^(1/3))-ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
)/((b*x^3+a)^2)^(1/2)/b/(a/b)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((b*x^3 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282005, size = 134, normalized size = 0.66

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2 - ab + \left(-ab^2\right)^{\frac{2}{3}}x\right) - 2\sqrt{3}\log\left(ab + \left(-ab^2\right)^{\frac{2}{3}}x\right) + 6\arctan\left(-\frac{\sqrt{3}ab - 2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}x}{3ab}\right)\right)}{18\left(-ab^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((b*x^3 + a)^2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^
(2/3)*x) - 2*sqrt(3)*log(a*b + (-a*b^2)^(2/3)*x) + 6*arctan(-1/3*
(sqrt(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)))/(-a*b^2)^(1/3)
```

Sympy [A] time = 0.431876, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3ab^2 + 1, \left(t \mapsto t\log\left(9t^2ab + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

GIAC/XCAS [A] time = 0.28277, size = 184, normalized size = 0.91

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sign}\left(bx^3 + a\right)}{3a} - \frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sign}\left(bx^3 + a\right)}{3ab^2} + \frac{\left(-ab^2\right)^{\frac{2}{3}}\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sign}\left(bx^3 + a\right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((b*x^3 + a)^2),x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(2/3)*ln(abs(x - (-a/b)^(1/3)))*sign(b*x^3 + a)/a - 1
/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))*sign(b*x^3 + a)/(a*b^2) + 1/6*(-a*b^2)^(2/3)*ln(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sign(b*x^3 + a)/(a*b^2)
```

3.93 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a
 + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi [A] time = 0.254339, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a
 + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(a+bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x**3)**2), x)
```

Mathematica [A] time = 0.039963, size = 109, normalized size = 0.54

$$-\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] - 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.006, size = 97, normalized size = 0.5

$$\frac{bx^3+a}{6\,b}\left(-2\,\sqrt{3}\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)+2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)\frac{1}{\sqrt{\left(bx^3+a\right)^2}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(
a/b)^(1/3))+2*ln(x+(a/b)^(1/3))-ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
)/((b*x^3+a)^2)^(1/2)/b/(a/b)^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^3 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259872, size = 120, normalized size = 0.59

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-2\sqrt{3}\log\left(\left(a^2b\right)^{\frac{1}{3}}x+a\right)-6\arctan\left(\frac{2\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}x-\sqrt{3}a}{3a}\right)\right)}{18\left(a^2b\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^3 + a)^2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x
+ a^2) - 2*sqrt(3)*log((a^2*b)^(1/3)*x + a) - 6*arctan(1/3*(2*sqr
t(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a))/(a^2*b)^(1/3)
```

Sympy [A] time = 0.467147, size = 20, normalized size = 0.1

$$\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

GIAC/XCAS [A] time = 0.293867, size = 165, normalized size = 0.82

$$-\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right)\operatorname{sign}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^3 + a)^2),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a*b) - (-a*b^2)^(1/3)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
*b))*sign(b*x^3 + a)
```

3.94 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
b*x^3)*Log[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0961869, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
b*x^3)*Log[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\left(a+bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a + b*x**3)**2)), x)
```

Mathematica [A] time = 0.0235559, size = 42, normalized size = 0.52

$$\frac{\left(a+bx^3\right)\left(3\log(x)-\log\left(a+bx^3\right)\right)}{3a\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*(3*Log[x] - Log[a + b*x^3]))/(3*a*Sqrt[(a + b*x^3)^2
])
```

Maple [A] time = 0.013, size = 37, normalized size = 0.5

$$-\frac{(bx^3+a)\left(\ln\left(bx^3+a\right)-3\ln(x)\right)}{3a}\frac{1}{\sqrt{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/3*(b*x^3+a)*(ln(b*x^3+a)-3*ln(x))/((b*x^3+a)^2)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274306, size = 24, normalized size = 0.3

$$-\frac{\log\left(bx^3+a\right)-3\log(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x),x, algorithm="fricas")
```

```
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [A] time = 0.721008, size = 15, normalized size = 0.19

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/((b*x**3+a)**2)**(1/2),x)

[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

GIAC/XCAS [A] time = 0.277561, size = 43, normalized size = 0.54

$$-\frac{1}{3}\left(\frac{\ln\left(|bx^3 + a|\right)}{a} - \frac{3\ln\left(|x|\right)}{a}\right)\operatorname{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x),x, algorithm="giac")

[Out] -1/3*(ln(abs(b*x^3 + a))/a - 3*ln(abs(x))/a)*sign(b*x^3 + a)
```

3.95 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=238

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(
a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.246815, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(
a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{(a+bx^3)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a + b*x**3)**2)), x)
```

Mathematica [A] time = 0.0651402, size = 133, normalized size = 0.56

$$-\frac{(a+bx^3)\left(\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{a}\right)}{6a^{4/3}x\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x]
+ b^(1/3)*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a
^(4/3)*x*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.013, size = 111, normalized size = 0.5

$$-\frac{bx^3+a}{6\,ax}\left(-2\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x-2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x+\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x+6\,\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(-2*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1
/3))*3^(1/2)*x-2*ln(x+(a/b)^(1/3))*x+ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))*x+6*(a/b)^(1/3))/((b*x^3+a)^2)^(1/2)/a/(a/b)^(1/3)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271098, size = 171, normalized size = 0.72

$$-\frac{\sqrt{3}\left(\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)-6x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}bx-\sqrt{3}a\left(\frac{b}{a}\right)^{\frac{2}{3}}}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)+ 6\sqrt{3}\right)}{18\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3)
+ a*(b/a)^(1/3)) - 2*sqrt(3)*x*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3
)) - 6*x*(b/a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*x - sqrt(3)*a*(b/a)
^(2/3))/(a*(b/a)^(2/3))) + 6*sqrt(3))/(a*x)
```

Sympy [A] time = 1.45133, size = 29, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3a^4-b,\left(t\mapsto t\log\left(\frac{9t^2a^3}{b}+x\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))
) - 1/(a*x)
```

GIAC/XCAS [A] time = 0.279593, size = 177, normalized size = 0.74

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b}-\frac{6}{ax}\right)\mathrm{sign}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(2/3)*ln(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a^2*b) - (-a*b^2)^(2/3)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a^2*b) - 6/(a*x))*sign(b*x^3 + a)
```

3.96 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=240

$$-\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -(a + b*x^3)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)
*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a +
b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.242496, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)
*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a +
b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\left(a+bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a + b*x**3)**2)), x)
```

Mathematica [A] time = 0.0683583, size = 140, normalized size = 0.58

$$-\frac{\left(a+bx^3\right)\left(-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+3a^{2/3}+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{5/3}x^2\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)
*x] - b^(2/3)*x^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
)/(6*a^(5/3)*x^2*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.013, size = 118, normalized size = 0.5

$$-\frac{bx^3+a}{6\,ax^2}\left(-2\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x^2+2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^2-\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^2+3\,\left(\frac{a}{b}\right)^{2/3}\right)\frac{1}{\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(-2*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1
/3))*3^(1/2)*x^2+2*ln(x+(a/b)^(1/3))*x^2-ln(x^2-x*(a/b)^(1/3)+(a/
b)^(2/3))*x^2+3*(a/b)^(2/3))/((b*x^3+a)^2)^(1/2)/a/(a/b)^(2/3)/x^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264275, size = 217, normalized size = 0.9

$$-\frac{\sqrt{3}\left(\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+6x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\cdots\right.\right.}{18\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(
-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 2*sqrt(3)*x^2*(-b^2/a^2
)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 6*x^2*(-b^2/a^2)^(1/3)*ar
ctan(1/3*(2*sqrt(3)*b*x + sqrt(3)*a*(-b^2/a^2)^(1/3))/(a*(-b^2/a^
2)^(1/3))) + 3*sqrt(3))/(a*x^2)
```

Sympy [A] time = 1.5714, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(-\frac{3ta^2}{b}+x\right)\right)\right)-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x)
)) - 1/(2*a*x**2)
```

GIAC/XCAS [A] time = 0.280367, size = 169, normalized size = 0.7

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{2\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2}-\frac{3}{ax^2}\right)\mathrm{sig}$$
$$+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(
-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/a^2 - (-a*b^2)^(1/3)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a
^2 - 3/(a*x^2))*sign(b*x^3 + a)
```

3.97 $\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=122

$$-\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -(a + b*x^3)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a +
b*x^3)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*
x^3)*Log[a + b*x^3])/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.124639, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a +
b*x^3)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*
x^3)*Log[a + b*x^3])/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{\left(a+bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt((a + b*x**3)**2)), x)
```

Mathematica [A] time = 0.0280344, size = 54, normalized size = 0.44

$$-\frac{(a+bx^3)\left(-bx^3\log(a+bx^3)+a+3bx^3\log(x)\right)}{3a^2x^3\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)*(a + 3*b*x^3*Log[x] - b*x^3*Log[a + b*x^3]))/(3*a^2
*x^3*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.018, size = 52, normalized size = 0.4

$$\frac{(bx^3+a)\left(b\ln(bx^3+a)x^3-3b\ln(x)x^3-a\right)}{3a^2x^3}\frac{1}{\sqrt{(bx^3+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^3+a)*(b*ln(b*x^3+a)*x^3-3*b*ln(x)*x^3-a)/((b*x^3+a)^2)^(
1/2)/a^2/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260216, size = 45, normalized size = 0.37

$$\frac{bx^3\log(bx^3+a)-3bx^3\log(x)-a}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

Sympy [A] time = 1.94945, size = 31, normalized size = 0.25

$$-\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b} + x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] -1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)
```

GIAC/XCAS [A] time = 0.279935, size = 68, normalized size = 0.56

$$\frac{1}{3}\left(\frac{b\ln\left(\left|bx^3 + a\right|\right)}{a^2} - \frac{3\,b\ln\left(|x|\right)}{a^2} + \frac{bx^3 - a}{a^2x^3}\right)\operatorname{sign}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^3 + a)^2)*x^4),x, algorithm="giac")
```

```
[Out] 1/3*(b*ln(abs(b*x^3 + a))/a^2 - 3*b*ln(abs(x))/a^2 + (b*x^3 - a)/
(a^2*x^3))*sign(b*x^3 + a)
```

3.98 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3
)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(1/3) + b^(1/
3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a
 + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(
4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.303458, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3
)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(1/3) + b^(1/
3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a
 + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(
4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.144227, size = 235, normalized size = 0.84

$$\frac{-3a^{4/3}b^{2/3}x^2+2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a +
b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x
^3)^2*Log[a^(1/3) + b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2] + 2*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.024, size = 299, normalized size = 1.1

$$\frac{bx^3+a}{54\,ab^2}\left(-2\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x^6b^2-2\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^6b^2+\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^6b^2+6\,\sqrt[3]{\frac{a}{b}}x^5b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-2*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/
2)*x^6*b^2-2*ln(x+(a/b)^(1/3))*x^6*b^2+ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))*x^6*b^2+6*(a/b)^(1/3)*x^5*b^2-4*arctan(1/3*(-2*x+(a/b)^(1
/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^3*a*b-4*ln(x+(a/b)^(1/3))*x^3
*a*b+2*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a*b-3*(a/b)^(1/3)*x^
2*a*b-2*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2
)*a^2-2*ln(x+(a/b)^(1/3))*a^2+ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a
^2)*(b*x^3+a)/(a/b)^(1/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278142, size = 278, normalized size = 0.99

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2-ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)-2\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)+6\,(\ldots}{162\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\left(-ab^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2)*log((-a*b^2)^
(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 2*sqrt(3)*(b^2*x^6 + 2*a*
b*x^3 + a^2)*log(a*b + (-a*b^2)^(2/3)*x) + 6*(b^2*x^6 + 2*a*b*x^3
 + a^2)*arctan(-1/3*(sqrt(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a
*b)) - 3*sqrt(3)*(2*b*x^5 - a*x^2)*(-a*b^2)^(1/3))/((a*b^3*x^6 +
2*a^2*b^2*x^3 + a^3*b)*(-a*b^2)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.642954, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.99 $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3
)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.293501, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3
)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.136401, size = 235, normalized size = 0.85

$$\frac{3a^{2/3}b^{4/3}x^4 - 2abx^3\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) - b^2x^6\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) - 6a^{5/3}\sqrt[3]{b}x - a^2\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\right.}{54a^{5/3}b^{4/3}\left(a + bx^3\right)\sqrt{\left(a + bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*
x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3
)^2*Log[a^(1/3) + b^(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2] - 2*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.023, size = 299, normalized size = 1.1

$$\frac{bx^3 + a}{54\,ab^2}\left(-2\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x + \sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x^6b^2 + 2\,\ln\left(x + \sqrt[3]{\frac{a}{b}}\right)x^6b^2 - \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^6b^2 + 3\,\left(\frac{a}{b}\right)^{2/3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-2*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/
2)*x^6*b^2+2*ln(x+(a/b)^(1/3))*x^6*b^2-ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))*x^6*b^2+3*(a/b)^(2/3)*x^4*b^2-4*arctan(1/3*(-2*x+(a/b)^(1
/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^3*a*b+4*ln(x+(a/b)^(1/3))*x^3
*a*b-2*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a*b-6*(a/b)^(2/3)*x*
a*b-2*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*
a^2+2*ln(x+(a/b)^(1/3))*a^2-ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^2
)*(b*x^3+a)/(a/b)^(2/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270987, size = 259, normalized size = 0.94

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-2\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(\left(a^2b\right)^{\frac{1}{3}}x+a\right)-6\left(b^2x^6+\right.\right.}{162\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\left(a^2b\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2)*log((a^2*b)^(
2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 2*sqrt(3)*(b^2*x^6 + 2*a*b*
x^3 + a^2)*log((a^2*b)^(1/3)*x + a) - 6*(b^2*x^6 + 2*a*b*x^3 + a^
2)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt
(3)*(b*x^4 - 2*a*x)*(a^2*b)^(1/3))/((a*b^3*x^6 + 2*a^2*b^2*x^3 +
a^3*b)*(a^2*b)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.708416, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.100 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0716736, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] -1/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [A] time = 9.28141, size = 36, normalized size = 0.95

$$-\frac{2a+2bx^3}{12b\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] -(2*a + 2*b*x**3)/(12*b*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2))
```

Mathematica [A] time = 0.0196767, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{6b\left(\left(a+bx^3\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -(a + b*x^3)/(6*b*((a + b*x^3)^2)^(3/2))
```

Maple [A] time = 0.01, size = 24, normalized size = 0.6

$$-\frac{bx^3+a}{6b}\left(\left(bx^3+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/6*(b*x^3+a)/b/((b*x^3+a)^2)^(3/2)
```

Maxima [A] time = 0.806821, size = 24, normalized size = 0.63

$$-\frac{1}{6\left(x^3+\frac{a}{b}\right)^2\left(b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6/((x^3 + a/b)^2*(b^2)^(3/2))
```

Fricas [A] time = 0.261232, size = 35, normalized size = 0.92

$$-\frac{1}{6\left(b^3x^6+2\,ab^2x^3+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.740985, size = 4, normalized size = 0.11

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x
```

3.101 $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\, dx$

Optimal. Leaf size=277

$$\frac{x^2}{6a\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b
*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2
/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*Log[a^(1/3)
 + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.29403, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{6a\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b
*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2
/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*Log[a^(1/3)
 + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.135443, size = 237, normalized size = 0.86

$$\frac{21a^{4/3}b^{2/3}x^2 + 4abx^3\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) + 2b^2x^6\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) + 2a^2\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{7/3}b^{2/3}\left(a + bx^3\right)\sqrt{\left(a + bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a +
 b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*
x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2] + 4*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.014, size = 301, normalized size = 1.1

$$\frac{bx^3 + a}{54\,a^2b}\left(-4\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x + \sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x^6b^2 - 4\,\ln\left(x + \sqrt[3]{\frac{a}{b}}\right)x^6b^2 + 2\,\ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{2/3}\right)x^6b^2 + 12\sqrt[3]{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-4*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/
2)*x^6*b^2-4*ln(x+(a/b)^(1/3))*x^6*b^2+2*ln(x^2-x*(a/b)^(1/3)+(a/
b)^(2/3))*x^6*b^2+12*(a/b)^(1/3)*x^5*b^2-8*arctan(1/3*(-2*x+(a/b)
^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^3*a*b-8*ln(x+(a/b)^(1/3))*
x^3*a*b+4*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a*b+21*(a/b)^(1/3
```

```
)*x^2*a*b-4*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^
(1/2)*a^2-4*ln(x+(a/b)^(1/3))*a^2+2*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2
/3))*a^2)*(b*x^3+a)/(a/b)^(1/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27214, size = 277, normalized size = 1.

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2-ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)-4\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)+1\right.}{162\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)\left(-ab^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(2*sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2)*log((-a*b^2
)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 4*sqrt(3)*(b^2*x^6 + 2*
a*b*x^3 + a^2)*log(a*b + (-a*b^2)^(2/3)*x) + 12*(b^2*x^6 + 2*a*b*
x^3 + a^2)*arctan(-1/3*(sqrt(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)
/(a*b)) - 3*sqrt(3)*(4*b*x^5 + 7*a*x^2)*(-a*b^2)^(1/3))/((a^2*b^2
*x^6 + 2*a^3*b*x^3 + a^4)*(-a*b^2)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.733697, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.102 $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a
 + b*x^3)^2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a +
 b*x^3)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*S
qrt[3]*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a
 + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^
3 + b^2*x^6)^(3/2))
```

Rubi [A] time = 0.306345, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```


```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a
 + b*x^3)^2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a +
 b*x^3)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*S
qrt[3]*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a
 + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^
3 + b^2*x^6)^(3/2))
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.147629, size = 235, normalized size = 0.82

$$\frac{15a^{2/3}b^{4/3}x^4 - 10abx^3\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) - 5b^2x^6\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) + 24a^{5/3}\sqrt[3]{b}x - 5a^2\log\left(a^{2/3} - \right.}{54a^{8/3}\sqrt[3]{b}\left(a + bx^3\right)\sqrt{\left(a + bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a +
b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*
x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2] - 10*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.012, size = 299, normalized size = 1.1

$$\frac{bx^3 + a}{54\,a^2b}\left(-10\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x + \sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x^6b^2 + 10\,\ln\left(x + \sqrt[3]{\frac{a}{b}}\right)x^6b^2 - 5\,\ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{2/3}\right)x^6b^2 + 15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-10*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1
/2)*x^6*b^2+10*ln(x+(a/b)^(1/3))*x^6*b^2-5*ln(x^2-x*(a/b)^(1/3)+(
a/b)^(2/3))*x^6*b^2+15*(a/b)^(2/3)*x^4*b^2-20*arctan(1/3*(-2*x+(a
/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^3*a*b+20*ln(x+(a/b)^(1/
3))*x^3*a*b-10*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a*b+24*(a/b)
^(2/3)*x*a*b-10*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3)
)*3^(1/2)*a^2+10*ln(x+(a/b)^(1/3))*a^2-5*ln(x^2-x*(a/b)^(1/3)+(a/
b)^(2/3))*a^2)*(b*x^3+a)/(a/b)^(2/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273105, size = 259, normalized size = 0.91

$$-\frac{\sqrt{3}\left(5\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-10\sqrt{3}\left(b^2x^6+2\,abx^3+a^2\right)\log\left(\left(a^2b\right)^{\frac{1}{3}}x+a\right)-30\,(b^2\cdots\right.}{162\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)\left(a^2b\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(5*sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2)*log((a^2*b)
^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 10*sqrt(3)*(b^2*x^6 + 2*a
*b*x^3 + a^2)*log((a^2*b)^(1/3)*x + a) - 30*(b^2*x^6 + 2*a*b*x^3
+ a^2)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*
sqrt(3)*(5*b*x^4 + 8*a*x)*(a^2*b)^(1/3))/((a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4)*(a^2*b)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

GIAC/XCAS [A]   time = 0.707593, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(-3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.103 $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.183795, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [A] time = 28.1173, size = 144, normalized size = 0.98

$$\frac{2a+2bx^3}{12a\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt{a^2+2abx^3+b^2x^6}\log\left(x^3\right)}{3a^3\left(a+bx^3\right)}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\log\left(a+bx^3\right)}{3a^3\left(a+bx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] (2*a + 2*b*x**3)/(12*a*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)) +
1/(3*a**2*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)) + sqrt(a**2 + 2*a*
```

```
b*x**3 + b**2*x**6)*log(x**3)/(3*a**3*(a + b*x**3)) - sqrt(a**2 +
 2*a*b*x**3 + b**2*x**6)*log(a + b*x**3)/(3*a**3*(a + b*x**3))
```

Mathematica [A] time = 0.0508981, size = 74, normalized size = 0.5

$$\frac{a\left(3a+2bx^3\right)+6\log(x)\left(a+bx^3\right)^2-2\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^3) + 6*(a + b*x^3)^2*Log[x] - 2*(a + b*x^3)^2*Log
[a + b*x^3])/(6*a^3*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.02, size = 107, normalized size = 0.7

$$-\frac{\left(2\ln\left(bx^3+a\right)x^6b^2-6\ln(x)x^6b^2+4\ln\left(bx^3+a\right)x^3ab-12\ln(x)x^3ab-2abx^3+2\ln\left(bx^3+a\right)a^2-6a^2\ln(x)-3a^2\right.}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/6*(2*ln(b*x^3+a)*x^6*b^2-6*ln(x)*x^6*b^2+4*ln(b*x^3+a)*x^3*a*b
-12*ln(x)*x^3*a*b-2*a*b*x^3+2*ln(b*x^3+a)*a^2-6*a^2*ln(x)-3*a^2)*
(b*x^3+a)/a^3/((b*x^3+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264618, size = 122, normalized size = 0.83

$$\frac{2\,abx^3 + 3\,a^2 - 2\left(b^2x^6 + 2\,abx^3 + a^2\right)\log\left(bx^3 + a\right) + 6\left(b^2x^6 + 2\,abx^3 + a^2\right)\log\left(x\right)}{6\left(a^3b^2x^6 + 2\,a^4bx^3 + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3
+ a) + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4
*b*x^3 + a^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.721415, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.104 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}+\frac{14\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{14\,(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^
3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*Log[a^(
1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
- (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.353361, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346

$$\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}+\frac{14\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{14\,(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^
3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*Log[a^(
1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

```
- (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.17722, size = 260, normalized size = 0.82

$$\frac{-14b^{7/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-28ab^{4/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-147a^{4/3}bx^3-54a^{7/3}-14a^2\sqrt[3]{b}x\log\left(\right.}{54a^{10/3}x\left(a+bx^3\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3
]*b^(1/3)*x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + 28*b^(1/3)*x*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a
^2*b^(1/3)*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*
a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 14
*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*
a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.027, size = 316, normalized size = 1.

$$-\frac{bx^3+a}{54\,xa^3}\left(-28\,\arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x^7b^2-28\,\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^7b^2+14\,\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{2/3}\right)x^7b^2+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-28*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(
1/2)*x^7*b^2-28*ln(x+(a/b)^(1/3))*x^7*b^2+14*ln(x^2-x*(a/b)^(1/3)
+(a/b)^(2/3))*x^7*b^2+84*(a/b)^(1/3)*x^6*b^2-56*arctan(1/3*(-2*x+
(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^4*a*b-56*ln(x+(a/b)^(
1/3))*x^4*a*b+28*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^4*a*b+147*(a
/b)^(1/3)*x^3*a*b-28*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^
(1/3))*3^(1/2)*x*a^2-28*ln(x+(a/b)^(1/3))*x*a^2+14*ln(x^2-x*(a/b)
^(1/3)+(a/b)^(2/3))*x*a^2+54*(a/b)^(1/3)*a^2)*(b*x^3+a)/x/(a/b)^(
1/3)/a^3/((b*x^3+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265524, size = 306, normalized size = 0.97

$$-\frac{\sqrt{3}\left(14\sqrt{3}\left(b^{2}x^{7}+2\,abx^{4}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\sqrt{3}\left(b^{2}x^{7}+2\,abx^{4}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right.\right.\right.}{162\left(a^{3}b^{2}x^{7}+2\,a^{4}bx^{4}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(14*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1
/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*sqrt(3)*(b^
2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) -
 84*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*arctan(-1/3*(2*sqrt
(3)*b*x - sqrt(3)*a*(b/a)^(2/3))/(a*(b/a)^(2/3))) + 3*sqrt(3)*(28
*b^2*x^6 + 49*a*b*x^3 + 18*a^2))/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5
*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.70722, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.105 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{20b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b
*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*
x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11
/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (20*b^(2/3)*(a + b*x^3)*Lo
g[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.356949, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{20b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b
*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*
x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11
/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (20*b^(2/3)*(a + b*x^3)*Lo
g[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
```

```
^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.197154, size = 266, normalized size = 0.84

$$\frac{20b^{8/3}x^8\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+40ab^{5/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-60a^{2/3}b^2x^6-96a^{5/3}bx^3-27a^{8/3}+20a}{54a^{11/3}x^2\left(a+bx^3\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]
*b^(2/3)*x^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] - 40*b^(2/3)*x^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 2
0*a^2*b^(2/3)*x^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
+ 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
 + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.027, size = 322, normalized size = 1.

$$-\frac{bx^3+a}{54\,x^2a^3}\left(-40\ \arctan\left(1/3\,\sqrt{3}\left(-2\,x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\sqrt{3}x^8b^2+40\ \ln\left(x+\sqrt[3]{\frac{a}{b}}\right)x^8b^2-20\ \ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{2/3}\right)x^8b^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-40*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(
1/2)*x^8*b^2+40*ln(x+(a/b)^(1/3))*x^8*b^2-20*ln(x^2-x*(a/b)^(1/3)
+(a/b)^(2/3))*x^8*b^2+60*(a/b)^(2/3)*x^6*b^2-80*arctan(1/3*(-2*x+
(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^5*a*b+80*ln(x+(a/b)^(
1/3))*x^5*a*b-40*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^5*a*b+96*(a/
b)^(2/3)*x^3*a*b-40*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(
1/3))*3^(1/2)*x^2*a^2+40*ln(x+(a/b)^(1/3))*x^2*a^2-20*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))*x^2*a^2+27*(a/b)^(2/3)*a^2)*(b*x^3+a)/x^2/
(a/b)^(2/3)/a^3/((b*x^3+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273772, size = 355, normalized size = 1.12

$$-\frac{\sqrt{3}\left(20\sqrt{3}\left(b^2x^8+2\,abx^5+a^2x^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-40\sqrt{3}\left(b^2x^8+2\,abx^5+a^2x^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\cdots\right.}{162\left(a^3b^2x^8+2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(20*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/
a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^
(2/3)) - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1
/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 120*(b^2*x^8 + 2*a*b*x^5 + a^
2*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x + sqrt(3)*a*(-b
^2/a^2)^(1/3))/(a*(-b^2/a^2)^(1/3))) + 3*sqrt(3)*(20*b^2*x^6 + 32
*a*b*x^3 + 9*a^2))/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left((a + bx^3)^2 \right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.731587, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3),x, algorithm="giac")

[Out] sage0*x
```

3.106 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*
x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])/(
a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.214629, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*
x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])/(
a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [A] time = 35.7636, size = 180, normalized size = 0.96

$$\frac{2a+2bx^3}{12ax^3\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}+\frac{1}{2a^2x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log\left(x^3\right)}{a^4\left(a+bx^3\right)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log\left(a+bx^3\right)}{a^4\left(a+bx^3\right)}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] (2*a + 2*b*x**3)/(12*a*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2
)) + 1/(2*a**2*x**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)) - b*sqrt
(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x**3)/(a**4*(a + b*x**3)) + b
*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(a + b*x**3)/(a**4*(a + b
*x**3)) - sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(a**4*x**3)
```

Mathematica [A] time = 0.0613676, size = 97, normalized size = 0.52

$$\frac{-a\left(2a^2+9abx^3+6b^2x^6\right)-18bx^3\log(x)\left(a+bx^3\right)^2+6bx^3\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^4x^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6)) - 18*b*x^3*(a + b*x^3)^2*Lo
g[x] + 6*b*x^3*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^4*x^3*(a + b*x^
3)*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.023, size = 133, normalized size = 0.7

$$\frac{\left(6\ln\left(bx^3+a\right)x^9b^3-18\,b^3\ln(x)\,x^9+12\ln\left(bx^3+a\right)x^6ab^2-36\,ab^2\ln(x)\,x^6-6\,ax^6b^2+6\ln\left(bx^3+a\right)x^3a^2b-18\,a^2b\ln}{6\,x^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/6*(6*ln(b*x^3+a)*x^9*b^3-18*b^3*ln(x)*x^9+12*ln(b*x^3+a)*x^6*a*
b^2-36*a*b^2*ln(x)*x^6-6*a*x^6*b^2+6*ln(b*x^3+a)*x^3*a^2*b-18*a^2
*b*ln(x)*x^3-9*x^3*a^2*b-2*a^3)*(b*x^3+a)/x^3/a^4/((b*x^3+a)^2)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274706, size = 161, normalized size = 0.86

$$-\frac{6\,ab^2x^6 + 9\,a^2bx^3 + 2\,a^3 - 6\,\left(b^3x^9 + 2\,ab^2x^6 + a^2bx^3\right)\log\left(bx^3 + a\right) + 18\,\left(b^3x^9 + 2\,ab^2x^6 + a^2bx^3\right)\log\left(x\right)}{6\,(a^4b^2x^9 + 2\,a^5bx^6 + a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^
6 + a^2*b*x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b
*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.69738, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.107 $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^4}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(
a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3
)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(8/3)*b^(7/3
)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3)*Log[a^(1/3) +
 b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(1458*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.411326, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^4}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(
a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3
)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(8/3)*b^(7/3
)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3)*Log[a^(1/3) +
 b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(1458*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.265722, size = 218, normalized size = 0.61

$$\frac{(a+bx^3)\left(-\frac{10(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}}+\frac{20(a+bx^3)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}+\frac{20\sqrt{3}(a+bx^3)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{8/3}}+\frac{30\sqrt[3]{b}x(a+bx^3)^3}{a^2}+18\cdots\right.}{2916b^{7/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b
^(1/3)*x*(a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (2
0*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/a^(8/3) + (20*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/
a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.028, size = 519, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-20*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^
(1/2)*x^12*b^4+20*ln(x+(a/b)^(1/3))*x^12*b^4-10*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))*x^12*b^4+30*(a/b)^(2/3)*x^10*b^4-80*arctan(1/3*(
-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^9*a*b^3+80*ln(x+
(a/b)^(1/3))*x^9*a*b^3-40*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^9*a
*b^3+108*(a/b)^(2/3)*x^7*a*b^3-120*arctan(1/3*(-2*x+(a/b)^(1/3))*
3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^6*a^2*b^2+120*ln(x+(a/b)^(1/3))*x^
6*a^2*b^2-60*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^6*a^2*b^2-225*(a
/b)^(2/3)*x^4*a^2*b^2-80*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a
/b)^(1/3))*3^(1/2)*x^3*a^3*b+80*ln(x+(a/b)^(1/3))*x^3*a^3*b-40*ln
(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a^3*b-60*(a/b)^(2/3)*x*a^3*b-
20*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*a^4
+20*ln(x+(a/b)^(1/3))*a^4-10*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^
4)*(b*x^3+a)/(a/b)^(2/3)/b^3/a^2/((b*x^3+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274164, size = 416, normalized size = 1.16

$$-\frac{\sqrt{3}\Big(10\sqrt{3}\left(b^4x^{12}+4ab^3x^9+6a^2b^2x^6+4a^3bx^3+a^4\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-20\sqrt{3}\left(b^4x^{12}+4ab^3x^9+6a^2b^2\cdots\right.}{8748\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8748*sqrt(3)*(10*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x
^6 + 4*a^3*b*x^3 + a^4)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x
 + a^2) - 20*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*
a^3*b*x^3 + a^4)*log((a^2*b)^(1/3)*x + a) - 60*(b^4*x^12 + 4*a*b^
3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*arctan(1/3*(2*sqrt(3)*
(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(10*b^3*x^10 + 36*a*b
^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)*(a^2*b)^(1/3))/((a^2*b^6*x^12 +
 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2)*(a^2*b)
^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 1.10826, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.108 $\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{a}{12b^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b
^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.124285, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{a}{12b^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b
^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [A] time = 14.879, size = 66, normalized size = 0.85

$$\frac{a(2a+2bx^3)}{24b^2(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}-\frac{1}{9b^2(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] a*(2*a + 2*b*x**3)/(24*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2
)) - 1/(9*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2))
```

Mathematica [A] time = 0.0335301, size = 39, normalized size = 0.5

$$\frac{-a-4bx^3}{36b^2(a+bx^3)^3\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (-a - 4*b*x^3)/(36*b^2*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.012, size = 32, normalized size = 0.4

$$-\frac{(bx^3+a)\,(4\,bx^3+a)}{36\,b^2}\left((bx^3+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(b*x^3+a)*(4*b*x^3+a)/b^2/((b*x^3+a)^2)^(5/2)
```

Maxima [A] time = 0.78543, size = 65, normalized size = 0.83

$$-\frac{1}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2}+\frac{a}{12\left(x^3+\frac{a}{b}\right)^4(b^2)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b
)^4*(b^2)^(5/2)*b)
```

Fricas [A] time = 0.262302, size = 78, normalized size = 1.

$$-\frac{4\,bx^3+a}{36\,(b^6x^{12}+4\,ab^5x^9+6\,a^2b^4x^6+4\,a^3b^3x^3+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a
^3*b^3*x^3 + a^4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.734401, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.109 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=368

$$\frac{7x^2}{324a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(
a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a +
b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a
 + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(a + b*x^3)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(10/
3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(a + b*x^3)*Log[
a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rubi [A] time = 0.424529, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346

$$\frac{7x^2}{324a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(
a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a +
b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a
 + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(a + b*x^3)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(10/
3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(a + b*x^3)*Log[
a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.269733, size = 229, normalized size = 0.62

$$\frac{\left(a+bx^3\right)\left(-243a^{10/3}b^{2/3}x^2+63a^{4/3}b^{2/3}x^2\left(a+bx^3\right)^2+54a^{7/3}b^{2/3}x^2\left(a+bx^3\right)+14\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x\right.\right.}{2916a^{10/3}b^{5/3}\left(\left(a+bx^3\right)^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*
(a + b*x^3) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b
^(2/3)*x^2*(a + b*x^3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1
/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 28*(a + b*x^3)^4*Log[a^(1
/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.028, size = 521, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-28*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^
(1/2)*x^12*b^4-28*ln(x+(a/b)^(1/3))*x^12*b^4+14*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))*x^12*b^4+84*(a/b)^(1/3)*x^11*b^4-112*arctan(1/3*
(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^9*a*b^3-112*ln(
x+(a/b)^(1/3))*x^9*a*b^3+56*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^9
*a*b^3+315*(a/b)^(1/3)*x^8*a*b^3-168*arctan(1/3*(-2*x+(a/b)^(1/3)
)*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^6*a^2*b^2-168*ln(x+(a/b)^(1/3))*
x^6*a^2*b^2+84*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^6*a^2*b^2+432*
(a/b)^(1/3)*x^5*a^2*b^2-112*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)
/(a/b)^(1/3))*3^(1/2)*x^3*a^3*b-112*ln(x+(a/b)^(1/3))*x^3*a^3*b+5
6*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a^3*b-42*(a/b)^(1/3)*x^2*
a^3*b-28*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/
2)*a^4-28*ln(x+(a/b)^(1/3))*a^4+14*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*a^4)*(b*x^3+a)/(a/b)^(1/3)/b^2/a^3/((b*x^3+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280486, size = 431, normalized size = 1.17

$$-\frac{\sqrt{3}\left(14\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2-ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)-28\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8748*sqrt(3)*(14*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x
^6 + 4*a^3*b*x^3 + a^4)*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)
^(2/3)*x) - 28*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 +
4*a^3*b*x^3 + a^4)*log(a*b + (-a*b^2)^(2/3)*x) + 84*(b^4*x^12 + 4
*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*arctan(-1/3*(sqrt
(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(28*b^3*
x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)*(-a*b^2)^(1/3)
)/((a^3*b^5*x^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3
+ a^7*b)*(-a*b^2)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.710137, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.110 $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=360

$$\frac{x}{81a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a +
b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3
)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(11/3)*b^(4/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)*Log[a^(1/3) +
b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.406346, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{x}{81a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a +
b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3
)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(11/3)*b^(4/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)*Log[a^(1/3) +
b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.278651, size = 221, normalized size = 0.61

$$\frac{(a+bx^3)\left(-20\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+60a^{2/3}\sqrt[3]{b}x\left(a+bx^3\right)^3+36a^{5/3}\sqrt[3]{b}x\left(a+bx^3\right)^2+27a^{8/3}\sqrt[3]{b}x\,(a+}{2916a^{11/3}b^{4/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a +
 b*x^3) + 36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)
*x*(a + b*x^3)^3 + 40*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 40*(a + b*x^3)^4*Log[a^(1/3) + b^
(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.027, size = 519, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-40*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^
(1/2)*x^12*b^4+40*ln(x+(a/b)^(1/3))*x^12*b^4-20*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))*x^12*b^4+60*(a/b)^(2/3)*x^10*b^4-160*arctan(1/3*
(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^9*a*b^3+160*ln(
x+(a/b)^(1/3))*x^9*a*b^3-80*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^9
*a*b^3+216*(a/b)^(2/3)*x^7*a*b^3-240*arctan(1/3*(-2*x+(a/b)^(1/3)
)*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^6*a^2*b^2+240*ln(x+(a/b)^(1/3))*
x^6*a^2*b^2-120*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^6*a^2*b^2+279
*(a/b)^(2/3)*x^4*a^2*b^2-160*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2
)/(a/b)^(1/3))*3^(1/2)*x^3*a^3*b+160*ln(x+(a/b)^(1/3))*x^3*a^3*b-
80*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a^3*b-120*(a/b)^(2/3)*x*
a^3*b-40*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/
2)*a^4+40*ln(x+(a/b)^(1/3))*a^4-20*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b^2/a^3/((b*x^3+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274966, size = 413, normalized size = 1.15

$$-\frac{\sqrt{3}\left(20\,\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-40\,\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2\right.\right.}{8748}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8748*sqrt(3)*(20*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x
^6 + 4*a^3*b*x^3 + a^4)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x
 + a^2) - 40*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*
a^3*b*x^3 + a^4)*log((a^2*b)^(1/3)*x + a) - 120*(b^4*x^12 + 4*a*b
```

```
^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*arctan(1/3*(2*sqrt(3)
*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(20*b^3*x^10 + 72*a*
b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)*(a^2*b)^(1/3))/((a^3*b^5*x^12
+ 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b)*(a^2*b)^
(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.724192, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.111 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{12b\,(a+bx^3)\,(a^2+2abx^3+b^2x^6)^{3/2}}$$

```
[Out] -1/(12*b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rubi [A] time = 0.0716311, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{1}{12b\,(a+bx^3)\,(a^2+2abx^3+b^2x^6)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/(12*b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rubi in Sympy [A] time = 9.26603, size = 36, normalized size = 0.95

$$-\frac{2a+2bx^3}{24b\,(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] -(2*a + 2*b*x**3)/(24*b*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2))
```

Mathematica [A] time = 0.0215019, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{12b\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a + b*x^3)/(12*b*((a + b*x^3)^2)^(5/2))
```

Maple [A] time = 0.011, size = 24, normalized size = 0.6

$$-\frac{bx^3+a}{12\,b}\left(\left(bx^3+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/12*(b*x^3+a)/b/((b*x^3+a)^2)^(5/2)
```

Maxima [A] time = 0.791476, size = 24, normalized size = 0.63

$$-\frac{1}{12\left(x^3+\frac{a}{b}\right)^4(b^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12/((x^3 + a/b)^4*(b^2)^(5/2))
```

Fricas [A] time = 0.264969, size = 65, normalized size = 1.71

$$-\frac{1}{12\left(b^5x^{12}+4\,ab^4x^9+6\,a^2b^3x^6+4\,a^3b^2x^3+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a
^4*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.708335, size = 4, normalized size = 0.11

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.112 $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a
 + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a
 + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*
(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (35*(a + b*x^3)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(
13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (35*(a + b*x^3)*
Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi [A] time = 0.398218, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a
 + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a
 + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*
(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (35*(a + b*x^3)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(
13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (35*(a + b*x^3)*
Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.235952, size = 219, normalized size = 0.61

$$\frac{(a+bx^3)\left(\frac{70(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+315a^{4/3}x^2\left(a+bx^3\right)^2+270a^{7/3}x^2\left(a+bx^3\right)+243a^{10/3}x^2-\frac{140(a+bx^3)^4\log\left(\sqrt[3]{a}\cdots\right.}{b^{2/3}}\cdots\right)}{2916a^{13/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 31
5*a^(4/3)*x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (14
0*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/b^(2/3) - (140*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])
/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.015, size = 521, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-140*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3
^(1/2)*x^12*b^4-140*ln(x+(a/b)^(1/3))*x^12*b^4+70*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))*x^12*b^4+420*(a/b)^(1/3)*x^11*b^4-560*arctan(1
/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^9*a*b^3-560*
ln(x+(a/b)^(1/3))*x^9*a*b^3+280*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
*x^9*a*b^3+1575*(a/b)^(1/3)*x^8*a*b^3-840*arctan(1/3*(-2*x+(a/b)^
(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^6*a^2*b^2-840*ln(x+(a/b)^(1
/3))*x^6*a^2*b^2+420*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^6*a^2*b^
2+2160*(a/b)^(1/3)*x^5*a^2*b^2-560*arctan(1/3*(-2*x+(a/b)^(1/3))*
3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^3*a^3*b-560*ln(x+(a/b)^(1/3))*x^3*
a^3*b+280*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a^3*b+1248*(a/b)^
(1/3)*x^2*a^3*b-140*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(
1/3))*3^(1/2)*a^4-140*ln(x+(a/b)^(1/3))*a^4+70*ln(x^2-x*(a/b)^(1/
3)+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(1/3)/b/a^4/((b*x^3+a)^2)^(5
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278379, size = 425, normalized size = 1.18

$$-\frac{\sqrt{3}\left(70\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2-ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)-140\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8748*sqrt(3)*(70*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x
^6 + 4*a^3*b*x^3 + a^4)*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)
^(2/3)*x) - 140*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 +
 4*a^3*b*x^3 + a^4)*log(a*b + (-a*b^2)^(2/3)*x) + 420*(b^4*x^12 +
 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*arctan(-1/3*(sq
rt(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(140*b
^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)*(-a*b^2)^(
1/3))/((a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^
3 + a^8)*(-a*b^2)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.721539, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.113 $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=364

$$\frac{11x(a+bx^3)^2}{108a^2(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{x(a+bx^3)}{12a(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}-\frac{110(a+bx^3)^5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}-\frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{55x(a+bx^3)^4}{243a^4(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{11x(a+bx^3)^3}{81a^3(a^2+2abx^3+b^2x^6)^{5/2}}$$

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*
(a + b*x^3)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*
x*(a + b*x^3)^3)/(81*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55
*x*(a + b*x^3)^4)/(243*a^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (
110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/
2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(14/3)*
b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/
3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))
```

Rubi [A] time = 0.406254, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{11x(a+bx^3)^2}{108a^2(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{x(a+bx^3)}{12a(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}-\frac{110(a+bx^3)^5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}-\frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{55x(a+bx^3)^4}{243a^4(a^2+2abx^3+b^2x^6)^{5/2}}+\frac{11x(a+bx^3)^3}{81a^3(a^2+2abx^3+b^2x^6)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*
(a + b*x^3)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*
x*(a + b*x^3)^3)/(81*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55
```

```
*x*(a + b*x^3)^4)/(243*a^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (
110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/
2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(14/3)*
b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/
3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.23057, size = 211, normalized size = 0.58

$$\frac{(a+bx^3)\left(-\frac{220(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+660a^{2/3}x\left(a+bx^3\right)^3+396a^{5/3}x\left(a+bx^3\right)^2+297a^{8/3}x\left(a+bx^3\right)+243a^{11/}\right.}{2916a^{14/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^
(5/3)*x*(a + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3
]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/b^(1/3) + (440*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
 - (220*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)^(5/2))
```

Maple [A] time = 0.013, size = 519, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-440*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3
^(1/2)*x^12*b^4+440*ln(x+(a/b)^(1/3))*x^12*b^4-220*ln(x^2-x*(a/b)
^(1/3)+(a/b)^(2/3))*x^12*b^4+660*(a/b)^(2/3)*x^10*b^4-1760*arctan
(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^9*a*b^3+17
60*ln(x+(a/b)^(1/3))*x^9*a*b^3-880*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*x^9*a*b^3+2376*(a/b)^(2/3)*x^7*a*b^3-2640*arctan(1/3*(-2*x+(a
/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^6*a^2*b^2+2640*ln(x+(a/
b)^(1/3))*x^6*a^2*b^2-1320*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^6*
a^2*b^2+3069*(a/b)^(2/3)*x^4*a^2*b^2-1760*arctan(1/3*(-2*x+(a/b)^
(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^3*a^3*b+1760*ln(x+(a/b)^(1/
3))*x^3*a^3*b-880*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^3*a^3*b+159
6*(a/b)^(2/3)*x*a^3*b-440*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(
a/b)^(1/3))*3^(1/2)*a^4+440*ln(x+(a/b)^(1/3))*a^4-220*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b/a^4/((b*x^3+a
)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(-5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27057, size = 408, normalized size = 1.12

$$-\frac{\sqrt{3}\left(220\,\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-440\,\sqrt{3}\left(b^4x^{12}+4\,ab^3x^9+6\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/8748*sqrt(3)*(220*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*
x^6 + 4*a^3*b*x^3 + a^4)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*
x + a^2) - 440*sqrt(3)*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 +
```

```
4*a^3*b*x^3 + a^4)*log((a^2*b)^(1/3)*x + a) - 1320*(b^4*x^12 + 4*
a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*arctan(1/3*(2*sqrt
(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(220*b^3*x^10 + 7
92*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)*(a^2*b)^(1/3))/((a^4*b
^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)*(a^2
*b)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.659759, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(-5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.114 $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^5*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.267277, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^5*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [A] time = 38.0024, size = 211, normalized size = 0.95

$$\frac{2a+2bx^3}{24a\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}+\frac{1}{9a^2\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}+\frac{2a+2bx^3}{12a^3\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}+\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt{a^2+2abx^3+b^2x^6}\log\left(x^3\right)}{3a^5\left(a+bx^3\right)}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\log\left(a+bx^3\right)}{3a^5\left(a+bx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] (2*a + 2*b*x**3)/(24*a*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2)) +
1/(9*a**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)) + (2*a + 2*b*x*
*3)/(12*a**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)) + 1/(3*a**4*
sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)) + sqrt(a**2 + 2*a*b*x**3 + b
**2*x**6)*log(x**3)/(3*a**5*(a + b*x**3)) - sqrt(a**2 + 2*a*b*x**
3 + b**2*x**6)*log(a + b*x**3)/(3*a**5*(a + b*x**3))
```

Mathematica [A] time = 0.0832497, size = 96, normalized size = 0.43

$$\frac{a\left(25a^3+52a^2bx^3+42ab^2x^6+12b^3x^9\right)+36\log(x)\left(a+bx^3\right)^4-12\left(a+bx^3\right)^4\log\left(a+bx^3\right)}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a +
b*x^3)^4*Log[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b
*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Maple [A] time = 0.022, size = 193, normalized size = 0.9

$$-\frac{\left(12\ln\left(bx^3+a\right)x^{12}b^4-36\ln(x)x^{12}b^4+48\ln\left(bx^3+a\right)x^9ab^3-144\ln(x)x^9ab^3-12x^9ab^3+72\ln\left(bx^3+a\right)x^6a^2b^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(12*ln(b*x^3+a)*x^12*b^4-36*ln(x)*x^12*b^4+48*ln(b*x^3+a)*x
^9*a*b^3-144*ln(x)*x^9*a*b^3-12*x^9*a*b^3+72*ln(b*x^3+a)*x^6*a^2*
b^2-216*ln(x)*x^6*a^2*b^2-42*x^6*a^2*b^2+48*ln(b*x^3+a)*x^3*a^3*b
-144*ln(x)*x^3*a^3*b-52*x^3*a^3*b+12*ln(b*x^3+a)*a^4-36*a^4*ln(x)
-25*a^4)*(b*x^3+a)/a^5/((b*x^3+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280096, size = 240, normalized size = 1.08

$$\frac{12\,ab^3x^9 + 42\,a^2b^2x^6 + 52\,a^3bx^3 + 25\,a^4 - 12\,\left(b^4x^{12} + 4\,ab^3x^9 + 6\,a^2b^2x^6 + 4\,a^3bx^3 + a^4\right)\log\left(bx^3 + a\right) + 36\,\left(b^4x^{12} + 4\,ab^3x\right.}{36\,(a^5b^4x^{12} + 4\,a^6b^3x^9 + 6\,a^7b^2x^6 + 4\,a^8bx^3 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*
(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(
b*x^3 + a) + 36*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b
*x^3 + a^4)*log(x))/(a^5*b^4*x^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6
 + 4*a^8*b*x^3 + a^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.676181, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.115 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^2}+\frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^3}+\frac{455\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455\sqrt[3]{b}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455\,(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}$$

```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a +
b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^
3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (455*(a + b*x^3))/(243*a^5*
x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*x^3)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(1
6/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*x^3)*
Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.477252, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^2}+\frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^3}+\frac{455\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455\sqrt[3]{b}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455\,(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a +
b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^
3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (455*(a + b*x^3))/(243*a^5*
x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*x^3)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(1
6/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*x^3)*
Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.276389, size = 242, normalized size = 0.61

$$\frac{(a+bx^3)\left(-910\sqrt[3]{b}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-243a^{10/3}bx^2-1179a^{4/3}bx^2\left(a+bx^3\right)^2-594a^{7/3}bx^2\left(a+bx\right.\right.}{2916a^{16/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3)
 - 1179*a^(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x
^3)^3 - (2916*a^(1/3)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a
+ b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1
820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 910*b^(1/3)*
(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2
916*a^(16/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.033, size = 536, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-1820*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))
*3^(1/2)*x^13*b^4-1820*ln(x+(a/b)^(1/3))*x^13*b^4+910*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))*x^13*b^4+5460*(a/b)^(1/3)*x^12*b^4-7280*ar
ctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^10*a*b
^3-7280*ln(x+(a/b)^(1/3))*x^10*a*b^3+3640*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))*x^10*a*b^3+20475*(a/b)^(1/3)*x^9*a*b^3-10920*arctan(1/
3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^7*a^2*b^2-109
20*ln(x+(a/b)^(1/3))*x^7*a^2*b^2+5460*ln(x^2-x*(a/b)^(1/3)+(a/b)^
(2/3))*x^7*a^2*b^2+28080*(a/b)^(1/3)*x^6*a^2*b^2-7280*arctan(1/3*
(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^4*a^3*b-7280*ln
(x+(a/b)^(1/3))*x^4*a^3*b+3640*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*
x^4*a^3*b+16224*(a/b)^(1/3)*x^3*a^3*b-1820*arctan(1/3*(-2*x+(a/b)
^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x*a^4-1820*ln(x+(a/b)^(1/3))
*x*a^4+910*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x*a^4+2916*(a/b)^(1/
3)*a^4)*(b*x^3+a)/x/(a/b)^(1/3)/a^5/((b*x^3+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268049, size = 455, normalized size = 1.14

$$-\frac{\sqrt{3}\left(910\,\sqrt{3}\left(b^4x^{13} + 4\,ab^3x^{10} + 6\,a^2b^2x^7 + 4\,a^3bx^4 + a^4x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 1820\,\sqrt{3}\left(b^4x^{13} + 4\,ab^3x^1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] -1/8748*sqrt(3)*(910*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2
*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/
```

```
3) + a*(b/a)^(1/3)) - 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a
^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(
2/3)) - 5460*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x
^4 + a^4*x)*(b/a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*x - sqrt(3)*a*(b
/a)^(2/3))/(a*(b/a)^(2/3))) + 3*sqrt(3)*(1820*b^4*x^12 + 6825*a*b
^3*x^9 + 9360*a^2*b^2*x^6 + 5408*a^3*b*x^3 + 972*a^4))/(a^5*b^4*x
^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4 + a^9*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.657459, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.116 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^2}+\frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^3}-\frac{770b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{770b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{385b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}$$

```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(
a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a
+ b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a
+ b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (385*(a + b*x^3))/(24
3*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (770*b^(2/3)*(a + b*
x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt
[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)*(a +
 b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi [A] time = 0.487287, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^2}+\frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^3}-\frac{770b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{770b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{385b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(
a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a
+ b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a
+ b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (385*(a + b*x^3))/(24
3*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (770*b^(2/3)*(a + b*
x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt
[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)*(a +
 b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.278541, size = 234, normalized size = 0.59

$$\frac{(a+bx^3)\left(1540b^{2/3}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-3162a^{2/3}bx\left(a+bx^3\right)^3-1314a^{5/3}bx\left(a+bx^3\right)^2-621a^{8/3}b}{2916a^{17/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1
314*a^(5/3)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 -
(1458*a^(2/3)*(a + b*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^
3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^
(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 1540*b^(2/3)*(a +
b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a
^(17/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] time = 0.033, size = 542, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-3080*arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))
*3^(1/2)*x^14*b^4+3080*ln(x+(a/b)^(1/3))*x^14*b^4-1540*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))*x^14*b^4+4620*(a/b)^(2/3)*x^12*b^4-12320*
arctan(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^11*a
*b^3+12320*ln(x+(a/b)^(1/3))*x^11*a*b^3-6160*ln(x^2-x*(a/b)^(1/3)
+(a/b)^(2/3))*x^11*a*b^3+16632*(a/b)^(2/3)*x^9*a*b^3-18480*arctan
(1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^8*a^2*b^2+
18480*ln(x+(a/b)^(1/3))*x^8*a^2*b^2-9240*ln(x^2-x*(a/b)^(1/3)+(a/
b)^(2/3))*x^8*a^2*b^2+21483*(a/b)^(2/3)*x^6*a^2*b^2-12320*arctan(
1/3*(-2*x+(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^5*a^3*b+123
20*ln(x+(a/b)^(1/3))*x^5*a^3*b-6160*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2
/3))*x^5*a^3*b+11172*(a/b)^(2/3)*x^3*a^3*b-3080*arctan(1/3*(-2*x+
(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3))*3^(1/2)*x^2*a^4+3080*ln(x+(a/b)
^(1/3))*x^2*a^4-1540*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*x^2*a^4+14
58*(a/b)^(2/3)*a^4)*(b*x^3+a)/x^2/(a/b)^(2/3)/a^5/((b*x^3+a)^2)^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276656, size = 504, normalized size = 1.27

$$-\frac{\sqrt{3}\left(1540\,\sqrt{3}\left(b^{4}x^{14}+4\,ab^{3}x^{11}+6\,a^{2}b^{2}x^{8}+4\,a^{3}bx^{5}+a^{4}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-3080\,\sqrt{3}\left(b^{4}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/8748*sqrt(3)*(1540*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^
2*x^8 + 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b
*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 3080*sqrt(3)*(b^4*x
^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a^3*b*x^5 + a^4*x^2)*(-b^2
/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 9240*(b^4*x^14 + 4*a*
b^3*x^11 + 6*a^2*b^2*x^8 + 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3
)*arctan(1/3*(2*sqrt(3)*b*x + sqrt(3)*a*(-b^2/a^2)^(1/3))/(a*(-b^
2/a^2)^(1/3))) + 3*sqrt(3)*(1540*b^4*x^12 + 5544*a*b^3*x^9 + 7161
*a^2*b^2*x^6 + 3724*a^3*b*x^3 + 486*a^4))/(a^5*b^4*x^14 + 4*a^6*b
^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.699327, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.117 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5b\log(x)\left(a+bx^3\right)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}-\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b
*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^
3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)*Log[x])/(a^6*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b*x^3])/(3*a^6*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.311404, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5b\log(x)\left(a+bx^3\right)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}-\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b
*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^
3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)*Log[x])/(a^6*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b*x^3])/(3*a^6*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [A] time = 46.0664, size = 264, normalized size = 0.98

$$\frac{2a+2bx^3}{24ax^3\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}+\frac{5}{36a^2x^3\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}+\frac{5\left(2a+2bx^3\right)}{36a^3x^3\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}+\frac{5}{6a^4x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5b\sqrt{a^2+2abx^3+b^2x^6}\log\left(x^3\right)}{3a^6\left(a+bx^3\right)}+\frac{5b\sqrt{a^2+2abx^3+b^2x^6}\log\left(a+bx^3\right)}{3a^6\left(a+bx^3\right)}-\frac{5\sqrt{a^2+2abx^3+b^2x^6}}{3a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] (2*a + 2*b*x**3)/(24*a*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2
)) + 5/(36*a**2*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2)) + 5*
(2*a + 2*b*x**3)/(36*a**3*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**(
3/2)) + 5/(6*a**4*x**3*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)) - 5*b
*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(x**3)/(3*a**6*(a + b*x**
3)) + 5*b*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*log(a + b*x**3)/(3*
a**6*(a + b*x**3)) - 5*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(3*a**
6*x**3)
```

Mathematica [A] time = 0.0958391, size = 119, normalized size = 0.44

$$\frac{-a\left(12a^4+125a^3bx^3+260a^2b^2x^6+210ab^3x^9+60b^4x^{12}\right)-180bx^3\log(x)\left(a+bx^3\right)^4+60bx^3\left(a+bx^3\right)^4\log\left(a+bx^3\right)}{36a^6x^3\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 +
60*b^4*x^12)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*
x^3)^4*Log[a + b*x^3])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)
^2])
```

Maple [A] time = 0.026, size = 219, normalized size = 0.8

$$\left(60\ln\left(bx^3+a\right)x^{15}b^5-180\,b^5\ln(x)\,x^{15}+240\ln\left(bx^3+a\right)x^{12}ab^4-720\,ab^4\ln(x)\,x^{12}-60\,ab^4x^{12}+360\ln\left(bx^3+a\right)x^9a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/36*(60*ln(b*x^3+a)*x^15*b^5-180*b^5*ln(x)*x^15+240*ln(b*x^3+a)*
x^12*a*b^4-720*a*b^4*ln(x)*x^12-60*a*b^4*x^12+360*ln(b*x^3+a)*x^9
*a^2*b^3-1080*a^2*b^3*ln(x)*x^9-210*a^2*b^3*x^9+240*ln(b*x^3+a)*x
^6*a^3*b^2-720*a^3*b^2*ln(x)*x^6-260*a^3*b^2*x^6+60*ln(b*x^3+a)*x
^3*a^4*b-180*a^4*b*ln(x)*x^3-125*a^4*b*x^3-12*a^5)*(b*x^3+a)/x^3/
a^6/((b*x^3+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26863, size = 279, normalized size = 1.04

$$-\frac{60\,ab^4x^{12} + 210\,a^2b^3x^9 + 260\,a^3b^2x^6 + 125\,a^4bx^3 + 12\,a^5 - 60\,\left(b^5x^{15} + 4\,ab^4x^{12} + 6\,a^2b^3x^9 + 4\,a^3b^2x^6 + a^4bx^3\right)\log\left(bx^3 + \right.}{36\,(a^6b^4x^{15} + 4\,a^7b^3x^{12} + 6\,a^8b^2x^9 + 4\,a^9bx^6 + a^{10}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^4),x, algorithm="fricas")
```

```
[Out] -1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^
4*b*x^3 + 12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 +
4*a^3*b^2*x^6 + a^4*b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b
^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6
*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b^2*x^9 + 4*a^9*b*x^6 + a^10*x
^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.659481, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.118 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=313

$$\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+16}}{d^{16}(m+16)(a+bx^3)}+\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+13}}{d^{13}(m+13)(a+bx^3)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)(a+bx^3)}+\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)(a+bx^3)}+\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)(a+bx^3)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)(a+bx^3)}$$

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a
 + b*x^3)) + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(d^4*(4 + m)*(a + b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7 + m)*(a + b*x^3)) + (10*a^2*b^3
*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^10*(10 + m)*(
a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rubi [A] time = 0.319728, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+16}}{d^{16}(m+16)(a+bx^3)}+\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+13}}{d^{13}(m+13)(a+bx^3)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)(a+bx^3)}+\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)(a+bx^3)}+\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)(a+bx^3)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```


```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a
 + b*x^3)) + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(d^4*(4 + m)*(a + b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7 + m)*(a + b*x^3)) + (10*a^2*b^3
*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^10*(10 + m)*(
a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rubi in Sympy [A] time = 66.403, size = 298, normalized size = 0.95

$$\frac{29160a^5(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}}{d(a+bx^3)(m+1)(m+4)(m+7)(m+10)(m+13)(m+16)} + \frac{9720a^4(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}}{d(m+4)(m+7)(m+10)(m+13)(m+16)} + \frac{1620a^3(dx)^{m+1}\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{d(m+7)(m+10)(m+13)(m+16)} + \frac{180a^2(dx)^{m+1}\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{d(m+10)(m+13)(m+16)} + \frac{15a(dx)^{m+1}\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{d(m+13)(m+16)} + \frac{(dx)^{m+1}\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}{d(m+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] 29160*a**5*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(d*
(a + b*x**3)*(m + 1)*(m + 4)*(m + 7)*(m + 10)*(m + 13)*(m + 16))
+ 9720*a**4*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(d
*(m + 4)*(m + 7)*(m + 10)*(m + 13)*(m + 16)) + 1620*a**3*(d*x)**(
m + 1)*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(d*(m + 7
)*(m + 10)*(m + 13)*(m + 16)) + 180*a**2*(d*x)**(m + 1)*(a**2 + 2
*a*b*x**3 + b**2*x**6)**(3/2)/(d*(m + 10)*(m + 13)*(m + 16)) + 15
*a*(d*x)**(m + 1)*(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**(
3/2)/(d*(m + 13)*(m + 16)) + (d*x)**(m + 1)*(a**2 + 2*a*b*x**3 +
b**2*x**6)**(5/2)/(d*(m + 16))
```

Mathematica [A] time = 0.12096, size = 111, normalized size = 0.35

$$\frac{\left(\left(a+bx^3\right)^2\right)^{5/2}(dx)^m\left(\frac{a^5x}{m+1}+\frac{5a^4bx^4}{m+4}+\frac{10a^3b^2x^7}{m+7}+\frac{10a^2b^3x^{10}}{m+10}+\frac{5ab^4x^{13}}{m+13}+\frac{b^5x^{16}}{m+16}\right)}{\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((d*x)^m*((a + b*x^3)^2)^(5/2)*((a^5*x)/(1 + m) + (5*a^4*b*x^4)/(
4 + m) + (10*a^3*b^2*x^7)/(7 + m) + (10*a^2*b^3*x^10)/(10 + m) +
(5*a*b^4*x^13)/(13 + m) + (b^5*x^16)/(16 + m)))/(a + b*x^3)^5
```

Maple [A] time = 0.011, size = 453, normalized size = 1.5

$$\left(b^5m^5x^{15} + 35\,b^5m^4x^{15} + 445\,b^5m^3x^{15} + 5\,ab^4m^5x^{12} + 2485\,b^5m^2x^{15} + 190\,ab^4m^4x^{12} + 5714\,b^5mx^{15} + 2555\,ab^4m^3x^{12} + 3640\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12
+2485*b^5*m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*
m^3*x^12+3640*b^5*x^15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+41
0*a^2*b^3*m^4*x^9+34840*a*b^4*m*x^12+5950*a^2*b^3*m^3*x^9+22400*a
*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a^2*b^3*m^2*x^9+440*a^3*b^2*m^
4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^6+58240*a^2*b^3*x^9+
5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^3+123920*a^
3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a^
4*b*m^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b
*x^3+8650*a^5*m^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5
/2)/(1+m)/(4+m)/(7+m)/(10+m)/(13+m)/(16+m)/(b*x^3+a)^5
```

Maxima [A] time = 0.802013, size = 328, normalized size = 1.05

$$\left(\left(m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640\right)b^5d^mx^{16} + 5\,\left(m^5 + 38\,m^4 + 511\,m^3 + 2962\,m^2 + 6968\,m + 4480\right)ab^4d^mx^1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] ((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16
 + 5*(m^5 + 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^
m*x^13 + 10*(m^5 + 41*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a
^2*b^3*d^m*x^10 + 10*(m^5 + 44*m^4 + 697*m^3 + 4726*m^2 + 12392*m
 + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m^4 + 817*m^3 + 6337*m^2 +
 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 + 955*m^3 + 8650*
m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m^4 +
9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Fricas [A] time = 0.280178, size = 498, normalized size = 1.59

$$\left(\left(b^5m^5 + 35\,b^5m^4 + 445\,b^5m^3 + 2485\,b^5m^2 + 5714\,b^5m + 3640\,b^5\right)x^{16} + 5\,\left(ab^4m^5 + 38\,ab^4m^4 + 511\,ab^4m^3 + 2962\,ab^4m^2 + 69\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m
+ 3640*b^5)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 +
2962*a*b^4*m^2 + 6968*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^
5 + 41*a^2*b^3*m^4 + 595*a^2*b^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^
2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b^2*m^5 + 44*a^3*b^2*m^4 +
 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^2*m + 8320*a^3*
b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 6337*a^4
*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4
 + 955*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)
^m/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58
240)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.318317, size = 1312, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b^5*m^5*x^16*e^(m*ln(d*x))*sign(b*x^3 + a) + 35*b^5*m^4*x^16*e^(
m*ln(d*x))*sign(b*x^3 + a) + 445*b^5*m^3*x^16*e^(m*ln(d*x))*sign(
b*x^3 + a) + 5*a*b^4*m^5*x^13*e^(m*ln(d*x))*sign(b*x^3 + a) + 248
5*b^5*m^2*x^16*e^(m*ln(d*x))*sign(b*x^3 + a) + 190*a*b^4*m^4*x^13
*e^(m*ln(d*x))*sign(b*x^3 + a) + 5714*b^5*m*x^16*e^(m*ln(d*x))*si
gn(b*x^3 + a) + 2555*a*b^4*m^3*x^13*e^(m*ln(d*x))*sign(b*x^3 + a)
 + 3640*b^5*x^16*e^(m*ln(d*x))*sign(b*x^3 + a) + 10*a^2*b^3*m^5*x
^10*e^(m*ln(d*x))*sign(b*x^3 + a) + 14810*a*b^4*m^2*x^13*e^(m*ln(
d*x))*sign(b*x^3 + a) + 410*a^2*b^3*m^4*x^10*e^(m*ln(d*x))*sign(b
```

```
*x^3 + a) + 34840*a*b^4*m*x^13*e^(m*ln(d*x))*sign(b*x^3 + a) + 59
50*a^2*b^3*m^3*x^10*e^(m*ln(d*x))*sign(b*x^3 + a) + 22400*a*b^4*x
^13*e^(m*ln(d*x))*sign(b*x^3 + a) + 10*a^3*b^2*m^5*x^7*e^(m*ln(d*
x))*sign(b*x^3 + a) + 36550*a^2*b^3*m^2*x^10*e^(m*ln(d*x))*sign(b
*x^3 + a) + 440*a^3*b^2*m^4*x^7*e^(m*ln(d*x))*sign(b*x^3 + a) + 8
9240*a^2*b^3*m*x^10*e^(m*ln(d*x))*sign(b*x^3 + a) + 6970*a^3*b^2*
m^3*x^7*e^(m*ln(d*x))*sign(b*x^3 + a) + 58240*a^2*b^3*x^10*e^(m*l
n(d*x))*sign(b*x^3 + a) + 5*a^4*b*m^5*x^4*e^(m*ln(d*x))*sign(b*x^
3 + a) + 47260*a^3*b^2*m^2*x^7*e^(m*ln(d*x))*sign(b*x^3 + a) + 23
5*a^4*b*m^4*x^4*e^(m*ln(d*x))*sign(b*x^3 + a) + 123920*a^3*b^2*m*
x^7*e^(m*ln(d*x))*sign(b*x^3 + a) + 4085*a^4*b*m^3*x^4*e^(m*ln(d*
x))*sign(b*x^3 + a) + 83200*a^3*b^2*x^7*e^(m*ln(d*x))*sign(b*x^3
+ a) + a^5*m^5*x*e^(m*ln(d*x))*sign(b*x^3 + a) + 31685*a^4*b*m^2*
x^4*e^(m*ln(d*x))*sign(b*x^3 + a) + 50*a^5*m^4*x*e^(m*ln(d*x))*si
gn(b*x^3 + a) + 100630*a^4*b*m*x^4*e^(m*ln(d*x))*sign(b*x^3 + a)
+ 955*a^5*m^3*x*e^(m*ln(d*x))*sign(b*x^3 + a) + 72800*a^4*b*x^4*e
^(m*ln(d*x))*sign(b*x^3 + a) + 8650*a^5*m^2*x*e^(m*ln(d*x))*sign(
b*x^3 + a) + 36824*a^5*m*x*e^(m*ln(d*x))*sign(b*x^3 + a) + 58240*
a^5*x*e^(m*ln(d*x))*sign(b*x^3 + a))/(m^6 + 51*m^5 + 1005*m^4 + 9
605*m^3 + 45474*m^2 + 95064*m + 58240)
```

3.119 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)(a+bx^3)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)(a+bx^3)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)(a+bx^3)}+\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)(a+bx^3)}$$

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a
 + b*x^3)) + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(d^4*(4 + m)*(a + b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(d^7*(7 + m)*(a + b*x^3)) + (b^3*(d*x)^(10
 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^10*(10 + m)*(a + b*x^3)
)
```

Rubi [A] time = 0.199124, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)(a+bx^3)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)(a+bx^3)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)(a+bx^3)}+\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a
 + b*x^3)) + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(d^4*(4 + m)*(a + b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(d^7*(7 + m)*(a + b*x^3)) + (b^3*(d*x)^(10
 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^10*(10 + m)*(a + b*x^3)
)
```

Rubi in Sympy [A] time = 30.6307, size = 182, normalized size = 0.89

$$\frac{162a^3(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}}{d(a+bx^3)(m+1)(m+4)(m+7)(m+10)}+\frac{54a^2(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}}{d(m+4)(m+7)(m+10)}+\frac{9a(dx)^{m+1}\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{d(m+7)(m+10)}+\frac{(dx)^{m+1}\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}{d(m+10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] 162*a**3*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(d*(a
 + b*x**3)*(m + 1)*(m + 4)*(m + 7)*(m + 10)) + 54*a**2*(d*x)**(m
+ 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(d*(m + 4)*(m + 7)*(m +
10)) + 9*a*(d*x)**(m + 1)*(a + b*x**3)*sqrt(a**2 + 2*a*b*x**3 + b
**2*x**6)/(d*(m + 7)*(m + 10)) + (d*x)**(m + 1)*(a**2 + 2*a*b*x**
3 + b**2*x**6)**(3/2)/(d*(m + 10))
```

Mathematica [A] time = 0.0704961, size = 79, normalized size = 0.39

$$\frac{\left(\left(a+bx^3\right)^2\right)^{3/2}(dx)^m\left(\frac{a^3x}{m+1}+\frac{3a^2bx^4}{m+4}+\frac{3ab^2x^7}{m+7}+\frac{b^3x^{10}}{m+10}\right)}{\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((d*x)^m*((a + b*x^3)^2)^(3/2)*((a^3*x)/(1 + m) + (3*a^2*b*x^4)/(
4 + m) + (3*a*b^2*x^7)/(7 + m) + (b^3*x^10)/(10 + m)))/(a + b*x^3
)^3
```

Maple [A] time = 0.009, size = 199, normalized size = 1.

$$\frac{\left(b^3m^3x^9+12\,b^3m^2x^9+39\,b^3mx^9+3\,ab^2m^3x^6+28\,b^3x^9+45\,ab^2m^2x^6+162\,ab^2mx^6+3\,a^2bm^3x^3+120\,ax^6b^2+54\,a^2bm^2x\right.}{(10+m)\,(7+m)\,(4+m)\,(1+m)\,(bx^3+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3
*x^9+45*a*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x
^6+54*a^2*b*m^2*x^3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*
m^2+138*a^3*m+280*a^3)*(d*x)^m*((b*x^3+a)^2)^(3/2)/(10+m)/(7+m)/(
4+m)/(1+m)/(b*x^3+a)^3
```

Maxima [A] time = 0.79959, size = 161, normalized size = 0.79

$$\frac{\left(\left(m^3+12\,m^2+39\,m+28\right)b^3d^mx^{10}+3\,\left(m^3+15\,m^2+54\,m+40\right)ab^2d^mx^7+3\,\left(m^3+18\,m^2+87\,m+70\right)a^2bd^mx^4+\left(m^3+2\right.\right.}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] ((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m
 + 40)*a*b^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4
 + (m^3 + 21*m^2 + 138*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 15
9*m^2 + 418*m + 280)
```

Fricas [A] time = 0.267396, size = 215, normalized size = 1.05

$$\frac{\left(\left(b^3m^3+12\,b^3m^2+39\,b^3m+28\,b^3\right)x^{10}+3\,\left(ab^2m^3+15\,ab^2m^2+54\,ab^2m+40\,ab^2\right)x^7+3\,\left(a^2bm^3+18\,a^2bm^2+87\,a^2bm+7\right.\right.}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 +
 15*a*b^2*m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^
2*b*m^2 + 87*a^2*b*m + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 13
8*a^3*m + 280*a^3)*x)*(d*x)^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 2
80)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.301742, size = 562, normalized size = 2.74

$$\frac{b^3m^3x^{10}e^{(m\ln(dx))}\mathrm{sign}\left(bx^3+a\right)+12\,b^3m^2x^{10}e^{(m\ln(dx))}\mathrm{sign}\left(bx^3+a\right)+39\,b^3mx^{10}e^{(m\ln(dx))}\mathrm{sign}\left(bx^3+a\right)+3\,ab^2m^3x^7e^{(m\ln(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^10*e^(m*ln(d*x))*sign(b*x^3 + a) + 12*b^3*m^2*x^10*e^(
m*ln(d*x))*sign(b*x^3 + a) + 39*b^3*m*x^10*e^(m*ln(d*x))*sign(b*x
^3 + a) + 3*a*b^2*m^3*x^7*e^(m*ln(d*x))*sign(b*x^3 + a) + 28*b^3*
x^10*e^(m*ln(d*x))*sign(b*x^3 + a) + 45*a*b^2*m^2*x^7*e^(m*ln(d*x
))*sign(b*x^3 + a) + 162*a*b^2*m*x^7*e^(m*ln(d*x))*sign(b*x^3 + a
) + 3*a^2*b*m^3*x^4*e^(m*ln(d*x))*sign(b*x^3 + a) + 120*a*b^2*x^7
*e^(m*ln(d*x))*sign(b*x^3 + a) + 54*a^2*b*m^2*x^4*e^(m*ln(d*x))*s
ign(b*x^3 + a) + 261*a^2*b*m*x^4*e^(m*ln(d*x))*sign(b*x^3 + a) +
a^3*m^3*x*e^(m*ln(d*x))*sign(b*x^3 + a) + 210*a^2*b*x^4*e^(m*ln(d
*x))*sign(b*x^3 + a) + 21*a^3*m^2*x*e^(m*ln(d*x))*sign(b*x^3 + a)
 + 138*a^3*m*x*e^(m*ln(d*x))*sign(b*x^3 + a) + 280*a^3*x*e^(m*ln(
d*x))*sign(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

3.120 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a +
 b*x^3)) + (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4
*(4 + m)*(a + b*x^3))
```

Rubi [A] time = 0.0928917, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a +
 b*x^3)) + (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4
*(4 + m)*(a + b*x^3))
```

Rubi in Sympy [A] time = 10.8222, size = 80, normalized size = 0.82

$$\frac{3a\,(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}}{d\left(a+bx^3\right)(m+1)\,(m+4)}+\frac{(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}}{d\,(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] 3*a*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)/(d*(a + b*
x**3)*(m + 1)*(m + 4)) + (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 +
b**2*x**6)/(d*(m + 4))
```

Mathematica [A] time = 0.0437938, size = 47, normalized size = 0.48

$$\frac{\sqrt{(a+bx^3)^2}(dx)^m\left(\frac{ax}{m+1}+\frac{bx^4}{m+4}\right)}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((d*x)^m*Sqrt[(a + b*x^3)^2]*((a*x)/(1 + m) + (b*x^4)/(4 + m)))/(
a + b*x^3)
```

Maple [A] time = 0.005, size = 56, normalized size = 0.6

$$\frac{\left(bmx^3+bx^3+am+4\,a\right)\,x\,(dx)^m}{(4+m)\,(1+m)\,(bx^3+a)}\sqrt{(bx^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

```
[Out] x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(4+m)/(1+m)
/(b*x^3+a)
```

Maxima [A] time = 0.795317, size = 47, normalized size = 0.48

$$\frac{\left(bd^m(m+1)x^4+ad^m(m+4)x\right)x^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

Fricas [A] time = 0.275705, size = 47, normalized size = 0.48

$$\frac{\left((bm+b)x^4+(am+4\,a)x\right)\,(dx)^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{(a + bx^3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

GIAC/XCAS [A] time = 0.314577, size = 123, normalized size = 1.27

$$\frac{bmx^4 e^{(m\ln(dx))}\mathrm{sign}\left(bx^3 + a\right) + bx^4 e^{(m\ln(dx))}\mathrm{sign}\left(bx^3 + a\right) + amxe^{(m\ln(dx))}\mathrm{sign}\left(bx^3 + a\right) + 4\,axe^{(m\ln(dx))}\mathrm{sign}\left(bx^3 + a\right)}{m^2 + 5\,m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b*m*x^4*e^(m*ln(d*x))*sign(b*x^3 + a) + b*x^4*e^(m*ln(d*x))*sign
(b*x^3 + a) + a*m*x*e^(m*ln(d*x))*sign(b*x^3 + a) + 4*a*x*e^(m*ln
(d*x))*sign(b*x^3 + a))/(m^2 + 5*m + 4)
```

3.121 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=73

$$\frac{(a+bx^3)\,(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi [A] time = 0.0883383, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{(a+bx^3)\,(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rubi in Sympy [A] time = 16.506, size = 61, normalized size = 0.84

$$\frac{(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{ad\,(a+bx^3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*hyper((1, m/3
+ 1/3), (m/3 + 4/3,), -b*x**3/a)/(a*d*(a + b*x**3)*(m + 1))
```

Mathematica [A] time = 0.0487395, size = 62, normalized size = 0.85

$$\frac{x\left(a+bx^3\right)(dx)^m\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{a(m+1)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m
)/3, -((b*x^3)/a)])/(a*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{b^2x^6+2\,abx^3+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^6+2\,abx^3+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^6+2\,abx^3+a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{(a + bx^3)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**3)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.122 $\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(a+bx^3)\,(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rubi [A] time = 0.0870962, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{(a+bx^3)\,(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rubi in Sympy [A] time = 16.2306, size = 63, normalized size = 0.86

$$\frac{(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}\,{}_2F_1\left(\begin{matrix}3,\frac{m}{3}+\frac{1}{3}\\\frac{m}{3}+\frac{4}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{a^3d\,(a+bx^3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*hyper((3, m/3
+ 1/3), (m/3 + 4/3,), -b*x**3/a)/(a**3*d*(a + b*x**3)*(m + 1))
```

Mathematica [A] time = 0.0556844, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^3\right)(dx)^m\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3(m+1)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3,
 -((b*x^3)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx)^m\left(b^2x^6+2\,abx^3+a^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

3.123 $\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}} \, dx$

Optimal. Leaf size=73

$$\frac{(a+bx^3)(dx)^{m+1} {}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rubi [A] time = 0.0850732, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{(a+bx^3)(dx)^{m+1} {}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rubi in Sympy [A] time = 16.1735, size = 63, normalized size = 0.86

$$\frac{(dx)^{m+1}\sqrt{a^2+2abx^3+b^2x^6}\,{}_2F_1\left(\begin{matrix}5, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle| -\frac{bx^3}{a}\right)}{a^5d\,(a+bx^3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**3 + b**2*x**6)*hyper((5, m/3
+ 1/3), (m/3 + 4/3,), -b*x**3/a)/(a**5*d*(a + b*x**3)*(m + 1))
```

Mathematica [A] time = 0.0637733, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^3\right)(dx)^m\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5(m+1)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3,
 -((b*x^3)/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (dx)^m\left(b^2x^6+2\,abx^3+a^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4)\sqrt{b^2x^6+2\,abx^3+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/((b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3
*b*x^3 + a^4)*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

3.124 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=77

$$\frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1
 + m)/3, -2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/
a)^(2*p))
```

Rubi [A] time = 0.0627538, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1
 + m)/3, -2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/
a)^(2*p))
```

Rubi in Sympy [A] time = 19.4657, size = 66, normalized size = 0.86

$$\frac{(dx)^{m+1}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\begin{matrix} -2p, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| -\frac{bx^3}{a}\right)}{d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] (d*x)**(m + 1)*(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x
**6)**p*hyper((-2*p, m/3 + 1/3), (m/3 + 4/3,), -b*x**3/a)/(d*(m +
 1))
```

Mathematica [A] time = 0.0637755, size = 66, normalized size = 0.86

$$\frac{x(dx)^m \left((a+bx^3)^2\right)^p \left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(\frac{m+1}{3}, -2p; \frac{m+1}{3}+1; -\frac{bx^3}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^p*Hypergeometric2F1[(1 + m)/3, -2*p, 1
 + (1 + m)/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int (dx)^m \left(b^2x^6 + 2\,ab x^3 + a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,ab x^3 + a^2\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2\,ab x^3 + a^2\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

3.125 $\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=172

$$\frac{\left(a+bx^3\right)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-\frac{a^3\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(2p+1)}$$

```
[Out] -(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^4*(1 + 2*p)
) + (a^2*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 +
 p)) - (a*(a + b*x^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 +
2*p)) + ((a + b*x^3)^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 +
 p))
```

Rubi [A] time = 0.22608, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(a+bx^3\right)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-\frac{a^3\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^4*(1 + 2*p)
) + (a^2*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 +
 p)) - (a*(a + b*x^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 +
2*p)) + ((a + b*x^3)^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 +
 p))
```

Rubi in Sympy [A] time = 45.8687, size = 184, normalized size = 1.07

$$-\frac{a^3\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4\left(p+2\right)\left(2p+1\right)\left(2p+3\right)}+\frac{a^2\left(a^2+2abx^3+b^2x^6\right)^{p+1}}{2b^4\left(p+1\right)\left(p+2\right)\left(2p+3\right)}-\frac{ax^6\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{4b^2\left(p+2\right)\left(2p+3\right)}+\frac{x^9\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{12b\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] -a**3*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(2*b**4
*(p + 2)*(2*p + 1)*(2*p + 3)) + a**2*(a**2 + 2*a*b*x**3 + b**2*x*
*6)**(p + 1)/(2*b**4*(p + 1)*(p + 2)*(2*p + 3)) - a*x**6*(2*a + 2
*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(4*b**2*(p + 2)*(2*p
+ 3)) + x**9*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/
(12*b*(p + 2))
```

Mathematica [A] time = 0.0725597, size = 110, normalized size = 0.64

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-3a^3+3a^2b(2p+1)x^3-3ab^2\left(2p^2+3p+1\right)x^6+b^3\left(4p^3+12p^2+11p+3\right)x^9\right)}{6b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 -
3*a*b^2*(1 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x
^9))/(6*b^4*(1 + p)*(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.012, size = 150, normalized size = 0.9

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p\left(-4\,b^3p^3x^9-12\,b^3p^2x^9-11\,b^3px^9-3\,b^3x^9+6\,ab^2p^2x^6+9\,ab^2px^6+3\,ax^6b^2-6\,a^2bpx^3-3\,x^3a^2b\right)}{6\,b^4\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*
b^3*p*x^9-3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a
^2*b*p*x^3-3*a^2*b*x^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+
25*p+6)
```

Maxima [A] time = 0.794362, size = 155, normalized size = 0.9

$$\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^{12}+2\left(2\,p^3+3\,p^2+p\right)ab^3x^9-3\left(2\,p^2+p\right)a^2b^2x^6+6\,a^3bpx^3-3\,a^4\right)\left(bx^3+a\right)^{2\,p}}{6\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^11,x, algorithm="maxima")
```

```
[Out] 1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)
*a*b^3*x^9 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(
b*x^3 + a)^(2*p)/((4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Fricas [A] time = 0.268128, size = 220, normalized size = 1.28

$$\frac{\left(\left(4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4\right)x^{12}+2\left(2\,ab^3p^3+3\,ab^3p^2+ab^3p\right)x^9+6\,a^3bpx^3-3\left(2\,a^2b^2p^2+a^2b^2p\right)x^6-3\,a^4\right)\left(b^2x^6+\right.}{6\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^11,x, algorithm="fricas")
```

```
[Out] 1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^
3*p^3 + 3*a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2
*p^2 + a^2*b^2*p)*x^6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b
^4*p^4 + 20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.296167, size = 536, normalized size = 3.12

$$\frac{4\,b^4p^3x^{12}e^{\left(p\ln\left(b^2x^6+2\,abx^3+a^2\right)\right)}+12\,b^4p^2x^{12}e^{\left(p\ln\left(b^2x^6+2\,abx^3+a^2\right)\right)}+11\,b^4px^{12}e^{\left(p\ln\left(b^2x^6+2\,abx^3+a^2\right)\right)}+4\,ab^3p^3x^9e^{\left(p\ln\left(b^2x^6+2\,abx^3\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^11,x, algorithm="giac")

[Out] 1/6*(4*b^4*p^3*x^12*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + 12*b^4*
p^2*x^12*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + 11*b^4*p*x^12*e^(p
*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + 4*a*b^3*p^3*x^9*e^(p*ln(b^2*x^6
 + 2*a*b*x^3 + a^2)) + 3*b^4*x^12*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a
^2)) + 6*a*b^3*p^2*x^9*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + 2*a*
b^3*p*x^9*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) - 6*a^2*b^2*p^2*x^6
*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) - 3*a^2*b^2*p*x^6*e^(p*ln(b^
2*x^6 + 2*a*b*x^3 + a^2)) + 6*a^3*b*p*x^3*e^(p*ln(b^2*x^6 + 2*a*b
*x^3 + a^2)) - 3*a^4*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)))/(4*b^4*
p^4 + 20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

3.126 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p))
 - (a*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)
) + ((a + b*x^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p
))
```

Rubi [A] time = 0.169132, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p))
 - (a*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)
) + ((a + b*x^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p
))
```

Rubi in Sympy [A] time = 29.1102, size = 128, normalized size = 0.98

$$\frac{a^2\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3\left(2p+1\right)\left(2p+3\right)}-\frac{a\left(a^2+2abx^3+b^2x^6\right)^{p+1}}{3b^3\left(p+1\right)\left(2p+3\right)}+\frac{x^6\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{6b\left(2p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] a**2*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(3*b**3*
(2*p + 1)*(2*p + 3)) - a*(a**2 + 2*a*b*x**3 + b**2*x**6)**(p + 1)
/(3*b**3*(p + 1)*(2*p + 3)) + x**6*(2*a + 2*b*x**3)*(a**2 + 2*a*b
*x**3 + b**2*x**6)**p/(6*b*(2*p + 3))
```

Mathematica [A] time = 0.0480362, size = 77, normalized size = 0.59

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(a^2-ab(2p+1)x^3+b^2\left(2p^2+3p+1\right)x^6\right)}{3b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1
+ 3*p + 2*p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.01, size = 96, normalized size = 0.7

$$\frac{\left(2\,b^2p^2x^6+3\,b^2px^6+b^2x^6-2\,abpx^3-abx^3+a^2\right)\left(bx^3+a\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{3\,b^3\left(4\,p^3+12\,p^2+11\,p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*
x^3+a^2)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Maxima [A] time = 0.795901, size = 107, normalized size = 0.82

$$\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^9+\left(2\,p^2+p\right)ab^2x^6-2\,a^2bpx^3+a^3\right)\left(bx^3+a\right)^{2\,p}}{3\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^8,x, algorithm="maxima")
```

```
[Out] 1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*
p*x^3 + a^3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A] time = 0.285186, size = 146, normalized size = 1.12

$$\frac{\left(\left(2\, b^{3} p^{2} + 3\, b^{3} p + b^{3}\right) x^{9} - 2\, a^{2} b p x^{3} + \left(2\, a b^{2} p^{2} + a b^{2} p\right) x^{6} + a^{3}\right) \left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)^{p}}{3 \left(4\, b^{3} p^{3} + 12\, b^{3} p^{2} + 11\, b^{3} p + 3\, b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^8,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p
^2 + a*b^2*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3
 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.269938, size = 336, normalized size = 2.58

$$\frac{2\, b^{3} p^{2} x^{9} e^{\left(p \ln\left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)\right)} + 3\, b^{3} p x^{9} e^{\left(p \ln\left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)\right)} + b^{3} x^{9} e^{\left(p \ln\left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)\right)} + 2\, a b^{2} p^{2} x^{6} e^{\left(p \ln\left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)\right)} + ab}{3 \left(4\, b^{3} p^{3} + 12\, b^{3} p^{2} + 11\, b^{3} p + 3\, b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^8,x, algorithm="giac")
```

```
[Out] 1/3*(2*b^3*p^2*x^9*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + 3*b^3*p*
x^9*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + b^3*x^9*e^(p*ln(b^2*x^6
 + 2*a*b*x^3 + a^2)) + 2*a*b^2*p^2*x^6*e^(p*ln(b^2*x^6 + 2*a*b*x^
3 + a^2)) + a*b^2*p*x^6*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) - 2*a
^2*b*p*x^3*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + a^3*e^(p*ln(b^2*
x^6 + 2*a*b*x^3 + a^2)))/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b
^3)
```

3.127 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

```
[Out] -(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p))
+ ((a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rubi [A] time = 0.124175, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p))
+ ((a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rubi in Sympy [A] time = 17.1614, size = 71, normalized size = 0.85

$$-\frac{a\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2\left(2p+1\right)}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{p+1}}{6b^2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] -a*(2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b**2*(2
*p + 1)) + (a**2 + 2*a*b*x**3 + b**2*x**6)**(p + 1)/(6*b**2*(p +
1))
```

Mathematica [A] time = 0.0288948, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p \left(b(2p+1)x^3-a\right)}{6b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1
+ p)*(1 + 2*p))
```

Maple [A] time = 0.009, size = 60, normalized size = 0.7

$$-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}\left(-2\,x^{3}pb-bx^{3}+a\right)\left(bx^{3}+a\right)}{6\,b^{2}\left(2\,p^{2}+3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-2*b*p*x^3-b*x^3+a)*(b*x^3+a)/b^2
/(2*p^2+3*p+1)
```

Maxima [A] time = 0.777646, size = 73, normalized size = 0.87

$$\frac{\left(b^{2}(2\,p+1)x^{6}+2\,abpx^{3}-a^{2}\right)\left(bx^{3}+a\right)^{2\,p}}{6\,(2\,p^{2}+3\,p+1)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^5,x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2
*p^2 + 3*p + 1)*b^2)
```

Fricas [A] time = 0.267309, size = 95, normalized size = 1.13

$$\frac{\left(\left(2\,b^{2}p+b^{2}\right)x^{6}+2\,abpx^{3}-a^{2}\right)\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{6\,(2\,b^{2}p^{2}+3\,b^{2}p+b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^5,x, algorithm="fricas")
```

```
[Out] 1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^
3 + a^2)^p/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.28051, size = 189, normalized size = 2.25

$$\frac{2\,b^2px^6e^{\left(p\ln\left(b^2x^6+2\,abx^3+a^2\right)\right)}+b^2x^6e^{\left(p\ln\left(b^2x^6+2\,abx^3+a^2\right)\right)}+2\,abpx^3e^{\left(p\ln\left(b^2x^6+2\,abx^3+a^2\right)\right)}-a^2e^{\left(p\ln\left(b^2x^6+2\,abx^3+a^2\right)\right)}}{6\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^5,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*p*x^6*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + b^2*x^6*e^
(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + 2*a*b*p*x^3*e^(p*ln(b^2*x^6 +
 2*a*b*x^3 + a^2)) - a^2*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)))/(2*
b^2*p^2 + 3*b^2*p + b^2)
```

3.128 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

```
[Out] (x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8
/3, -((b*x^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0549996, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8
/3, -((b*x^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.4698, size = 53, normalized size = 0.88

$$\frac{x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\begin{matrix}-2p,\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] x**5*(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*hy
per((-2*p, 5/3), (8/3,), -b*x**3/a)/5
```

Mathematica [A] time = 0.0286049, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*((a + b*x^3)^2)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x^3
)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x^{4} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^{2} x^{6}+2 a b x^{3}+a^{2}\right)^{p} x^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2} x^{6}+2 a b x^{3}+a^{2}\right)^{p} x^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4,x, algorithm="giac")

[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

3.129 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

```
[Out] (x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7
/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0551676, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7
/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Rubi in Sympy [A] time = 18.287, size = 53, normalized size = 0.88

$$\frac{x^4\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\begin{matrix}-2p,\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] x**4*(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*hy
per((-2*p, 4/3), (7/3,), -b*x**3/a)/4
```

Mathematica [A] time = 0.0205704, size = 51, normalized size = 0.85

$$\frac{1}{4}x^4\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*((a + b*x^3)^2)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x^3
)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int x^{3}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}x^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

3.130 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rubi [A] time = 0.0612889, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rubi in Sympy [A] time = 10.4896, size = 37, normalized size = 0.9

$$\frac{\left(2a+2bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{6b\,(2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] (2*a + 2*b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*(2*p + 1
))
```

Mathematica [A] time = 0.017983, size = 30, normalized size = 0.73

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p}{3(2bp+b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p)/(3*(b + 2*b*p))
```

Maple [A] time = 0.008, size = 40, normalized size = 1.

$$\frac{\left(bx^{3}+a\right)\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{3\,b\,(1+2\,p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

Maxima [A] time = 0.772493, size = 41, normalized size = 1.

$$\frac{\left(bx^{3}+a\right)\left(bx^{3}+a\right)^{2\,p}}{3\,b(2\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

Fricas [A] time = 0.268102, size = 50, normalized size = 1.22

$$\frac{\left(bx^{3}+a\right)\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{p}}{3\,(2\,bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.282747, size = 84, normalized size = 2.05

$$\frac{bx^3 e^{\left(p \ln\left(b^2 x^6 + 2\,abx^3 + a^2\right)\right)} + ae^{\left(p \ln\left(b^2 x^6 + 2\,abx^3 + a^2\right)\right)}}{3\,(2\,bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*x^3*e^(p*ln(b^2*x^6 + 2*a*b*x^3 + a^2)) + a*e^(p*ln(b^2*x^
6 + 2*a*b*x^3 + a^2)))/(2*b*p + b)
```

3.131 $\int x \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=58

$$\frac{x^2 \left(a+bx^3\right) \left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+\frac{5}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] (x^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[
1, 5/3 + 2*p, 5/3, -((b*x^3)/a)])/(2*a)
```

Rubi [A] time = 0.0433727, antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}x^2\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2/3, -2*p, 5
/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

Rubi in Sympy [A] time = 15.6771, size = 53, normalized size = 0.91

$$\frac{x^2\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\begin{matrix}-2p,\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] x**2*(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*hy
per((-2*p, 2/3), (5/3,), -b*x**3/a)/2
```

Mathematica [A] time = 0.0182643, size = 51, normalized size = 0.88

$$\frac{1}{2}x^2\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*((a + b*x^3)^2)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x^3
)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int x \left(b^2 x^6 + 2\,ab x^3 + a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^6 + 2\,ab x^3 + a^2\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 x^6 + 2\,ab x^3 + a^2\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6+2\,abx^3+a^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

3.132 $\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(1,2p+\frac{4}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a}$$

```
[Out] (x*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1,
 4/3 + 2*p, 4/3, -((b*x^3)/a)])/a
```

Rubi [A] time = 0.0378543, antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$x\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1/3, -2*p, 4/3
, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2*p)
```

Rubi in Sympy [A] time = 7.03083, size = 49, normalized size = 0.92

$$x\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\begin{matrix}-2p,\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] x*(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*hyper
((-2*p, 1/3), (4/3,), -b*x**3/a)
```

Mathematica [C] time = 0.315211, size = 204, normalized size = 3.85

$$\frac{4^{-p}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-2p}\left(\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-2p}\left(\left(a+bx^3\right)^2\right)^p F_1\left(2p+1;-2p,-2p;2(p+1);-\frac{i\left(\sqrt[3]{b}x+(-1)^{2/3}\sqrt[3]{a}\right.}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt[3]{b}(2p+1)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a + b*x^3)^2)^p*AppellF1[1 +
2*p, -2*p, -2*p, 2*(1 + p), ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x
))/(Sqrt[3]*a^(1/3)), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(
3*I + Sqrt[3])])/(4^p*b^(1/3)*(1 + 2*p)*((a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^(2*p)*((I*(1 + (b^(1/3)*x)/a
^(1/3)))/(3*I + Sqrt[3]))^(2*p))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

3.133 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x} dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

```
[Out] -((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1,
1 + 2*p, 2*(1 + p), 1 + (b*x^3)/a])/(3*a*(1 + 2*p))
```

Rubi [A] time = 0.080366, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] -((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1,
1 + 2*p, 2*(1 + p), 1 + (b*x^3)/a])/(3*a*(1 + 2*p))
```

Rubi in Sympy [A] time = 18.1519, size = 76, normalized size = 1.21

$$-\frac{(ab+b^2x^3)^{-2p}\,(ab+b^2x^3)^{2p+1}\,(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(\begin{matrix}1,2p+1\\2p+2\end{matrix}\middle|1+\frac{bx^3}{a}\right)}{3ab\,(2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
```

```
[Out] -(a*b + b**2*x**3)**(-2*p)*(a*b + b**2*x**3)**(2*p + 1)*(a**2 + 2
*a*b*x**3 + b**2*x**6)**p*hyper((1, 2*p + 1), (2*p + 2,), 1 + b*x
**3/a)/(3*a*b*(2*p + 1))
```

Mathematica [A] time = 0.0363654, size = 53, normalized size = 0.84

$$\frac{\left(\frac{a}{bx^3}+1\right)^{-2p}\left(\left(a+bx^3\right)^2\right)^p\,{}_2F_1\left(-2p,-2p;1-2p;-\frac{a}{bx^3}\right)}{6p}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] (((a + b*x^3)^2)^p*Hypergeometric2F1[-2*p, -2*p, 1 - 2*p, -(a/(b*
x^3))])/(6*p*(1 + a/(b*x^3))^(2*p))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

3.134 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

```
[Out] -(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/
3, -((b*x^3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Rubi [A] time = 0.0496543, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/
3, -((b*x^3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Rubi in Sympy [A] time = 17.2192, size = 53, normalized size = 0.91

$$-\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\left.\begin{matrix}-2p,-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right|-\frac{bx^3}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
```

```
[Out] -(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*hyper(
(-2*p, -1/3), (2/3,), -b*x**3/a)/x
```

Mathematica [A] time = 0.0294807, size = 49, normalized size = 0.84

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -((((a + b*x^3)^2)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^3)
/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

3.135 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3
, -((b*x^3)/a)])/(2*x^2*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.0487008, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3
, -((b*x^3)/a)])/(2*x^2*(1 + (b*x^3)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.1874, size = 56, normalized size = 0.93

$$-\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\begin{matrix}-2p,-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
```

```
[Out] -(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*hyper(
(-2*p, -2/3), (1/3,), -b*x**3/a)/(2*x**2)
```

Mathematica [A] time = 0.0238109, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -(((a + b*x^3)^2)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3)/
a)])/(2*x^2*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

3.136 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx$

Optimal. Leaf size=64

$$\frac{b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(2,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

```
[Out] (b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2,
 1 + 2*p, 2*(1 + p), 1 + (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Rubi [A] time = 0.0850723, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(2,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2,
 1 + 2*p, 2*(1 + p), 1 + (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Rubi in Sympy [A] time = 18.1037, size = 75, normalized size = 1.17

$$\frac{\left(ab+b^2x^3\right)^{-2p}\left(ab+b^2x^3\right)^{2p+1}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\begin{matrix}2,2p+1\\2p+2\end{matrix}\middle|1+\frac{bx^3}{a}\right)}{3a^2\,(2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)
```

```
[Out] (a*b + b**2*x**3)**(-2*p)*(a*b + b**2*x**3)**(2*p + 1)*(a**2 + 2*
a*b*x**3 + b**2*x**6)**p*hyper((2, 2*p + 1), (2*p + 2,), 1 + b*x*
*3/a)/(3*a**2*(2*p + 1))
```

Mathematica [A] time = 0.028491, size = 62, normalized size = 0.97

$$\frac{\left(\frac{a}{bx^3}+1\right)^{-2p}\left(\left(a+bx^3\right)^2\right)^p\,{}_2F_1\left(1-2p,-2p;2-2p;-\frac{a}{bx^3}\right)}{3(2p-1)x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (((a + b*x^3)^2)^p*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, -(a/
(b*x^3))])/(3*(-1 + 2*p)*(1 + a/(b*x^3))^(2*p)*x^3)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4,x, algorithm="fricas")

[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^{2} x^{6} + 2\, a b x^{3} + a^{2}\right)^{p}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4,x, algorithm="giac")

[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

3.137 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/
3, -((b*x^3)/a)])/(4*x^4*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] time = 0.04904, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/
3, -((b*x^3)/a)])/(4*x^4*(1 + (b*x^3)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.2184, size = 58, normalized size = 0.97

$$-\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p{}_2F_1\left(\begin{matrix}-2p,-\frac{4}{3}\\ -\frac{1}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
```

```
[Out] -(1 + b*x**3/a)**(-2*p)*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*hyper(
(-2*p, -4/3), (-1/3,), -b*x**3/a)/(4*x**4)
```

Mathematica [A] time = 0.0310892, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -(((a + b*x^3)^2)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^3)
/a)])/(4*x^4*(1 + (b*x^3)/a)^(2*p))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2 x^6 + 2\,abx^3 + a^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

3.138 $\int \frac{x^8}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c
]])/(3*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2
)
```

Rubi [A] time = 0.164693, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c
]])/(3*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2
)
```

Rubi in Sympy [A] time = 29.6385, size = 73, normalized size = 0.9

$$-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**6+b*x**3+a),x)
```

```
[Out] -b*log(a + b*x**3 + c*x**6)/(6*c**2) + x**3/(3*c) - (-2*a*c + b**
2)*atanh((b + 2*c*x**3)/sqrt(-4*a*c + b**2))/(3*c**2*sqrt(-4*a*c
+ b**2))
```

Mathematica [A] time = 0.0940408, size = 78, normalized size = 0.96

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^3+cx^6\right)+2cx^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Maple [A] time = 0.006, size = 111, normalized size = 1.4

$$\frac{x^3}{3\,c}-\frac{b\ln\left(cx^6+bx^3+a\right)}{6\,c^2}-\frac{2\,a}{3\,c}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{3\,c^2}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-2/3/c/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a+1/3/c^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275022, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)\log\left(\frac{2\,\left(b^2c-4\,ac^2\right)x^3+b^3-4\,abc+\left(2\,c^2x^6+2\,bcx^3+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(2\,cx^3-b\log\left(cx^6+bx^3+a\right)\right)\sqrt{b^2-4\,ac}}{6\,\sqrt{b^2-4\,ac}c^2},\ \frac{2\,(b^2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 2*a*c)*log((2*(b^2*c - 4*a*c^2)*x^3 + b^3 - 4*a*b*c
 + (2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^
6 + b*x^3 + a)) - (2*c*x^3 - b*log(c*x^6 + b*x^3 + a))*sqrt(b^2 -
 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), 1/6*(2*(b^2 - 2*a*c)*arctan(-(2
*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*x^3 - b*log(
c*x^6 + b*x^3 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 8.01714, size = 316, normalized size = 3.9

$$\left(-\frac{b}{6c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\left(-\frac{b}{6c^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c
- b**2)))*log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c
 + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(
```

```
6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**
2))))/(2*a*c - b**2)) + (-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 + (-a*b - 12*a*c**2*(-
b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c -
b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

GIAC/XCAS [A] time = 0.278579, size = 101, normalized size = 1.25

$$\frac{x^3}{3\,c} - \frac{b\ln\left(cx^6 + bx^3 + a\right)}{6\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/c - 1/6*b*ln(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*a
rctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.139 $\int \frac{x^5}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^3 + c*x^6]/(6*c)
```

Rubi [A] time = 0.112803, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^3 + c*x^6]/(6*c)
```

Rubi in Sympy [A] time = 19.092, size = 54, normalized size = 0.86

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3c\sqrt{-4ac+b^2}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**6+b*x**3+a),x)
```

```
[Out] b*atanh((b + 2*c*x**3)/sqrt(-4*a*c + b**2))/(3*c*sqrt(-4*a*c + b*
*2)) + log(a + b*x**3 + c*x**6)/(6*c)
```

Mathematica [A] time = 0.0423322, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^3+cx^6\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c] + Log[a + b*x^3 + c*x^6])/(6*c)
```

Maple [A] time = 0.004, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^6+bx^3+a\right)}{6\,c}-\frac{b}{3\,c}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3
+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275205, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{2\,(b^2c-4\,ac^2)\,x^3+b^3-4\,abc+(2\,c^2x^6+2\,bcx^3+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\sqrt{b^2-4\,ac}\log\left(cx^6+bx^3+a\right)}{6\,\sqrt{b^2-4\,ac}c},\ -\frac{2\,b\arctan\left(-\frac{(2\,cx^3+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\log\left(cx^6+bx^3+a\right)}{6\,\sqrt{-b^2+4\,ac}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(b*log((2*(b^2*c - 4*a*c^2)*x^3 + b^3 - 4*a*b*c + (2*c^2*x^6
 + 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a
)) + sqrt(b^2 - 4*a*c)*log(c*x^6 + b*x^3 + a))/(sqrt(b^2 - 4*a*c)
*c), -1/6*(2*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)) - sqrt(-b^2 + 4*a*c)*log(c*x^6 + b*x^3 + a))/(sqrt(-b^2 + 4
*a*c)*c)]
```

Sympy [A] time = 4.17719, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6c\,(4ac-b^2)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6c\,(4ac-b^2)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3
+ (-12*a*c*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)
) + 2*a + 3*b**2*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1
/(6*c)))/b) + (b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*
c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2
)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c -
 b**2)) + 1/(6*c)))/b)
```

GIAC/XCAS [A] time = 0.310209, size = 80, normalized size = 1.27

$$-\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c}+\frac{\ln\left(cx^6+bx^3+a\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*c) + 1/6*ln(c*x^6 + b*x^3 + a)/c
```

3.140 $\int \frac{x^2}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c
])
```

Rubi [A] time = 0.0695368, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c
])
```

Rubi in Sympy [A] time = 10.9272, size = 37, normalized size = 0.97

$$-\frac{2\,\text{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] -2*atanh((b + 2*c*x**3)/sqrt(-4*a*c + b**2))/(3*sqrt(-4*a*c + b**
2))
```

Mathematica [A] time = 0.0164807, size = 42, normalized size = 1.11

$$\frac{2\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c
])
```

Maple [A] time = 0.002, size = 37, normalized size = 1.

$$\frac{2}{3}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] 2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270991, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{2\,(b^2c-4\,ac^2)\,x^3+b^3-4\,abc-(2\,c^2x^6+2\,bcx^3+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{3\,\sqrt{b^2-4\,ac}},\ \frac{2\,\arctan\left(-\frac{(2\,cx^3+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{3\,\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/3*log(-(2*(b^2*c - 4*a*c^2)*x^3 + b^3 - 4*a*b*c - (2*c^2*x^6 +
 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a))
/sqrt(b^2 - 4*a*c), 2/3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/
(b^2 - 4*a*c))/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 1.93442, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c
- b**2))*log(x**3 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1
/(4*a*c - b**2)) + b)/(2*c))/3
```

GIAC/XCAS [A] time = 0.297027, size = 49, normalized size = 1.29

$$\frac{2\arctan\left(\frac{2cx^3+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] 2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.141 $\int \frac{1}{x(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rubi [A] time = 0.134114, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rubi in Sympy [A] time = 26.8782, size = 63, normalized size = 0.91

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3a\sqrt{-4ac+b^2}}+\frac{\log\left(x^3\right)}{3a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**6+b*x**3+a),x)
```

```
[Out] b*atanh((b + 2*c*x**3)/sqrt(-4*a*c + b**2))/(3*a*sqrt(-4*a*c + b*
*2)) + log(x**3)/(3*a) - log(a + b*x**3 + c*x**6)/(6*a)
```

Mathematica [C] time = 0.0399524, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log
[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A] time = 0.007, size = 66, normalized size = 1.

$$\frac{\ln(x)}{a}-\frac{\ln\left(cx^6+bx^3+a\right)}{6a}-\frac{b}{3a}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^6+b*x^3+a),x)
```

```
[Out] ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a-1/3/a*b/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28225, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(\frac{2\,(b^2c-4\,ac^2)\,x^3+b^3-4\,abc+\left(2\,c^2x^6+2\,bcx^3+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\sqrt{b^2-4\,ac}\left(\log\left(cx^6+bx^3+a\right)-6\,\log\left(x\right)\right)}{6\,\sqrt{b^2-4\,ac}a},\right.$$
$$\left.-\frac{2\,b\arctan\left(-\frac{\left(2\,cx^3+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(\log\left(cx^6+bx^3+a\right)-6\,\log\left(x\right)\right)}{6\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/6*(b*log((2*(b^2*c - 4*a*c^2)*x^3 + b^3 - 4*a*b*c + (2*c^2*x^6
 + 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a
)) - sqrt(b^2 - 4*a*c)*(log(c*x^6 + b*x^3 + a) - 6*log(x)))/(sqrt
(b^2 - 4*a*c)*a), -1/6*(2*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*x^6 + b*x^3 + a) -
 6*log(x)))/(sqrt(-b^2 + 4*a*c)*a)]
```

Sympy [A] time = 12.1341, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)$$
$$+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3
+ (-12*a**2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6
*a)) + 3*a*b**2*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/
(6*a)) - 2*a*c + b**2)/(b*c)) + (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*
```

```
c - b**2)) - 1/(6*a))*log(x**3 + (-12*a**2*c*(b*sqrt(-4*a*c + b**
2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(b*sqrt(-4*a*c + b*
*2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c)) + log(
x)/a
```

GIAC/XCAS [A] time = 0.263246, size = 89, normalized size = 1.29

$$-\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a}-\frac{\ln\left(cx^6+bx^3+a\right)}{6\,a}+\frac{\ln\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - 1/6*ln(c*x^6 + b*x^3 + a)/a + ln(abs(x))/a
```

3.142 $\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*
a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^3 + c*x^6])/(6*a^2)
```

Rubi [A] time = 0.258512, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(3*a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*
a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^3 + c*x^6])/(6*a^2)
```

Rubi in Sympy [A] time = 42.9007, size = 87, normalized size = 0.98

$$-\frac{1}{3ax^3}-\frac{b\log\left(x^3\right)}{3a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{(-2ac+b^2)\,\mathrm{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] -1/(3*a*x**3) - b*log(x**3)/(3*a**2) + b*log(a + b*x**3 + c*x**6)
/(6*a**2) - (-2*a*c + b**2)*atanh((b + 2*c*x**3)/sqrt(-4*a*c + b*
*2))/(3*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0533079, size = 92, normalized size = 1.03

$$\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(3*a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (
b^2*Log[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*
c*#1^3) & ]/(3*a^2)
```

Maple [A] time = 0.007, size = 119, normalized size = 1.3

$$-\frac{1}{3\,ax^3}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{2\,c}{3\,a}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{3\,a^2}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-2/3/a/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*c+1/3/a^2/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.323047, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)x^3\log\left(\frac{2\,\left(b^2c-4\,ac^2\right)x^3+b^3-4\,abc+\left(2\,c^2x^6+2\,bcx^3+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(bx^3\log\left(cx^6+bx^3+a\right)-6\,bx^3\log\left(x\right)-2\,a\right)\cdots}{6\,\sqrt{b^2-4\,ac}a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 2*a*c)*x^3*log((2*(b^2*c - 4*a*c^2)*x^3 + b^3 - 4*a
*b*c + (2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(
c*x^6 + b*x^3 + a)) - (b*x^3*log(c*x^6 + b*x^3 + a) - 6*b*x^3*log
(x) - 2*a)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x^3), 1/6*(2
*(b^2 - 2*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)) + (b*x^3*log(c*x^6 + b*x^3 + a) - 6*b*x^3*log(x) - 2*a)
*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.263225, size = 126, normalized size = 1.42

$$\frac{b\ln\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{b\ln\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^3-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^4),x, algorithm="giac")
```

```
[Out] 1/6*b*ln(c*x^6 + b*x^3 + a)/a^2 - b*ln(abs(x))/a^2 + 1/3*(b^2 - 2
*a*c)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*a^2) + 1/3*(b*x^3 - a)/(a^2*x^3)
```

3.143 $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=636

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2
*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(
2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2
- 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b
+ Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b
 + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2
^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
*c^(1/3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) -
((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c]
)^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/
3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)
) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*
a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(
```

```
1/3))
```

Rubi [A] time = 2.61173, antiderivative size = 636, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2
*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(
2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2
- 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b
+ Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b
 + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2
^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
```

```
*c^(1/3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) -
((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c]
)^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/
3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)
) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*
a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(
1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0532375, size = 70, normalized size = 0.11

$$\frac{3x^2-2\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log
[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Maple [C] time = 0.119, size = 61, normalized size = 0.1

$$\frac{x^2}{2\,c}-\frac{1}{3\,c}\sum_{\_R=RootOf\left(c\_Z^6+b\_Z^3+a\right)}\frac{\left(\_R^4b+\_R\,a\right)\ln\left(x-\_R\right)}{2\,\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=R
ootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2\,c} - \frac{\int \frac{bx^4+ax}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [A] time = 0.609276, size = 7657, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*
b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*arctan(-(1/2)^(2
/3)*(sqrt(3)*(b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b
^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^
2*b^2*c^12 - 64*a^3*c^13)) - sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*
b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4))*((b^4 - 3*a*b^2*c + a
^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6
*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3)/((
1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 +
 60*a^4*b^2*c^4 - (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*
a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^
2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b
^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a
^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b
^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - 4*(a^3*b^5
- 5*a^4*b^3*c + 5*a^5*b*c^2)*x - 4*sqrt(1/2)*(a^3*b^5 - 5*a^4*b^3
*c + 5*a^5*b*c^2)*sqrt((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x
^2 + (1/2)^(2/3)*((b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*
```

a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c + 52*a^2*b
^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c +
a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) -
(1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c
^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*sqrt((b^10 - 10
*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 -
3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c
^6))^(1/3))/(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)))) - 4*sqrt(3)*
(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*s
qrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*arctan(-(1/2)^(2/3)*(sqrt(3)*(b^
8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*
c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3
*b^4*c^3 + 60*a^4*b^2*c^4))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^
5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^
4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^
12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3)/((1/2)^(2/3)*(b^10
- 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4
+ (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64
*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c
^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6
)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*
a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - 4*(a^3*b^5 - 5*a^4*b^3*c +
5*a^5*b*c^2)*x - 4*sqrt(1/2)*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2
)*sqrt((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^(2/3)
*((b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64
*a^4*c^9)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4
*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)) + (b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*
b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c
^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b
^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^
2*b^7 - 9*a^3*b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^
5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*sqrt((b^10 - 10*a*b^8*c + 35*a^
2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*
c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^
2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3))/(a^3
*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)))) + (1/2)^(1/3)*c*((b^4 - 3*a*
b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 3

```
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))
^(1/3)*log(2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^(2
/3)*((b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 +
 64*a^4*c^9)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*
b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a
^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^
2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^
3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*
(a^2*b^7 - 9*a^3*b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5
*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*sqrt((b^10 - 10*a*b^8*c + 35
*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a
^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6
*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)) +
 (1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*
sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^
4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(2*(a^3*b^5 - 5*a^4*b^3*c +
5*a^5*b*c^2)*x^2 - (1/2)^(2/3)*((b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*
b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^10 - 12*a*b^
8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4
- 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8
*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a
*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 25*a^4*b^3*c^
2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*s
qrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^1
0 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4
)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b
^2*c^5 - 4*a*c^6))^(1/3)) - 2*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a
^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6
*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*lo
g((1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^
3 + 60*a^4*b^2*c^4 - (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 1
12*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6
*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 +
 (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^
2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 2*(a^3*b
^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) - 2*(1/2)^(1/3)*c*((b^4 - 3*a*
b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))
^(1/3)*log((1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a
```

```
^3*b^4*c^3 + 60*a^4*b^2*c^4 + (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^
4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c +
a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) +
 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) - 3*x^2)/c
```

Sympy [A] time = 13.0958, size = 279, normalized size = 0.44

$$\text{RootSum}\left(t^6\left(46656a^3c^8 - 34992a^2b^2c^7 + 8748ab^4c^6 - 729b^6c^5\right) + t^3\left(432a^4c^4 - 1512a^3b^2c^3 + 1107a^2b^4c^2 - 297ab^6c + 27b\right.\right.$$
$$+ \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b*
*4*c**6 - 729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*
c**3 + 1107*a**2*b**4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lamb
da(_t, _t*log(x + (-15552*_t**5*a**4*c**9 + 27216*_t**5*a**3*b**2
*c**8 - 14580*_t**5*a**2*b**4*c**7 + 3159*_t**5*a*b**6*c**6 - 243
*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_t**2*a**4*b**2*c**4
- 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2 - 108*_t**2
*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**3*
b**5)))) + x**2/(2*c)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

3.144 $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*S
qrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt
[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqr
t[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)
*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(
2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((
b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 2.48005, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*S
qrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt
[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqr
t[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)
*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(
2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((
b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0552879, size = 70, normalized size = 0.11

$$\frac{x}{c}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] time = 0.007, size = 59, normalized size = 0.1

$$\frac{x}{c}+\frac{1}{3\,c}\sum_{\_R=RootOf(\_Z^6c+\_Z^3b+a)}\frac{\left(-\_R^3b-a\right)\ln\left(x-\_R\right)}{2\,\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^6+b*x^3+a),x)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c}-\frac{\int\frac{bx^3+a}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [A] time = 0.418858, size = 6885, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^
5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^
4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))
/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan((1/2)^(1/3)*(sqrt(3)*(b^5*c^4
- 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*
c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^
2*c^2 - 8*a^3*c^3))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(1/3)/(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + 4*sqr
t(1/2)*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*sqrt((2*(a^2*b^4 - 4*a^3
*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*
b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 +
 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*
a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11))
)/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^
3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*
c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*
a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 -
 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c
^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) - (1/2)^(1/3)*(b
^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6
*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 -
 4*a*c^5))^(1/3))) - 4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (
b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
```

```
3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
- 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan((1/2)^(1/3)*(s
qrt(3)*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a
*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^6 - 8*a*b
^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3))*(-(b^3 - 2*a*b*c - (b^2*c^4 -
 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*
c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)/(4*(a*b^4 - 4*a^2*b^2*c + 2*a^
3*c^2)*x + 4*sqrt(1/2)*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*sqrt((2*
(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a
*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4
- 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c -
(b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^
5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*
a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5
)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/
(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2))
- (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + (b^
5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4
*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^
11)))/(b^2*c^4 - 4*a*c^5))^(1/3))) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b
*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2
*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4
- 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c +
34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*a*b^
5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^
9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4
- 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^
3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3
*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8
*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a
^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c
^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^
6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
- 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 -
4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c
^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c +
2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 -
```

```
 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b
^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt(
(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^
4 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 1
6*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64
*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*
x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3
)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt(
(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^
4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (
1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - (b^5*c
^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48
*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*
c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*
a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)
))/(b^2*c^4 - 4*a*c^5))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c
 - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*
b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48
*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) + 6*x)
/c
```

Sympy [A] time = 8.91244, size = 196, normalized size = 0.31

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{3}c^{7}-34992a^{2}b^{2}c^{6}+8748ab^{4}c^{5}-729b^{6}c^{4}\right)+t^{3}\left(864a^{3}bc^{3}-864a^{2}b^{3}c^{2}+270ab^{5}c-27b^{7}\right)+a^{4},\left(t\mapsto\right.\right.$$

$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b*
*4*c**5 - 729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3
```

```
*c**2 + 270*a*b**5*c - 27*b**7) + a**4, Lambda(_t, _t*log(x + (12
96*_t**4*a**2*b*c**6 - 648*_t**4*a*b**3*c**5 + 81*_t**4*b**5*c**4
 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**2 - 21*_t*a*b**4*c + 3*_t
*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) + x/c
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

3.145 $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}$$

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)
*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 -
(2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*
c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])
^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2
/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)
*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(
1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*
c])
```

Rubi [A] time = 1.27545, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)
*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 -
(2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*
c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])
^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2
/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)
*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqr
```

```
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(
1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*
c])
```

Rubi in Sympy [A] time = 149.312, size = 529, normalized size = 0.95

$$\frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{-4ac+b^2}}\right)}{6c^{\frac{2}{3}}\sqrt{-4ac+b^2}} - \frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{12c^{\frac{2}{3}}\sqrt{-4ac+b^2}} + \frac{\sqrt[3]{2}\sqrt{3}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{6c^{\frac{2}{3}}\sqrt{-4ac+b^2}} - \frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{-4ac+b^2}}\right)}{6c^{\frac{2}{3}}\sqrt{-4ac+b^2}} + \frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b+\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{12c^{\frac{2}{3}}\sqrt{-4ac+b^2}} - \frac{\sqrt[3]{2}\sqrt{3}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{6c^{\frac{2}{3}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] 2**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)*log(2**(1/3)*c**(1/3)*x
 + (b - sqrt(-4*a*c + b**2))**(1/3))/(6*c**(2/3)*sqrt(-4*a*c + b*
*2)) - 2**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)*log(c**(2/3)*x**
2 - 2**(2/3)*c**(1/3)*x*(b - sqrt(-4*a*c + b**2))**(1/3)/2 + 2**(
1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)/2)/(12*c**(2/3)*sqrt(-4*a*c
 + b**2)) + 2**(1/3)*sqrt(3)*(b - sqrt(-4*a*c + b**2))**(2/3)*ata
n(sqrt(3)*(-2*2**(1/3)*c**(1/3)*x/(3*(b - sqrt(-4*a*c + b**2))**(
1/3)) + 1/3))/(6*c**(2/3)*sqrt(-4*a*c + b**2)) - 2**(1/3)*(b + sq
rt(-4*a*c + b**2))**(2/3)*log(2**(1/3)*c**(1/3)*x + (b + sqrt(-4*
```

```
a*c + b**2))**(1/3))/(6*c**(2/3)*sqrt(-4*a*c + b**2)) + 2**(1/3)*
(b + sqrt(-4*a*c + b**2))**(2/3)*log(c**(2/3)*x**2 - 2**(2/3)*c**
(1/3)*x*(b + sqrt(-4*a*c + b**2))**(1/3)/2 + 2**(1/3)*(b + sqrt(-
4*a*c + b**2))**(2/3)/2)/(12*c**(2/3)*sqrt(-4*a*c + b**2)) - 2**(
1/3)*sqrt(3)*(b + sqrt(-4*a*c + b**2))**(2/3)*atan(sqrt(3)*(-2*2*
*(1/3)*c**(1/3)*x/(3*(b + sqrt(-4*a*c + b**2))**(1/3)) + 1/3))/(6
*c**(2/3)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0301178, size = 44, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^2\log(x-\#1)}{2\#1^3c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3)
 & ]/3
```

Maple [C] time = 0.005, size = 43, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6c+\_Z^3b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

Fricas [A] time = 0.329221, size = 5165, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a
^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)*arctan(-(1/2)^(2/3)*(sqr
t(3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt(
(b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^
2*c^6 - 64*a^3*c^7)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2))*(
-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4
 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4
*a*c^3))^(2/3)/((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - (b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 6
4*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b
)/(b^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x - 4*sqrt(1/2
)*(a*b^2 - 2*a^2*c)*sqrt((2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^
4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c
^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-((b^2*c^
2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b
^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^
(2/3) - 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sq
rt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2
*b^2*c^6 - 64*a^3*c^7))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^
3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3))/(a*b^2 - 2*a^2*c)))) + 2
/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*
c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*arctan(-(1/2)^(2/3)*(sqrt(3
)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^
4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c
^6 - 64*a^3*c^7)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2))*(((b
^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 1
2*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c
^3))^(2/3)/((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 + (b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^
3*c^7)))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^
2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x - 4*sqrt(1/2)*(a*
b^2 - 2*a^2*c)*sqrt((2*(a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((b^6*
```

c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3))/(a*b^2 - 2*a^2*c)))) - 1/6*(1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log(-2*(a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)) - 1/6*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log(-2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)) + 1/3*(1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c)*x) + 1/3*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c)*x)

Sympy [A] time = 5.78376, size = 175, normalized size = 0.31

$$\text{RootSum}\left(t^6\left(46656a^3c^5-34992a^2b^2c^4+8748ab^4c^3-729b^6c^2\right)+t^3\left(-432a^2bc^2+216ab^3c-27b^5\right)+a^2,\left(t\mapsto t\log\left(x+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b*
*4*c**3 - 729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c
 - 27*b**5) + a**2, Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5
- 11664*_t**5*a**2*b**2*c**4 + 2916*_t**5*a*b**4*c**3 - 243*_t**5
*b**6*c**2 - 108*_t**2*a**2*b*c**2 + 63*_t**2*a*b**3*c - 9*_t**2*
b**5)/(2*a**2*c - a*b**2))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

3.146 $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)
*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 -
(2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*
c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*a*c])
^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1
/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)
*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(
1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*
c])
```

Rubi [A] time = 1.29445, antiderivative size = 558, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$-\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$+\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)
*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 -
(2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*
c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*a*c])
^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1
/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)
*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(
1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*
c])
```

Rubi in Sympy [A] time = 139.29, size = 529, normalized size = 0.95

$$-\frac{2^{\frac{2}{3}}\sqrt[3]{b-\sqrt{-4ac+b^2}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{-4ac+b^2}}\right)}{6\sqrt[3]{c}\sqrt{-4ac+b^2}}+\frac{2^{\frac{2}{3}}\sqrt[3]{b-\sqrt{-4ac+b^2}}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{12\sqrt[3]{c}\sqrt{-4ac+b^2}}+\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[3]{b-\sqrt{-4ac+b^2}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{c}\sqrt{-4ac+b^2}}+\frac{2^{\frac{2}{3}}\sqrt[3]{b+\sqrt{-4ac+b^2}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{-4ac+b^2}}\right)}{6\sqrt[3]{c}\sqrt{-4ac+b^2}}-\frac{2^{\frac{2}{3}}\sqrt[3]{b+\sqrt{-4ac+b^2}}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b+\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{12\sqrt[3]{c}\sqrt{-4ac+b^2}}-\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[3]{b+\sqrt{-4ac+b^2}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] -2**(2/3)*(b - sqrt(-4*a*c + b**2))**(1/3)*log(2**(1/3)*c**(1/3)*
x + (b - sqrt(-4*a*c + b**2))**(1/3))/(6*c**(1/3)*sqrt(-4*a*c + b
**2)) + 2**(2/3)*(b - sqrt(-4*a*c + b**2))**(1/3)*log(c**(2/3)*x*
*2 - 2**(2/3)*c**(1/3)*x*(b - sqrt(-4*a*c + b**2))**(1/3)/2 + 2**
(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)/2)/(12*c**(1/3)*sqrt(-4*a*
c + b**2)) + 2**(2/3)*sqrt(3)*(b - sqrt(-4*a*c + b**2))**(1/3)*at
an(sqrt(3)*(-2*2**(1/3)*c**(1/3)*x/(3*(b - sqrt(-4*a*c + b**2))**
(1/3)) + 1/3))/(6*c**(1/3)*sqrt(-4*a*c + b**2)) + 2**(2/3)*(b + s
qrt(-4*a*c + b**2))**(1/3)*log(2**(1/3)*c**(1/3)*x + (b + sqrt(-4
*a*c + b**2))**(1/3))/(6*c**(1/3)*sqrt(-4*a*c + b**2)) - 2**(2/3)
*(b + sqrt(-4*a*c + b**2))**(1/3)*log(c**(2/3)*x**2 - 2**(2/3)*c*
*(1/3)*x*(b + sqrt(-4*a*c + b**2))**(1/3)/2 + 2**(1/3)*(b + sqrt(
-4*a*c + b**2))**(2/3)/2)/(12*c**(1/3)*sqrt(-4*a*c + b**2)) - 2**
(2/3)*sqrt(3)*(b + sqrt(-4*a*c + b**2))**(1/3)*atan(sqrt(3)*(-2*2
**(1/3)*c**(1/3)*x/(3*(b + sqrt(-4*a*c + b**2))**(1/3)) + 1/3))/(
```

```
6*c**(1/3)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0273745, size = 42, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1\log(x-\#1)}{2\#1^3c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) &
 ]/3
```

Maple [C] time = 0.004, size = 43, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf(\_Z^6c+\_Z^3b+a)}\frac{\_R^3\ln(x-\_R)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Fricas [A] time = 0.293198, size = 3260, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^
2))^(1/3)*arctan(-sqrt(3)*(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)/((1/2)^
(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a
*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sqrt
(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)
/(b^2*c - 4*a*c^2))^(1/3) - 2*b*x - 2*b*sqrt(-((1/2)^(1/3)*(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c -
4*a*c^2))^(1/3) - b*x^2 - (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c -
4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3))/b))) + 2/3*sqrt(3)*(1/2)^
(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*arctan(s
qrt(3)*(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((b^2*c -
 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)/((1/2)^(1/3)*(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2
))^(1/3) + 2*b*x + 2*b*sqrt(((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5))*x*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3) +
 b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4*a*c^2)*sqrt(b^
2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b
^2*c - 4*a*c^2))^(2/3))/b))) - 1/6*(1/2)^(1/3)*(((b^2*c - 4*a*c^2
)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a
*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^
(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c - 4*a*c^2)*s
qrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) +
 1)/(b^2*c - 4*a*c^2))^(2/3)) - 1/6*(1/2)^(1/3)*(-((b^2*c - 4*a*c
^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^
5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12
```

```
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2)
)^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4*a*c^2
)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
) - 1)/(b^2*c - 4*a*c^2))^(2/3)) + 1/3*(1/2)^(1/3)*(((b^2*c - 4*a
*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))
^(1/3) + b*x) + 1/3*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c
- 4*a*c^2))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3) + b*x)
```

Sympy [A] time = 6.32845, size = 122, normalized size = 0.22

$$\text{RootSum}\left(t^6\left(46656a^3c^4 - 34992a^2b^2c^3 + 8748ab^4c^2 - 729b^6c\right) + t^3\left(432a^2c^2 - 216ab^2c + 27b^4\right) + a, \left(t \mapsto t\log\left(x + \frac{2592\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b*
*4*c**2 - 729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*
b**4) + a, Lambda(_t, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t*
*4*a*b**2*c**2 + 162*_t**4*b**4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

3.147 $\int \frac{x}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])^(1/3))) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt
[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (2^(1/3)*c^(1/3)*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^
2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)*Log[
(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 -
 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b
^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
)*x + 2^(2/3)*c^(2/3)*x^2]])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2]])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^
(1/3))
```

Rubi [A] time = 1.23441, antiderivative size = 558, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])^(1/3))) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt
[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (2^(1/3)*c^(1/3)*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^
2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)*Log[
(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 -
 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b
^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^
(1/3))
```

Rubi in Sympy [A] time = 147.85, size = 529, normalized size = 0.95

$$\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{-4ac+b^2}}\right)}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b+\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{6\sqrt[3]{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} + \frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{-4ac+b^2}}\right)}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} + \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{6\sqrt[3]{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**6+b*x**3+a),x)
```

```
[Out] 2**(1/3)*c**(1/3)*log(2**(1/3)*c**(1/3)*x + (b + sqrt(-4*a*c + b*
*2))**(1/3))/(3*(b + sqrt(-4*a*c + b**2))**(1/3)*sqrt(-4*a*c + b*
*2)) - 2**(1/3)*c**(1/3)*log(c**(2/3)*x**2 - 2**(2/3)*c**(1/3)*x*
(b + sqrt(-4*a*c + b**2))**(1/3)/2 + 2**(1/3)*(b + sqrt(-4*a*c +
b**2))**(2/3)/2)/(6*(b + sqrt(-4*a*c + b**2))**(1/3)*sqrt(-4*a*c
+ b**2)) + 2**(1/3)*sqrt(3)*c**(1/3)*atan(sqrt(3)*(-2*2**(1/3)*c*
*(1/3)*x/(3*(b + sqrt(-4*a*c + b**2))**(1/3)) + 1/3))/(3*(b + sqr
t(-4*a*c + b**2))**(1/3)*sqrt(-4*a*c + b**2)) - 2**(1/3)*c**(1/3)
*log(2**(1/3)*c**(1/3)*x + (b - sqrt(-4*a*c + b**2))**(1/3))/(3*(
b - sqrt(-4*a*c + b**2))**(1/3)*sqrt(-4*a*c + b**2)) + 2**(1/3)*c
**(1/3)*log(c**(2/3)*x**2 - 2**(2/3)*c**(1/3)*x*(b - sqrt(-4*a*c
+ b**2))**(1/3)/2 + 2**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)/2)/
(6*(b - sqrt(-4*a*c + b**2))**(1/3)*sqrt(-4*a*c + b**2)) - 2**(1/
3)*sqrt(3)*c**(1/3)*atan(sqrt(3)*(-2*2**(1/3)*c**(1/3)*x/(3*(b -
sqrt(-4*a*c + b**2))**(1/3)) + 1/3))/(3*(b - sqrt(-4*a*c + b**2))
**(1/3)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0301338, size = 43, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\log(x-\#1)}{2\#1^4c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]
/3
```

Maple [C] time = 0.004, size = 41, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf(\_Z^6c+\_Z^3b+a)}\frac{\_R\ln(x-\_R)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

Fricas [A] time = 0.303701, size = 4020, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*
c))^(1/3)*arctan(-(1/2)^(2/3)*(sqrt(3)*(a*b^6 - 12*a^2*b^4*c + 48
*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3)) - sqrt(3)*(b^4 - 4*a*b^2*c))*(-((a*b^2
 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3)/(4*b*c*x + 4*sqrt(1/2)*
b*c*sqrt((2*b*c*x^2 + (1/2)^(2/3)*((a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^2*c)*x)*(-((a*b^2 - 4*a^2
*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c -
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b
^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b
^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(
a*b^2 - 4*a^2*c))^(1/3))/(b*c)) - (1/2)^(2/3)*(b^4 - 4*a*b^2*c -
(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 -
 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3))) - 2/3*sqrt(3)*(1/2)^(1/
3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(-(1/2
)^(2/3)*(sqrt(3)*(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*
c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3)) + sqrt(3)*(b^4 - 4*a*b^2*c))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a
^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2
 - 4*a^2*c))^(2/3)/(4*b*c*x + 4*sqrt(1/2)*b*c*sqrt((2*b*c*x^2 - (
1/2)^(2/3)*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*
x + (b^4 - 4*a*b^2*c)*x)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*
c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c + (a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3))/
(b*c)) - (1/2)^(2/3)*(b^4 - 4*a*b^2*c + (a*b^6 - 12*a^2*b^4*c + 4
8*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*
a^2*c))^(2/3))) - 1/6*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^
2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2 + (1/2)^(2/3)*((a*b^6 - 12*a^2*
b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b
^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^2*c)*x)*(-(
(a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^
3 - 4*a*b*c - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4
*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
```

```
5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)) - 1/6*(1/2)^(1/3)*(((a*b^2
 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2 - (1/2)^(
2/3)*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b
^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x + (b
^4 - 4*a*b^2*c)*x)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2
/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c + (a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)) + 1/3*
(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*lo
g(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*c - (a*b^6 - 12*a^2*b^4*c
+ 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2
- 4*a^2*c))^(2/3)) + 1/3*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2
/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*
b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*c
+ (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(((a*b^2
- 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3))
```

Sympy [A] time = 4.72364, size = 158, normalized size = 0.28

$$\operatorname{RootSum}\left(t^6\left(46656a^4c^3 - 34992a^3b^2c^2 + 8748a^2b^4c - 729ab^6\right) + t^3\left(-432a^2c^2 + 216ab^2c - 27b^4\right) + c, \left(t \mapsto t\log\left(x + \frac{-15\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2
*b**4*c - 729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27
*b**4) + c, Lambda(_t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664
*_t**5*a**3*b**2*c**2 - 2916*_t**5*a**2*b**4*c + 243*_t**5*a*b**6
 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b**2*c + 9*_t**2*b**4)/(b*c)))
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^6 + b*x^3 + a),x, algorithm="giac")

[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

3.148 $\int \frac{1}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])^(2/3))) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt
[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2^(2/3)*c^(2/3)*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^
2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)*Log[
(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 -
 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b
^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqr
t[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^
(2/3))
```

Rubi [A] time = 1.26151, antiderivative size = 558, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])^(2/3))) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt
[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2^(2/3)*c^(2/3)*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^
2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)*Log[
(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 -
 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b
^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqr
t[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^
(2/3))
```

Rubi in Sympy [A] time = 133.387, size = 529, normalized size = 0.95

$$
\begin{aligned}
&-\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{-4ac+b^2}}\right)}{3\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} \\
&+\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b+\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{6\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} \\
&+\frac{2^{\frac{2}{3}}\sqrt{3}c^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{3\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{-4ac+b^2}}\right)}{3\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} \\
&-\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{6\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} \\
&-\frac{2^{\frac{2}{3}}\sqrt{3}c^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{3\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**6+b*x**3+a),x)
```

```
[Out] -2**(2/3)*c**(2/3)*log(2**(1/3)*c**(1/3)*x + (b + sqrt(-4*a*c + b
**2))**(1/3))/(3*(b + sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b
**2)) + 2**(2/3)*c**(2/3)*log(c**(2/3)*x**2 - 2**(2/3)*c**(1/3)*x
*(b + sqrt(-4*a*c + b**2))**(1/3)/2 + 2**(1/3)*(b + sqrt(-4*a*c +
 b**2))**(2/3)/2)/(6*(b + sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*c
 + b**2)) + 2**(2/3)*sqrt(3)*c**(2/3)*atan(sqrt(3)*(-2*2**(1/3)*c
**(1/3)*x/(3*(b + sqrt(-4*a*c + b**2))**(1/3)) + 1/3))/(3*(b + sq
rt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b**2)) + 2**(2/3)*c**(2/3
)*log(2**(1/3)*c**(1/3)*x + (b - sqrt(-4*a*c + b**2))**(1/3))/(3*
(b - sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b**2)) - 2**(2/3)*
c**(2/3)*log(c**(2/3)*x**2 - 2**(2/3)*c**(1/3)*x*(b - sqrt(-4*a*c
 + b**2))**(1/3)/2 + 2**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)/2)
/(6*(b - sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b**2)) - 2**(2
/3)*sqrt(3)*c**(2/3)*atan(sqrt(3)*(-2*2**(1/3)*c**(1/3)*x/(3*(b -
 sqrt(-4*a*c + b**2))**(1/3)) + 1/3))/(3*(b - sqrt(-4*a*c + b**2)
```

```
)**(2/3)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0297783, size = 45, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\log(x - \#1)}{2\#1^5c + \#1^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(-1),x]

[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) &
 ]/3
```

Maple [C] time = 0.004, size = 40, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf(\_Z^6c+\_Z^3b+a)}\frac{\ln(x - \_R)}{2\_R^5c + \_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^6+b*x^3+a),x)

[Out] 1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^6 + b*x^3 + a),x, algorithm="maxima")

[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

Fricas [A] time = 0.358855, size = 5284, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*arctan(-(1/2)^(1/3)*(sqrt(
3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c +
 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3
)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2))*(((a^2*b^2 - 4*a^3*c)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*
a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)/(4*(b^
2*c - 2*a*c^2)*x + 4*sqrt(1/2)*(b^2*c - 2*a*c^2)*sqrt((2*(b^2*c^2
 - 2*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 -
 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b
*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c
+ 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4
- 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2
 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a
^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c
+ 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^
3)) - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a^3*c)*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a
^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3))/(b^2*c
^2 - 2*a*c^3)) + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 - (a^2*
b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c
^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a
^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 1
2*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3
*c))^(1/3))) - 2/3*sqrt(3)*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqr
t((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*arctan(-(1
/2)^(1/3)*(sqrt(3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b
^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*
c^2 - 64*a^7*c^3)) + sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2))*(-((a
^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 1
2*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3
*c))^(1/3)/(4*(b^2*c - 2*a*c^2)*x + 4*sqrt(1/2)*(b^2*c - 2*a*c^2)
*sqrt((2*(b^2*c^2 - 2*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c +
 20*a^2*b^2*c^2 - 16*a^3*c^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b
^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^
6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4
*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*
c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3)
 + (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt
((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)) + (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(-((
a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 -
```

```
12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^
3*c))^(1/3))/(b^2*c^2 - 2*a*c^3)) + (1/2)^(1/3)*(b^4 - 6*a*b^2*c
+ 8*a^2*c^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 -
4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 -
 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a
^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) -
 b)/(a^2*b^2 - 4*a^3*c))^(1/3))) - 1/6*(1/2)^(1/3)*(((a^2*b^2 - 4
*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*
c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)
*log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)^(2/3)*(b^6 - 8*a*b^4*c +
20*a^2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^
3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6
 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a
^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c
+ 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) +
 (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((
b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(((a^2
*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c
))^(1/3)) - 1/6*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 6
4*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c^2 - 2*a
*c^3)*x^2 - (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^
3*c^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a
^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a
*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64
*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^
5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a
^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) +
 (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt
((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)) + 1/3*(1/2
)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/
(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2
*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(
b^4 - 6*a*b^2*c + 8*a^2*c^2 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c
^2)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 -
4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 -
 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) + 1/3*(1/2)^(1/3)*
(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^
6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 -
4*a^3*c))^(1/3)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6
*a*b^2*c + 8*a^2*c^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqr
t((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^
7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
```

Sympy [A] time = 9.29303, size = 155, normalized size = 0.28

$$\mathrm{RootSum}\left(t^6\left(46656a^5c^3-34992a^4b^2c^2+8748a^3b^4c-729a^2b^6\right)+t^3\left(432a^2bc^2-216ab^3c+27b^5\right)+c^2,\left(t\mapsto t\log\left(x+-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3
*b**4*c - 729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c
+ 27*b**5) + c**2, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2
 + 648*_t**4*a**3*b**3*c - 81*_t**4*a**2*b**5 + 12*_t*a**2*c**2 -
 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c**2 - b**2*c))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

3.149 $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=610

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^
(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3
)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt
[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 -
4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c
])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2
- 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sq
rt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sq
rt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[
(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4
*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b - Sqrt[b^2
```

```
 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + S
qrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])
^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rubi [A] time = 1.7032, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^
(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3
```

```
)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt
[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 -
4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c
])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2
- 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sq
rt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sq
rt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[
(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4
*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b - Sqrt[b^2
 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + S
qrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])
^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0541206, size = 71, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{3a}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*L
og[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Maple [C] time = 0.008, size = 61, normalized size = 0.1

$$-\frac{1}{ax}-\frac{1}{3\,a}\sum_{\_R=RootOf\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^4c+\_R\,b\right)\ln\left(x-\_R\right)}{2\,\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/a/x-1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=Root
Of(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{cx^4+bx}{cx^6+bx^3+a}\,dx}{a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)
```

Fricas [A] time = 0.550153, size = 7272, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^
5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*
a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)
))/(a^4*b^2 - 4*a^5*c))^(1/3)*arctan(-(1/2)^(2/3)*(sqrt(3)*(a^4*b
^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4
)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) -
 sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24
*a^4*b*c^4))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 -
 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5
*c))^(2/3)/((1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a
^3*b^3*c^3 + 24*a^4*b*c^4 - (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*
a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
```

```
4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^
3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - 4*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^
2*c^5)*x - 4*sqrt(1/2)*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*sqrt((
2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x^2 + (1/2)^(2/3)*((a^4*b^8
 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*
x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) -
 (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c
^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*
c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^
9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(
2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*
b*c^4 - (a^4*b^6*c - 10*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4
)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*
((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3))/(
b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)))) - 4*sqrt(3)*(1/2)^(1/3)*a*x
*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*
c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*a
rctan(-(1/2)^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42
*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*((b^3 - 2*a*b*c -
(a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2
 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3)/((1/2)^(2/3)*(b^9 -
11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 + (a^
4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*
c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*
a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)
))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^
4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3)
 - 4*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x - 4*sqrt(1/2)*(b^4*c^3
 - 4*a*b^2*c^4 + 2*a^2*c^5)*sqrt((2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^
2*c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^
2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)) + (b^9 - 11*a*b^7*c + 42*a^2*b^5
*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c - (a^4*b
^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2
*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*
a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*c - 8*a
*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c - 10*a^5*b^4
*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*
a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^
3)))/(a^4*b^2 - 4*a^5*c))^(1/3))/(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c
```

^5)))) + (1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sq
rt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4
)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4
*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*
x^2 + (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112
*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c
^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)) - (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 -
62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*
a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^
3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^
2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 - (a^4*b^6*c - 10*a^5*b^4*c^2 +
32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c
^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a
^4*b^2 - 4*a^5*c))^(1/3)) + (1/2)^(1/3)*a*x*((b^3 - 2*a*b*c - (a^
4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c^3 - 4*a*b^
2*c^4 + 2*a^2*c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 6
0*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12
*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) + (b^9 - 11*a*b^7*c
+ 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*
b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b
^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*
(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c
- 10*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12
*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c - (
a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2
- 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)) - 2*(1/2)^(1/3)*a*x*
((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*lo
g((1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3
 + 24*a^4*b*c^4 - (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*
a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b
^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*
b^2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x)
- 2*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^
8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2
- 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*
c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 + (a^4*b^8 - 13*a^5*b^6*c + 6
0*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6

```
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a
^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c - (a^
4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a*b^2*
c^4 + 2*a^2*c^5)*x) + 6)/(a*x)
```

Sympy [A] time = 8.32987, size = 252, normalized size = 0.41

$$\operatorname{RootSum}\left(t^6\left(46656a^7c^3 - 34992a^6b^2c^2 + 8748a^5b^4c - 729a^4b^6\right) + t^3\left(-864a^3bc^3 + 864a^2b^3c^2 - 270ab^5c + 27b^7\right) + c^4, \left(t \mapsto\right.\right.$$

$$-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5
*b**4*c - 729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**
3*c**2 - 270*a*b**5*c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-
15552*_t**5*a**8*c**4 + 27216*_t**5*a**7*b**2*c**3 - 14580*_t**5*
a**6*b**4*c**2 + 3159*_t**5*a**5*b**6*c - 243*_t**5*a**4*b**8 + 2
52*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c**3 + 378*_t**2*a**2*
b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*c**5 - 4*a*
b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^6 + bx^3 + a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

3.150 $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=612

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1
/3)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sq
rt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2
- 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a
*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^
2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b +
Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b +
Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
```

```
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b - Sqrt[b
^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b +
 Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c
])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4
*a*c])^(2/3))
```

Rubi [A] time = 1.94986, antiderivative size = 612, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(2*a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1
/3)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sq
rt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2
- 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a
*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^
2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b +
Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b +
Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b - Sqrt[b
^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b +
 Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c
])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4
*a*c])^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0563509, size = 75, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^6 c+\#1^3 b+a\&,\frac{\#1^3 c \log(x-\#1)+b \log(x-\#1)}{2\#1^5 c+\#1^2 b}\&\right]}{3a}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(2*a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c
*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Maple [C] time = 0.009, size = 62, normalized size = 0.1

$$\frac{1}{3a} \sum_{\_R = RootOf\left(\_Z^6c + \_Z^3b + a\right)} \frac{\left(-\_R^3c - b\right)\ln\left(x - \_R\right)}{2\_R^5c + \_R^2b} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c
+_Z^3*b+a))-1/2/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a}\,dx}{a} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)
```

Fricas [A] time = 0.494311, size = 7645, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 +
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*arctan(-(1/
2)^(1/3)*(sqrt(3)*(a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a
^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3
 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
64*a^13*c^3)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^
3*b^2*c^3))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sq
rt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*
```

```
b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^2 - 4*a^6*c))^(1/3)/(4*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2
*b*c^4)*x + 4*sqrt(1/2)*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*sqr
t((2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^1
1 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b^3*c
^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 16
0*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b
^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^
2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) + (1/2)
^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c
^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)) - (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^
2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt
((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c
^6)) - (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2
*c^3 - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqr
t((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b
^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3
)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)
/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^
5*b^2 - 4*a^6*c))^(1/3))) - 4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 -
3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6
*c))^(1/3)*arctan(-(1/2)^(1/3)*(sqrt(3)*(a^5*b^6 - 10*a^6*b^4*c +
 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + sqrt(3)*(b^8 - 9*a*b^6*c +
25*a^2*b^4*c^2 - 20*a^3*b^2*c^3))*(-(b^4 - 3*a*b^2*c + a^2*c^2 -
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)/(4*(b^5*c^2
 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x + 4*sqrt(1/2)*(b^5*c^2 - 5*a*b^3*
c^3 + 5*a^2*b*c^4)*sqrt((2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*
x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b
^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9 - 14*a^6*b^7*c
 + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 -
 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^
4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b
^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4
*a^6*c))^(2/3) - (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*
a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^2 - 9*a*b^6*c^3 + 25*a
^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a
```

```
^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a
^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^5*c^4 - 5
*a*b^3*c^5 + 5*a^2*b*c^6)) - (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^
2*b^4*c^2 - 20*a^3*b^2*c^3 + (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2
*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*
a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2
 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4
*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))) + (1/2)^(1/3)*a*x
^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5
*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6
)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3
*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7
*c + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)
/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(
b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a
*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 -
 4*a^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 3
2*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - (b^8*c^2 - 9*a*b^6*c^3 + 25
*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 +
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)) + (1/2)^(1
/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt
((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^
2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 -
138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9 - 14*
a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqr
t((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b
^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3
)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)
/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^
5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*
c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35
*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^2 - 9*a*b^6*c
^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2
*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c
^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)) - 2
*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^
6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
```

```
a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c
^3 + 5*a^2*b*c^4)*x + (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c
^2 - 20*a^3*b^2*c^3 - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 -
32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4
*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^
6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)) - 2*(1/2)^(1/3)*a*x^2*(-(
b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a
*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 -
 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x +
(1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 +
(a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)
/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(
b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a
*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 -
 4*a^6*c))^(1/3)) + 3)/(a*x^2)
```

Sympy [A] time = 18.5479, size = 241, normalized size = 0.39

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{8}c^{3}-34992a^{7}b^{2}c^{2}+8748a^{6}b^{4}c-729a^{5}b^{6}\right)+t^{3}\left(-432a^{4}c^{4}+1512a^{3}b^{2}c^{3}-1107a^{2}b^{4}c^{2}+297ab^{6}c-27\right.\right.$$

$$-\frac{1}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**8*c**3 - 34992*a**7*b**2*c**2 + 8748*a**6
*b**4*c - 729*a**5*b**6) + _t**3*(-432*a**4*c**4 + 1512*a**3*b**2
*c**3 - 1107*a**2*b**4*c**2 + 297*a*b**6*c - 27*b**8) + c**5, Lam
bda(_t, _t*log(x + (-2592*_t**4*a**8*c**3 + 2592*_t**4*a**7*b**2*
c**2 - 810*_t**4*a**6*b**4*c + 81*_t**4*a**5*b**6 + 12*_t*a**4*c*
*4 - 75*_t*a**3*b**2*c**3 + 78*_t*a**2*b**4*c**2 - 27*_t*a*b**6*c
 + 3*_t*b**8)/(5*a**2*b*c**4 - 5*a*b**3*c**3 + b**5*c**2)))) - 1/
(2*a*x**2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{6}+bx^{3}+a)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

3.151 $\int \frac{x^{11}}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=35

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rubi [A] time = 0.0561295, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4x^3}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{9\log\left(x^3+3\right)}{2}+\frac{\int^{x^3} x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(x**6+4*x**3+3),x)
```

```
[Out] -4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2 + Integral(x, (x,
 x**3))/3
```

Mathematica [A] time = 0.010818, size = 35, normalized size = 1.

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Maple [A] time = 0.009, size = 28, normalized size = 0.8

$$-\frac{4x^{3}}{3}+\frac{x^{6}}{6}-\frac{\ln\left(x^{3}+1\right)}{6}+\frac{9\ln\left(x^{3}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(x^6+4*x^3+3),x)
```

```
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Maxima [A] time = 0.78124, size = 36, normalized size = 1.03

$$\frac{1}{6}x^{6}-\frac{4}{3}x^{3}+\frac{9}{2}\log\left(x^{3}+3\right)-\frac{1}{6}\log\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Fricas [A] time = 0.261441, size = 36, normalized size = 1.03

$$\frac{1}{6}x^{6}-\frac{4}{3}x^{3}+\frac{9}{2}\log\left(x^{3}+3\right)-\frac{1}{6}\log\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.256044, size = 29, normalized size = 0.83

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{\log\left(x^3+1\right)}{6} + \frac{9\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(x**6+4*x**3+3),x)

[Out] x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

GIAC/XCAS [A] time = 0.285126, size = 39, normalized size = 1.11

$$\frac{1}{6}\,x^6 - \frac{4}{3}\,x^3 + \frac{9}{2}\ln\left(\left|x^3+3\right|\right) - \frac{1}{6}\ln\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^6 + 4*x^3 + 3),x, algorithm="giac")

[Out] 1/6*x^6 - 4/3*x^3 + 9/2*ln(abs(x^3 + 3)) - 1/6*ln(abs(x^3 + 1))
```

3.152 $\int \frac{x^8}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=28

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rubi [A] time = 0.0469661, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(3 + 4*x^3 + x^6),x]

[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rubi in Sympy [A] time = 11.1333, size = 22, normalized size = 0.79

$$\frac{x^3}{3} + \frac{\log\left(x^3+1\right)}{6} - \frac{3\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**8/(x**6+4*x**3+3),x)

[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

Mathematica [A] time = 0.00915247, size = 28, normalized size = 1.

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(3 + 4*x^3 + x^6),x]

[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Maple [A] time = 0.007, size = 23, normalized size = 0.8

$$\frac{x^3}{3} + \frac{\ln\left(x^3+1\right)}{6} - \frac{3\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^6+4*x^3+3),x)

[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Maxima [A] time = 0.779735, size = 30, normalized size = 1.07

$$\frac{1}{3}x^3 - \frac{3}{2}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^6 + 4*x^3 + 3),x, algorithm="maxima")

[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Fricas [A] time = 0.263143, size = 30, normalized size = 1.07

$$\frac{1}{3}x^3 - \frac{3}{2}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^6 + 4*x^3 + 3),x, algorithm="fricas")

[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.266127, size = 22, normalized size = 0.79

$$\frac{x^3}{3} + \frac{\log\left(x^3+1\right)}{6} - \frac{3\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**6+4*x**3+3),x)
```

```
[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

GIAC/XCAS [A] time = 0.277353, size = 32, normalized size = 1.14

$$\frac{1}{3}\,x^3 - \frac{3}{2}\ln\left(\left|x^3+3\right|\right) + \frac{1}{6}\ln\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 3/2*ln(abs(x^3 + 3)) + 1/6*ln(abs(x^3 + 1))
```

3.153 $\int \frac{x^5}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

```
[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Rubi [A] time = 0.0360109, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(3 + 4*x^3 + x^6),x]

[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Rubi in Sympy [A] time = 7.919, size = 15, normalized size = 0.71

$$-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**6+4*x**3+3),x)

[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2
```

Mathematica [A] time = 0.00720602, size = 21, normalized size = 1.

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(3 + 4*x^3 + x^6),x]
```

```
[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Maple [A] time = 0.007, size = 18, normalized size = 0.9

$$-\frac{\ln\left(x^3+1\right)}{6}+\frac{\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Maxima [A] time = 0.795143, size = 23, normalized size = 1.1

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Fricas [A] time = 0.25847, size = 23, normalized size = 1.1

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.23647, size = 15, normalized size = 0.71

$$-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2
```

GIAC/XCAS [A] time = 0.300991, size = 26, normalized size = 1.24

$$\frac{1}{2}\ln\left(\left|x^3+3\right|\right)-\frac{1}{6}\ln\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(x^3 + 3)) - 1/6*ln(abs(x^3 + 1))
```

3.154 $\int \frac{x^2}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\tanh^{-1}\left(x^3+2\right)$$

```
[Out] -ArcTanh[2 + x^3]/3
```

Rubi [B] time = 0.0295754, antiderivative size = 21, normalized size of antiderivative = 2.1, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(3 + 4*x^3 + x^6),x]

[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Rubi in Sympy [A] time = 5.73303, size = 15, normalized size = 1.5

$$\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**6+4*x**3+3),x)

[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6
```

Mathematica [B] time = 0.00635518, size = 21, normalized size = 2.1

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Maple [B] time = 0.007, size = 18, normalized size = 1.8

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{\ln\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x^3+1)-1/6*ln(x^3+3)
```

Maxima [A] time = 0.782302, size = 23, normalized size = 2.3

$$-\frac{1}{6}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Fricas [A] time = 0.253812, size = 23, normalized size = 2.3

$$-\frac{1}{6}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] time = 0.215654, size = 15, normalized size = 1.5

$$\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6
```

GIAC/XCAS [A] time = 0.261742, size = 26, normalized size = 2.6

$$-\frac{1}{6}\ln\left(\left|x^3+3\right|\right)+\frac{1}{6}\ln\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] -1/6*ln(abs(x^3 + 3)) + 1/6*ln(abs(x^3 + 1))
```

3.155 $\int \frac{1}{x(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rubi [A] time = 0.0449717, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(3 + 4*x^3 + x^6)),x]

[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rubi in Sympy [A] time = 10.9894, size = 22, normalized size = 0.81

$$\frac{\log\left(x^3\right)}{9}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**6+4*x**3+3),x)

[Out] log(x**3)/9 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

Mathematica [A] time = 0.0101607, size = 27, normalized size = 1.

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Maple [A] time = 0.013, size = 31, normalized size = 1.2

$$\frac{\ln\left(x^3+3\right)}{18}-\frac{\ln\left(1+x\right)}{6}+\frac{\ln\left(x\right)}{3}-\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^6+4*x^3+3),x)
```

```
[Out] 1/18*ln(x^3+3)-1/6*ln(1+x)+1/3*ln(x)-1/6*ln(x^2-x+1)
```

Maxima [A] time = 0.789482, size = 31, normalized size = 1.15

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{9}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x),x, algorithm="maxima")
```

```
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

Fricas [A] time = 0.252718, size = 28, normalized size = 1.04

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x),x, algorithm="fricas")
```

```
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

Sympy [A] time = 0.299551, size = 20, normalized size = 0.74

$$\frac{\log\left(x\right)}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**6+4*x**3+3),x)
```

```
[Out] log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

GIAC/XCAS [A] time = 0.288446, size = 32, normalized size = 1.19

$$\frac{1}{18}\ln\left(\left|x^3+3\right|\right)-\frac{1}{6}\ln\left(\left|x^3+1\right|\right)+\frac{1}{3}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x),x, algorithm="giac")
```

```
[Out] 1/18*ln(abs(x^3 + 3)) - 1/6*ln(abs(x^3 + 1)) + 1/3*ln(abs(x))
```

3.156 $\int \frac{1}{x^4(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rubi [A] time = 0.0757246, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rubi in Sympy [A] time = 16.4955, size = 31, normalized size = 0.91

$$-\frac{4\log\left(x^3\right)}{27}+\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{54}-\frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**6+4*x**3+3),x)
```

```
[Out] -4*log(x**3)/27 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

Mathematica [A] time = 0.00963341, size = 34, normalized size = 1.

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Maple [A] time = 0.013, size = 36, normalized size = 1.1

$$-\frac{\ln\left(x^3+3\right)}{54}+\frac{\ln\left(1+x\right)}{6}-\frac{1}{9\,x^3}-\frac{4\,\ln\left(x\right)}{9}+\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^6+4*x^3+3),x)
```

```
[Out] -1/54*ln(x^3+3)+1/6*ln(1+x)-1/9/x^3-4/9*ln(x)+1/6*ln(x^2-x+1)
```

Maxima [A] time = 0.761698, size = 38, normalized size = 1.12

$$-\frac{1}{9\,x^3}-\frac{1}{54}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)-\frac{4}{27}\,\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^4),x, algorithm="maxima")
```

```
[Out] -1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

Fricas [A] time = 0.253134, size = 47, normalized size = 1.38

$$-\frac{x^3\log\left(x^3+3\right)-9\,x^3\log\left(x^3+1\right)+24\,x^3\log\left(x\right)+6}{54\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^4),x, algorithm="fricas")
```

```
[Out] -1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)
/x^3
```

Sympy [A] time = 0.421574, size = 29, normalized size = 0.85

$$-\frac{4\log(x)}{9}+\frac{\log\left(x^{3}+1\right)}{6}-\frac{\log\left(x^{3}+3\right)}{54}-\frac{1}{9x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**6+4*x**3+3),x)
```

```
[Out] -4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

GIAC/XCAS [A] time = 0.283973, size = 49, normalized size = 1.44

$$\frac{4\,x^{3}-3}{27\,x^{3}}-\frac{1}{54}\ln\left(\left|x^{3}+3\right|\right)+\frac{1}{6}\ln\left(\left|x^{3}+1\right|\right)-\frac{4}{9}\ln\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^4),x, algorithm="giac")
```

```
[Out] 1/27*(4*x^3 - 3)/x^3 - 1/54*ln(abs(x^3 + 3)) + 1/6*ln(abs(x^3 + 1
)) - 4/9*ln(abs(x))
```

3.157 $\int \frac{1}{x^7(3+4x^3+x^6)} \, dx$

Optimal. Leaf size=41

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[
3 + x^3]/162
```

Rubi [A] time = 0.0852755, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[
3 + x^3]/162
```

Rubi in Sympy [A] time = 15.1415, size = 37, normalized size = 0.9

$$\frac{13\log\left(x^3\right)}{81}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{162}+\frac{4}{27x^3}-\frac{1}{18x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(x**6+4*x**3+3),x)
```

```
[Out] 13*log(x**3)/81 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + 4/(27*x**
3) - 1/(18*x**6)
```

Mathematica [A] time = 0.00949006, size = 41, normalized size = 1.

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[
3 + x^3]/162
```

Maple [A] time = 0.013, size = 41, normalized size = 1.

$$\frac{\ln\left(x^3+3\right)}{162}-\frac{\ln\left(1+x\right)}{6}-\frac{1}{18\,x^6}+\frac{4}{27\,x^3}+\frac{13\,\ln\left(x\right)}{27}-\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^6+4*x^3+3),x)
```

```
[Out] 1/162*ln(x^3+3)-1/6*ln(1+x)-1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(
x^2-x+1)
```

Maxima [A] time = 0.772754, size = 47, normalized size = 1.15

$$\frac{8\,x^3-3}{54\,x^6}+\frac{1}{162}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)+\frac{13}{81}\,\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^7),x, algorithm="maxima")
```

```
[Out] 1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13
/81*log(x^3)
```

Fricas [A] time = 0.249315, size = 54, normalized size = 1.32

$$\frac{x^6\log\left(x^3+3\right)-27\,x^6\log\left(x^3+1\right)+78\,x^6\log\left(x\right)+24\,x^3-9}{162\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^7),x, algorithm="fricas")
```

```
[Out] 1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 2
4*x^3 - 9)/x^6
```

Sympy [A] time = 0.532243, size = 34, normalized size = 0.83

$$\frac{13\log(x)}{27} - \frac{\log(x^3+1)}{6} + \frac{\log(x^3+3)}{162} + \frac{8x^3-3}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**6+4*x**3+3),x)
```

```
[Out] 13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)
/(54*x**6)
```

GIAC/XCAS [A] time = 0.303019, size = 55, normalized size = 1.34

$$-\frac{13\,x^6 - 8\,x^3 + 3}{54\,x^6} + \frac{1}{162}\ln\left(\left|x^3+3\right|\right) - \frac{1}{6}\ln\left(\left|x^3+1\right|\right) + \frac{13}{27}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^7),x, algorithm="giac")
```

```
[Out] -1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*ln(abs(x^3 + 3)) - 1/6*ln(
abs(x^3 + 1)) + 13/27*ln(abs(x))
```

3.158 $\int \frac{x^{10}}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=124

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/
6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)
*Log[3^(1/3) + x])/2 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/
3) - 3^(1/3)*x + x^2])/4
```

Rubi [A] time = 0.243442, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/
6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)
*Log[3^(1/3) + x])/2 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/
3) - 3^(1/3)*x + x^2])/4
```

Rubi in Sympy [A] time = 34.9788, size = 119, normalized size = 0.96

$$\frac{x^5}{5}-2x^2+\frac{\log(x+1)}{6}-\frac{3\cdot 3^{\frac{2}{3}}\log\left(x+\sqrt[3]{3}\right)}{2}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{3\cdot 3^{\frac{2}{3}}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{4}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{9\sqrt[6]{3}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(x**6+4*x**3+3),x)
```

```
[Out] x**5/5 - 2*x**2 + log(x + 1)/6 - 3*3**(2/3)*log(x + 3**(1/3))/2 -
 log(x**2 - x + 1)/12 + 3*3**(2/3)*log(x**2 - 3**(1/3)*x + 3**(2/
3))/4 - sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/6 - 9*3**(1/6)*atan(s
qrt(3)*(-2*3**(2/3)*x/9 + 1/3))/2
```

Mathematica [A] time = 0.094674, size = 118, normalized size = 0.95

$$\frac{1}{60}\left(12x^5-120x^2-5\log\left(x^2-x+1\right)+45\,3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+10\log(x+1)-90\,3^{2/3}\log\left(3^{2/3}x+3\right)-270\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-10\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)]
- 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2
/3)*Log[3 + 3^(2/3)*x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 -
3^(2/3)*x + 3^(1/3)*x^2])/60
```

Maple [A] time = 0.014, size = 94, normalized size = 0.8

$$\frac{x^5}{5}-2\,x^2-\frac{3\,3^{2/3}\ln\left(\sqrt[3]{3}+x\right)}{2}+\frac{3\,3^{2/3}\ln\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4}+\frac{9\sqrt[6]{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)+\frac{\ln(1+x)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(x^6+4*x^3+3),x)
```

```
[Out] 1/5*x^5-2*x^2-3/2*3^(2/3)*ln(3^(1/3)+x)+3/4*3^(2/3)*ln(3^(2/3)-3^
(1/3)*x+x^2)+9/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/
6*ln(1+x)-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2)
)
```

Maxima [A] time = 0.857923, size = 127, normalized size = 1.02

$$\frac{1}{5}x^5 - 2x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2 - x + 1\right) + \frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/
2*3^(2/3)*log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x
- 1)) + 9/2*3^(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*lo
g(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 0.25815, size = 163, normalized size = 1.31

$$-\frac{1}{180}\sqrt{3}\left(45\sqrt{3}(-9)^{\frac{1}{3}}\log\left(3x^2 - (-9)^{\frac{2}{3}}x - 3(-9)^{\frac{1}{3}}\right) - 90\sqrt{3}(-9)^{\frac{1}{3}}\log\left(3x + (-9)^{\frac{2}{3}}\right) - 12\sqrt{3}\left(x^5 - 10x^2\right) + 5\sqrt{3}\log\left(x^2 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] -1/180*sqrt(3)*(45*sqrt(3)*(-9)^(1/3)*log(3*x^2 - (-9)^(2/3)*x -
3*(-9)^(1/3)) - 90*sqrt(3)*(-9)^(1/3)*log(3*x + (-9)^(2/3)) - 12*
sqrt(3)*(x^5 - 10*x^2) + 5*sqrt(3)*log(x^2 - x + 1) - 10*sqrt(3)*
log(x + 1) - 270*(-9)^(1/3)*arctan(1/27*sqrt(3)*(-9)^(1/3)*(6*x -
 (-9)^(2/3))) + 30*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 2.22207, size = 144, normalized size = 1.16

$$\frac{x^5}{5} - 2x^2 + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3872\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{3281} + \frac{3188648\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{88587}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3188648\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{88587} + \frac{3872\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{3281}\right) + \operatorname{RootSum}\left(8t^3 + 243, \left(t \mapsto t\log\left(\frac{3872t^5}{3281} + \frac{3188648t^2}{88587} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(x**6+4*x**3+3),x)
```

```
[Out] x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3
872*(-1/12 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/1
2)**2/88587) + (-1/12 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sq
rt(3)*I/12)**2/88587 + 3872*(-1/12 + sqrt(3)*I/12)**5/3281) + Roo
tSum(8*_t**3 + 243, Lambda(_t, _t*log(3872*_t**5/3281 + 3188648*_
t**2/88587 + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.159 $\int \frac{x^9}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=122

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)
*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*L
og[3^(1/3) + x])/2 + Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3)
 - 3^(1/3)*x + x^2])/4
```

Rubi [A] time = 0.201544, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)
*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*L
og[3^(1/3) + x])/2 + Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3)
 - 3^(1/3)*x + x^2])/4
```

Rubi in Sympy [A] time = 36.2379, size = 117, normalized size = 0.96

$$\frac{x^4}{4}-4x-\frac{\log(x+1)}{6}+\frac{3\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)}{2}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{3\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{4}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{3\cdot 3^{\frac{5}{6}}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**6+4*x**3+3),x)
```

```
[Out] x**4/4 - 4*x - log(x + 1)/6 + 3*3**(1/3)*log(x + 3**(1/3))/2 + lo
g(x**2 - x + 1)/12 - 3*3**(1/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))
/4 - sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/6 - 3*3**(5/6)*atan(sqrt
(3)*(-2*3**(2/3)*x/9 + 1/3))/2
```

Mathematica [A] time = 0.0522935, size = 114, normalized size = 0.93

$$\frac{1}{12}\left(3x^4+\log\left(x^2-x+1\right)-9\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-48x-2\log(x+1)\right.$$
$$\left.+18\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-18\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*S
qrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log
[3 + 3^(2/3)*x] + Log[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x
+ 3^(1/3)*x^2])/12
```

Maple [A] time = 0.01, size = 92, normalized size = 0.8

$$\frac{x^4}{4}-4\,x+\frac{3\,\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{2}-\frac{3\,\sqrt[3]{3}\ln\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4}+\frac{3\,3^{5/6}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)$$
$$-\frac{\ln\left(1+x\right)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2\,x-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^6+4*x^3+3),x)
```

```
[Out] 1/4*x^4-4*x+3/2*3^(1/3)*ln(3^(1/3)+x)-3/4*3^(1/3)*ln(3^(2/3)-3^(1
/3)*x+x^2)+3/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*
ln(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.841271, size = 124, normalized size = 1.02

$$\frac{1}{4}x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - 4x + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*s
qrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1
/3)*x + 3^(2/3)) + 3/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(
x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 0.258288, size = 140, normalized size = 1.15

$$-\frac{1}{36}\sqrt{3}\left(9\cdot 3^{\frac{5}{6}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - 18\cdot 3^{\frac{5}{6}}\log\left(x + 3^{\frac{1}{3}}\right) - 3\sqrt{3}\left(x^4 - 16x\right) - \sqrt{3}\log\left(x^2 - x + 1\right) + 2\sqrt{3}\log\left(x+1\right) + 54\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(9*3^(5/6)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 18*3^(5
/6)*log(x + 3^(1/3)) - 3*sqrt(3)*(x^4 - 16*x) - sqrt(3)*log(x^2 -
 x + 1) + 2*sqrt(3)*log(x + 1) + 54*3^(1/3)*arctan(-1/3*3^(1/6)*(
2*x - 3^(1/3))) + 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 2.20625, size = 129, normalized size = 1.06

$$\frac{x^4}{4} - 4x - \frac{\log(x+1)}{6} + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} - \frac{9841\sqrt{3}i}{19692} + \frac{360\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{547}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} + \frac{360\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{547} + \frac{9841\sqrt{3}i}{19692}\right) + \operatorname{RootSum}\left(8t^3 - 81, \left(t \mapsto t\log\left(\frac{360t^4}{547} - \frac{9841t}{1641} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**6+4*x**3+3),x)
```

```
[Out] x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/
19692 - 9841*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547)
+ (1/12 - sqrt(3)*I/12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*
I/12)**4/547 + 9841*sqrt(3)*I/19692) + RootSum(8*_t**3 - 81, Lamb
da(_t, _t*log(360*_t**4/547 - 9841*_t/1641 + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.160 $\int \frac{x^7}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=119

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

```
[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan
[(3^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3
) + x])/2 + Log[1 - x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*
x + x^2])/4
```

Rubi [A] time = 0.167437, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan
[(3^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3
) + x])/2 + Log[1 - x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*
x + x^2])/4
```

Rubi in Sympy [A] time = 27.7032, size = 110, normalized size = 0.92

$$\frac{x^2}{2}-\frac{\log(x+1)}{6}+\frac{3^{\frac{2}{3}}\log\left(x+\sqrt[3]{3}\right)}{2}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{3^{\frac{2}{3}}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{3\sqrt[6]{3}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**6+4*x**3+3),x)
```

```
[Out] x**2/2 - log(x + 1)/6 + 3**(2/3)*log(x + 3**(1/3))/2 + log(x**2 -
 x + 1)/12 - 3**(2/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/4 + sqrt(
3)*atan(sqrt(3)*(2*x/3 - 1/3))/6 + 3*3**(1/6)*atan(sqrt(3)*(-2*3*
*(2/3)*x/9 + 1/3))/2
```

Mathematica [A] time = 0.0484352, size = 111, normalized size = 0.93

$$\frac{1}{12}\left(6x^2+\log\left(x^2-x+1\right)-3\,3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+6\,3^{2/3}\log\left(3^{2/3}x+3\right)+18\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*A
rcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2
/3)*x] + Log[1 - x + x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)
*x^2])/12
```

Maple [A] time = 0.009, size = 89, normalized size = 0.8

$$\frac{x^2}{2}+\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{4}-\frac{3\sqrt[6]{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)-\frac{\ln(1+x)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^6+4*x^3+3),x)
```

```
[Out] 1/2*x^2+1/2*3^(2/3)*ln(3^(1/3)+x)-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*
x+x^2)-3/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1
+x)+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.867801, size = 120, normalized size = 1.01

$$\frac{1}{2}x^2 - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3
)*log(x + 3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) -
3/2*3^(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 -
x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 0.278836, size = 154, normalized size = 1.29

$$\frac{1}{36}\sqrt{3}\left(6\sqrt{3}x^2 - 3\cdot 9^{\frac{1}{3}}\sqrt{3}\log\left(3x^2 - 9^{\frac{2}{3}}x + 3\cdot 9^{\frac{1}{3}}\right) + 6\cdot 9^{\frac{1}{3}}\sqrt{3}\log\left(3x + 9^{\frac{2}{3}}\right) + \sqrt{3}\log\left(x^2 - x + 1\right) - 2\sqrt{3}\log\left(x+1\right) + 18\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*(6*sqrt(3)*x^2 - 3*9^(1/3)*sqrt(3)*log(3*x^2 - 9^(2/
3)*x + 3*9^(1/3)) + 6*9^(1/3)*sqrt(3)*log(3*x + 9^(2/3)) + sqrt(3
)*log(x^2 - x + 1) - 2*sqrt(3)*log(x + 1) + 18*9^(1/3)*arctan(-1/
27*9^(1/3)*sqrt(3)*(6*x - 9^(2/3))) + 6*arctan(1/3*sqrt(3)*(2*x -
 1)))
```

Sympy [A] time = 2.24524, size = 134, normalized size = 1.13

$$\frac{x^2}{2} - \frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{6562\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{183} - \frac{1872\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{1872\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{61} + \frac{6562\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{183}\right) + \operatorname{RootSum}\left(8t^3 - 9, \left(t \mapsto t\log\left(-\frac{1872t^5}{61} + \frac{6562t^2}{183} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**6+4*x**3+3),x)
```

```
[Out] x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12
- sqrt(3)*I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/1
2 + sqrt(3)*I/12)*log(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562
*(1/12 + sqrt(3)*I/12)**2/183) + RootSum(8*_t**3 - 9, Lambda(_t,
_t*log(-1872*_t**5/61 + 6562*_t**2/183 + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.161 $\int \frac{x^6}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

```
[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1
/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x]
)/2 - Log[1 - x + x^2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^
2])/4
```

Rubi [A] time = 0.146049, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1
/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x]
)/2 - Log[1 - x + x^2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^
2])/4
```

Rubi in Sympy [A] time = 28.8504, size = 105, normalized size = 0.93

$$x+\frac{\log(x+1)}{6}-\frac{\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)}{2}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{3^{\frac{5}{6}}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**6+4*x**3+3),x)
```

```
[Out] x + log(x + 1)/6 - 3**(1/3)*log(x + 3**(1/3))/2 - log(x**2 - x +
1)/12 + 3**(1/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/4 + sqrt(3)*at
an(sqrt(3)*(2*x/3 - 1/3))/6 + 3**(5/6)*atan(sqrt(3)*(-2*3**(2/3)*
x/9 + 1/3))/2
```

Mathematica [A] time = 0.0505266, size = 111, normalized size = 0.98

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+12x+2\log(x+1)-6\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+6\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] (12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3
)*x] - Log[1 - x + x^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x
^2])/12
```

Maple [A] time = 0.009, size = 85, normalized size = 0.8

$$x-\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{2}+\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{4}-\frac{3^{\frac{5}{6}}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)+\frac{\ln(1+x)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^6+4*x^3+3),x)
```

```
[Out] x-1/2*3^(1/3)*ln(3^(1/3)+x)+1/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
-1/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-1/
12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.852332, size = 115, normalized size = 1.02

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+x-\frac{1}{12}\,\log\left(x^2-x+1\right)+\frac{1}{6}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3
^(2/3)) - 1/2*3^(1/3)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1
) + 1/6*log(x + 1)
```

Fricas [A] time = 0.265852, size = 142, normalized size = 1.26

$$-\frac{1}{36}\,\sqrt{3}\left(3\,\sqrt{3}\,(-3)^{\frac{1}{3}}\log\left(x^2+(-3)^{\frac{1}{3}}\,x+(-3)^{\frac{2}{3}}\right)-6\,\sqrt{3}\,(-3)^{\frac{1}{3}}\log\left(x-(-3)^{\frac{1}{3}}\right)-12\,\sqrt{3}x+\sqrt{3}\log\left(x^2-x+1\right)-2\,\sqrt{3}\log\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(3*sqrt(3)*(-3)^(1/3)*log(x^2 + (-3)^(1/3)*x + (-3)
^(2/3)) - 6*sqrt(3)*(-3)^(1/3)*log(x - (-3)^(1/3)) - 12*sqrt(3)*x
 + sqrt(3)*log(x^2 - x + 1) - 2*sqrt(3)*log(x + 1) + 18*(-3)^(1/3
)*arctan(-1/9*sqrt(3)*(-3)^(2/3)*(2*x + (-3)^(1/3))) - 6*arctan(1
/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 2.21808, size = 126, normalized size = 1.12

$$x+\frac{\log\left(x+1\right)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}-\frac{121\sqrt{3}i}{246}+\frac{864\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}+\frac{864\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{41}+\frac{121\sqrt{3}i}{246}\right)+\operatorname{RootSum}\left(8t^3+3,\left(t\mapsto t\log\left(\frac{864t^4}{41}+\frac{242t}{41}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**6+4*x**3+3),x)
```

```
[Out] x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*s
qrt(3)*I/246 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(
3)*I/12)*log(x - 121/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121
*sqrt(3)*I/246) + RootSum(8*_t**3 + 3, Lambda(_t, _t*log(864*_t**
4/41 + 242*_t/41 + x)))
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.162 $\int \frac{x^4}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3)
- 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3))
- Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3)
)
```

Rubi [A] time = 0.135999, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3)
- 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3))
- Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3)
)
```

Rubi in Sympy [A] time = 21.6207, size = 104, normalized size = 0.93

$$\frac{\log\left(x+1\right)}{6}-\frac{3^{\frac{2}{3}}\log\left(x+\sqrt[3]{3}\right)}{6}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{3^{\frac{2}{3}}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{12}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{\sqrt[6]{3}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 - 3**(2/3)*log(x + 3**(1/3))/6 - log(x**2 - x + 1)/1
2 + 3**(2/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/12 - sqrt(3)*atan(
sqrt(3)*(2*x/3 - 1/3))/6 - 3**(1/6)*atan(sqrt(3)*(-2*3**(2/3)*x/9
 + 1/3))/2
```

Mathematica [A] time = 0.0461831, size = 107, normalized size = 0.96

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+2\log(x+1)-2\,3^{2/3}\log\left(3^{2/3}x+3\right)-6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-
1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] -
 Log[1 - x + x^2] + 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A] time = 0.009, size = 84, normalized size = 0.8

$$-\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{12}+\frac{\sqrt[6]{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)+\frac{\ln(1+x)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*3^(2/3)*ln(3^(1/3)+x)+1/12*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
+1/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-1/
12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.845666, size = 113, normalized size = 1.01

$$\frac{1}{12} \cdot 3^{\frac{2}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{6} \cdot 3^{\frac{2}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2 - x + 1\right) + \frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x +
 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/
6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) +
1/6*log(x + 1)
```

Fricas [A] time = 0.267202, size = 163, normalized size = 1.46

$$-\frac{1}{36} \cdot 3^{\frac{1}{6}}\left(3\sqrt{3}(-1)^{\frac{1}{3}}\log\left(-3^{\frac{2}{3}}(-1)^{\frac{2}{3}}x + 3^{\frac{1}{3}}x^2 - 3(-1)^{\frac{1}{3}}\right) + 3^{\frac{5}{6}}\log\left(x^2 - x + 1\right) - 6\sqrt{3}(-1)^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}x + 3(-1)^{\frac{2}{3}}\right) - 2 \cdot 3^{\frac{5}{6}}\log(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] -1/36*3^(1/6)*(3*sqrt(3)*(-1)^(1/3)*log(-3^(2/3)*(-1)^(2/3)*x + 3
^(1/3)*x^2 - 3*(-1)^(1/3)) + 3^(5/6)*log(x^2 - x + 1) - 6*sqrt(3)
*(-1)^(1/3)*log(3^(2/3)*x + 3*(-1)^(2/3)) - 2*3^(5/6)*log(x + 1)
+ 18*(-1)^(1/3)*arctan(-1/9*sqrt(3)*(-1)^(1/3)*(2*3^(2/3)*x - 3*(
-1)^(2/3))) + 6*3^(1/3)*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 2.18963, size = 134, normalized size = 1.2

$$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{2592\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{5} + \frac{168\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{5}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{168\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{5} + \frac{2592\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{5}\right) + \operatorname{RootSum}\left(24t^3 + 1, \left(t \mapsto t\log\left(\frac{2592t^5}{5} + \frac{168t^2}{5} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(
3)*I/12)**5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(
3)*I/12)*log(x + 168*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 +
sqrt(3)*I/12)**5/5) + RootSum(24*_t**3 + 1, Lambda(_t, _t*log(259
2*_t**5/5 + 168*_t**2/5 + x)))
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.163 $\int \frac{x^3}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^
(5/6)]/(2*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3))
+ Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3)
)
```

Rubi [A] time = 0.132702, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^
(5/6)]/(2*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3))
+ Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3)
)
```

Rubi in Sympy [A] time = 22.8163, size = 104, normalized size = 0.93

$$-\frac{\log\left(x+1\right)}{6}+\frac{\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)}{6}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{3^{\frac{5}{6}}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + 3**(1/3)*log(x + 3**(1/3))/6 + log(x**2 - x + 1)/
12 - 3**(1/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/12 - sqrt(3)*atan
(sqrt(3)*(2*x/3 - 1/3))/6 - 3**(5/6)*atan(sqrt(3)*(-2*3**(2/3)*x/
9 + 1/3))/6
```

Mathematica [A] time = 0.0455173, size = 106, normalized size = 0.95

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-
1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] +
 Log[1 - x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A] time = 0.009, size = 84, normalized size = 0.8

$$\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{6}-\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{12}+\frac{3^{\frac{5}{6}}}{6}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)-\frac{\ln(1+x)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*3^(1/3)*ln(3^(1/3)+x)-1/12*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+
1/6*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)+1/1
2*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.844276, size = 113, normalized size = 1.01

$$\frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-\frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3
^(2/3)) + 1/6*3^(1/3)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) -
1/6*log(x + 1)
```

Fricas [A] time = 0.266945, size = 139, normalized size = 1.24

$$\frac{1}{324}\cdot 9^{\frac{2}{3}}\sqrt{3}\left(9^{\frac{1}{3}}\sqrt{3}\log\left(x^2-x+1\right)-2\cdot 9^{\frac{1}{3}}\sqrt{3}\log\left(x+1\right)-3\,\sqrt{3}\log\left(9^{\frac{2}{3}}x^2-3\cdot 9^{\frac{1}{3}}x+9\right)+6\,\sqrt{3}\log\left(9^{\frac{1}{3}}x+3\right)-6\cdot 9^{\frac{1}{3}}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] 1/324*9^(2/3)*sqrt(3)*(9^(1/3)*sqrt(3)*log(x^2 - x + 1) - 2*9^(1/
3)*sqrt(3)*log(x + 1) - 3*sqrt(3)*log(9^(2/3)*x^2 - 3*9^(1/3)*x +
 9) + 6*sqrt(3)*log(9^(1/3)*x + 3) - 6*9^(1/3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 18*arctan(1/9*sqrt(3)*(2*9^(1/3)*x - 3)))
```

Sympy [A] time = 2.19262, size = 110, normalized size = 0.98

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4+\frac{\sqrt{3}i}{4}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4-\frac{\sqrt{3}i}{4}\right)+\operatorname{RootSum}\left(72t^3-1,\left(t\mapsto t\log\left(648t^4-3t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - s
qrt(3)*I/12)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/
4 + 648*(1/12 + sqrt(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**
3 - 1, Lambda(_t, _t*log(648*_t**4 - 3*_t + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.164 $\int \frac{x}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(2*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3))
 + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/
3))
```

Rubi [A] time = 0.128514, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(2*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3))
 + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/
3))
```

Rubi in Sympy [A] time = 20.7523, size = 104, normalized size = 0.93

$$-\frac{\log(x+1)}{6}+\frac{3^{\frac{2}{3}}\log\left(x+\sqrt[3]{3}\right)}{18}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{3^{\frac{2}{3}}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{36}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt[6]{3}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + 3**(2/3)*log(x + 3**(1/3))/18 + log(x**2 - x + 1)
/12 - 3**(2/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/36 + sqrt(3)*ata
n(sqrt(3)*(2*x/3 - 1/3))/6 + 3**(1/6)*atan(sqrt(3)*(-2*3**(2/3)*x
/9 + 1/3))/6
```

Mathematica [A] time = 0.0446223, size = 108, normalized size = 0.96

$$\frac{1}{36}\left(3\log\left(x^2-x+1\right)-3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-6\log(x+1)+2\,3^{2/3}\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1
 + 2*x)/Sqrt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] +
3*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [A] time = 0.008, size = 84, normalized size = 0.8

$$\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{18}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{36}-\frac{\sqrt[6]{3}}{6}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)-\frac{\ln(1+x)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^6+4*x^3+3),x)
```

```
[Out] 1/18*3^(2/3)*ln(3^(1/3)+x)-1/36*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
-1/6*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)+1/
12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.875692, size = 113, normalized size = 1.01

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x
 + 3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(
1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1)
- 1/6*log(x + 1)
```

Fricas [A] time = 0.273926, size = 126, normalized size = 1.12

$$\frac{1}{36}\cdot 3^{\frac{1}{6}}\left(3^{\frac{5}{6}}\log\left(x^2-x+1\right)-2\cdot 3^{\frac{5}{6}}\log\left(x+1\right)-\sqrt{3}\log\left(3^{\frac{1}{3}}x^2-3^{\frac{2}{3}}x+3\right)+2\sqrt{3}\log\left(3^{\frac{2}{3}}x+3\right)+6\cdot 3^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] 1/36*3^(1/6)*(3^(5/6)*log(x^2 - x + 1) - 2*3^(5/6)*log(x + 1) - s
qrt(3)*log(3^(1/3)*x^2 - 3^(2/3)*x + 3) + 2*sqrt(3)*log(3^(2/3)*x
 + 3) + 6*3^(1/3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*arctan(2/3*3^
(1/6)*x - 1/3*sqrt(3)))
```

Sympy [A] time = 4.49702, size = 119, normalized size = 1.06

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2+11664\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+11664\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5+90\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2\right)+\operatorname{RootSum}\left(648t^3-1,\left(t\mapsto t\log\left(11664t^5+90t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*
I/12)**2 + 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12
)*log(x + 11664*(1/12 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/1
2)**2) + RootSum(648*_t**3 - 1, Lambda(_t, _t*log(11664*_t**5 + 9
0*_t**2 + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.165 $\int \frac{1}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3))
 - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/
3))
```

Rubi [A] time = 0.117778, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3))
 - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/
3))
```

Rubi in Sympy [A] time = 17.9273, size = 104, normalized size = 0.93

$$\frac{\log\left(x+1\right)}{6}-\frac{\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)}{18}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{3^{\frac{5}{6}}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 - 3**(1/3)*log(x + 3**(1/3))/18 - log(x**2 - x + 1)/
12 + 3**(1/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/36 + sqrt(3)*atan
(sqrt(3)*(2*x/3 - 1/3))/6 + 3**(5/6)*atan(sqrt(3)*(-2*3**(2/3)*x/
9 + 1/3))/18
```

Mathematica [A]   time = 0.0499714, size = 107, normalized size = 0.96

$$\frac{1}{36}\left(-3\log\left(x^2-x+1\right)+\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+6\log(x+1)\right.$$
$$\left.-2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+2\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] (2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1
 + 2*x)/Sqrt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] -
3*Log[1 - x + x^2] + 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [A]   time = 0.009, size = 84, normalized size = 0.8

$$-\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{18}+\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{36}-\frac{3^{\frac{5}{6}}}{18}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)$$
$$+\frac{\ln(1+x)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6+4*x^3+3),x)
```

```
[Out] -1/18*3^(1/3)*ln(3^(1/3)+x)+1/36*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)-1/18*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-
1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.867097, size = 113, normalized size = 1.01

$$-\frac{1}{18} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2\,x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\,\sqrt{3} \arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x - 1)\right) + \frac{1}{36} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{18} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{12} \log\left(x^2 - x + 1\right) + \frac{1}{6} \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + 4*x^3 + 3),x, algorithm="maxima")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x +
 3^(2/3)) - 1/18*3^(1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1)
 + 1/6*log(x + 1)
```

Fricas [A] time = 0.263153, size = 178, normalized size = 1.59

$$-\frac{1}{324} \cdot 9^{\frac{2}{3}}\sqrt{3}\left(\sqrt{3}\,(-1)^{\frac{1}{3}} \log\left(9^{\frac{2}{3}}x^2 + 3 \cdot 9^{\frac{1}{3}}\,(-1)^{\frac{1}{3}}\,x + 9\,(-1)^{\frac{2}{3}}\right) + 9^{\frac{1}{3}}\sqrt{3}\log\left(x^2 - x + 1\right) - 2\,\sqrt{3}\,(-1)^{\frac{1}{3}} \log\left(9^{\frac{1}{3}}x - 3\,(-1)^{\frac{1}{3}}\right) - 2 \cdot 9^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + 4*x^3 + 3),x, algorithm="fricas")
```

```
[Out] -1/324*9^(2/3)*sqrt(3)*(sqrt(3)*(-1)^(1/3)*log(9^(2/3)*x^2 + 3*9^
(1/3)*(-1)^(1/3)*x + 9*(-1)^(2/3)) + 9^(1/3)*sqrt(3)*log(x^2 - x
+ 1) - 2*sqrt(3)*(-1)^(1/3)*log(9^(1/3)*x - 3*(-1)^(1/3)) - 2*9^(
1/3)*sqrt(3)*log(x + 1) + 6*(-1)^(1/3)*arctan(-1/9*(-1)^(2/3)*(2*
9^(1/3)*sqrt(3)*x + 3*sqrt(3)*(-1)^(1/3))) - 6*9^(1/3)*arctan(1/3
*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 4.6758, size = 124, normalized size = 1.11

$$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{13}{10} - \frac{13\sqrt{3}i}{10} + \frac{23328\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{5}\right) + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{13}{10} + \frac{23328\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{5} + \frac{13\sqrt{3}i}{10}\right) + \mathrm{RootSum}\left(1944t^3 + 1, \left(t \mapsto t\log\left(\frac{23328t^4}{5} - \frac{78t}{5} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*
I/10 + 23328*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12
)*log(x + 13/10 + 23328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*
I/10) + RootSum(1944*_t**3 + 1, Lambda(_t, _t*log(23328*_t**4/5 -
 78*_t/5 + x)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + 4*x^3 + 3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.166 $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}$$

```
[Out] -1/(3*x) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3
) - 2*x)/3^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(
18*3^(1/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2
]/(36*3^(1/3))
```

Rubi [A] time = 0.165755, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(3*x) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3
) - 2*x)/3^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(
18*3^(1/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2
]/(36*3^(1/3))
```

Rubi in Sympy [A] time = 27.9204, size = 109, normalized size = 0.92

$$\frac{\log\left(x+1\right)}{6}-\frac{3^{\frac{2}{3}}\log\left(x+\sqrt[3]{3}\right)}{54}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{3^{\frac{2}{3}}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{108}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{\sqrt[6]{3}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{18}-\frac{1}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 - 3**(2/3)*log(x + 3**(1/3))/54 - log(x**2 - x + 1)/
12 + 3**(2/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/108 - sqrt(3)*ata
n(sqrt(3)*(2*x/3 - 1/3))/6 - 3**(1/6)*atan(sqrt(3)*(-2*3**(2/3)*x
/9 + 1/3))/18 - 1/(3*x)
```

Mathematica [A] time = 0.0855583, size = 118, normalized size = 0.99

$$-\frac{9x\log\left(x^2-x+1\right)-3^{2/3}x\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18x\log(x+1)+2\,3^{2/3}x\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}x\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+18}{108x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x
*ArcTan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3
 + 3^(2/3)*x] + 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*
x + 3^(1/3)*x^2])/(108*x)
```

Maple [A] time = 0.011, size = 89, normalized size = 0.8

$$-\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{54}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{108}+\frac{\sqrt[6]{3}}{18}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)+\frac{\ln(1+x)}{6}-\frac{1}{3\,x}-\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^6+4*x^3+3),x)
```

```
[Out] -1/54*3^(2/3)*ln(3^(1/3)+x)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^
2)+1/18*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)
-1/3/x-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.871209, size = 120, normalized size = 1.01

$$\frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{3x}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x
 + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^
(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2
- x + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 0.259007, size = 181, normalized size = 1.52

$$-\frac{3^{\frac{1}{6}}\left(\sqrt{3}(-1)^{\frac{1}{3}}x\log\left(-3^{\frac{2}{3}}(-1)^{\frac{2}{3}}x+3^{\frac{1}{3}}x^2-3(-1)^{\frac{1}{3}}\right)+3\cdot 3^{\frac{5}{6}}x\log\left(x^2-x+1\right)-2\sqrt{3}(-1)^{\frac{1}{3}}x\log\left(3^{\frac{2}{3}}x+3(-1)^{\frac{2}{3}}\right)-6\cdot 3^{\frac{5}{6}}\right.}{108x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] -1/108*3^(1/6)*(sqrt(3)*(-1)^(1/3)*x*log(-3^(2/3)*(-1)^(2/3)*x +
3^(1/3)*x^2 - 3*(-1)^(1/3)) + 3*3^(5/6)*x*log(x^2 - x + 1) - 2*sq
rt(3)*(-1)^(1/3)*x*log(3^(2/3)*x + 3*(-1)^(2/3)) - 6*3^(5/6)*x*lo
g(x + 1) + 18*3^(1/3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*(-1)^(1
/3)*x*arctan(1/3*(-1)^(1/3)*(sqrt(3)*(-1)^(2/3) - 2*3^(1/6)*x)) +
 12*3^(5/6))/x
```

Sympy [A] time = 4.46155, size = 139, normalized size = 1.17

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{8188128\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{41}+\frac{39384\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{39384\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{41}-\frac{8188128\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{41}\right)+\operatorname{RootSum}\left(17496t^3+1,\left(t\mapsto t\log\left(-\frac{8188128t^5}{41}+\frac{39384t^2}{41}+x\right)\right)\right)-\frac{1}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sq
rt(3)*I/12)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12
+ sqrt(3)*I/12)*log(x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188
128*(-1/12 + sqrt(3)*I/12)**5/41) + RootSum(17496*_t**3 + 1, Lamb
da(_t, _t*log(-8188128*_t**5/41 + 39384*_t**2/41 + x))) - 1/(3*x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.167 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

```
[Out] -1/(6*x^2) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1
/3) - 2*x)/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x
]/(18*3^(2/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x +
x^2]/(36*3^(2/3))
```

Rubi [A] time = 0.14598, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(6*x^2) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1
/3) - 2*x)/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x
]/(18*3^(2/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x +
x^2]/(36*3^(2/3))
```

Rubi in Sympy [A] time = 27.4788, size = 110, normalized size = 0.92

$$-\frac{\log\left(x+1\right)}{6}+\frac{\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)}{54}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{108}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{3^{\frac{5}{6}}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{54}-\frac{1}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + 3**(1/3)*log(x + 3**(1/3))/54 + log(x**2 - x + 1)
/12 - 3**(1/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/108 - sqrt(3)*at
an(sqrt(3)*(2*x/3 - 1/3))/6 - 3**(5/6)*atan(sqrt(3)*(-2*3**(2/3)*
x/9 + 1/3))/54 - 1/(6*x**2)
```

Mathematica [A] time = 0.105209, size = 113, normalized size = 0.95

$$\frac{1}{108}\left(-\frac{18}{x^2}+9\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-18\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]
*ArcTan[(-1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3
^(2/3)*x] + 9*Log[1 - x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1
/3)*x^2])/108
```

Maple [A] time = 0.011, size = 89, normalized size = 0.8

$$\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{54}-\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{108}+\frac{3^{\frac{5}{6}}}{54}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)-\frac{\ln\left(1+x\right)}{6}-\frac{1}{6\,x^2}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^6+4*x^3+3),x)
```

```
[Out] 1/54*3^(1/3)*ln(3^(1/3)+x)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/54*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)-
1/6/x^2+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.856451, size = 120, normalized size = 1.01

$$\frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-\frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6\,x^2}+\frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^3),x, algorithm="maxima")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x +
 3^(2/3)) + 1/54*3^(1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^
2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 0.279468, size = 182, normalized size = 1.53

$$\frac{9^{\frac{2}{3}}\sqrt{3}\left(3\cdot 9^{\frac{1}{3}}\sqrt{3}x^2\log\left(x^2-x+1\right)-6\cdot 9^{\frac{1}{3}}\sqrt{3}x^2\log\left(x+1\right)-\sqrt{3}x^2\log\left(9^{\frac{2}{3}}x^2-3\cdot 9^{\frac{1}{3}}x+9\right)+2\,\sqrt{3}x^2\log\left(9^{\frac{1}{3}}x+3\right)-18\cdot 9\right.}{972\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^3),x, algorithm="fricas")
```

```
[Out] 1/972*9^(2/3)*sqrt(3)*(3*9^(1/3)*sqrt(3)*x^2*log(x^2 - x + 1) - 6
*9^(1/3)*sqrt(3)*x^2*log(x + 1) - sqrt(3)*x^2*log(9^(2/3)*x^2 - 3
*9^(1/3)*x + 9) + 2*sqrt(3)*x^2*log(9^(1/3)*x + 3) - 18*9^(1/3)*x
^2*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*x^2*arctan(2/9*9^(1/3)*sqrt(
3)*x - 1/3*sqrt(3)) - 6*9^(1/3)*sqrt(3))/x^2
```

Sympy [A] time = 4.45838, size = 128, normalized size = 1.08

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}-\frac{1093\sqrt{3}i}{244}+\frac{787320\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}+\frac{787320\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{61}+\frac{1093\sqrt{3}i}{244}\right)+\operatorname{RootSum}\left(52488t^3-1,\left(t\mapsto t\log\left(\frac{787320t^4}{61}+\frac{3279t}{61}+x\right)\right)\right)-\frac{1}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqr
t(3)*I/244 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3
)*I/12)*log(x + 1093/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1
093*sqrt(3)*I/244) + RootSum(52488*_t**3 - 1, Lambda(_t, _t*log(7
87320*_t**4/61 + 3279*_t/61 + x))) - 1/(6*x**2)
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.168 $\int \frac{1}{x^5(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}$$

```
[Out] -1/(12*x^4) + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + A
rcTan[(3^(1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[
3^(1/3) + x]/(54*3^(1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3
^(1/3)*x + x^2]/(108*3^(1/3))
```

Rubi [A] time = 0.210512, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(12*x^4) + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + A
rcTan[(3^(1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[
3^(1/3) + x]/(54*3^(1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3
^(1/3)*x + x^2]/(108*3^(1/3))
```

Rubi in Sympy [A] time = 34.4844, size = 116, normalized size = 0.92

$$-\frac{\log(x+1)}{6}+\frac{3^{\frac{2}{3}}\log\left(x+\sqrt[3]{3}\right)}{162}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{3^{\frac{2}{3}}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{324}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt[6]{3}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{54}+\frac{4}{9x}-\frac{1}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + 3**(2/3)*log(x + 3**(1/3))/162 + log(x**2 - x + 1
)/12 - 3**(2/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/324 + sqrt(3)*a
tan(sqrt(3)*(2*x/3 - 1/3))/6 + 3**(1/6)*atan(sqrt(3)*(-2*3**(2/3)
*x/9 + 1/3))/54 + 4/(9*x) - 1/(12*x**4)
```

Mathematica [A] time = 0.0996501, size = 118, normalized size = 0.94

$$\frac{1}{324}\left(-\frac{27}{x^4}+27\log\left(x^2-x+1\right)-3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+\frac{144}{x}-54\log(x+1)+2\,3^{2/3}\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+54\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54
*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*L
og[3 + 3^(2/3)*x] + 27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)
*x + 3^(1/3)*x^2])/324
```

Maple [A] time = 0.014, size = 94, normalized size = 0.8

$$\frac{3^{\frac{2}{3}}\ln\left(\sqrt[3]{3}+x\right)}{162}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{324}-\frac{\sqrt[6]{3}}{54}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)-\frac{\ln(1+x)}{6}-\frac{1}{12\,x^4}+\frac{4}{9\,x}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^6+4*x^3+3),x)
```

```
[Out] 1/162*3^(2/3)*ln(3^(1/3)+x)-1/324*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^
2)-1/54*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)
-1/12/x^4+4/9/x+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3
^(1/2))
```

Maxima [A] time = 0.866539, size = 130, normalized size = 1.03

$$-\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^5),x, algorithm="maxima")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log
(x + 3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*
3^(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x
^4 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A] time = 0.263886, size = 171, normalized size = 1.36

$$\frac{3^{\frac{1}{6}}\left(9\cdot 3^{\frac{5}{6}}x^4\log\left(x^2-x+1\right)-18\cdot 3^{\frac{5}{6}}x^4\log\left(x+1\right)-\sqrt{3}x^4\log\left(3^{\frac{1}{3}}x^2-3^{\frac{2}{3}}x+3\right)+2\sqrt{3}x^4\log\left(3^{\frac{2}{3}}x+3\right)+54\cdot 3^{\frac{1}{3}}x^4\arctan\right.}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^5),x, algorithm="fricas")
```

```
[Out] 1/324*3^(1/6)*(9*3^(5/6)*x^4*log(x^2 - x + 1) - 18*3^(5/6)*x^4*lo
g(x + 1) - sqrt(3)*x^4*log(3^(1/3)*x^2 - 3^(2/3)*x + 3) + 2*sqrt(
3)*x^4*log(3^(2/3)*x + 3) + 54*3^(1/3)*x^4*arctan(1/3*sqrt(3)*(2*
x - 1)) - 6*x^4*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) + 3*3^(5/6)*(
16*x^3 - 3))/x^4
```

Sympy [A] time = 4.61768, size = 141, normalized size = 1.12

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{4782978\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{547}+\frac{1028869776\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{547}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1028869776\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{547}+\frac{4782978\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{547}\right)+\operatorname{RootSum}\left(472392t^3-1,\left(t\mapsto t\log\left(\frac{1028869776t^5}{547}+\frac{4782978t^2}{547}+x\right)\right)\right)+\frac{16x^3-3}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqr
t(3)*I/12)**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1
/12 + sqrt(3)*I/12)*log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/5
47 + 4782978*(1/12 + sqrt(3)*I/12)**2/547) + RootSum(472392*_t**3
 - 1, Lambda(_t, _t*log(1028869776*_t**5/547 + 4782978*_t**2/547
+ x))) + (16*x**3 - 3)/(36*x**4)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^5),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.169 $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}$$

```
[Out] -1/(15*x^5) + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) +
 ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Lo
g[3^(1/3) + x]/(54*3^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) -
 3^(1/3)*x + x^2]/(108*3^(2/3))
```

Rubi [A] time = 0.202867, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$

$$-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(15*x^5) + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) +
 ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Lo
g[3^(1/3) + x]/(54*3^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) -
 3^(1/3)*x + x^2]/(108*3^(2/3))
```

Rubi in Sympy [A] time = 34.2344, size = 117, normalized size = 0.93

$$\frac{\log(x+1)}{6}-\frac{\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)}{162}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{\frac{2}{3}}\right)}{324}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{3^{\frac{5}{6}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\cdot 3^{\frac{2}{3}}x}{9}+\frac{1}{3}\right)\right)}{162}+\frac{2}{9x^2}-\frac{1}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 - 3**(1/3)*log(x + 3**(1/3))/162 - log(x**2 - x + 1)
/12 + 3**(1/3)*log(x**2 - 3**(1/3)*x + 3**(2/3))/324 + sqrt(3)*at
an(sqrt(3)*(2*x/3 - 1/3))/6 + 3**(5/6)*atan(sqrt(3)*(-2*3**(2/3)*
x/9 + 1/3))/162 + 2/(9*x**2) - 1/(15*x**5)
```

Mathematica [A] time = 0.110343, size = 118, normalized size = 0.94

$$\frac{-\frac{108}{x^5}+\frac{360}{x^2}-135\log\left(x^2-x+1\right)+5\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+270\log(x+1)-10\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+10\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{}}{}\right.}{1620}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)]
+ 270*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^
(1/3)*Log[3 + 3^(2/3)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3
 - 3^(2/3)*x + 3^(1/3)*x^2])/1620
```

Maple [A] time = 0.014, size = 94, normalized size = 0.8

$$-\frac{\sqrt[3]{3}\ln\left(\sqrt[3]{3}+x\right)}{162}+\frac{\sqrt[3]{3}\ln\left(3^{\frac{2}{3}}-\sqrt[3]{3}x+x^2\right)}{324}-\frac{3^{\frac{5}{6}}}{162}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}x}{3}-1\right)\right)+\frac{\ln(1+x)}{6}-\frac{1}{15\,x^5}+\frac{2}{9\,x^2}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^6+4*x^3+3),x)
```

```
[Out] -1/162*3^(1/3)*ln(3^(1/3)+x)+1/324*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x
^2)-1/162*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+
x)-1/15/x^5+2/9/x^2-1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-
1)*3^(1/2))
```

Maxima [A] time = 0.883354, size = 130, normalized size = 1.03

$$-\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{10\,x^3-3}{45\,x^5}-\frac{1}{12}\,\log\left(x^2-x+1\right)+\frac{1}{6}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^6),x, algorithm="maxima")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x
 + 3^(2/3)) - 1/162*3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/
x^5 - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 0.270312, size = 230, normalized size = 1.83

$$-\frac{9^{\frac{2}{3}}\sqrt{3}\left(5\,\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^5\log\left(9^{\frac{2}{3}}x^2+3\cdot 9^{\frac{1}{3}}\,(-1)^{\frac{1}{3}}\,x+9\,(-1)^{\frac{2}{3}}\right)+45\cdot 9^{\frac{1}{3}}\,\sqrt{3}x^5\log\left(x^2-x+1\right)-10\,\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^5\log\left(9^{\frac{1}{3}}x-3\,(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^6),x, algorithm="fricas")
```

```
[Out] -1/14580*9^(2/3)*sqrt(3)*(5*sqrt(3)*(-1)^(1/3)*x^5*log(9^(2/3)*x^
2 + 3*9^(1/3)*(-1)^(1/3)*x + 9*(-1)^(2/3)) + 45*9^(1/3)*sqrt(3)*x
^5*log(x^2 - x + 1) - 10*sqrt(3)*(-1)^(1/3)*x^5*log(9^(1/3)*x - 3
*(-1)^(1/3)) - 90*9^(1/3)*sqrt(3)*x^5*log(x + 1) + 30*(-1)^(1/3)*
x^5*arctan(-1/9*(-1)^(2/3)*(2*9^(1/3)*sqrt(3)*x + 3*sqrt(3)*(-1)^
(1/3))) - 270*9^(1/3)*x^5*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*9^(1
/3)*sqrt(3)*(10*x^3 - 3))/x^5
```

Sympy [A] time = 4.36458, size = 136, normalized size = 1.08

$$\frac{\log\left(x+1\right)}{6}+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{88573}{6562}-\frac{88573\sqrt{3}i}{6562}+\frac{119042784\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{3281}\right)+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{88573}{6562}+\frac{119042784\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{3281}+\frac{88573\sqrt{3}i}{6562}\right)+\operatorname{RootSum}\left(1417176t^3+1,\left(t\mapsto t\log\left(\frac{119042784t^4}{3281}-\frac{531438t}{3281}+x\right)\right)\right)+\frac{10x^3-3}{45x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*
sqrt(3)*I/6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/
12 - sqrt(3)*I/12)*log(x + 88573/6562 + 119042784*(-1/12 - sqrt(3
)*I/12)**4/3281 + 88573*sqrt(3)*I/6562) + RootSum(1417176*_t**3 +
 1, Lambda(_t, _t*log(119042784*_t**4/3281 - 531438*_t/3281 + x))
) + (10*x**3 - 3)/(45*x**5)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 4*x^3 + 3)*x^6),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.170 $\int \frac{x^6}{1-x^3+x^6}\,dx$

Optimal. Leaf size=412

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+x+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/S
qrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTa
n[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 +
 I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) -
 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])
*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*Sqrt[3
])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sq
rt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3)
)
```

Rubi [A] time = 0.93431, antiderivative size = 412, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+x+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/S
qrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTa
n[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 +
 I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) -
 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])
*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*Sqrt[3
])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sq
rt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3)
)
```

Rubi in Sympy [A] time = 103.886, size = 354, normalized size = 0.86

$$x+\frac{2^{\frac{2}{3}}\left(3-\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(3+\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\left(3-\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\left(3+\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\left(\sqrt{3}-i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\left(\sqrt{3}+i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**6-x**3+1),x)
```

```
[Out] x + 2**(2/3)*(3 - sqrt(3)*I)*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1
/3))/(18*(1 - sqrt(3)*I)**(2/3)) + 2**(2/3)*(3 + sqrt(3)*I)*log(2
**(1/3)*x - (1 + sqrt(3)*I)**(1/3))/(18*(1 + sqrt(3)*I)**(2/3)) -
 2**(2/3)*(3 - sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**
(1/3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/(36*(1 - sqrt(3)*I)*
*(2/3)) - 2**(2/3)*(3 + sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 + sqr
t(3)*I)**(1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/(36*(1 + sq
rt(3)*I)**(2/3)) - 2**(2/3)*(sqrt(3) - I)*atan(sqrt(3)*(2*2**(1/3
)*x/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3))/(6*(1 - sqrt(3)*I)**(2/3))
 - 2**(2/3)*(sqrt(3) + I)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 + sqrt
(3)*I)**(1/3)) + 1/3))/(6*(1 + sqrt(3)*I)**(2/3))
```

Mathematica [C] time = 0.0234896, size = 59, normalized size = 0.14

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)
/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.009, size = 44, normalized size = 0.1

$$x+\frac{1}{3}\sum_{\_R=RootOf(\_Z^6-\_Z^3+1)}\frac{(\_R^3-1)\ln(x-\_R)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^6-x^3+1),x)
```

```
[Out] x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x+\int\frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.281193, size = 884, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2))) + sin(2
/3*arctan(1/(sqrt(3) + 2))))*arctan((sqrt(3)*sin(2/3*arctan(1/(sq
rt(3) + 2))) + cos(2/3*arctan(1/(sqrt(3) + 2))))/(sqrt(3)*cos(2/3
```

```
*arctan(1/(sqrt(3) + 2))) + 2*x + 2*sqrt(sqrt(3)*x*cos(2/3*arctan
(1/(sqrt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x
*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2
)))^2) - sin(2/3*arctan(1/(sqrt(3) + 2))))) + 4*(sqrt(3)*cos(2/3*
arctan(1/(sqrt(3) + 2))) - sin(2/3*arctan(1/(sqrt(3) + 2))))*arct
an(cos(2/3*arctan(1/(sqrt(3) + 2)))/(x + sqrt(x^2 + cos(2/3*arcta
n(1/(sqrt(3) + 2)))^2 + 2*x*sin(2/3*arctan(1/(sqrt(3) + 2))) + si
n(2/3*arctan(1/(sqrt(3) + 2)))^2) + sin(2/3*arctan(1/(sqrt(3) + 2
))))) + (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) - cos(2/3*arcta
n(1/(sqrt(3) + 2))))*log(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2)
)) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2/3*arctan(
1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2) + 2*cos(2
/3*arctan(1/(sqrt(3) + 2)))*log(-sqrt(3)*x*cos(2/3*arctan(1/(sqrt
(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2/3
*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2) -
 (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + cos(2/3*arctan(1/(sq
rt(3) + 2))))*log(x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*x*
sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)
))^2) + 8*arctan(-(sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) - cos
(2/3*arctan(1/(sqrt(3) + 2))))/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3)
 + 2))) - 2*x - 2*sqrt(-sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2))
) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2/3*arctan(1
/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2) + sin(2/3*
arctan(1/(sqrt(3) + 2)))))*sin(2/3*arctan(1/(sqrt(3) + 2))) + 6*s
qrt(3)*x)
```

Sympy [A] time = 0.46664, size = 26, normalized size = 0.06

$$x + \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(729t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**6-x**3+1),x)
```

```
[Out] x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t
**4 - 9*_t + x)))
```

GIAC/XCAS [A] time = 0.287918, size = 861, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)
^2 + sqrt(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(
4/9*pi)*sin(4/9*pi)^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*ar
ctan(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/
9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*si
n(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*sin(2/9*pi)
 - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*sin(2/
9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i +
1)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9
*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*si
n(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) +
 2*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt
(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*
pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos
(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi)
- 2*cos(4/9*pi))*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^
2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*co
s(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9
*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*l
n(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*s
qrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*
pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*p
i)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*ln((sqrt(3)*i*cos(1
/9*pi) + cos(1/9*pi))*x + x^2 + 1) + x
```

3.171 $\int \frac{x^5}{1-x^3+x^6} \, dx$

Optimal. Leaf size=39

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.074078, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 - x^3 + x^6),x]

[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rubi in Sympy [A] time = 9.67574, size = 34, normalized size = 0.87

$$\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**6-x**3+1),x)

[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x**3/3 - 1/3))/9
```

Mathematica [A] time = 0.0188092, size = 39, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] time = 0.004, size = 33, normalized size = 0.9

$$\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^6-x^3+1),x)
```

```
[Out] 1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.865741, size = 43, normalized size = 1.1

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)+\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 +
 1)
```

Fricas [A] time = 0.254006, size = 49, normalized size = 1.26

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^{6}-x^{3}+1\right)+2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*log(x^6 - x^3 + 1) + 2*arctan(1/3*sqrt(3)*(
2*x^3 - 1)))
```

Sympy [A] time = 0.270352, size = 37, normalized size = 0.95

$$\frac{\log\left(x^6 - x^3 + 1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

GIAC/XCAS [A] time = 0.27841, size = 43, normalized size = 1.1

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right) + \frac{1}{6}\ln\left(x^6 - x^3 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*ln(x^6 - x^3 +
1)
```

3.172 $\int \frac{x^4}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1
 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*S
qrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log
[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(
1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sq
rt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)
) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3
]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.714557, antiderivative size = 411, normalized size of antiderivative = 1., number

of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - x^3 + x^6),x]
```

```
[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1
 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*S
qrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log
[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(
1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sq
rt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)
) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3
]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi in Sympy [A] time = 99.8014, size = 335, normalized size = 0.82

$$\frac{\sqrt[3]{2}\sqrt{3}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18}-\frac{\sqrt[3]{2}\sqrt{3}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18}-\frac{\sqrt[3]{2}\sqrt{3}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}+\frac{\sqrt[3]{2}\sqrt{3}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}+\frac{\sqrt[3]{2}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}-\frac{\sqrt[3]{2}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**6-x**3+1),x)
```

```
[Out] 2**(1/3)*sqrt(3)*I*(1 - sqrt(3)*I)**(2/3)*log(2**(1/3)*x - (1 - s
qrt(3)*I)**(1/3))/18 - 2**(1/3)*sqrt(3)*I*(1 + sqrt(3)*I)**(2/3)*
log(2**(1/3)*x - (1 + sqrt(3)*I)**(1/3))/18 - 2**(1/3)*sqrt(3)*I*
(1 - sqrt(3)*I)**(2/3)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/
3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/36 + 2**(1/3)*sqrt(3)*I
*(1 + sqrt(3)*I)**(2/3)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(1
/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/36 + 2**(1/3)*I*(1 - s
qrt(3)*I)**(2/3)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**(
1/3)) + 1/3))/6 - 2**(1/3)*I*(1 + sqrt(3)*I)**(2/3)*atan(sqrt(3)*
(2*2**(1/3)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/6
```

Mathematica [C] time = 0.0158241, size = 41, normalized size = 0.1

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/
3
```

Maple [C] time = 0.006, size = 40, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf(\_Z^6-\_Z^3+1)}\frac{\_R^4\ln(x-\_R)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.271635, size = 1436, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) - sin(2
/3*arctan(1/(sqrt(3) - 2))))*arctan(-2*cos(2/3*arctan(1/(sqrt(3)
- 2)))*sin(2/3*arctan(1/(sqrt(3) - 2)))/(cos(2/3*arctan(1/(sqrt(3
) - 2)))^2 - sin(2/3*arctan(1/(sqrt(3) - 2)))^2 - x - sqrt(cos(2/
3*arctan(1/(sqrt(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4
 - 2*x*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + 2*(cos(2/3*arctan(1/(
sqrt(3) - 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2)))
 - 4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1
/(sqrt(3) - 2))))*arctan(-(2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) -
2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + 3*cos(2/3*arctan(1/(sqrt(
```

```
3) - 2)))^2 - 3*sin(2/3*arctan(1/(sqrt(3) - 2)))^2)/(sqrt(3)*cos(
2/3*arctan(1/(sqrt(3) - 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3
) - 2)))^2 - 6*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/
(sqrt(3) - 2))) + 2*sqrt(3)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(
sqrt(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 - 2*sqrt(3)
*x*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2
))) + x*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/
(sqrt(3) - 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2))
) + 2*cos(2/3*arctan(1/(sqrt(3) - 2)))*log(cos(2/3*arctan(1/(sqrt
(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 + 2*sqrt(3)*x*c
os(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2)))
+ x*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqr
t(3) - 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2) + (s
qrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) - cos(2/3*arctan(1/(sqrt(
3) - 2))))*log(cos(2/3*arctan(1/(sqrt(3) - 2)))^4 + sin(2/3*arcta
n(1/(sqrt(3) - 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2
)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*cos(2/3*arctan(1/(sqrt(3
) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - x)*sin(2/3*a
rctan(1/(sqrt(3) - 2)))^2 + x^2) - (sqrt(3)*sin(2/3*arctan(1/(sqr
t(3) - 2))) + cos(2/3*arctan(1/(sqrt(3) - 2))))*log(cos(2/3*arcta
n(1/(sqrt(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 - 2*x*
cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + 2*(cos(2/3*arctan(1/(sqrt(3)
 - 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2) - 8*arct
an((2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(
sqrt(3) - 2))) - 3*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + 3*sin(2/3
*arctan(1/(sqrt(3) - 2)))^2)/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) -
 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + 6*cos(2/3*
arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + 2*sqr
t(3)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(sqrt(3) - 2)))^4 + sin(
2/3*arctan(1/(sqrt(3) - 2)))^4 + 2*sqrt(3)*x*cos(2/3*arctan(1/(sq
rt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*cos(2/3*arctan(
1/(sqrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - x)*
sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2)))*sin(2/3*arctan(1/(sqr
t(3) - 2))))
```

Sympy [A] time = 0.473822, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(6561t^5 + 54t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5
 + 54*_t**2 + x)))
```

GIAC/XCAS [A] time = 0.276396, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.173 $\int \frac{x^3}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(
1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*
Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^
(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Lo
g[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*Sqrt[3])^
(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*S
qrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3
)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[
3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.742176, antiderivative size = 411, normalized size of antiderivative = 1., number

of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 - x^3 + x^6),x]
```

```
[Out] -((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(
1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*
Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^
(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Lo
g[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*Sqrt[3])^
(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*S
qrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3
)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[
3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi in Sympy [A] time = 98.8125, size = 335, normalized size = 0.82

$$\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1-\sqrt{3}i}\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18}-\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1+\sqrt{3}i}\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18}-\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1-\sqrt{3}i}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}+\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1+\sqrt{3}i}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}-\frac{2^{\frac{2}{3}}i\sqrt[3]{1-\sqrt{3}i}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}+\frac{2^{\frac{2}{3}}i\sqrt[3]{1+\sqrt{3}i}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**6-x**3+1),x)
```

```
[Out] 2**(2/3)*sqrt(3)*I*(1 - sqrt(3)*I)**(1/3)*log(2**(1/3)*x - (1 - s
qrt(3)*I)**(1/3))/18 - 2**(2/3)*sqrt(3)*I*(1 + sqrt(3)*I)**(1/3)*
log(2**(1/3)*x - (1 + sqrt(3)*I)**(1/3))/18 - 2**(2/3)*sqrt(3)*I*
(1 - sqrt(3)*I)**(1/3)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/
3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/36 + 2**(2/3)*sqrt(3)*I
*(1 + sqrt(3)*I)**(1/3)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(1
/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/36 - 2**(2/3)*I*(1 - s
qrt(3)*I)**(1/3)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**(
1/3)) + 1/3))/6 + 2**(2/3)*I*(1 + sqrt(3)*I)**(1/3)*atan(sqrt(3)*
(2*2**(1/3)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/6
```

Mathematica [C] time = 0.0136854, size = 39, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Maple [C] time = 0.007, size = 40, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.271637, size = 873, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) - sin(2
/3*arctan(1/(sqrt(3) - 2))))*arctan(-(sqrt(3)*sin(2/3*arctan(1/(s
qrt(3) - 2))) + cos(2/3*arctan(1/(sqrt(3) - 2))))/(sqrt(3)*cos(2/
3*arctan(1/(sqrt(3) - 2))) - 2*x - 2*sqrt(-sqrt(3)*x*cos(2/3*arct
an(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 +
 x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) -
 2)))^2) - sin(2/3*arctan(1/(sqrt(3) - 2))))) + 4*(sqrt(3)*cos(2/
3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2))))*ar
ctan((sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) - cos(2/3*arctan(1
/(sqrt(3) - 2))))/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) + 2*x
```

```
 + 2*sqrt(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2))) + x^2 + cos(
2/3*arctan(1/(sqrt(3) - 2)))^2 + x*sin(2/3*arctan(1/(sqrt(3) - 2)
)) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2) + sin(2/3*arctan(1/(sqrt
(3) - 2))))) + (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) - cos(2/
3*arctan(1/(sqrt(3) - 2))))*log(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(
3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x*sin(2/3*
arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2) -
(sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) + cos(2/3*arctan(1/(sqr
t(3) - 2))))*log(-sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2))) + x^
2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x*sin(2/3*arctan(1/(sqrt
(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2) + 2*cos(2/3*arct
an(1/(sqrt(3) - 2)))*log(x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2
 - 2*x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(
3) - 2)))^2) - 8*arctan(cos(2/3*arctan(1/(sqrt(3) - 2)))/(x + sqr
t(x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - 2*x*sin(2/3*arctan(1
/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2) - sin(2/3*
arctan(1/(sqrt(3) - 2)))))*sin(2/3*arctan(1/(sqrt(3) - 2))))
```

Sympy [A] time = 0.478794, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**
4 - 9*_t + x)))
```

GIAC/XCAS [A] time = 0.285892, size = 860, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*
pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8
*cos(4/9*pi)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*a
rctan(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4
/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^
2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(2/9*pi)^3*sin(2
/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) + sin(
```

```
2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i
+ 1)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*co
s(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*p
i)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*
pi) + sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((s
qrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4
/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 +
12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/
9*pi) - cos(4/9*pi))*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x
+ x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3
)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*
sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*p
i))*ln(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/18
*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin
(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2
*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*ln((sqrt(3)*i
*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

3.174 $\int \frac{x^2}{1-x^3+x^6}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0494201, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 - x^3 + x^6),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 6.25011, size = 24, normalized size = 1.04

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**6-x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*(2*x**3/3 - 1/3))/9
```

Mathematica [A] time = 0.0100737, size = 23, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 - x^3 + x^6),x]

[Out] (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] time = 0.004, size = 19, normalized size = 0.8

$$\frac{2\sqrt{3}}{9}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^6-x^3+1),x)

[Out] 2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.843112, size = 24, normalized size = 1.04

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 - x^3 + 1),x, algorithm="maxima")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Fricas [A] time = 0.250419, size = 24, normalized size = 1.04

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 - x^3 + 1),x, algorithm="fricas")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A] time = 0.243834, size = 27, normalized size = 1.17

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**6-x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.260693, size = 24, normalized size = 1.04

$$\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.175 $\int \frac{x}{1-x^3+x^6}\,dx$

Optimal. Leaf size=375

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1
 - I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3]
)/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 -
 I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3
)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1
+ I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 - I*Sq
rt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1
/3))
```

Rubi [A] time = 0.563947, antiderivative size = 375, normalized size of antiderivative = 1., number

of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 - x^3 + x^6),x]
```

```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1
 - I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3]
)/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 -
 I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3
)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1
+ I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 - I*Sq
rt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1
/3))
```

Rubi in Sympy [A] time = 95.3515, size = 335, normalized size = 0.89

$$\frac{\sqrt[3]{2}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{9\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{9\sqrt[3]{1+\sqrt{3}i}}-\frac{\sqrt[3]{2}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\sqrt[3]{1-\sqrt{3}i}}+\frac{\sqrt[3]{2}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\sqrt[3]{1+\sqrt{3}i}}+\frac{\sqrt[3]{2}i\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}i\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\sqrt[3]{1+\sqrt{3}i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**6-x**3+1),x)
```

```
[Out] 2**(1/3)*sqrt(3)*I*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1/3))/(9*(1
 - sqrt(3)*I)**(1/3)) - 2**(1/3)*sqrt(3)*I*log(2**(1/3)*x - (1 +
sqrt(3)*I)**(1/3))/(9*(1 + sqrt(3)*I)**(1/3)) - 2**(1/3)*sqrt(3)*
I*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/3)/2 + 2**(1/3)*(1 -
sqrt(3)*I)**(2/3)/2)/(18*(1 - sqrt(3)*I)**(1/3)) + 2**(1/3)*sqrt(
3)*I*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(1/3)/2 + 2**(1/3)*(1
 + sqrt(3)*I)**(2/3)/2)/(18*(1 + sqrt(3)*I)**(1/3)) + 2**(1/3)*I*
atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3))/(3*
(1 - sqrt(3)*I)**(1/3)) - 2**(1/3)*I*atan(sqrt(3)*(2*2**(1/3)*x/(
3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/(3*(1 + sqrt(3)*I)**(1/3))
```

Mathematica [C] time = 0.0156337, size = 40, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Maple [C] time = 0.007, size = 38, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf(\_Z^6-\_Z^3+1)}\frac{\_R\ln(x-\_R)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.268505, size = 1436, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2))) + sin(
2/3*arctan(1/(sqrt(3) + 2))))*arctan(-2*cos(2/3*arctan(1/(sqrt(3)
 + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2)))/(cos(2/3*arctan(1/(sqrt(
3) + 2)))^2 - sin(2/3*arctan(1/(sqrt(3) + 2)))^2 - x - sqrt(cos(2
/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^
4 - 2*x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*(cos(2/3*arctan(1/
(sqrt(3) + 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2))
) + 4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2))) - sin(2/3*arctan(
1/(sqrt(3) + 2))))*arctan((2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) +
2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) - 3*cos(2/3*arctan(1/(sqrt(
3) + 2)))^2 + 3*sin(2/3*arctan(1/(sqrt(3) + 2)))^2)/(sqrt(3)*cos(
2/3*arctan(1/(sqrt(3) + 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3
```

```
) + 2)))^2 + 6*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/
(sqrt(3) + 2))) + 2*sqrt(3)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(
sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 + 2*sqrt(3)
*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2
))) + x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/
(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2))
) + (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + cos(2/3*arctan(1/
(sqrt(3) + 2))))*log(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3
*arctan(1/(sqrt(3) + 2)))^4 + 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(
3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*arctan(1/(
sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x)*sin
(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2) - 2*cos(2/3*arctan(1/(sqrt
(3) + 2)))*log(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arcta
n(1/(sqrt(3) + 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2
)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*arctan(1/(sqrt(3
) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x)*sin(2/3*a
rctan(1/(sqrt(3) + 2)))^2 + x^2) - (sqrt(3)*sin(2/3*arctan(1/(sqr
t(3) + 2))) - cos(2/3*arctan(1/(sqrt(3) + 2))))*log(cos(2/3*arcta
n(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*x*
cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*(cos(2/3*arctan(1/(sqrt(3)
 + 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2) - 8*arct
an(-(2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/
(sqrt(3) + 2))) + 3*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - 3*sin(2/
3*arctan(1/(sqrt(3) + 2)))^2)/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3)
+ 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 - 6*cos(2/3
*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + 2*sq
rt(3)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin
(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(s
qrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*arctan
(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x)
*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2)))*sin(2/3*arctan(1/(sq
rt(3) + 2))))
```

Sympy [A] time = 0.463552, size = 26, normalized size = 0.07

$$\text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5
 - 27*_t**2 + x)))
```

GIAC/XCAS [A] time = 0.325885, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.176 $\int \frac{1}{1-x^3+x^6}\,dx$

Optimal. Leaf size=186

$$-\frac{(-1)^{5/18}\left(3\log\left(\sqrt[9]{-1}-x\right)+\log(2)\right)}{9\sqrt{3}}+\frac{(-1)^{13/18}\log\left(-\sqrt[3]{2}\left(x+(-1)^{8/9}\right)\right)}{3\sqrt{3}}-\frac{(-1)^{13/18}\log\left(-2^{2/3}\left(\left((-1)^{8/9}-x\right)x+(-1)^{7/9}\right)\right)}{6\sqrt{3}}+\frac{(-1)^{5/18}\log\left(2^{2/3}\left(x\left(x+\sqrt[9]{-1}\right)+(-1)^{2/9}\right)\right)}{6\sqrt{3}}-\frac{1}{3}(-1)^{13/18}\tan^{-1}\left(\frac{2\sqrt[9]{-1}x+1}{\sqrt{3}}\right)+\frac{1}{3}(-1)^{5/18}\tan^{-1}\left(\frac{1-2(-1)^{8/9}x}{\sqrt{3}}\right)$$

```
[Out] -((-1)^(13/18)*ArcTan[(1 + 2*(-1)^(1/9)*x)/Sqrt[3]])/3 + ((-1)^(5
/18)*ArcTan[(1 - 2*(-1)^(8/9)*x)/Sqrt[3]])/3 - ((-1)^(5/18)*(Log[
2] + 3*Log[(-1)^(1/9) - x]))/(9*Sqrt[3]) + ((-1)^(13/18)*Log[-(2^
(1/3)*((-1)^(8/9) + x))])/(3*Sqrt[3]) - ((-1)^(13/18)*Log[-(2^(2/
3)*((-1)^(7/9) + ((-1)^(8/9) - x)*x))])/(6*Sqrt[3]) + ((-1)^(5/18
)*Log[2^(2/3)*((-1)^(2/9) + x*((-1)^(1/9) + x))])/(6*Sqrt[3])
```

Rubi [C] time = 0.572612, antiderivative size = 375, normalized size of antiderivative = 2.02, number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] ((-I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((
1 - I*Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3
])/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1
- I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/
3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1
 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3) + (2*(
1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 - I*S
qrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqr
t[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(
2/3))
```

Rubi in Sympy [A] time = 88.5601, size = 335, normalized size = 1.8

$$\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{9\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{9\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}i\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}i\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-x**3+1),x)
```

```
[Out] 2**(2/3)*sqrt(3)*I*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1/3))/(9*(1
 - sqrt(3)*I)**(2/3)) - 2**(2/3)*sqrt(3)*I*log(2**(1/3)*x - (1 +
sqrt(3)*I)**(1/3))/(9*(1 + sqrt(3)*I)**(2/3)) - 2**(2/3)*sqrt(3)*
I*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/3)/2 + 2**(1/3)*(1 -
sqrt(3)*I)**(2/3)/2)/(18*(1 - sqrt(3)*I)**(2/3)) + 2**(2/3)*sqrt(
3)*I*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(1/3)/2 + 2**(1/3)*(1
 + sqrt(3)*I)**(2/3)/2)/(18*(1 + sqrt(3)*I)**(2/3)) - 2**(2/3)*I*
atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3))/(3*
(1 - sqrt(3)*I)**(2/3)) + 2**(2/3)*I*atan(sqrt(3)*(2*2**(1/3)*x/(
3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/(3*(1 + sqrt(3)*I)**(2/3))
```

Mathematica [C] time = 0.0141045, size = 42, normalized size = 0.23

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3 + x^6)^(-1),x]

[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.006, size = 37, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6-\_Z^3+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-x^3+1),x)

[Out] 1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3 + 1),x, algorithm="maxima")

[Out] integrate(1/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.28018, size = 876, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2))) - sin(2
/3*arctan(1/(sqrt(3) + 2))))*arctan(-(sqrt(3)*sin(2/3*arctan(1/(s
qrt(3) + 2))) - cos(2/3*arctan(1/(sqrt(3) + 2))))/(sqrt(3)*cos(2/
3*arctan(1/(sqrt(3) + 2))) - 2*x - 2*sqrt(-sqrt(3)*x*cos(2/3*arct
an(1/(sqrt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 -
 x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) +
 2)))^2) + sin(2/3*arctan(1/(sqrt(3) + 2))))) - 4*(sqrt(3)*cos(2/
3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2))))*ar
ctan(cos(2/3*arctan(1/(sqrt(3) + 2)))/(x + sqrt(x^2 + cos(2/3*arc
tan(1/(sqrt(3) + 2)))^2 + 2*x*sin(2/3*arctan(1/(sqrt(3) + 2))) +
sin(2/3*arctan(1/(sqrt(3) + 2)))^2) + sin(2/3*arctan(1/(sqrt(3) +
 2))))) + 2*cos(2/3*arctan(1/(sqrt(3) + 2)))*log(sqrt(3)*x*cos(2/
3*arctan(1/(sqrt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)
))^2 - x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqr
t(3) + 2)))^2) - (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + cos(
2/3*arctan(1/(sqrt(3) + 2))))*log(-sqrt(3)*x*cos(2/3*arctan(1/(sq
rt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2
/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2)
 + (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) - cos(2/3*arctan(1/(
sqrt(3) + 2))))*log(x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*
x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) +
2)))^2) - 8*arctan((sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + co
s(2/3*arctan(1/(sqrt(3) + 2))))/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3
) + 2))) + 2*x + 2*sqrt(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2))
) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2/3*arctan(1
/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2) - sin(2/3*
arctan(1/(sqrt(3) + 2)))))*sin(2/3*arctan(1/(sqrt(3) + 2))))
```

Sympy [A] time = 0.467334, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(729t^4 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4
+ x)))
```

GIAC/XCAS [A] time = 0.291177, size = 849, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)
^2 + sqrt(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(
4/9*pi)*sin(4/9*pi)^3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan
(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi
))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/
9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*sin(2/9*pi) - 4
*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*pi))*a
rctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*s
in(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi
) + 4*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*p
i))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*s
in(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt
(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*s
in(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4
*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9
*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*ln(-(sqrt(3)*i*cos(2/
9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3
*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^
4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1
/9*pi) + cos(1/9*pi))*ln((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x
+ x^2 + 1)
```

3.177 $\int \frac{1}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 +
 x^6]/6
```

Rubi [A] time = 0.0784621, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 +
 x^6]/6
```

Rubi in Sympy [A] time = 13.6936, size = 41, normalized size = 1.

$$\frac{\log\left(x^3\right)}{3}-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**6-x**3+1),x)
```

```
[Out] log(x**3)/3 - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x*
*3/3 - 1/3))/9
```

Mathematica [C] time = 0.0196479, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*
#1^3)/(-1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}}{9}\arctan\left(\frac{(2x^3 - 1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^6-x^3+1),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.851076, size = 51, normalized size = 1.24

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\log(x^6 - x^3 + 1) + \frac{1}{3}\log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 +
 1) + 1/3*log(x^3)
```

Fricas [A] time = 0.258769, size = 58, normalized size = 1.41

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log(x^6 - x^3 + 1) - 6\sqrt{3}\log(x) - 2\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^6 - x^3 + 1) - 6*sqrt(3)*log(x) - 2*
arctan(1/3*sqrt(3)*(2*x^3 - 1)))
```

Sympy [A] time = 0.319142, size = 41, normalized size = 1.

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 -
 sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.277337, size = 47, normalized size = 1.15

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\ln(x^6 - x^3 + 1) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*ln(x^6 - x^3 +
1) + ln(abs(x))
```

3.178 $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1
/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])
*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(
1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 + I*Sq
rt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*
Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2
*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt
[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
+ I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])
^(1/3))
```

Rubi [A] time = 0.662124, antiderivative size = 416, normalized size of antiderivative = 1., number

of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1
/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])
*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(
1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 + I*Sq
rt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*
Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2
*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt
[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
+ I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])
^(1/3))
```

Rubi in Sympy [A] time = 105.556, size = 355, normalized size = 0.85

$$-\frac{\sqrt[3]{2}\left(3-\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}\left(3+\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18\sqrt[3]{1+\sqrt{3}i}}+\frac{\sqrt[3]{2}\left(3-\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\sqrt[3]{1-\sqrt{3}i}}+\frac{\sqrt[3]{2}\left(3+\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\sqrt[3]{1+\sqrt{3}i}}-\frac{\sqrt[3]{2}\left(\sqrt{3}-i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}\left(\sqrt{3}+i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{1+\sqrt{3}i}}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] -2**(1/3)*(3 - sqrt(3)*I)*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1/3)
)/(18*(1 - sqrt(3)*I)**(1/3)) - 2**(1/3)*(3 + sqrt(3)*I)*log(2**(
1/3)*x - (1 + sqrt(3)*I)**(1/3))/(18*(1 + sqrt(3)*I)**(1/3)) + 2*
*(1/3)*(3 - sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/
3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/(36*(1 - sqrt(3)*I)**(1
/3)) + 2**(1/3)*(3 + sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3
)*I)**(1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/(36*(1 + sqrt(
3)*I)**(1/3)) - 2**(1/3)*(sqrt(3) - I)*atan(sqrt(3)*(2*2**(1/3)*x
/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3))/(6*(1 - sqrt(3)*I)**(1/3)) -
2**(1/3)*(sqrt(3) + I)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 + sqrt(3)
*I)**(1/3)) + 1/3))/(6*(1 + sqrt(3)*I)**(1/3)) - 1/x
```

Mathematica [C] time = 0.0228045, size = 61, normalized size = 0.15

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]
*#1^3)/(-#1 + 2*#1^4) & ]/3
```

Maple [C] time = 0.011, size = 50, normalized size = 0.1

$$-\frac{1}{3} \sum_{\_R=RootOf(\_Z^6-\_Z^3+1)} \frac{\left(\_R^4-\_R\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2} - x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - \int \frac{x^4 - x}{x^6 - x^3 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.286828, size = 1442, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(2*x*cos(2/3*arctan(1/(sqrt(3) - 2)))*log(cos(2/3*ar
ctan(1/(sqrt(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 - 2
```

```
*x*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + 2*(cos(2/3*arctan(1/(sqrt
(3) - 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2) + 8*x
*arctan(-2*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqr
t(3) - 2)))/(cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - sin(2/3*arctan(
1/(sqrt(3) - 2)))^2 - x - sqrt(cos(2/3*arctan(1/(sqrt(3) - 2)))^4
 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 - 2*x*cos(2/3*arctan(1/(sqr
t(3) - 2)))^2 + 2*(cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x)*sin(2/
3*arctan(1/(sqrt(3) - 2)))^2 + x^2)))*sin(2/3*arctan(1/(sqrt(3) -
 2))) + 4*(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2))) + x*sin(2/3
*arctan(1/(sqrt(3) - 2))))*arctan(-(sqrt(3)*cos(2/3*arctan(1/(sqr
t(3) - 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 - 2*co
s(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))))/
(2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqr
t(3) - 2))) + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - sin(2/3*arctan
(1/(sqrt(3) - 2)))^2 + 2*x + 2*sqrt(cos(2/3*arctan(1/(sqrt(3) - 2
)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 + 2*sqrt(3)*x*cos(2/3*
arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*cos
(2/3*arctan(1/(sqrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) -
2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2))) + 4*(sqrt
(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2))) - x*sin(2/3*arctan(1/(sqrt
(3) - 2))))*arctan((sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 -
sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + 2*cos(2/3*arctan(1/(
sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))))/(2*sqrt(3)*cos(2
/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) - co
s(2/3*arctan(1/(sqrt(3) - 2)))^2 + sin(2/3*arctan(1/(sqrt(3) - 2)
))^2 - 2*x - 2*sqrt(cos(2/3*arctan(1/(sqrt(3) - 2)))^4 + sin(2/3*
arctan(1/(sqrt(3) - 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3
) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*cos(2/3*arctan(1/(s
qrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - x)*sin(
2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2))) + (sqrt(3)*x*sin(2/3*arct
an(1/(sqrt(3) - 2))) - x*cos(2/3*arctan(1/(sqrt(3) - 2))))*log(co
s(2/3*arctan(1/(sqrt(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)
))^4 + 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arcta
n(1/(sqrt(3) - 2))) + x*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + (2*c
os(2/3*arctan(1/(sqrt(3) - 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3)
- 2)))^2 + x^2) - (sqrt(3)*x*sin(2/3*arctan(1/(sqrt(3) - 2))) + x
*cos(2/3*arctan(1/(sqrt(3) - 2))))*log(cos(2/3*arctan(1/(sqrt(3)
- 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 - 2*sqrt(3)*x*cos(2
/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*
cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3)
 - 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2) - 6*sqrt
(3))/x
```

Sympy [A] time = 0.539674, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-27t^2 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2
+ x))) - 1/x
```

GIAC/XCAS [A] time = 0.306222, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^2),x, algorithm="giac")
```

```
[Out] Done
```

3.179 $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/(2*x^2) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[
3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(
1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3]
)^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((3 + I
*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 +
 I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) +
 (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*S
qrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*
(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[
3])^(2/3))
```

Rubi [A] time = 0.754704, antiderivative size = 418, normalized size of antiderivative = 1., number

of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[
3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(
1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3]
)^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((3 + I
*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 +
 I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) +
 (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*S
qrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*
(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[
3])^(2/3))
```

Rubi in Sympy [A] time = 115.871, size = 359, normalized size = 0.86

$$-\frac{2^{\frac{2}{3}}\left(3-\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\left(3+\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(3-\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(3+\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(\sqrt{3}-i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(\sqrt{3}+i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**6-x**3+1),x)
```

```
[Out] -2**(2/3)*(3 - sqrt(3)*I)*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1/3)
)/(18*(1 - sqrt(3)*I)**(2/3)) - 2**(2/3)*(3 + sqrt(3)*I)*log(2**(
1/3)*x - (1 + sqrt(3)*I)**(1/3))/(18*(1 + sqrt(3)*I)**(2/3)) + 2*
*(2/3)*(3 - sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/
3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/(36*(1 - sqrt(3)*I)**(2
/3)) + 2**(2/3)*(3 + sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3
)*I)**(1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/(36*(1 + sqrt(
3)*I)**(2/3)) + 2**(2/3)*(sqrt(3) - I)*atan(sqrt(3)*(2*2**(1/3)*x
/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3))/(6*(1 - sqrt(3)*I)**(2/3)) +
2**(2/3)*(sqrt(3) + I)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 + sqrt(3)
*I)**(1/3)) + 1/3))/(6*(1 + sqrt(3)*I)**(2/3)) - 1/(2*x**2)
```

Mathematica [C] time = 0.0205042, size = 65, normalized size = 0.16

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x -
#1]*#1^3)/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.012, size = 50, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))-
1/2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2x^2}-\int\frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.277702, size = 927, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(2*x^2*cos(2/3*arctan(1/(sqrt(3) - 2)))*log(sqrt(3)*
x*cos(2/3*arctan(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt
```

```
(3) - 2)))^2 + x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arcta
n(1/(sqrt(3) - 2)))^2) - 8*x^2*arctan((sqrt(3)*sin(2/3*arctan(1/(
sqrt(3) - 2))) - cos(2/3*arctan(1/(sqrt(3) - 2))))/(sqrt(3)*cos(2
/3*arctan(1/(sqrt(3) - 2))) + 2*x + 2*sqrt(sqrt(3)*x*cos(2/3*arct
an(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 +
 x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) -
 2)))^2) + sin(2/3*arctan(1/(sqrt(3) - 2)))))*sin(2/3*arctan(1/(s
qrt(3) - 2))) - 4*(sqrt(3)*x^2*cos(2/3*arctan(1/(sqrt(3) - 2))) +
 x^2*sin(2/3*arctan(1/(sqrt(3) - 2))))*arctan(-(sqrt(3)*sin(2/3*a
rctan(1/(sqrt(3) - 2))) + cos(2/3*arctan(1/(sqrt(3) - 2))))/(sqrt
(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) - 2*x - 2*sqrt(-sqrt(3)*x*co
s(2/3*arctan(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3)
- 2)))^2 + x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/
(sqrt(3) - 2)))^2) - sin(2/3*arctan(1/(sqrt(3) - 2))))) - 4*(sqrt
(3)*x^2*cos(2/3*arctan(1/(sqrt(3) - 2))) - x^2*sin(2/3*arctan(1/(
sqrt(3) - 2))))*arctan(cos(2/3*arctan(1/(sqrt(3) - 2)))/(x + sqrt
(x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - 2*x*sin(2/3*arctan(1/
(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2) - sin(2/3*a
rctan(1/(sqrt(3) - 2))))) + (sqrt(3)*x^2*sin(2/3*arctan(1/(sqrt(3
) - 2))) - x^2*cos(2/3*arctan(1/(sqrt(3) - 2))))*log(-sqrt(3)*x*c
os(2/3*arctan(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3)
 - 2)))^2 + x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1
/(sqrt(3) - 2)))^2) - (sqrt(3)*x^2*sin(2/3*arctan(1/(sqrt(3) - 2)
)) + x^2*cos(2/3*arctan(1/(sqrt(3) - 2))))*log(x^2 + cos(2/3*arct
an(1/(sqrt(3) - 2)))^2 - 2*x*sin(2/3*arctan(1/(sqrt(3) - 2))) + s
in(2/3*arctan(1/(sqrt(3) - 2)))^2) - 3*sqrt(3))/x^2
```

Sympy [A] time = 0.567722, size = 31, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 + 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4
+ 9*_t + x))) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.323989, size = 867, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^
2 + sqrt(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4
/9*pi)*sin(4/9*pi)^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arc
tan(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9
*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin
(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*sin(2/9*pi)
- 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*sin(2/9
*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1
)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin
(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) +
2*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(
3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*p
i) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(
4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2
 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos
(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*
pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*ln
(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*p
i)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi
)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*ln((sqrt(3)*i*cos(1/
9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

3.180 $\int \frac{1}{x^4(1-x^3+x^6)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - L
og[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.116309, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - L
og[1 - x^3 + x^6]/6
```

Rubi in Sympy [A] time = 15.8757, size = 48, normalized size = 1.

$$\frac{\log\left(x^3\right)}{3}-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**6-x**3+1),x)
```

```
[Out] log(x**3)/3 - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(sqrt(3)*(2*x*
*3/3 - 1/3))/9 - 1/(3*x**3)
```

Mathematica [C] time = 0.0234762, size = 51, normalized size = 1.06

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1
^3)/(-1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.009, size = 40, normalized size = 0.8

$$-\frac{1}{3x^{3}}+\ln\left(x\right)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^6-x^3+1),x)
```

```
[Out] -1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)
*3^(1/2))
```

Maxima [A] time = 0.871716, size = 58, normalized size = 1.21

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)-\frac{1}{3x^{3}}-\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)+\frac{1}{3}\log\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(
x^6 - x^3 + 1) + 1/3*log(x^3)
```

Fricas [A] time = 0.260065, size = 81, normalized size = 1.69

$$-\frac{\sqrt{3}\left(\sqrt{3}x^{3}\log\left(x^{6}-x^{3}+1\right)-6\sqrt{3}x^{3}\log\left(x\right)+2x^{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)+2\sqrt{3}\right)}{18x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*x^3*log(x^6 - x^3 + 1) - 6*sqrt(3)*x^3*log
(x) + 2*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 2*sqrt(3))/x^3
```

Sympy [A] time = 0.434192, size = 48, normalized size = 1.

$$\log(x) - \frac{\log\left(x^6 - x^3 + 1\right)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 -
 sqrt(3)/3)/9 - 1/(3*x**3)
```

GIAC/XCAS [A] time = 0.281414, size = 61, normalized size = 1.27

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right) - \frac{x^3 + 1}{3x^3} - \frac{1}{6}\ln\left(x^6 - x^3 + 1\right) + \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^4),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3
- 1/6*ln(x^6 - x^3 + 1) + ln(abs(x))
```

3.181 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

Optimal. Leaf size=423

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/(4*x^4) - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((
I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]
])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 -
I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3))
- ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(
2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)
*(1 - I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2
/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1
+ I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.827119, antiderivative size = 423, normalized size of antiderivative = 1., number

of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(4*x^4) - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((
I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]
])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 -
I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3))
- ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(
2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)
*(1 - I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2
/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1
+ I*Sqrt[3])^(1/3))
```

Rubi in Sympy [A] time = 134.623, size = 345, normalized size = 0.82

$$-\frac{\sqrt[3]{2}\sqrt{3}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18}+\frac{\sqrt[3]{2}\sqrt{3}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18}+\frac{\sqrt[3]{2}\sqrt{3}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}-\frac{\sqrt[3]{2}\sqrt{3}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}-\frac{\sqrt[3]{2}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}+\frac{\sqrt[3]{2}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}-\frac{1}{x}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**6-x**3+1),x)
```

```
[Out] -2**(1/3)*sqrt(3)*I*(1 - sqrt(3)*I)**(2/3)*log(2**(1/3)*x - (1 -
sqrt(3)*I)**(1/3))/18 + 2**(1/3)*sqrt(3)*I*(1 + sqrt(3)*I)**(2/3)
*log(2**(1/3)*x - (1 + sqrt(3)*I)**(1/3))/18 + 2**(1/3)*sqrt(3)*I
*(1 - sqrt(3)*I)**(2/3)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1
/3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/36 - 2**(1/3)*sqrt(3)*
I*(1 + sqrt(3)*I)**(2/3)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(
1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/36 - 2**(1/3)*I*(1 -
sqrt(3)*I)**(2/3)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**
(1/3)) + 1/3))/6 + 2**(1/3)*I*(1 + sqrt(3)*I)**(2/3)*atan(sqrt(3)
*(2*2**(1/3)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/6 - 1/x - 1/(4*
x**4)
```

Mathematica [C] time = 0.0248704, size = 54, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{4x^4}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(4*x^4) - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1
^2)/(-1 + 2*#1^3) & ]/3
```

Maple [C] time = 0.012, size = 51, normalized size = 0.1

$$-\frac{1}{4\,x^4}-x^{-1}-\frac{1}{3}\sum_{\_R\,=RootOf(\_Z^6-\_Z^3+1)}\frac{\_R^4\ln(x-\_R)}{2\,\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^6-x^3+1),x)
```

```
[Out] -1/4/x^4-1/x-1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_
Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,x^3+1}{4\,x^4}-\int\frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^5),x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.290066, size = 1501, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^5),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*(4*x^4*cos(2/3*arctan(1/(sqrt(3) + 2)))*log(cos(2/3*
arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 +
```

```
 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(s
qrt(3) + 2))) + x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3
*arctan(1/(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))
^2 + x^2) - 16*x^4*arctan((2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) +
2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) - 3*cos(2/3*arctan(1/(sqrt(
3) + 2)))^2 + 3*sin(2/3*arctan(1/(sqrt(3) + 2)))^2)/(sqrt(3)*cos(
2/3*arctan(1/(sqrt(3) + 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3
) + 2)))^2 + 6*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/
(sqrt(3) + 2))) + 2*sqrt(3)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(
sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 + 2*sqrt(3)
*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2
))) + x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/
(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2))
)*sin(2/3*arctan(1/(sqrt(3) + 2))) + 8*(sqrt(3)*x^4*cos(2/3*arcta
n(1/(sqrt(3) + 2))) - x^4*sin(2/3*arctan(1/(sqrt(3) + 2))))*arcta
n(-2*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) +
 2)))/(cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - sin(2/3*arctan(1/(sqr
t(3) + 2)))^2 - x - sqrt(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin
(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*x*cos(2/3*arctan(1/(sqrt(3) +
 2)))^2 + 2*(cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + x)*sin(2/3*arct
an(1/(sqrt(3) + 2)))^2 + x^2))) - 8*(sqrt(3)*x^4*cos(2/3*arctan(1
/(sqrt(3) + 2))) + x^4*sin(2/3*arctan(1/(sqrt(3) + 2))))*arctan(-
(2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqr
t(3) + 2))) + 3*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - 3*sin(2/3*ar
ctan(1/(sqrt(3) + 2)))^2)/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2)
))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 - 6*cos(2/3*arc
tan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + 2*sqrt(3
)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3
*arctan(1/(sqrt(3) + 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(
3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*arctan(1/(
sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x)*sin
(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2))) + 2*(sqrt(3)*x^4*sin(2/3
*arctan(1/(sqrt(3) + 2))) - x^4*cos(2/3*arctan(1/(sqrt(3) + 2))))
*log(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(
3) + 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/
3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2
 + (2*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1/(s
qrt(3) + 2)))^2 + x^2) - 2*(sqrt(3)*x^4*sin(2/3*arctan(1/(sqrt(3)
 + 2))) + x^4*cos(2/3*arctan(1/(sqrt(3) + 2))))*log(cos(2/3*arcta
n(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*x*
cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*(cos(2/3*arctan(1/(sqrt(3)
 + 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2) - 3*sqrt
(3)*(4*x^3 + 1))/x^4
```

Sympy [A] time = 0.628326, size = 37, normalized size = 0.09

$$\text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 + 54t^2 + x\right)\right)\right) - \frac{4x^3 + 1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**
5 + 54*_t**2 + x))) - (4*x**3 + 1)/(4*x**4)
```

GIAC/XCAS [A] time = 0.311377, size = 1, normalized size = 0.

$$\textit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 - x^3 + 1)*x^5),x, algorithm="giac")
```

```
[Out] Done
```

3.182 $\int \frac{1}{2+x^3+x^6}\,dx$

Optimal. Leaf size=381

$$\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}-\frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[2
1]*((1 - I*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt
[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - (
(I/3)*Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sq
rt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])
/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 - I*Sqrt[7])
^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqr
t[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) -
(2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 +
 I*Sqrt[7])^(2/3))
```

Rubi [A] time = 0.888122, antiderivative size = 381, normalized size of antiderivative = 1., number

of steps used = 13, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$

$$\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[2
1]*((1 - I*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt
[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - (
(I/3)*Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sq
rt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])
/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 - I*Sqrt[7])
^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqr
t[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) -
(2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 +
 I*Sqrt[7])^(2/3))
```

Rubi in Sympy [A] time = 96.7762, size = 345, normalized size = 0.91

$$-\frac{2^{\frac{2}{3}}\sqrt{7}i\log\left(\sqrt[3]{2}x+\sqrt[3]{1-\sqrt{7}i}\right)}{21\left(1-\sqrt{7}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\sqrt{7}i\log\left(\sqrt[3]{2}x+\sqrt[3]{1+\sqrt{7}i}\right)}{21\left(1+\sqrt{7}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\sqrt{7}i\log\left(x^2-\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{7}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{7}i\right)^{\frac{2}{3}}}{2}\right)}{42\left(1-\sqrt{7}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\sqrt{7}i\log\left(x^2-\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{7}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{7}i\right)^{\frac{2}{3}}}{2}\right)}{42\left(1+\sqrt{7}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\sqrt{21}i\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{7}i}}+\frac{1}{3}\right)\right)}{21\left(1-\sqrt{7}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\sqrt{21}i\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{7}i}}+\frac{1}{3}\right)\right)}{21\left(1+\sqrt{7}i\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6+x**3+2),x)
```

```
[Out] -2**(2/3)*sqrt(7)*I*log(2**(1/3)*x + (1 - sqrt(7)*I)**(1/3))/(21*
(1 - sqrt(7)*I)**(2/3)) + 2**(2/3)*sqrt(7)*I*log(2**(1/3)*x + (1
+ sqrt(7)*I)**(1/3))/(21*(1 + sqrt(7)*I)**(2/3)) + 2**(2/3)*sqrt(
7)*I*log(x**2 - 2**(2/3)*x*(1 - sqrt(7)*I)**(1/3)/2 + 2**(1/3)*(1
 - sqrt(7)*I)**(2/3)/2)/(42*(1 - sqrt(7)*I)**(2/3)) - 2**(2/3)*sq
rt(7)*I*log(x**2 - 2**(2/3)*x*(1 + sqrt(7)*I)**(1/3)/2 + 2**(1/3)
*(1 + sqrt(7)*I)**(2/3)/2)/(42*(1 + sqrt(7)*I)**(2/3)) + 2**(2/3)
*sqrt(21)*I*atan(sqrt(3)*(-2*2**(1/3)*x/(3*(1 - sqrt(7)*I)**(1/3)
) + 1/3))/(21*(1 - sqrt(7)*I)**(2/3)) - 2**(2/3)*sqrt(21)*I*atan(
sqrt(3)*(-2*2**(1/3)*x/(3*(1 + sqrt(7)*I)**(1/3)) + 1/3))/(21*(1
+ sqrt(7)*I)**(2/3))
```

Mathematica [C] time = 0.0146363, size = 38, normalized size = 0.1

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\log(x-\#1)}{2\#1^5+\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.008, size = 33, normalized size = 0.1

$$\frac{1}{3} \sum_{\_R = RootOf(\_Z^6 + \_Z^3 + 2)} \frac{\ln(x - \_R)}{2\_R^5 + \_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(1/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 + x^3 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + x^3 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

Fricas [A] time = 0.277735, size = 1477, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + x^3 + 2),x, algorithm="fricas")
```

```
[Out] 1/8232*1372^(5/6)*(4*(sqrt(3)*cos(2/3*arctan(3/(sqrt(7) - 4))) -
sin(2/3*arctan(3/(sqrt(7) - 4))))*arctan(7*((sqrt(7)*sqrt(3) - 1)
*cos(2/3*arctan(3/(sqrt(7) - 4))) - (sqrt(7) + sqrt(3))*sin(2/3*a
rctan(3/(sqrt(7) - 4))))/(7*(sqrt(7) + sqrt(3))*cos(2/3*arctan(3/
(sqrt(7) - 4))) + 7*(sqrt(7)*sqrt(3) - 1)*sin(2/3*arctan(3/(sqrt(
7) - 4))) - 2*1372^(1/6)*sqrt(7)*sqrt(1/14)*sqrt(4^(2/3)*(14*4^(1
/3)*x^2 - 1372^(1/6)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(3/(
sqrt(7) - 4))) + 1372^(1/6)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*arc
tan(3/(sqrt(7) - 4))) + 28*cos(2/3*arctan(3/(sqrt(7) - 4)))^2 + 2
8*sin(2/3*arctan(3/(sqrt(7) - 4)))^2)) - 4*1372^(1/6)*sqrt(7)*x))
```

```
 - 4*(sqrt(3)*cos(2/3*arctan(3/(sqrt(7) - 4))) + sin(2/3*arctan(3
/(sqrt(7) - 4))))*arctan(-7*((sqrt(7)*sqrt(3) + 1)*cos(2/3*arctan
(3/(sqrt(7) - 4))) + (sqrt(7) - sqrt(3))*sin(2/3*arctan(3/(sqrt(7
) - 4))))/(7*(sqrt(7) - sqrt(3))*cos(2/3*arctan(3/(sqrt(7) - 4)))
 - 7*(sqrt(7)*sqrt(3) + 1)*sin(2/3*arctan(3/(sqrt(7) - 4))) - 2*1
372^(1/6)*sqrt(7)*sqrt(1/14)*sqrt(4^(2/3)*(14*4^(1/3)*x^2 + 1372^
(1/6)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(2/3*arctan(3/(sqrt(7) - 4)))
+ 1372^(1/6)*(sqrt(7)*x + 7*sqrt(3)*x)*sin(2/3*arctan(3/(sqrt(7)
- 4))) + 28*cos(2/3*arctan(3/(sqrt(7) - 4)))^2 + 28*sin(2/3*arcta
n(3/(sqrt(7) - 4)))^2)) - 4*1372^(1/6)*sqrt(7)*x)) + 2*cos(2/3*ar
ctan(3/(sqrt(7) - 4)))*log(-1372^(1/6)*sqrt(7)*x*sin(2/3*arctan(3
/(sqrt(7) - 4))) + 7*4^(1/3)*x^2 + 7*1372^(1/6)*x*cos(2/3*arctan(
3/(sqrt(7) - 4))) + 14*cos(2/3*arctan(3/(sqrt(7) - 4)))^2 + 14*si
n(2/3*arctan(3/(sqrt(7) - 4)))^2) - (sqrt(3)*sin(2/3*arctan(3/(sq
rt(7) - 4))) + cos(2/3*arctan(3/(sqrt(7) - 4))))*log(14*4^(1/3)*x
^2 - 1372^(1/6)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(3/(sqrt(
7) - 4))) + 1372^(1/6)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*arctan(3
/(sqrt(7) - 4))) + 28*cos(2/3*arctan(3/(sqrt(7) - 4)))^2 + 28*sin
(2/3*arctan(3/(sqrt(7) - 4)))^2) + (sqrt(3)*sin(2/3*arctan(3/(sqr
t(7) - 4))) - cos(2/3*arctan(3/(sqrt(7) - 4))))*log(14*4^(1/3)*x^
2 + 1372^(1/6)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(2/3*arctan(3/(sqrt(7
) - 4))) + 1372^(1/6)*(sqrt(7)*x + 7*sqrt(3)*x)*sin(2/3*arctan(3/
(sqrt(7) - 4))) + 28*cos(2/3*arctan(3/(sqrt(7) - 4)))^2 + 28*sin(
2/3*arctan(3/(sqrt(7) - 4)))^2) - 8*arctan(7*(sqrt(7)*sin(2/3*arc
tan(3/(sqrt(7) - 4))) + cos(2/3*arctan(3/(sqrt(7) - 4))))/(1372^(
1/6)*sqrt(7)*sqrt(1/7)*sqrt(-4^(2/3)*(1372^(1/6)*sqrt(7)*x*sin(2/
3*arctan(3/(sqrt(7) - 4))) - 7*4^(1/3)*x^2 - 7*1372^(1/6)*x*cos(2
/3*arctan(3/(sqrt(7) - 4))) - 14*cos(2/3*arctan(3/(sqrt(7) - 4)))
^2 - 14*sin(2/3*arctan(3/(sqrt(7) - 4)))^2)) + 2*1372^(1/6)*sqrt(
7)*x + 7*sqrt(7)*cos(2/3*arctan(3/(sqrt(7) - 4))) - 7*sin(2/3*arc
tan(3/(sqrt(7) - 4)))))*sin(2/3*arctan(3/(sqrt(7) - 4))))
```

Sympy [A] time = 0.364926, size = 24, normalized size = 0.06

$$\text{RootSum}\left(1000188t^6 + 1323t^3 + 1, \left(t \mapsto t\log\left(-5292t^4 + 7t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6+x**3+2),x)
```

```
[Out] RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_
t**4 + 7*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 + x^3 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + x^3 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

3.183 $\int \frac{x^2}{2+x^3+x^6} \, dx$

Optimal. Leaf size=23

$$\frac{2 \tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rubi [A] time = 0.0488604, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{2 \tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(2 + x^3 + x^6),x]

[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rubi in Sympy [A] time = 5.27161, size = 24, normalized size = 1.04

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\sqrt{7}\left(\frac{2x^3}{7}+\frac{1}{7}\right)\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**6+x**3+2),x)

[Out] 2*sqrt(7)*atan(sqrt(7)*(2*x**3/7 + 1/7))/21
```

Mathematica [A] time = 0.013718, size = 23, normalized size = 1.

$$\frac{2 \tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(2 + x^3 + x^6),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Maple [A] time = 0.004, size = 19, normalized size = 0.8

$$\frac{2\sqrt{7}}{21}\arctan\left(\frac{(2x^3+1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^6+x^3+2),x)
```

```
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 0.84682, size = 24, normalized size = 1.04

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^3+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 + x^3 + 2),x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Fricas [A] time = 0.255343, size = 24, normalized size = 1.04

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^3+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 + x^3 + 2),x, algorithm="fricas")
```

```
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Sympy [A] time = 0.238163, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**6+x**3+2),x)

[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

GIAC/XCAS [A] time = 0.28272, size = 24, normalized size = 1.04

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 + x^3 + 2),x, algorithm="giac")

[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

3.184 $\int \frac{x^3}{2+x^3+x^6}\,dx$

Optimal. Leaf size=399

$$-\frac{\left(7+i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}+\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}$$

```
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7]
)/2)^(1/3))/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*Arc
Tan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((
7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3
)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[7])*Log[(1 + I*Sqrt[7])^(
1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/3)) - ((7 + I*S
qrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x +
 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(
2/3)*x^2])/(42*2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rubi [A] time = 0.74257, antiderivative size = 399, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\left(7+i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}+\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(2 + x^3 + x^6),x]
```

```
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7]
)/2)^(1/3))/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*Arc
Tan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((
7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3
)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[7])*Log[(1 + I*Sqrt[7])^(
1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/3)) - ((7 + I*S
qrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x +
 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(
2/3)*x^2])/(42*2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rubi in Sympy [A] time = 108.427, size = 345, normalized size = 0.86

$$\frac{2^{\frac{2}{3}}\sqrt{7}i\sqrt[3]{1-\sqrt{7}i}\log\left(\sqrt[3]{2}x+\sqrt[3]{1-\sqrt{7}i}\right)}{42}-\frac{2^{\frac{2}{3}}\sqrt{7}i\sqrt[3]{1+\sqrt{7}i}\log\left(\sqrt[3]{2}x+\sqrt[3]{1+\sqrt{7}i}\right)}{42}-\frac{2^{\frac{2}{3}}\sqrt{7}i\sqrt[3]{1-\sqrt{7}i}\log\left(x^2-\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{7}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{7}i\right)^{\frac{2}{3}}}{2}\right)}{84}+\frac{2^{\frac{2}{3}}\sqrt{7}i\sqrt[3]{1+\sqrt{7}i}\log\left(x^2-\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{7}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{7}i\right)^{\frac{2}{3}}}{2}\right)}{84}-\frac{2^{\frac{2}{3}}\sqrt{21}i\sqrt[3]{1-\sqrt{7}i}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{7}i}}+\frac{1}{3}\right)\right)}{42}+\frac{2^{\frac{2}{3}}\sqrt{21}i\sqrt[3]{1+\sqrt{7}i}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{7}i}}+\frac{1}{3}\right)\right)}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**6+x**3+2),x)
```

```
[Out] 2**(2/3)*sqrt(7)*I*(1 - sqrt(7)*I)**(1/3)*log(2**(1/3)*x + (1 - s
qrt(7)*I)**(1/3))/42 - 2**(2/3)*sqrt(7)*I*(1 + sqrt(7)*I)**(1/3)*
log(2**(1/3)*x + (1 + sqrt(7)*I)**(1/3))/42 - 2**(2/3)*sqrt(7)*I*
(1 - sqrt(7)*I)**(1/3)*log(x**2 - 2**(2/3)*x*(1 - sqrt(7)*I)**(1/
3)/2 + 2**(1/3)*(1 - sqrt(7)*I)**(2/3)/2)/84 + 2**(2/3)*sqrt(7)*I
*(1 + sqrt(7)*I)**(1/3)*log(x**2 - 2**(2/3)*x*(1 + sqrt(7)*I)**(1
/3)/2 + 2**(1/3)*(1 + sqrt(7)*I)**(2/3)/2)/84 - 2**(2/3)*sqrt(21)
*I*(1 - sqrt(7)*I)**(1/3)*atan(sqrt(3)*(-2*2**(1/3)*x/(3*(1 - sqr
t(7)*I)**(1/3)) + 1/3))/42 + 2**(2/3)*sqrt(21)*I*(1 + sqrt(7)*I)*
*(1/3)*atan(sqrt(3)*(-2*2**(1/3)*x/(3*(1 + sqrt(7)*I)**(1/3)) + 1
/3))/42
```

Mathematica [C] time = 0.0151688, size = 37, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\#1\log(x-\#1)}{2\#1^3+1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(2 + x^3 + x^6),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

Maple [C] time = 0.007, size = 36, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf(\_Z^6+\_Z^3+2)}\frac{\_R^3\ln(x-\_R)}{2\_R^5+\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{x^6+x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 + x^3 + 2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

Fricas [A] time = 0.277145, size = 1454, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 + x^3 + 2),x, algorithm="fricas")
```

```
[Out] 1/2058*343^(5/6)*(2*2^(1/6)*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)
)))*log(2*343^(1/6)*sqrt(7)*2^(1/6)*x*sin(2/3*arctan(1/(sqrt(7) -
 2*sqrt(2)))) + 7*2^(1/3)*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))
)^2 + 7*2^(1/3)*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 + 7*x^
```

```
2) + 8*2^(1/6)*arctan(7*2^(1/6)*cos(2/3*arctan(1/(sqrt(7) - 2*sqr
t(2))))/(343^(1/6)*sqrt(7)*x + 343^(1/6)*sqrt(7)*sqrt(2/7*343^(1/
6)*sqrt(7)*2^(1/6)*x*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) + 2
^(1/3)*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 + 2^(1/3)*sin(2
/3*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 + x^2) + 7*2^(1/6)*sin(2/3*
arctan(1/(sqrt(7) - 2*sqrt(2))))))*sin(2/3*arctan(1/(sqrt(7) - 2*
sqrt(2)))) - 4*(sqrt(3)*2^(1/6)*cos(2/3*arctan(1/(sqrt(7) - 2*sqr
t(2)))) - 2^(1/6)*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))))*arcta
n(7*(sqrt(3)*2^(1/6)*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) + 2
^(1/6)*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))))/(2*343^(1/6)*sqr
t(7)*x + 7*sqrt(3)*2^(1/6)*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))
)) + 2*343^(1/6)*sqrt(7)*sqrt(1/7*343^(1/6)*sqrt(7)*sqrt(3)*2^(1/
6)*x*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) - 1/7*343^(1/6)*sqr
t(7)*2^(1/6)*x*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) + 2^(1/3)
*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 + 2^(1/3)*sin(2/3*arc
tan(1/(sqrt(7) - 2*sqrt(2))))^2 + x^2) - 7*2^(1/6)*sin(2/3*arctan
(1/(sqrt(7) - 2*sqrt(2)))))) - 4*(sqrt(3)*2^(1/6)*cos(2/3*arctan(
1/(sqrt(7) - 2*sqrt(2)))) + 2^(1/6)*sin(2/3*arctan(1/(sqrt(7) - 2
*sqrt(2)))))*arctan(7*(sqrt(3)*2^(1/6)*sin(2/3*arctan(1/(sqrt(7)
- 2*sqrt(2)))) - 2^(1/6)*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))
)/(2*343^(1/6)*sqrt(7)*x - 7*sqrt(3)*2^(1/6)*cos(2/3*arctan(1/(sq
rt(7) - 2*sqrt(2)))) + 2*343^(1/6)*sqrt(7)*sqrt(-1/7*343^(1/6)*sq
rt(7)*sqrt(3)*2^(1/6)*x*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))
- 1/7*343^(1/6)*sqrt(7)*2^(1/6)*x*sin(2/3*arctan(1/(sqrt(7) - 2*s
qrt(2)))) + 2^(1/3)*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 +
2^(1/3)*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 + x^2) - 7*2^(
1/6)*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))))) - (sqrt(3)*2^(1/6
)*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) + 2^(1/6)*cos(2/3*arct
an(1/(sqrt(7) - 2*sqrt(2)))))*log(343^(1/6)*sqrt(7)*sqrt(3)*2^(1/
6)*x*cos(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) - 343^(1/6)*sqrt(7)
*2^(1/6)*x*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) + 7*2^(1/3)*c
os(2/3*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 + 7*2^(1/3)*sin(2/3*arc
tan(1/(sqrt(7) - 2*sqrt(2))))^2 + 7*x^2) + (sqrt(3)*2^(1/6)*sin(2
/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) - 2^(1/6)*cos(2/3*arctan(1/(s
qrt(7) - 2*sqrt(2)))))*log(-343^(1/6)*sqrt(7)*sqrt(3)*2^(1/6)*x*c
os(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) - 343^(1/6)*sqrt(7)*2^(1/
6)*x*sin(2/3*arctan(1/(sqrt(7) - 2*sqrt(2)))) + 7*2^(1/3)*cos(2/3
*arctan(1/(sqrt(7) - 2*sqrt(2))))^2 + 7*2^(1/3)*sin(2/3*arctan(1/
(sqrt(7) - 2*sqrt(2))))^2 + 7*x^2))
```

Sympy [A] time = 0.359883, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(250047t^6 + 1323t^3 + 2, \left(t \mapsto t\log\left(7938t^4 + 21t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**6+x**3+2),x)
```

```
[Out] RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t*
```

```
*4 + 21*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{x^6 + x^3 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^6 + x^3 + 2),x, algorithm="giac")

[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

3.185 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=231

$$-\frac{(b^2-4ac)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}+\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{\left(7b\left(15b^2-28ac\right)-6cx^3\left(21b^2-20ac\right)\right)\left(a+bx^3+cx^6\right)^{3/2}}{2880c^4}-\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c}$$

```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3
+ c*x^6])/(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2)
 + (x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*
c) - 6*c*(21*b^2 - 20*a*c)*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(2880*
c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*ArcTanh[
(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(11/2
))
```

Rubi [A] time = 0.661407, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{(b^2-4ac)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}+\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{\left(7b\left(15b^2-28ac\right)-6cx^3\left(21b^2-20ac\right)\right)\left(a+bx^3+cx^6\right)^{3/2}}{2880c^4}-\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```


```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3
+ c*x^6])/(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2)
 + (x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*
c) - 6*c*(21*b^2 - 20*a*c)*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(2880*
c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*ArcTanh[
```

```
(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(11/2
))
```

Rubi in Sympy [A] time = 51.703, size = 221, normalized size = 0.96

$$-\frac{bx^6\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{20c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{18c}-\frac{\left(\frac{21b\left(-28ac+15b^2\right)}{8}-\frac{9cx^3\left(-20ac+21b^2\right)}{4}\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{1080c^4}+\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(16a^2c^2-56ab^2c+21b^4\right)}{1536c^5}-\frac{\left(-4ac+b^2\right)\left(16a^2c^2-56ab^2c+21b^4\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -b*x**6*(a + b*x**3 + c*x**6)**(3/2)/(20*c**2) + x**9*(a + b*x**3
 + c*x**6)**(3/2)/(18*c) - (21*b*(-28*a*c + 15*b**2)/8 - 9*c*x**3
*(-20*a*c + 21*b**2)/4)*(a + b*x**3 + c*x**6)**(3/2)/(1080*c**4)
+ (b + 2*c*x**3)*sqrt(a + b*x**3 + c*x**6)*(16*a**2*c**2 - 56*a*b
**2*c + 21*b**4)/(1536*c**5) - (-4*a*c + b**2)*(16*a**2*c**2 - 56
*a*b**2*c + 21*b**4)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x
**3 + c*x**6)))/(3072*c**(11/2))
```

Mathematica [A] time = 0.203254, size = 203, normalized size = 0.88

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(16bc^2\left(113a^2-34acx^6+8c^2x^{12}\right)+160c^3x^3\left(-3a^2+2acx^6+8c^2x^{12}\right)+168b^3c\left(cx^6-10a\right)+16b^2c^2x^3}{46080c^{11/}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*
b^2*c^2*x^3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c
^2*(113*a^2 - 34*a*c*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*
a*c*x^6 + 8*c^2*x^12)) - 15*(b^2 - 4*a*c)*(21*b^4 - 56*a*b^2*c +
16*a^2*c^2)*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])
```

```
/(46080*c^(11/2))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^{14}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14*(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^14,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294425, size = 1, normalized size = 0.

$$\left[\frac{4\left(1280\,c^{5}x^{15}+128\,bc^{4}x^{12}-16\left(9\,b^{2}c^{3}-20\,ac^{4}\right)x^{9}+8\left(21\,b^{3}c^{2}-68\,abc^{3}\right)x^{6}+315\,b^{5}-1680\,ab^{3}c+1808\,a^{2}bc^{2}-2\left(105\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^14,x, algorithm="fricas")

[Out] [1/92160*(4*(1280*c^5*x^15 + 128*b*c^4*x^12 - 16*(9*b^2*c^3 - 20*
a*c^4)*x^9 + 8*(21*b^3*c^2 - 68*a*b*c^3)*x^6 + 315*b^5 - 1680*a*b
^3*c + 1808*a^2*b*c^2 - 2*(105*b^4*c - 448*a*b^2*c^2 + 240*a^2*c^
3)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) - 15*(21*b^6 - 140*a*b^4*
c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*log(-4*sqrt(c*x^6 + b*x^3 + a)*
(2*c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)
```

```
))/c^(11/2), 1/46080*(2*(1280*c^5*x^15 + 128*b*c^4*x^12 - 16*(9*b
^2*c^3 - 20*a*c^4)*x^9 + 8*(21*b^3*c^2 - 68*a*b*c^3)*x^6 + 315*b^
5 - 1680*a*b^3*c + 1808*a^2*b*c^2 - 2*(105*b^4*c - 448*a*b^2*c^2
+ 240*a^2*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c) - 15*(21*b^6
 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(1/2*(2*c*x^
3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^14,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

3.186 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=171

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c}$$

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c
 - 42*b*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 -
 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rubi [A] time = 0.341052, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c
 - 42*b*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 -
 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rubi in Sympy [A] time = 32.0307, size = 163, normalized size = 0.95

$$-\frac{b\left(b+2cx^3\right)\left(-12ac+7b^2\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{b\left(-12ac+7b^2\right)\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{\frac{9}{2}}}+\frac{x^6\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{15c}+\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\left(-8ac+\frac{35b^2}{4}-\frac{21bcx^3}{2}\right)}{180c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -b*(b + 2*c*x**3)*(-12*a*c + 7*b**2)*sqrt(a + b*x**3 + c*x**6)/(3
84*c**4) + b*(-12*a*c + 7*b**2)*(-4*a*c + b**2)*atanh((b + 2*c*x*
*3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(768*c**(9/2)) + x**6*
(a + b*x**3 + c*x**6)**(3/2)/(15*c) + (a + b*x**3 + c*x**6)**(3/2
)*(-8*a*c + 35*b**2/4 - 21*b*c*x**3/2)/(180*c**3)
```

Mathematica [A] time = 0.119824, size = 159, normalized size = 0.93

$$\frac{\sqrt{a+bx^3+cx^6}\left(128c^2\left(-2a^2+acx^6+3c^2x^{12}\right)+4b^2c\left(115a-14cx^6\right)+8bc^2x^3\left(6cx^6-29a\right)-105b^4+70b^3cx^3\right)}{5760c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{768c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^4 + 70*b^3*c*x^3 + 4*b^2*c*(115*
a - 14*c*x^6) + 8*b*c^2*x^3*(-29*a + 6*c*x^6) + 128*c^2*(-2*a^2 +
 a*c*x^6 + 3*c^2*x^12)))/(5760*c^4) + (b*(7*b^2 - 12*a*c)*(b^2 -
4*a*c)*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(768
*c^(9/2))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x^{11}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288798, size = 1, normalized size = 0.01

$$\left[\frac{4\left(384\,c^4x^{12}+48\,bc^3x^9-8\left(7\,b^2c^2-16\,ac^3\right)x^6-105\,b^4+460\,ab^2c-256\,a^2c^2+2\left(35\,b^3c-116\,abc^2\right)x^3\right)\sqrt{cx^6+bx^3+a}\sqrt{}}{23040\,c^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^11,x, algorithm="fricas")
```

```
[Out] [1/23040*(4*(384*c^4*x^12 + 48*b*c^3*x^9 - 8*(7*b^2*c^2 - 16*a*c^
3)*x^6 - 105*b^4 + 460*a*b^2*c - 256*a^2*c^2 + 2*(35*b^3*c - 116*
a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) + 15*(7*b^5 - 40*a*
b^3*c + 48*a^2*b*c^2)*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 +
 b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c^(9/2),
1/11520*(2*(384*c^4*x^12 + 48*b*c^3*x^9 - 8*(7*b^2*c^2 - 16*a*c^3
)*x^6 - 105*b^4 + 460*a*b^2*c - 256*a^2*c^2 + 2*(35*b^3*c - 116*a
*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c) + 15*(7*b^5 - 40*a*
b^3*c + 48*a^2*b*c^2)*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x
^6 + b*x^3 + a)*c)))/(sqrt(-c)*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}x^{11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^11,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

3.187 $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=153

$$-\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}+\frac{(5b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\,(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3\,(a+bx^3+cx^6)^{3/2}}{12c}$$

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3)
 - (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c
*x^6)^(3/2))/(12*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(7/2))
```

Rubi [A] time = 0.27897, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}+\frac{(5b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\,(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3\,(a+bx^3+cx^6)^{3/2}}{12c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3)
 - (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c
*x^6)^(3/2))/(12*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(7/2))
```

Rubi in Sympy [A] time = 29.9161, size = 141, normalized size = 0.92

$$-\frac{5b\,(a+bx^3+cx^6)^{\frac{3}{2}}}{72c^2}+\frac{x^3\,(a+bx^3+cx^6)^{\frac{3}{2}}}{12c}+\frac{(b+2cx^3)\,(-4ac+5b^2)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{(-4ac+b^2)\,(-4ac+5b^2)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -5*b*(a + b*x**3 + c*x**6)**(3/2)/(72*c**2) + x**3*(a + b*x**3 +
c*x**6)**(3/2)/(12*c) + (b + 2*c*x**3)*(-4*a*c + 5*b**2)*sqrt(a +
 b*x**3 + c*x**6)/(192*c**3) - (-4*a*c + b**2)*(-4*a*c + 5*b**2)*
atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(384*
c**(7/2))
```

Mathematica [A] time = 0.143219, size = 134, normalized size = 0.88

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(b\left(8c^2x^6-52ac\right)+24c^2x^3\left(a+2cx^6\right)+15b^3-10b^2cx^3\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\log\left(2\sqrt{c}\sqrt{a+bx\cdots}\right.}{1152c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 10*b^2*c*x^3 + 24*c^
2*x^3*(a + 2*c*x^6) + b*(-52*a*c + 8*c^2*x^6)) - 3*(5*b^4 - 24*a*
b^2*c + 16*a^2*c^2)*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 +
c*x^6]])/(1152*c^(7/2))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x^8\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283242, size = 1, normalized size = 0.01

$$\left[\frac{4\left(48\,c^{3}x^{9}+8\,bc^{2}x^{6}-2\left(5\,b^{2}c-12\,ac^{2}\right)x^{3}+15\,b^{3}-52\,abc\right)\sqrt{cx^{6}+bx^{3}+a}\sqrt{c}+3\left(5\,b^{4}-24\,ab^{2}c+16\,a^{2}c^{2}\right)\log\left(4\sqrt{cx^{6}+\right.}{2304\,c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^8,x, algorithm="fricas")
```

```
[Out] [1/2304*(4*(48*c^3*x^9 + 8*b*c^2*x^6 - 2*(5*b^2*c - 12*a*c^2)*x^3
 + 15*b^3 - 52*a*b*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) + 3*(5*b^4
- 24*a*b^2*c + 16*a^2*c^2)*log(4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x
^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c^(7/
2), 1/1152*(2*(48*c^3*x^9 + 8*b*c^2*x^6 - 2*(5*b^2*c - 12*a*c^2)*
x^3 + 15*b^3 - 52*a*b*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c) - 3*(5*
b^4 - 24*a*b^2*c + 16*a^2*c^2)*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/
(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{8}\sqrt{a+bx^{3}+cx^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{6}+bx^{3}+a}\,x^{8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^8,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

3.188 $\int x^5\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

```
[Out] -(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c^2) + (a + b*x^3
+ c*x^6)^(3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*
Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(5/2))
```

Rubi [A] time = 0.169687, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c^2) + (a + b*x^3
+ c*x^6)^(3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*
Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(5/2))
```

Rubi in Sympy [A] time = 17.2655, size = 97, normalized size = 0.9

$$-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{b\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{\frac{5}{2}}}+\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{9c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -b*(b + 2*c*x**3)*sqrt(a + b*x**3 + c*x**6)/(24*c**2) + b*(-4*a*c
 + b**2)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6
)))/(48*c**(5/2)) + (a + b*x**3 + c*x**6)**(3/2)/(9*c)
```

Mathematica [A] time = 0.141276, size = 97, normalized size = 0.9

$$\frac{(b^3-4abc)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{48c^{5/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(8c\left(a+cx^6\right)-3b^2+2bcx^3\right)}{72c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/
(72*c^2) + ((b^3 - 4*a*b*c)*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6]])/(48*c^(5/2))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int x^5\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282138, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8\,c^2x^6+2\,bcx^3-3\,b^2+8\,ac\right)\sqrt{cx^6+bx^3+a}\sqrt{c}-3\left(b^3-4\,abc\right)\log\left(4\sqrt{cx^6+bx^3+a}\left(2\,c^2x^3+bc\right)-\left(8\,c^2x^6+8\,bcx^3+\right.\right.}{288\,c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^5,x, algorithm="fricas")
```

```
[Out] [1/288*(4*(8*c^2*x^6 + 2*b*c*x^3 - 3*b^2 + 8*a*c)*sqrt(c*x^6 + b*
x^3 + a)*sqrt(c) - 3*(b^3 - 4*a*b*c)*log(4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(
c)))/c^(5/2), 1/144*(2*(8*c^2*x^6 + 2*b*c*x^3 - 3*b^2 + 8*a*c)*sq
rt(c*x^6 + b*x^3 + a)*sqrt(-c) + 3*(b^3 - 4*a*b*c)*arctan(1/2*(2*
c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [A] time = 0.325321, size = 132, normalized size = 1.22

$$\frac{1}{72}\sqrt{cx^6 + bx^3 + a}\left(2\left(4\,x^3 + \frac{b}{c}\right)x^3 - \frac{3\,b^2 - 8\,ac}{c^2}\right) - \frac{\left(b^3 - 4\,abc\right)\ln\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{48\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^5,x, algorithm="giac")
```

```
[Out] 1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*
c)/c^2) - 1/48*(b^3 - 4*a*b*c)*ln(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^
6 + b*x^3 + a))*sqrt(c) - b))/c^(5/2)
```

3.189 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^
(3/2))
```

Rubi [A] time = 0.11902, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^
(3/2))
```

Rubi in Sympy [A] time = 10.1775, size = 73, normalized size = 0.88

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(-4ac+b^2)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] (b + 2*c*x**3)*sqrt(a + b*x**3 + c*x**6)/(12*c) - (-4*a*c + b**2)
*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(24*
c**(3/2))
```

Mathematica [A] time = 0.0556396, size = 83, normalized size = 1.

$$\frac{2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*
Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(3/2)
)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int x^2\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269591, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-\left(b^2-4\,ac\right)\log\left(-4\sqrt{cx^6+bx^3+a}\left(2\,c^2x^3+bc\right)-\left(8\,c^2x^6+8\,bcx^3+b^2+4\,ac\right)\sqrt{c}\right)}{48\,c^{\frac{3}{2}}},\ 2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^2,x, algorithm="fricas")
```

```
[Out] [1/48*(4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - (b^2 - 4
*a*c)*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2*x
^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c^(3/2), 1/24*(2*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c) - (b^2 - 4*a*c)*arctan(1/2
*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [A] time = 0.286409, size = 103, normalized size = 1.24

$$\frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\ln\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{24\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^2,x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*l
n(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^
(3/2)
```

3.190 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*S
qrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*Sqrt[c])
```

Rubi [A] time = 0.252078, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*S
qrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*Sqrt[c])
```

Rubi in Sympy [A] time = 30.0251, size = 95, normalized size = 0.87

$$-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3}+\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}+\frac{\sqrt{a+bx^3+cx^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6
)))/3 + b*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**
6)))/(6*sqrt(c)) + sqrt(a + b*x**3 + c*x**6)/3
```

Mathematica [A] time = 0.272838, size = 109, normalized size = 1.

$$\frac{1}{6}\left(2\sqrt{a+bx^3+cx^6}+2\sqrt{a}\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)+\frac{b\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3 + c*x^6] + 2*Sqrt[a]*(Log[x^3] - Log[2*a + b*x^
3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]) + (b*Log[b + 2*c*x^3 + 2*
Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/Sqrt[c])/6
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299892, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(-4\sqrt{cx^6+bx^3+a}\left(2c^2x^3+bc\right)-\left(8c^2x^6+8bcx^3+b^2+4ac\right)\sqrt{c}\right)+2\sqrt{a}\sqrt{c}\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(b}{x^6}\right.}{12\sqrt{c}}\right.$$

$$-\frac{4\sqrt{-a}\sqrt{c}\arctan\left(\frac{bx^3+2a}{2\sqrt{cx^6+bx^3+a}\sqrt{-a}}\right)-b\log\left(-4\sqrt{cx^6+bx^3+a}\left(2c^2x^3+bc\right)-\left(8c^2x^6+8bcx^3+b^2+4ac\right)\sqrt{c}\right)-4\sqrt{cx^6}}{12\sqrt{c}}$$

$$\left.-\frac{2\sqrt{-a}\sqrt{-c}\arctan\left(\frac{bx^3+2a}{2\sqrt{cx^6+bx^3+a}\sqrt{-a}}\right)-b\arctan\left(\frac{(2cx^3+b)\sqrt{-c}}{2\sqrt{cx^6+bx^3+a}c}\right)-2\sqrt{cx^6+bx^3+a}\sqrt{-c}}{6\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x,x, algorithm="fricas")
```

```
[Out] [1/12*(b*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^
2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)) + 2*sqrt(a)*sqrt(c)*log
(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*sqrt(
c))/sqrt(c), 1/6*(b*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6
 + b*x^3 + a)*c)) + sqrt(a)*sqrt(-c)*log(-((b^2 + 4*a*c)*x^6 + 8*
a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2
)/x^6) + 2*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c))/sqrt(-c), -1/12*(4*s
qrt(-a)*sqrt(c)*arctan(1/2*(b*x^3 + 2*a)/(sqrt(c*x^6 + b*x^3 + a)
*sqrt(-a))) - b*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c)
- (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)) - 4*sqrt(c*x^6 +
 b*x^3 + a)*sqrt(c))/sqrt(c), -1/6*(2*sqrt(-a)*sqrt(-c)*arctan(1/
2*(b*x^3 + 2*a)/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))) - b*arctan(1/
2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)) - 2*sqrt(c*
x^6 + b*x^3 + a)*sqrt(-c))/sqrt(-c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

3.191 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sq
rt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(
b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rubi [A] time = 0.251808, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sq
rt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(
b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rubi in Sympy [A] time = 28.7965, size = 99, normalized size = 0.88

$$\frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3}-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\,\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(c)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)
))/3 - sqrt(a + b*x**3 + c*x**6)/(3*x**3) - b*atanh((2*a + b*x**3
)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(6*sqrt(a))
```

Mathematica [A] time = 0.21062, size = 112, normalized size = 1.

$$\frac{1}{6}\left(-\frac{2\sqrt{a+bx^3+cx^6}}{x^3}+\frac{b\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{\sqrt{a}}+2\sqrt{c}\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] ((-2*Sqrt[a + b*x^3 + c*x^6])/x^3 + (b*(Log[x^3] - Log[2*a + b*x^
3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/Sqrt[a] + 2*Sqrt[c]*Log[
b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/6
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29594, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{a}\sqrt{c}x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+bx^3\log\left(\frac{4\,\sqrt{cx^6+bx^3+a}(abx^3+2\,a^2)-((b^2+4\,ac)x}{x^6}\right.}{12\,\sqrt{a}x^3}\right.$$

$$-\frac{bx^3\arctan\left(\frac{(bx^3+2\,a)\sqrt{-a}}{2\,\sqrt{cx^6+bx^3+a}a}\right)-\sqrt{-a}\sqrt{c}x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+2\,\sqrt{cx^6+b}}{6\,\sqrt{-a}x^3}$$

$$\left.-\frac{bx^3\arctan\left(\frac{(bx^3+2\,a)\sqrt{-a}}{2\,\sqrt{cx^6+bx^3+a}a}\right)-2\,\sqrt{-a}\sqrt{-c}x^3\arctan\left(\frac{2\,cx^3+b}{2\,\sqrt{cx^6+bx^3+a}\sqrt{-c}}\right)+2\,\sqrt{cx^6+bx^3+a}\sqrt{-a}}{6\,\sqrt{-a}x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(a)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4
*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + b*x^3*l
og((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*
x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) - 4*sqrt(c*x^6 + b*x^3 + a
)*sqrt(a))/(sqrt(a)*x^3), 1/12*(4*sqrt(a)*sqrt(-c)*x^3*arctan(1/2
*(2*c*x^3 + b)/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-c))) + b*x^3*log((4
*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*x^6 +
 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*sqr
t(a))/(sqrt(a)*x^3), -1/6*(b*x^3*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a
)/(sqrt(c*x^6 + b*x^3 + a)*a)) - sqrt(-a)*sqrt(c)*x^3*log(-8*c^2*
x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(c) - 4*a*c) + 2*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*x
^3), -1/6*(b*x^3*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 +
b*x^3 + a)*a)) - 2*sqrt(-a)*sqrt(-c)*x^3*arctan(1/2*(2*c*x^3 + b)
/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-c))) + 2*sqrt(c*x^6 + b*x^3 + a)*
sqrt(-a))/(sqrt(-a)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

3.192 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7} \, dx$

Optimal. Leaf size=88

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

```
[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((b^2 - 4*a
*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(
24*a^(3/2))
```

Rubi [A] time = 0.154004, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((b^2 - 4*a
*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(
24*a^(3/2))
```

Rubi in Sympy [A] time = 18.8226, size = 76, normalized size = 0.86

$$-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{(-4ac+b^2)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] -(2*a + b*x**3)*sqrt(a + b*x**3 + c*x**6)/(12*a*x**6) + (-4*a*c +
 b**2)*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6))
)/(24*a**(3/2))
```

Mathematica [A] time = 0.224819, size = 93, normalized size = 1.06

$$-\frac{(b^2-4ac)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) - ((b^2 - 4*a
*c)*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^
6]]))/(24*a^(3/2))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^7}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278603, size = 1, normalized size = 0.01

$$\left[-\frac{(b^2-4\,ac)\,x^6\log\left(\frac{4\,\sqrt{cx^6+bx^3+a}\,(abx^3+2\,a^2)-((b^2+4\,ac)\,x^6+8\,abx^3+8\,a^2)\,\sqrt{a}}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\,\sqrt{a}}{48\,a^{\frac{3}{2}}x^6},\ \frac{(b^2-4\,ac)\,x^6\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*x^6*log((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3
 + 2*a^2) - ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6)
 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a))/(a^(3/2)*x^6)
, 1/24*((b^2 - 4*a*c)*x^6*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt
(c*x^6 + b*x^3 + a)*a)) - 2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(-a))/(sqrt(-a)*a*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

3.193 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x
^3 + c*x^6)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x
^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rubi [A] time = 0.209907, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x
^3 + c*x^6)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x
^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rubi in Sympy [A] time = 24.851, size = 104, normalized size = 0.9

$$-\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{9ax^9}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{b\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

```
[Out] -(a + b*x**3 + c*x**6)**(3/2)/(9*a*x**9) + b*(2*a + b*x**3)*sqrt(
a + b*x**3 + c*x**6)/(24*a**2*x**6) - b*(-4*a*c + b**2)*atanh((2*
a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(48*a**(5/2))
```

Mathematica [A] time = 0.377257, size = 113, normalized size = 0.97

$$\frac{b\left(b^2-4ac\right)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{48a^{5/2}}-\frac{\sqrt{a+bx^3+cx^6}\left(8a^2+2ax^3\left(b+4cx^3\right)-3b^2x^6\right)}{72a^2x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(8*a^2 - 3*b^2*x^6 + 2*a*x^3*(b + 4*c*x
^3)))/(72*a^2*x^9) + (b*(b^2 - 4*a*c)*(Log[x^3] - Log[2*a + b*x^3
 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(48*a^(5/2))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int\frac{1}{x^{10}}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291593, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^{3}-4\,abc\right)x^{9}\log\left(-\frac{4\,\sqrt{cx^{6}+bx^{3}+a}\left(abx^{3}+2\,a^{2}\right)+\left(\left(b^{2}+4\,ac\right)x^{6}+8\,abx^{3}+8\,a^{2}\right)\sqrt{a}}{x^{6}}\right)-4\left(\left(3\,b^{2}-8\,ac\right)x^{6}-2\,abx^{3}-8\,a^{2}\right)\sqrt{cx^{6}+bx^{3}+a}\sqrt{a}}{288\,a^{\frac{5}{2}}x^{9}},\;-\frac{3\left(b^{3}-4\,abc\right)x^{9}\arctan\left(\frac{\left(bx^{3}+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^{6}+bx^{3}+a}a}\right)-2\left(\left(3\,b^{2}-8\,ac\right)x^{6}-2\,abx^{3}-8\,a^{2}\right)\sqrt{cx^{6}+bx^{3}+a}\sqrt{-a}}{144\,\sqrt{-a}a^{2}x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*x^9*log(-(4*sqrt(c*x^6 + b*x^3 + a)*(a
*b*x^3 + 2*a^2) + ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a)
)/x^6) - 4*((3*b^2 - 8*a*c)*x^6 - 2*a*b*x^3 - 8*a^2)*sqrt(c*x^6 +
 b*x^3 + a)*sqrt(a))/(a^(5/2)*x^9), -1/144*(3*(b^3 - 4*a*b*c)*x^9
*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) -
 2*((3*b^2 - 8*a*c)*x^6 - 2*a*b*x^3 - 8*a^2)*sqrt(c*x^6 + b*x^3 +
 a)*sqrt(-a))/(sqrt(-a)*a^2*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^{3}+cx^{6}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{6}+bx^{3}+a}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

3.194 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

Optimal. Leaf size=161

$$\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}-\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}}$$

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*a^3
*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 +
 c*x^6)^(3/2))/(72*a^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcT
anh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(7
/2))
```

Rubi [A] time = 0.348684, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}-\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*a^3
*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 +
 c*x^6)^(3/2))/(72*a^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcT
anh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(7
/2))
```

Rubi in Sympy [A] time = 37.7016, size = 148, normalized size = 0.92

$$-\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{12ax^{12}}+\frac{5b\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{72a^2x^9}-\frac{\left(2a+bx^3\right)\left(-4ac+5b^2\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{\left(-4ac+b^2\right)\left(-4ac+5b^2\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

```
[Out] -(a + b*x**3 + c*x**6)**(3/2)/(12*a*x**12) + 5*b*(a + b*x**3 + c*
x**6)**(3/2)/(72*a**2*x**9) - (2*a + b*x**3)*(-4*a*c + 5*b**2)*sq
rt(a + b*x**3 + c*x**6)/(192*a**3*x**6) + (-4*a*c + b**2)*(-4*a*c
 + 5*b**2)*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x*
*6)))/(384*a**(7/2))
```

Mathematica [A] time = 0.191117, size = 141, normalized size = 0.88

$$-\frac{(b^2-4ac)\left(5b^2-4ac\right)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{384a^{7/2}}-\frac{\sqrt{a+bx^3+cx^6}\left(48a^3+8a^2x^3\left(b+3cx^3\right)-2abx^6\left(5b+26cx^3\right)+15b^3x^9\right)}{576a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(48*a^3 + 15*b^3*x^9 + 8*a^2*x^3*(b + 3
*c*x^3) - 2*a*b*x^6*(5*b + 26*c*x^3)))/(576*a^3*x^12) - ((b^2 - 4
*a*c)*(5*b^2 - 4*a*c)*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqr
t[a + b*x^3 + c*x^6]]))/(384*a^(7/2))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288404, size = 1, normalized size = 0.01

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)x^{12}\log\left(-\frac{4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)+\left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right)-4\left(\left(15\,b^3-52\,abc\right)x^9-2\left(5\,ab^2\right.\right.}{2304\,a^{\frac{7}{2}}x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^13,x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^12*log(-(4*sqrt(c*
x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) + ((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 + 8*a^2)*sqrt(a))/x^6) - 4*((15*b^3 - 52*a*b*c)*x^9 - 2*(5*a*b
^2 - 12*a^2*c)*x^6 + 8*a^2*b*x^3 + 48*a^3)*sqrt(c*x^6 + b*x^3 + a
)*sqrt(a))/(a^(7/2)*x^12), 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*a^2
*c^2)*x^12*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3
+ a)*a)) - 2*((15*b^3 - 52*a*b*c)*x^9 - 2*(5*a*b^2 - 12*a^2*c)*x^
6 + 8*a^2*b*x^3 + 48*a^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt
(-a)*a^3*x^12)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^13,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

3.195 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

Optimal. Leaf size=199

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9}+\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}}$$

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a
^4*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3
 + c*x^6)^(3/2))/(120*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 +
 c*x^6)^(3/2))/(720*a^3*x^9) - (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*
ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(768*
a^(9/2))
```

Rubi [A] time = 0.527924, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9}+\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a
^4*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3
 + c*x^6)^(3/2))/(120*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 +
 c*x^6)^(3/2))/(720*a^3*x^9) - (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*
ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(768*
a^(9/2))
```

Rubi in Sympy [A] time = 54.9897, size = 185, normalized size = 0.93

$$-\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{15ax^{15}}+\frac{7b\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{120a^2x^{12}}-\frac{\left(-32ac+35b^2\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{720a^3x^9}+\frac{b\left(2a+bx^3\right)\left(-12ac+7b^2\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{b\left(-12ac+7b^2\right)\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

```
[Out] -(a + b*x**3 + c*x**6)**(3/2)/(15*a*x**15) + 7*b*(a + b*x**3 + c*
x**6)**(3/2)/(120*a**2*x**12) - (-32*a*c + 35*b**2)*(a + b*x**3 +
 c*x**6)**(3/2)/(720*a**3*x**9) + b*(2*a + b*x**3)*(-12*a*c + 7*b
**2)*sqrt(a + b*x**3 + c*x**6)/(384*a**4*x**6) - b*(-12*a*c + 7*b
**2)*(-4*a*c + b**2)*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x
**3 + c*x**6)))/(768*a**(9/2))
```

Mathematica [A] time = 0.301364, size = 175, normalized size = 0.88

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{768a^{9/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-384a^4-16a^3\left(3bx^3+8cx^6\right)+8a^2x^6\left(7b^2+29bcx^3+32c^2x^6\right)-10ab^2x^9\left(7b+46cx^3\right)+105b^4x^{12}\right)}{5760a^4x^{15}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-384*a^4 + 105*b^4*x^12 - 10*a*b^2*x^9*
(7*b + 46*c*x^3) - 16*a^3*(3*b*x^3 + 8*c*x^6) + 8*a^2*x^6*(7*b^2
+ 29*b*c*x^3 + 32*c^2*x^6)))/(5760*a^4*x^15) + (b*(7*b^2 - 12*a*c
)*(b^2 - 4*a*c)*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a +
b*x^3 + c*x^6]]))/(768*a^(9/2))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x^{16}}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296472, size = 1, normalized size = 0.01

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)x^{15}\log\left(\frac{4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right)+4\left(\left(105\,b^4-460\,ab^2c+256\,a^2c^2\right.\right.}{23040\,a^{\frac{9}{2}}x^{15}}\right.$$

$$-\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)x^{15}\arctan\left(\frac{\left(bx^3+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^6+bx^3+a}a}\right)-2\left(\left(105\,b^4-460\,ab^2c+256\,a^2c^2\right)x^{12}-2\left(35\,ab^3-116\,a^2bc\right)x\right.}{11520\,\sqrt{-a}a^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^16,x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x^15*log((4*sqrt
(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*x^6 + 8*a*
b*x^3 + 8*a^2)*sqrt(a))/x^6) + 4*((105*b^4 - 460*a*b^2*c + 256*a^
2*c^2)*x^12 - 2*(35*a*b^3 - 116*a^2*b*c)*x^9 - 48*a^3*b*x^3 + 8*(
7*a^2*b^2 - 16*a^3*c)*x^6 - 384*a^4)*sqrt(c*x^6 + b*x^3 + a)*sqrt
(a))/(a^(9/2)*x^15), -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*
c^2)*x^15*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 +
 a)*a)) - 2*((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*x^12 - 2*(35*a
*b^3 - 116*a^2*b*c)*x^9 - 48*a^3*b*x^3 + 8*(7*a^2*b^2 - 16*a^3*c)
*x^6 - 384*a^4)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*a^4*x
^15)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)

[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^16,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

3.196 $\int x^3\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.529538, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{x^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 37.1961, size = 124, normalized size = 0.89

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,\mathrm{appellf}_1\left(\frac{4}{3},-\frac{1}{2},-\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] x**4*sqrt(a + b*x**3 + c*x**6)*appellf1(4/3, -1/2, -1/2, 7/3, -2*
c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b*
```

```
*2)))/(4*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 5.14938, size = 1043, normalized size = 7.45

$$\frac{336a^2c\left(2cx^3+b-\sqrt{b^2-4ac}\right)\left(2cx^3+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)x^4}{28aF_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{3};\frac{1}{2},\frac{3}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{3};\frac{3}{2},\frac{1}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (8*c*(3*b*x + 8*c*x^4)*(a + b*x^3 + c*x^6)^2 + (96*a^2*b*x*(b - S
qrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*App
ellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(-16*a*AppellF1[1/3, 1/2, 1/2,
4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3
/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt
[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2,
 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^
2 - 4*a*c])])) + (336*a^2*c*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (
-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])])/(28*a*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sq
rt[b^2 - 4*a*c])*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt
[b^2 - 4*a*c])*AppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) + (105*a*b^
2*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(-28*a*AppellF1[4/3
, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-
b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1
[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7
/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])])))/(448*c^2*(a + b*x^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^3\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6 + bx^3 + a}x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x^3,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

3.197 $\int x\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.354646, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 28.7392, size = 124, normalized size = 0.89

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,\mathrm{appellf}_1\left(\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] x**2*sqrt(a + b*x**3 + c*x**6)*appellf1(2/3, -1/2, -1/2, 5/3, -2*
c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b*
```

```
*2)))/(2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.82652, size = 701, normalized size = 5.01

$$x^2\left(\frac{75a^2\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{40acF_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-6cx^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{5}{3};\frac{1}{2},\frac{3}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{3};\frac{3}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*(5*(a + b*x^3 + c*x^6)^2 + (75*a^2*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2
, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])])/(40*a*c*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 6*c*
x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
+ (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) +
 (12*a*b*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(32*a*App
ellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])
*AppellF1[8/3, 1/2, 3/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*A
ppellF1[8/3, 3/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))))/(25*(a + b*x^3 + c*x^6)^
(3/2))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int x\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6 + bx^3 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*x,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

3.198 $\int \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
/(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.204733, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{x\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
/(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 37.389, size = 121, normalized size = 0.9

$$\frac{x\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] x*sqrt(a + b*x**3 + c*x**6)*appellf1(1/3, -1/2, -1/2, 4/3, -2*c*x
**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)
```

```
))/(sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**3/(b
 + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.07529, size = 702, normalized size = 5.2

$$x\left(\frac{24a^2\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{c\left(16aF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac-}}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(2*(a + b*x^3 + c*x^6)^2 + (24*a^2*(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2,
4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])])/(c*(16*a*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3
*((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (
b - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (
21*a*b*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(28*a*App
ellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])
*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*A
ppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))))/(8*(a + b*x^3 + c*x^6)^(
3/2))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6 + bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")

[Out] integral(sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

3.199 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)/(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi [A] time = 0.422217, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)/(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi in Sympy [A] time = 35.066, size = 124, normalized size = 0.9

$$-\frac{\sqrt{a+bx^3+cx^6}\,\operatorname{appellf}_1\left(-\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)*appellf1(-1/3, -1/2, -1/2, 2/3, -2*c*x
**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)
```

```
))/(x*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**3/
(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.08189, size = 702, normalized size = 5.09

$$\frac{75abx^3\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{4c\left(20aF_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{5}{3};\frac{1}{2},\frac{3}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{3};\frac{3}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] (-5*(a + b*x^3 + c*x^6)^2 + (75*a*b*x^3*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2
, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])])/(4*c*(20*a*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3
*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
 + (24*a*x^6*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(32*a*Appell
F1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Ap
pellF1[8/3, 1/2, 3/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
llF1[8/3, 3/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))/(5*x*(a + b*x^3 + c*x^6)^(3/2
))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6 + bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

3.200 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
/(2*x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.418836, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
/(2*x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 32.7198, size = 128, normalized size = 0.91

$$-\frac{\sqrt{a+bx^3+cx^6}\,\mathrm{appellf}_1\left(-\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)*appellf1(-2/3, -1/2, -1/2, 1/3, -2*c*x
**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)
```

```
))/(2*x**2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*
x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.11704, size = 702, normalized size = 5.01

$$\frac{6abx^3\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{c\left(16aF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] (-(a + b*x^3 + c*x^6)^2 + (6*a*b*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2, 4
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])])/(c*(16*a*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*
((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (2
1*a*x^6*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*(28*a*AppellF1
[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3
)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Appe
llF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(2*x^2*(a + b*x^3 + c*x^6)^(3/
2))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^6 + bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

3.201 $\int x^{14} \left(a+bx^3+cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=293

$$\frac{\left(b^2-4ac\right)^2 \left(16a^2c^2-72ab^2c+33b^4\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}} - \frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6} + \frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5} - \frac{\left(3b\left(77b^2-124ac\right)-10cx^3\left(33b^2-28ac\right)\right)\left(a+bx^3+cx^6\right)^{5/2}}{13440c^4} - \frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}$$

```
[Out] -((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*
Sqrt[a + b*x^3 + c*x^6])/(16384*c^6) + ((33*b^4 - 72*a*b^2*c + 16
*a^2*c^2)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (
11*b*x^6*(a + b*x^3 + c*x^6)^(5/2))/(336*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 - 124*a*c) - 10*c*(33*b^2 -
 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^4) + ((b^2 - 4*
a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```

Rubi [A] time = 0.799884, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{\left(b^2-4ac\right)^2 \left(16a^2c^2-72ab^2c+33b^4\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}} - \frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6} + \frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5} - \frac{\left(3b\left(77b^2-124ac\right)-10cx^3\left(33b^2-28ac\right)\right)\left(a+bx^3+cx^6\right)^{5/2}}{13440c^4} - \frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*
Sqrt[a + b*x^3 + c*x^6])/(16384*c^6) + ((33*b^4 - 72*a*b^2*c + 16
*a^2*c^2)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (
11*b*x^6*(a + b*x^3 + c*x^6)^(5/2))/(336*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 - 124*a*c) - 10*c*(33*b^2 -
 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^4) + ((b^2 - 4*
a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```

Rubi in Sympy [A] time = 67.2398, size = 284, normalized size = 0.97

$$-\frac{11bx^6\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{24c} - \frac{\left(\frac{9b(-124ac+77b^2)}{8}-\frac{15cx^3(-28ac+33b^2)}{4}\right)\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{5040c^4} + \frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\left(16a^2c^2-72ab^2c+33b^4\right)}{6144c^5} - \frac{\left(b+2cx^3\right)\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}\left(16a^2c^2-72ab^2c+33b^4\right)}{16384c^6} + \frac{\left(-4ac+b^2\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -11*b*x**6*(a + b*x**3 + c*x**6)**(5/2)/(336*c**2) + x**9*(a + b*
x**3 + c*x**6)**(5/2)/(24*c) - (9*b*(-124*a*c + 77*b**2)/8 - 15*c
*x**3*(-28*a*c + 33*b**2)/4)*(a + b*x**3 + c*x**6)**(5/2)/(5040*c
**4) + (b + 2*c*x**3)*(a + b*x**3 + c*x**6)**(3/2)*(16*a**2*c**2
- 72*a*b**2*c + 33*b**4)/(6144*c**5) - (b + 2*c*x**3)*(-4*a*c + b
**2)*sqrt(a + b*x**3 + c*x**6)*(16*a**2*c**2 - 72*a*b**2*c + 33*b
**4)/(16384*c**6) + (-4*a*c + b**2)**2*(16*a**2*c**2 - 72*a*b**2*
c + 33*b**4)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*
x**6)))/(32768*c**(13/2))
```

Mathematica [A] time = 0.313281, size = 289, normalized size = 0.99

$$105\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)-2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(16b^3c^2\left(5103a^2-78\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(3465*b^7 - 2310*b^6*c*x^3 +
24*b^4*c^2*x^3*(749*a - 66*c*x^6) + 84*b^5*c*(-365*a + 22*c*x^6)
- 32*b^2*c^3*x^3*(1181*a^2 - 284*a*c*x^6 + 40*c^2*x^12) + 16*b^3*
c^2*(5103*a^2 - 780*a*c*x^6 + 88*c^2*x^12) - 4480*c^4*x^3*(-3*a^3
 + 2*a^2*c*x^6 + 24*a*c^2*x^12 + 16*c^3*x^18) - 64*b*c^3*(919*a^3
 - 302*a^2*c*x^6 + 104*a*c^2*x^12 + 1360*c^3*x^18)) + 105*(b^2 -
4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b + 2*c*x^3 + 2*S
qrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3440640*c^(13/2))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int x^{14} \left(c x^6 + b x^3 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.334089, size = 1, normalized size = 0.

$$\left[\frac{4 \left(71680\, c^7 x^{21} + 87040\, b c^6 x^{18} + 1280 \left(b^2 c^5 + 84\, a c^6\right) x^{15} - 128 \left(11\, b^3 c^4 - 52\, a b c^5\right) x^{12} + 16 \left(99\, b^4 c^3 - 568\, a b^2 c^4 + 560\, a^2 c^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14,x, algorithm="fricas")
```

```
[Out] [1/6881280*(4*(71680*c^7*x^21 + 87040*b*c^6*x^18 + 1280*(b^2*c^5
+ 84*a*c^6)*x^15 - 128*(11*b^3*c^4 - 52*a*b*c^5)*x^12 + 16*(99*b^
4*c^3 - 568*a*b^2*c^4 + 560*a^2*c^5)*x^9 - 3465*b^7 + 30660*a*b^5
*c - 81648*a^2*b^3*c^2 + 58816*a^3*b*c^3 - 8*(231*b^5*c^2 - 1560*
a*b^3*c^3 + 2416*a^2*b*c^4)*x^6 + 2*(1155*b^6*c - 8988*a*b^4*c^2
+ 18896*a^2*b^2*c^3 - 6720*a^3*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a)*
sqrt(c) + 105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3
*b^2*c^3 + 256*a^4*c^4)*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3
 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c^(13/2
), 1/3440640*(2*(71680*c^7*x^21 + 87040*b*c^6*x^18 + 1280*(b^2*c^
5 + 84*a*c^6)*x^15 - 128*(11*b^3*c^4 - 52*a*b*c^5)*x^12 + 16*(99*
b^4*c^3 - 568*a*b^2*c^4 + 560*a^2*c^5)*x^9 - 3465*b^7 + 30660*a*b
^5*c - 81648*a^2*b^3*c^2 + 58816*a^3*b*c^3 - 8*(231*b^5*c^2 - 156
0*a*b^3*c^3 + 2416*a^2*b*c^4)*x^6 + 2*(1155*b^6*c - 8988*a*b^4*c^
2 + 18896*a^2*b^2*c^3 - 6720*a^3*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a
)*sqrt(-c) + 105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*
a^3*b^2*c^3 + 256*a^4*c^4)*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqr
t(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{14} \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^{14}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)
```

3.202 $\int x^{11} \left(a+bx^3+cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=223

$$-\frac{b \left(b^2-4ac\right)^2 \left(3b^2-4ac\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}+\frac{b \left(b^2-4ac\right) \left(3b^2-4ac\right) \left(b+2cx^3\right) \sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b \left(3b^2-4ac\right) \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{\left(-16ac+21b^2-30bcx^3\right) \left(a+bx^3+cx^6\right)^{5/2}}{840c^3}+\frac{x^6 \left(a+bx^3+cx^6\right)^{5/2}}{21c}$$

```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c
*x^6])/(1024*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 +
 c*x^6)^(3/2))/(384*c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c)
 + ((21*b^2 - 16*a*c - 30*b*c*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(84
0*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11/2))
```

Rubi [A] time = 0.43652, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{b \left(b^2-4ac\right)^2 \left(3b^2-4ac\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}+\frac{b \left(b^2-4ac\right) \left(3b^2-4ac\right) \left(b+2cx^3\right) \sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b \left(3b^2-4ac\right) \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{\left(-16ac+21b^2-30bcx^3\right) \left(a+bx^3+cx^6\right)^{5/2}}{840c^3}+\frac{x^6 \left(a+bx^3+cx^6\right)^{5/2}}{21c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```


```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c
*x^6])/(1024*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 +
 c*x^6)^(3/2))/(384*c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c)
 + ((21*b^2 - 16*a*c - 30*b*c*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(84
0*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11/2))
```

Rubi in Sympy [A] time = 41.3591, size = 214, normalized size = 0.96

$$-\frac{b\left(b+2cx^{3}\right)\left(-4ac+3b^{2}\right)\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}{384c^{4}}+\frac{b\left(b+2cx^{3}\right)\left(-4ac+b^{2}\right)\left(-4ac+3b^{2}\right)\sqrt{a+bx^{3}+cx^{6}}}{1024c^{5}}-\frac{b\left(-4ac+b^{2}\right)^{2}\left(-4ac+3b^{2}\right)\operatorname{atanh}\left(\frac{b+2cx^{3}}{2\sqrt{c}\sqrt{a+bx^{3}+cx^{6}}}\right)}{2048c^{\frac{11}{2}}}+\frac{x^{6}\left(a+bx^{3}+cx^{6}\right)^{\frac{5}{2}}}{21c}+\frac{\left(a+bx^{3}+cx^{6}\right)^{\frac{5}{2}}\left(-12ac+\frac{63b^{2}}{4}-\frac{45bcx^{3}}{2}\right)}{630c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -b*(b + 2*c*x**3)*(-4*a*c + 3*b**2)*(a + b*x**3 + c*x**6)**(3/2)/
(384*c**4) + b*(b + 2*c*x**3)*(-4*a*c + b**2)*(-4*a*c + 3*b**2)*s
qrt(a + b*x**3 + c*x**6)/(1024*c**5) - b*(-4*a*c + b**2)**2*(-4*a
*c + 3*b**2)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*
x**6)))/(2048*c**(11/2)) + x**6*(a + b*x**3 + c*x**6)**(5/2)/(21*
c) + (a + b*x**3 + c*x**6)**(5/2)*(-12*a*c + 63*b**2/4 - 45*b*c*x
**3/2)/(630*c**3)
```

Mathematica [A] time = 0.23929, size = 222, normalized size = 1.

$$2\sqrt{c}\sqrt{a+bx^{3}+cx^{6}}\left(16b^{2}c^{2}\left(343a^{2}-62acx^{6}+8c^{2}x^{12}\right)+32bc^{3}x^{3}\left(-73a^{2}+22acx^{6}+200c^{2}x^{12}\right)+168b^{4}c\left(cx^{6}-15a\right)+16b\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^6 - 210*b^5*c*x^3 + 16*
b^3*c^2*x^3*(91*a - 9*c*x^6) + 168*b^4*c*(-15*a + c*x^6) + 1024*c
^3*(a + c*x^6)^2*(-2*a + 5*c*x^6) + 16*b^2*c^2*(343*a^2 - 62*a*c*
x^6 + 8*c^2*x^12) + 32*b*c^3*x^3*(-73*a^2 + 22*a*c*x^6 + 200*c^2*
x^12)) - 105*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*Log[b + 2*c*x^3 +
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(215040*c^(11/2))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int x^{11} \left(c x^6 + b x^3 + a\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.311414, size = 1, normalized size = 0.

$$\left[\frac{4\left(5120\,c^6x^{18} + 6400\,bc^5x^{15} + 128\left(b^2c^4 + 64\,ac^5\right)x^{12} - 16\left(9\,b^3c^3 - 44\,abc^4\right)x^9 + 8\left(21\,b^4c^2 - 124\,ab^2c^3 + 128\,a^2c^4\right)x^6 + 31\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11,x, algorithm="fricas")

[Out] [1/430080*(4*(5120*c^6*x^18 + 6400*b*c^5*x^15 + 128*(b^2*c^4 + 64
*a*c^5)*x^12 - 16*(9*b^3*c^3 - 44*a*b*c^4)*x^9 + 8*(21*b^4*c^2 -
124*a*b^2*c^3 + 128*a^2*c^4)*x^6 + 315*b^6 - 2520*a*b^4*c + 5488*
a^2*b^2*c^2 - 2048*a^3*c^3 - 2*(105*b^5*c - 728*a*b^3*c^2 + 1168*
a^2*b*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) - 105*(3*b^7 - 28
*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*log(-4*sqrt(c*x^6 + b*x
^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)
*sqrt(c)))/c^(11/2), 1/215040*(2*(5120*c^6*x^18 + 6400*b*c^5*x^15
 + 128*(b^2*c^4 + 64*a*c^5)*x^12 - 16*(9*b^3*c^3 - 44*a*b*c^4)*x^
9 + 8*(21*b^4*c^2 - 124*a*b^2*c^3 + 128*a^2*c^4)*x^6 + 315*b^6 -
2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3 - 2*(105*b^5*c - 7
```

```
28*a*b^3*c^2 + 1168*a^2*b*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(
-c) - 105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*ar
ctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sq
rt(-c)*c^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11} \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^{11} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

3.203 $\int x^8 \left(a+bx^3+cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=204

$$\frac{\left(b^2-4ac\right)^2 \left(7b^2-4ac\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}} - \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3} - \frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2} + \frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}$$

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*
x^6])/(1536*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*
x^6)^(3/2))/(576*c^3) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2)
 + (x^3*(a + b*x^3 + c*x^6)^(5/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6])])/(3072*c^(9/2))
```

Rubi [A] time = 0.363897, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\left(b^2-4ac\right)^2 \left(7b^2-4ac\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}} - \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3} - \frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2} + \frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```


```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*
x^6])/(1536*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*
x^6)^(3/2))/(576*c^3) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2)
 + (x^3*(a + b*x^3 + c*x^6)^(5/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6])])/(3072*c^(9/2))
```

Rubi in Sympy [A] time = 39.2812, size = 190, normalized size = 0.93

$$-\frac{7b\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{180c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{18c}+\frac{\left(b+2cx^3\right)\left(-4ac+7b^2\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{576c^3}-\frac{\left(b+2cx^3\right)\left(-4ac+b^2\right)\left(-4ac+7b^2\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(-4ac+b^2\right)^2\left(-4ac+7b^2\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -7*b*(a + b*x**3 + c*x**6)**(5/2)/(180*c**2) + x**3*(a + b*x**3 +
 c*x**6)**(5/2)/(18*c) + (b + 2*c*x**3)*(-4*a*c + 7*b**2)*(a + b*
x**3 + c*x**6)**(3/2)/(576*c**3) - (b + 2*c*x**3)*(-4*a*c + b**2)
*(-4*a*c + 7*b**2)*sqrt(a + b*x**3 + c*x**6)/(1536*c**4) + (-4*a*
c + b**2)**2*(-4*a*c + 7*b**2)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sq
rt(a + b*x**3 + c*x**6)))/(3072*c**(9/2))
```

Mathematica [A] time = 0.194568, size = 194, normalized size = 0.95

$$\frac{15\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)-2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(-16bc^2\left(-81a^2+18acx^6+104c^2x^{12}\right.\right.}{46080c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(105*b^5 - 70*b^4*c*x^3 - 48*
b^2*c^2*x^3*(-9*a + c*x^6) + 8*b^3*c*(-95*a + 7*c*x^6) - 160*c^3*
x^3*(3*a^2 + 14*a*c*x^6 + 8*c^2*x^12) - 16*b*c^2*(-81*a^2 + 18*a*
c*x^6 + 104*c^2*x^12)) + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b
 + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(46080*c^(9/2))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x^8\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.310548, size = 1, normalized size = 0.

$$\left[\frac{4\left(1280\,c^5x^{15}+1664\,bc^4x^{12}+16\left(3\,b^2c^3+140\,ac^4\right)x^9-8\left(7\,b^3c^2-36\,abc^3\right)x^6-105\,b^5+760\,ab^3c-1296\,a^2bc^2+2\left(35\,b^4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] [1/92160*(4*(1280*c^5*x^15 + 1664*b*c^4*x^12 + 16*(3*b^2*c^3 + 14
0*a*c^4)*x^9 - 8*(7*b^3*c^2 - 36*a*b*c^3)*x^6 - 105*b^5 + 760*a*b
^3*c - 1296*a^2*b*c^2 + 2*(35*b^4*c - 216*a*b^2*c^2 + 240*a^2*c^3
)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) - 15*(7*b^6 - 60*a*b^4*c +
 144*a^2*b^2*c^2 - 64*a^3*c^3)*log(4*sqrt(c*x^6 + b*x^3 + a)*(2*c
^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c
^(9/2), 1/46080*(2*(1280*c^5*x^15 + 1664*b*c^4*x^12 + 16*(3*b^2*c
^3 + 140*a*c^4)*x^9 - 8*(7*b^3*c^2 - 36*a*b*c^3)*x^6 - 105*b^5 +
760*a*b^3*c - 1296*a^2*b*c^2 + 2*(35*b^4*c - 216*a*b^2*c^2 + 240*
a^2*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c) + 15*(7*b^6 - 60*a
*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(1/2*(2*c*x^3 + b)*s
qrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

3.204 $\int x^5 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=150

$$-\frac{b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}+\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}$$

```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3)
 - (b*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*
x^3 + c*x^6)^(5/2)/(15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(256*c^(7/2))
```

Rubi [A] time = 0.231477, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}+\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3)
 - (b*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*
x^3 + c*x^6)^(5/2)/(15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(256*c^(7/2))
```

Rubi in Sympy [A] time = 23.6512, size = 138, normalized size = 0.92

$$-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{48c^2}+\frac{b\left(b+2cx^3\right)\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(-4ac+b^2\right)^2 \operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{\frac{7}{2}}}+\frac{\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{15c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -b*(b + 2*c*x**3)*(a + b*x**3 + c*x**6)**(3/2)/(48*c**2) + b*(b +
 2*c*x**3)*(-4*a*c + b**2)*sqrt(a + b*x**3 + c*x**6)/(128*c**3) -
 b*(-4*a*c + b**2)**2*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*
x**3 + c*x**6)))/(256*c**(7/2)) + (a + b*x**3 + c*x**6)**(5/2)/(1
5*c)
```

Mathematica [A] time = 0.155749, size = 142, normalized size = 0.95

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(4b^2c\left(2cx^6-25a\right)+8bc^2x^3\left(7a+22cx^6\right)+128c^2\left(a+cx^6\right)^2+15b^4-10b^3cx^3\right)-15b\left(b^2-4ac\right)^2\log}{3840c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^4 - 10*b^3*c*x^3 + 128*c
^2*(a + c*x^6)^2 + 4*b^2*c*(-25*a + 2*c*x^6) + 8*b*c^2*x^3*(7*a +
 22*c*x^6)) - 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sq
rt[a + b*x^3 + c*x^6]])/(3840*c^(7/2))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x^5\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305644, size = 1, normalized size = 0.01

$$\left[\frac{4\left(128\,c^4x^{12} + 176\,bc^3x^9 + 8\left(b^2c^2 + 32\,ac^3\right)x^6 + 15\,b^4 - 100\,ab^2c + 128\,a^2c^2 - 2\left(5\,b^3c - 28\,abc^2\right)x^3\right)\sqrt{cx^6 + bx^3 + a}\sqrt{c} + 1}{7680\,c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [1/7680*(4*(128*c^4*x^12 + 176*b*c^3*x^9 + 8*(b^2*c^2 + 32*a*c^3)
*x^6 + 15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 2*(5*b^3*c - 28*a*b*c
^2)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) + 15*(b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*log(4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (
8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c^(7/2), 1/3840*(2
*(128*c^4*x^12 + 176*b*c^3*x^9 + 8*(b^2*c^2 + 32*a*c^3)*x^6 + 15*
b^4 - 100*a*b^2*c + 128*a^2*c^2 - 2*(5*b^3*c - 28*a*b*c^2)*x^3)*s
qrt(c*x^6 + b*x^3 + a)*sqrt(-c) - 15*(b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c
)))/(sqrt(-c)*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [A] time = 0.294707, size = 232, normalized size = 1.55

$$\frac{1}{1920}\sqrt{cx^6 + bx^3 + a}\left(2\left(4\left(2\left(8\,cx^3 + 11\,b\right)x^3 + \frac{b^2c^3 + 32\,ac^4}{c^4}\right)x^3 - \frac{5\,b^3c^2 - 28\,abc^3}{c^4}\right)x^3 + \frac{15\,b^4c - 100\,ab^2c^2 + 128\,a^2c^3}{c^4}\right)$$
$$+ \frac{\left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)\ln\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{256\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b
^2*c^3 + 32*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 +
 (15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*ln(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^
3 + a))*sqrt(c) - b))/c^(7/2)
```

3.205 $\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=124

$$\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*c^2) +
 ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c
)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
128*c^(5/2))
```

Rubi [A] time = 0.174603, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*c^2) +
 ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c
)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
128*c^(5/2))
```

Rubi in Sympy [A] time = 15.1663, size = 112, normalized size = 0.9

$$\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{24c}-\frac{\left(b+2cx^3\right)\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(-4ac+b^2\right)^2\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] (b + 2*c*x**3)*(a + b*x**3 + c*x**6)**(3/2)/(24*c) - (b + 2*c*x**
3)*(-4*a*c + b**2)*sqrt(a + b*x**3 + c*x**6)/(64*c**2) + (-4*a*c
+ b**2)**2*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x*
```

```
*6)))/(128*c**(5/2))
```

Mathematica [A] time = 0.106789, size = 111, normalized size = 0.9

$$\frac{2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{384c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*
x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^2 - 4*a*c)^2*Log[b + 2*c*x^3 +
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(5/2))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x^2\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294575, size = 1, normalized size = 0.01

$$\left[\frac{4\left(16\,c^3x^9+24\,bc^2x^6+2\left(b^2c+20\,ac^2\right)x^3-3\,b^3+20\,abc\right)\sqrt{cx^6+bx^3+a}\sqrt{c}+3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(-4\sqrt{cx^6+bx\right.}{768\,c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/768*(4*(16*c^3*x^9 + 24*b*c^2*x^6 + 2*(b^2*c + 20*a*c^2)*x^3 -
 3*b^3 + 20*a*b*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) + 3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 +
b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c^(5/2), 1
/384*(2*(16*c^3*x^9 + 24*b*c^2*x^6 + 2*(b^2*c + 20*a*c^2)*x^3 - 3
*b^3 + 20*a*b*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c) + 3*(b^4 - 8*a*
b^2*c + 16*a^2*c^2)*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6
 + b*x^3 + a)*c)))/(sqrt(-c)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [A] time = 0.290269, size = 182, normalized size = 1.47

$$\frac{1}{192}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\,cx^3+3\,b\right)x^3+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^3-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\ln\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^2,x, algorithm="giac")
```

[Out] 1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*c^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*ln(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(5/2)

3.206 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x} \, dx$

Optimal. Leaf size=155

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2}$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a +
 b*x^3 + c*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[
a]*Sqrt[a + b*x^3 + c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(3/2))
```

Rubi [A] time = 0.427794, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```


```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a +
 b*x^3 + c*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[
a]*Sqrt[a + b*x^3 + c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(3/2))
```

Rubi in Sympy [A] time = 46.631, size = 139, normalized size = 0.9

$$-\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3}-\frac{b\left(-12ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{\frac{3}{2}}}+\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{9}+\frac{\sqrt{a+bx^3+cx^6}\left(4ac+\frac{b^2}{2}+bcx^3\right)}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

```
[Out] -a**(3/2)*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**
6)))/3 - b*(-12*a*c + b**2)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(
a + b*x**3 + c*x**6)))/(48*c**(3/2)) + (a + b*x**3 + c*x**6)**(3/
2)/9 + sqrt(a + b*x**3 + c*x**6)*(4*a*c + b**2/2 + b*c*x**3)/(12*
c)
```

Mathematica [A] time = 0.473632, size = 181, normalized size = 1.17

$$\frac{1}{144}\left(48a^{3/2}\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)-\frac{3b^3\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{c^{3/2}}+\frac{2\sqrt{a+bx^3+cx^6}\left(8c\left(4a+cx^6\right)+3b^2+14bcx^3\right)}{c}+\frac{36ab\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^
6)))/c + 48*a^(3/2)*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6]]) - (3*b^3*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6]])/c^(3/2) + (36*a*b*Log[b + 2*c*x^3 + 2*Sqrt[c]*
Sqrt[a + b*x^3 + c*x^6]])/Sqrt[c])/144
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.395202, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^(3/2)*c^(3/2)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 -
4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4
*(8*c^2*x^6 + 14*b*c*x^3 + 3*b^2 + 32*a*c)*sqrt(c*x^6 + b*x^3 + a
)*sqrt(c) - 3*(b^3 - 12*a*b*c)*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*
c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/
c^(3/2), 1/144*(24*a^(3/2)*sqrt(-c)*c*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) + 2*(8*c^2*x^6 + 14*b*c*x^3 + 3*b^2 + 32*a*c)*sqrt(c*x^6
+ b*x^3 + a)*sqrt(-c) - 3*(b^3 - 12*a*b*c)*arctan(1/2*(2*c*x^3 +
b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c), -1/288*(9
6*sqrt(-a)*a*c^(3/2)*arctan(1/2*(b*x^3 + 2*a)/(sqrt(c*x^6 + b*x^3
 + a)*sqrt(-a))) - 4*(8*c^2*x^6 + 14*b*c*x^3 + 3*b^2 + 32*a*c)*sq
rt(c*x^6 + b*x^3 + a)*sqrt(c) + 3*(b^3 - 12*a*b*c)*log(-4*sqrt(c*
x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2
 + 4*a*c)*sqrt(c)))/c^(3/2), -1/144*(48*sqrt(-a)*a*sqrt(-c)*c*arc
tan(1/2*(b*x^3 + 2*a)/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))) - 2*(8*
c^2*x^6 + 14*b*c*x^3 + 3*b^2 + 32*a*c)*sqrt(c*x^6 + b*x^3 + a)*sq
rt(-c) + 3*(b^3 - 12*a*b*c)*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sq
rt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

3.207 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4} dx$

Optimal. Leaf size=150

$$\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)
^(3/2)/(3*x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*Sqrt[c])
```

Rubi [A] time = 0.414049, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)
^(3/2)/(3*x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*Sqrt[c])
```

Rubi in Sympy [A] time = 44.5705, size = 134, normalized size = 0.89

$$-\frac{\sqrt{a}b\,\mathrm{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2}+\frac{\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4}-\frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{3x^3}+\frac{\left(4ac+b^2\right)\mathrm{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

```
[Out] -sqrt(a)*b*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x*
*6)))/2 + (3*b + 2*c*x**3)*sqrt(a + b*x**3 + c*x**6)/4 - (a + b*x
**3 + c*x**6)**(3/2)/(3*x**3) + (4*a*c + b**2)*atanh((b + 2*c*x**
3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(8*sqrt(c))
```

Mathematica [A] time = 0.349401, size = 173, normalized size = 1.15

$$\frac{1}{8}\left(\frac{b^2 \log \left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{\sqrt{c}}+\frac{2\sqrt{a+bx^3+cx^6}\left(-4a+5bx^3+2cx^6\right)}{3x^3}+4\sqrt{a}b\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)+4a\sqrt{c}\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/(3*x^3) +
 4*Sqrt[a]*b*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x
^3 + c*x^6]]) + (b^2*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 +
 c*x^6]])/Sqrt[c] + 4*a*Sqrt[c]*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6]])/8
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.349202, size = 1, normalized size = 0.01

$$\left[\frac{12\sqrt{a}b\sqrt{c}x^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)+3\left(b^2+4ac\right)x^3\log\left(-4\sqrt{cx^6+bx^3+a}\left(2c^2x^3+bc\right)-\right.}{48\sqrt{c}x^3}\right.$$

$$-\frac{24\sqrt{-a}b\sqrt{c}x^3\arctan\left(\frac{bx^3+2a}{2\sqrt{cx^6+bx^3+a}\sqrt{-a}}\right)-3\left(b^2+4ac\right)x^3\log\left(-4\sqrt{cx^6+bx^3+a}\left(2c^2x^3+bc\right)-\left(8c^2x^6+8bcx^3+b^2+4\right.\right.}{48\sqrt{c}x^3}$$

$$-\frac{12\sqrt{-a}b\sqrt{-c}x^3\arctan\left(\frac{bx^3+2a}{2\sqrt{cx^6+bx^3+a}\sqrt{-a}}\right)-3\left(b^2+4ac\right)x^3\arctan\left(\frac{(2cx^3+b)\sqrt{-c}}{2\sqrt{cx^6+bx^3+ac}}\right)-2\left(2cx^6+5bx^3-4a\right)\sqrt{cx^6+bx^3+}}{24\sqrt{-c}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(a)*b*sqrt(c)*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6
) + 3*(b^2 + 4*a*c)*x^3*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3
 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)) + 4*(2*c
*x^6 + 5*b*x^3 - 4*a)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c))/(sqrt(c)*x
^3), 1/24*(6*sqrt(a)*b*sqrt(-c)*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a
*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)
/x^6) + 3*(b^2 + 4*a*c)*x^3*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sq
rt(c*x^6 + b*x^3 + a)*c)) + 2*(2*c*x^6 + 5*b*x^3 - 4*a)*sqrt(c*x^
6 + b*x^3 + a)*sqrt(-c))/(sqrt(-c)*x^3), -1/48*(24*sqrt(-a)*b*sqr
t(c)*x^3*arctan(1/2*(b*x^3 + 2*a)/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-
a))) - 3*(b^2 + 4*a*c)*x^3*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*
x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)) - 4*(
2*c*x^6 + 5*b*x^3 - 4*a)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c))/(sqrt(c
)*x^3), -1/24*(12*sqrt(-a)*b*sqrt(-c)*x^3*arctan(1/2*(b*x^3 + 2*a
)/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))) - 3*(b^2 + 4*a*c)*x^3*arcta
n(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)) - 2*(2*
c*x^6 + 5*b*x^3 - 4*a)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c))/(sqrt(-c
```

```
)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)

[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

3.208 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*x^3) - (a + b*x^3 + c
*x^6)^(3/2)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[
(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/2
```

Rubi [A] time = 0.401881, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] -((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*x^3) - (a + b*x^3 + c
*x^6)^(3/2)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[
(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/2
```

Rubi in Sympy [A] time = 44.1126, size = 136, normalized size = 0.9

$$\frac{b\sqrt{c}\,\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}-\frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{6x^6}-\frac{(4ac+b^2)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] b*sqrt(c)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**
6)))/2 - (b - 2*c*x**3)*sqrt(a + b*x**3 + c*x**6)/(4*x**3) - (a +
 b*x**3 + c*x**6)**(3/2)/(6*x**6) - (4*a*c + b**2)*atanh((2*a + b
*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(8*sqrt(a))
```

Mathematica [A] time = 0.6959, size = 149, normalized size = 0.99

$$\frac{1}{24}\left(\frac{3\log\left(x^3\right)\left(4ac+b^2\right)}{\sqrt{a}}-\frac{3\left(4ac+b^2\right)\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)}{\sqrt{a}}-\frac{2\sqrt{a+bx^3+cx^6}\left(2a+5bx^3-4cx^6\right)}{x^6}+12b\sqrt{c}\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] ((-2*(2*a + 5*b*x^3 - 4*c*x^6)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (3*
(b^2 + 4*a*c)*Log[x^3])/Sqrt[a] - (3*(b^2 + 4*a*c)*Log[2*a + b*x^
3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]])/Sqrt[a] + 12*b*Sqrt[c]*Lo
g[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/24
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^7}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.333714, size = 1, normalized size = 0.01

$$\left[\frac{12\sqrt{a}b\sqrt{c}x^6\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+3\left(b^2+4ac\right)x^6\log\left(\frac{4\sqrt{cx^6+bx^3+a}(abx^3+2}{}\right.}{48\sqrt{a}x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(a)*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2
- 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b
^2 + 4*a*c)*x^6*log((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2)
- ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) + 4*(4*c*
x^6 - 5*b*x^3 - 2*a)*sqrt(c*x^6 + b*x^3 + a)*sqrt(a))/(sqrt(a)*x^
6), 1/48*(24*sqrt(a)*b*sqrt(-c)*x^6*arctan(1/2*(2*c*x^3 + b)/(sqr
t(c*x^6 + b*x^3 + a)*sqrt(-c))) + 3*(b^2 + 4*a*c)*x^6*log((4*sqrt
(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*x^6 + 8*a*
b*x^3 + 8*a^2)*sqrt(a))/x^6) + 4*(4*c*x^6 - 5*b*x^3 - 2*a)*sqrt(c
*x^6 + b*x^3 + a)*sqrt(a))/(sqrt(a)*x^6), 1/24*(6*sqrt(-a)*b*sqrt
(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 +
 a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^2 + 4*a*c)*x^6*arctan(1
/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) + 2*(4*c*x
^6 - 5*b*x^3 - 2*a)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*x
^6), 1/24*(12*sqrt(-a)*b*sqrt(-c)*x^6*arctan(1/2*(2*c*x^3 + b)/(s
qrt(c*x^6 + b*x^3 + a)*sqrt(-c))) - 3*(b^2 + 4*a*c)*x^6*arctan(1/
2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) + 2*(4*c*x^
6 - 5*b*x^3 - 2*a)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*x^
6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

3.209 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=163

$$\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{(x^3(8ac+b^2)+2ab)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{(a+bx^3+cx^6)^{3/2}}{9x^9}$$

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a*x^6)
 - (a + b*x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2))
+ (c^(3/2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^
6])])/3
```

Rubi [A] time = 0.427679, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{(x^3(8ac+b^2)+2ab)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{(a+bx^3+cx^6)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a*x^6)
 - (a + b*x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2))
+ (c^(3/2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^
6])])/3
```

Rubi in Sympy [A] time = 45.878, size = 146, normalized size = 0.9

$$\frac{c^{\frac{3}{2}}\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3}-\frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{9x^9}-\frac{\left(ab+x^3\left(4ac+\frac{b^2}{2}\right)\right)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{b(-12ac+b^2)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

```
[Out] c**(3/2)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6
)))/3 - (a + b*x**3 + c*x**6)**(3/2)/(9*x**9) - (a*b + x**3*(4*a*
c + b**2/2))*sqrt(a + b*x**3 + c*x**6)/(12*a*x**6) + b*(-12*a*c +
 b**2)*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6))
)/(48*a**(3/2))
```

Mathematica [A] time = 0.471896, size = 174, normalized size = 1.07

$$\frac{-2\sqrt{a}\left(\sqrt{a+bx^3+cx^6}\left(8a^2+14abx^3+32acx^6+3b^2x^6\right)-24ac^{3/2}x^9\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)\right)-3bx^9\log\left(x^3\right)}{144a^{3/2}x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] (-3*b*(b^2 - 12*a*c)*x^9*Log[x^3] + 3*b*(b^2 - 12*a*c)*x^9*Log[2*
a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]] - 2*Sqrt[a]*(Sqrt[
a + b*x^3 + c*x^6]*(8*a^2 + 14*a*b*x^3 + 3*b^2*x^6 + 32*a*c*x^6)
- 24*a*c^(3/2)*x^9*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c
*x^6]]))/(144*a^(3/2)*x^9)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.361038, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^(3/2)*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 -
 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^
3 - 12*a*b*c)*x^9*log((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2
) - ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) - 4*((3
*b^2 + 32*a*c)*x^6 + 14*a*b*x^3 + 8*a^2)*sqrt(c*x^6 + b*x^3 + a)*
sqrt(a))/(a^(3/2)*x^9), 1/288*(96*a^(3/2)*sqrt(-c)*c*x^9*arctan(1
/2*(2*c*x^3 + b)/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-c))) - 3*(b^3 - 1
2*a*b*c)*x^9*log((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - (
(b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) - 4*((3*b^2
+ 32*a*c)*x^6 + 14*a*b*x^3 + 8*a^2)*sqrt(c*x^6 + b*x^3 + a)*sqrt(
a))/(a^(3/2)*x^9), 1/144*(24*sqrt(-a)*a*c^(3/2)*x^9*log(-8*c^2*x^
6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(c) - 4*a*c) + 3*(b^3 - 12*a*b*c)*x^9*arctan(1/2*(b*x^3 + 2*a)*s
qrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) - 2*((3*b^2 + 32*a*c)*x^6 +
14*a*b*x^3 + 8*a^2)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*a
*x^9), 1/144*(48*sqrt(-a)*a*sqrt(-c)*c*x^9*arctan(1/2*(2*c*x^3 +
b)/(sqrt(c*x^6 + b*x^3 + a)*sqrt(-c))) + 3*(b^3 - 12*a*b*c)*x^9*a
rctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) - 2
*((3*b^2 + 32*a*c)*x^6 + 14*a*b*x^3 + 8*a^2)*sqrt(c*x^6 + b*x^3 +
 a)*sqrt(-a))/(sqrt(-a)*a*x^9)]
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

3.210 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}+\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6
) - ((2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2
 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x
^6])])/(128*a^(5/2))
```

Rubi [A] time = 0.23522, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}+\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6
) - ((2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2
 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x
^6])])/(128*a^(5/2))
```

Rubi in Sympy [A] time = 28.0324, size = 119, normalized size = 0.89

$$-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{24ax^{12}}+\frac{\left(2a+bx^3\right)\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(-4ac+b^2\right)^2\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

```
[Out] -(2*a + b*x**3)*(a + b*x**3 + c*x**6)**(3/2)/(24*a*x**12) + (2*a
+ b*x**3)*(-4*a*c + b**2)*sqrt(a + b*x**3 + c*x**6)/(64*a**2*x**6
) - (-4*a*c + b**2)**2*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b
*x**3 + c*x**6)))/(128*a**(5/2))
```

Mathematica [A] time = 0.233658, size = 125, normalized size = 0.94

$$\frac{3\left(b^2-4ac\right)^2\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)-\frac{2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}\left(8a^2+8abx^3+20acx^6-3b^2x^6\right)}{x^{12}}}{384a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] ((-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 8*a*b
*x^3 - 3*b^2*x^6 + 20*a*c*x^6))/x^12 + 3*(b^2 - 4*a*c)^2*(Log[x^3
] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(384*a
^(5/2))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305055, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)x^{12}\log\left(\frac{4\sqrt{cx^{6}+bx^{3}+a}\left(abx^{3}+2\,a^{2}\right)-\left(\left(b^{2}+4\,ac\right)x^{6}+8\,abx^{3}+8\,a^{2}\right)\sqrt{a}}{x^{6}}\right)+4\left(\left(3\,b^{3}-20\,abc\right)x^{9}-2\left(ab^{2}+20\,a^{2}c\right.\right.}{768\,a^{\frac{5}{2}}x^{12}}\right.$$

$$-\frac{3\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)x^{12}\arctan\left(\frac{\left(bx^{3}+2\,a\right)\sqrt{-a}}{2\sqrt{cx^{6}+bx^{3}+a}a}\right)-2\left(\left(3\,b^{3}-20\,abc\right)x^{9}-2\left(ab^{2}+20\,a^{2}c\right)x^{6}-24\,a^{2}bx^{3}-16\,a^{3}\right)\sqrt{cx^{6}}}{384\sqrt{-a}a^{2}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*x^12*log((4*sqrt(c*x^6 +
 b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 +
8*a^2)*sqrt(a))/x^6) + 4*((3*b^3 - 20*a*b*c)*x^9 - 2*(a*b^2 + 20*
a^2*c)*x^6 - 24*a^2*b*x^3 - 16*a^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(
a))/(a^(5/2)*x^12), -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*x^12
*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) -
 2*((3*b^3 - 20*a*b*c)*x^9 - 2*(a*b^2 + 20*a^2*c)*x^6 - 24*a^2*b*
x^3 - 16*a^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*a^2*x^1
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

3.211 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=162

$$\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-\frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}}$$

```
[Out] -(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*a^3
*x^6) + (b*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12)
 - (a + b*x^3 + c*x^6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*Arc
Tanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(256*a^(
7/2))
```

Rubi [A] time = 0.296424, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-\frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*a^3
*x^6) + (b*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12)
 - (a + b*x^3 + c*x^6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*Arc
Tanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(256*a^(
7/2))
```

Rubi in Sympy [A] time = 34.3938, size = 148, normalized size = 0.91

$$-\frac{(a+bx^3+cx^6)^{\frac{5}{2}}}{15ax^{15}}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{\frac{3}{2}}}{48a^2x^{12}}-\frac{b(2a+bx^3)(-4ac+b^2)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(-4ac+b^2)^2\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

```
[Out] -(a + b*x**3 + c*x**6)**(5/2)/(15*a*x**15) + b*(2*a + b*x**3)*(a
+ b*x**3 + c*x**6)**(3/2)/(48*a**2*x**12) - b*(2*a + b*x**3)*(-4*
a*c + b**2)*sqrt(a + b*x**3 + c*x**6)/(128*a**3*x**6) + b*(-4*a*c
 + b**2)**2*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x
**6)))/(256*a**(7/2))
```

Mathematica [A] time = 0.271081, size = 163, normalized size = 1.01

$$-\frac{b\left(b^2-4ac\right)^2\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{256a^{7/2}}-\frac{\sqrt{a+bx^3+cx^6}\left(128a^4+16a^3\left(11bx^3+16cx^6\right)+8a^2x^6\left(b^2+7bcx^3+16c^2x^6\right)-10ab^2x^9\left(b+10cx^3\right)+15b^4x^{12}\right)}{1920a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(128*a^4 + 15*b^4*x^12 - 10*a*b^2*x^9*(
b + 10*c*x^3) + 16*a^3*(11*b*x^3 + 16*c*x^6) + 8*a^2*x^6*(b^2 + 7
*b*c*x^3 + 16*c^2*x^6)))/(1920*a^3*x^15) - (b*(b^2 - 4*a*c)^2*(Lo
g[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(
256*a^(7/2))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{x^{16}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.303809, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x^{15}\log\left(-\frac{4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)+\left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right) - 4\left(\left(15\,b^4 - 100\,ab^2c + 128\,a^2c^2\right)x\right.}{7680\,a^{\frac{7}{2}}x^{15}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^15*log(-(4*sqrt(c*
x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) + ((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 + 8*a^2)*sqrt(a))/x^6) - 4*((15*b^4 - 100*a*b^2*c + 128*a^2*c^
2)*x^12 - 2*(5*a*b^3 - 28*a^2*b*c)*x^9 + 176*a^3*b*x^3 + 8*(a^2*b
^2 + 32*a^3*c)*x^6 + 128*a^4)*sqrt(c*x^6 + b*x^3 + a)*sqrt(a))/(a
^(7/2)*x^15), 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^15*ar
ctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) - 2*
((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*x^12 - 2*(5*a*b^3 - 28*a^2*
b*c)*x^9 + 176*a^3*b*x^3 + 8*(a^2*b^2 + 32*a^3*c)*x^6 + 128*a^4)*
sqrt(c*x^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*a^3*x^15)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

3.212 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx$

Optimal. Leaf size=216

$$-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}+\frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}}-\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}}$$

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x
^6])/(1536*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 +
 c*x^6)^(3/2))/(576*a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x
^18) + (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*a^2*x^15) - ((b^2 - 4
*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
 b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rubi [A] time = 0.452807, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}+\frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}}-\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x
^6])/(1536*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 +
 c*x^6)^(3/2))/(576*a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x
^18) + (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*a^2*x^15) - ((b^2 - 4
*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
 b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rubi in Sympy [A] time = 48.431, size = 201, normalized size = 0.93

$$-\frac{\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{18ax^{18}}+\frac{7b\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{180a^2x^{15}}-\frac{\left(2a+bx^3\right)\left(-4ac+7b^2\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{576a^3x^{12}}+\frac{\left(2a+bx^3\right)\left(-4ac+b^2\right)\left(-4ac+7b^2\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{\left(-4ac+b^2\right)^2\left(-4ac+7b^2\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

```
[Out] -(a + b*x**3 + c*x**6)**(5/2)/(18*a*x**18) + 7*b*(a + b*x**3 + c*
x**6)**(5/2)/(180*a**2*x**15) - (2*a + b*x**3)*(-4*a*c + 7*b**2)*
(a + b*x**3 + c*x**6)**(3/2)/(576*a**3*x**12) + (2*a + b*x**3)*(-
4*a*c + b**2)*(-4*a*c + 7*b**2)*sqrt(a + b*x**3 + c*x**6)/(1536*a
**4*x**6) - (-4*a*c + b**2)**2*(-4*a*c + 7*b**2)*atanh((2*a + b*x
**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(3072*a**(9/2))
```

Mathematica [A] time = 0.352849, size = 206, normalized size = 0.95

$$\frac{15\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)-\frac{2\sqrt{a}\sqrt{a+bx^3+cx^6}\left(1280a^5+64a^4\left(26bx^3+35cx^6\right)+48a^3x}{}}{46080a^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] ((-2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(1280*a^5 - 105*b^5*x^15 + 1
0*a*b^3*x^12*(7*b + 76*c*x^3) + 64*a^4*(26*b*x^3 + 35*c*x^6) + 48
*a^3*x^6*(b^2 + 6*b*c*x^3 + 10*c^2*x^6) - 8*a^2*b*x^9*(7*b^2 + 54
*b*c*x^3 + 162*c^2*x^6)))/x^18 + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*
c)*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6
]]))/(46080*a^(9/2))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x^{19}}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^19,x)

[Out] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.337772, size = 1, normalized size = 0.

$$\left[-\frac{15\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right)x^{18}\log\left(-\frac{4\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)+\left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right)-4\left(\left(105\,b^5-760\cdots\right.\right.}{\cdots}\right.$$

$$-\frac{15\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right)x^{18}\arctan\left(\frac{\left(bx^3+2\,a\right)\sqrt{-a}}{2\sqrt{cx^6+bx^3+a}a}\right)-2\left(\left(105\,b^5-760\,ab^3c+1296\,a^2bc^2\right)x^{15}-2\left(35\,ab^4\cdots\right.\right.}{46080\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19,x, algorithm="fricas")

[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)
*x^18*log(-(4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) + ((b^2 +
 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) - 4*((105*b^5 - 76
0*a*b^3*c + 1296*a^2*b*c^2)*x^15 - 2*(35*a*b^4 - 216*a^2*b^2*c +
240*a^3*c^2)*x^12 + 8*(7*a^2*b^3 - 36*a^3*b*c)*x^9 - 1664*a^4*b*x
^3 - 16*(3*a^3*b^2 + 140*a^4*c)*x^6 - 1280*a^5)*sqrt(c*x^6 + b*x^
3 + a)*sqrt(a))/(a^(9/2)*x^18), -1/46080*(15*(7*b^6 - 60*a*b^4*c
+ 144*a^2*b^2*c^2 - 64*a^3*c^3)*x^18*arctan(1/2*(b*x^3 + 2*a)*sqr
t(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) - 2*((105*b^5 - 760*a*b^3*c +
1296*a^2*b*c^2)*x^15 - 2*(35*a*b^4 - 216*a^2*b^2*c + 240*a^3*c^2)
*x^12 + 8*(7*a^2*b^3 - 36*a^3*b*c)*x^9 - 1664*a^4*b*x^3 - 16*(3*a
^3*b^2 + 140*a^4*c)*x^6 - 1280*a^5)*sqrt(c*x^6 + b*x^3 + a)*sqrt(
-a))/(sqrt(-a)*a^4*x^18)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

3.213 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}-\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+\frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}}-\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}}+\frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}}$$

```
[Out] -(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 +
c*x^6])/(1024*a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*
x^3 + c*x^6)^(3/2))/(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(2
1*a*x^21) + (b*(a + b*x^3 + c*x^6)^(5/2))/(28*a^2*x^18) - ((21*b^
2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(840*a^3*x^15) + (b*(b^2 -
 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
 + b*x^3 + c*x^6])])/(2048*a^(11/2))
```

Rubi [A] time = 0.670914, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}-\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+\frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}}-\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}}+\frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 +
c*x^6])/(1024*a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*
x^3 + c*x^6)^(3/2))/(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(2
1*a*x^21) + (b*(a + b*x^3 + c*x^6)^(5/2))/(28*a^2*x^18) - ((21*b^
2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(840*a^3*x^15) + (b*(b^2 -
```

```
4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
 + b*x^3 + c*x^6])])/(2048*a^(11/2))
```

Rubi in Sympy [A] time = 70.7257, size = 238, normalized size = 0.93

$$-\frac{\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{21ax^{21}}+\frac{b\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{28a^2x^{18}}-\frac{\left(-16ac+21b^2\right)\left(a+bx^3+cx^6\right)^{\frac{5}{2}}}{840a^3x^{15}}+\frac{b\left(2a+bx^3\right)\left(-4ac+3b^2\right)\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{384a^4x^{12}}-\frac{b\left(2a+bx^3\right)\left(-4ac+b^2\right)\left(-4ac+3b^2\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+\frac{b\left(-4ac+b^2\right)^2\left(-4ac+3b^2\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

```
[Out] -(a + b*x**3 + c*x**6)**(5/2)/(21*a*x**21) + b*(a + b*x**3 + c*x*
*6)**(5/2)/(28*a**2*x**18) - (-16*a*c + 21*b**2)*(a + b*x**3 + c*
x**6)**(5/2)/(840*a**3*x**15) + b*(2*a + b*x**3)*(-4*a*c + 3*b**2
)*(a + b*x**3 + c*x**6)**(3/2)/(384*a**4*x**12) - b*(2*a + b*x**3
)*(-4*a*c + b**2)*(-4*a*c + 3*b**2)*sqrt(a + b*x**3 + c*x**6)/(10
24*a**5*x**6) + b*(-4*a*c + b**2)**2*(-4*a*c + 3*b**2)*atanh((2*a
 + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(2048*a**(11/2)
)
```

Mathematica [A] time = 0.401172, size = 247, normalized size = 0.97

$$-\frac{b\left(3b^2-4ac\right)\left(b^2-4ac\right)^2\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{2048a^{11/2}}-\frac{\sqrt{a+bx^3+cx^6}\left(5120a^6+256a^5\left(25bx^3+32cx^6\right)+64a^4x^6\left(2b^2+11bcx^3+16c^2x^6\right)-16a^3x^9\left(9b^3+62b^2cx^3+146bc^2x^6\right.\right.}{107520a^5x^{21}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(5120*a^6 + 315*b^6*x^18 - 210*a*b^4*x^
15*(b + 12*c*x^3) + 256*a^5*(25*b*x^3 + 32*c*x^6) + 64*a^4*x^6*(2
*b^2 + 11*b*c*x^3 + 16*c^2*x^6) + 56*a^2*b^2*x^12*(3*b^2 + 26*b*c
```

```
*x^3 + 98*c^2*x^6) - 16*a^3*x^9*(9*b^3 + 62*b^2*c*x^3 + 146*b*c^2
*x^6 + 128*c^3*x^9)))/(107520*a^5*x^21) - (b*(b^2 - 4*a*c)^2*(3*b
^2 - 4*a*c)*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^
3 + c*x^6]]))/(2048*a^(11/2))
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \frac{1}{x^{22}} \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.372054, size = 1, normalized size = 0.

$$\left[-\frac{105\left(3\,b^7 - 28\,ab^5c + 80\,a^2b^3c^2 - 64\,a^3bc^3\right)x^{21}\log\left(\frac{4\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right) - \left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right) + 4\left(\left(315\,b^6 - 2520\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22,x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c
^3)*x^21*log((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2
```

```
 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) + 4*((315*b^6 -
2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*x^18 - 2*(105*a*b
^5 - 728*a^2*b^3*c + 1168*a^3*b*c^2)*x^15 + 8*(21*a^2*b^4 - 124*a
^3*b^2*c + 128*a^4*c^2)*x^12 - 16*(9*a^3*b^3 - 44*a^4*b*c)*x^9 +
6400*a^5*b*x^3 + 128*(a^4*b^2 + 64*a^5*c)*x^6 + 5120*a^6)*sqrt(c*
x^6 + b*x^3 + a)*sqrt(a))/(a^(11/2)*x^21), 1/215040*(105*(3*b^7 -
 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^21*arctan(1/2*(b*x
^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) - 2*((315*b^6 - 2
520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*x^18 - 2*(105*a*b^
5 - 728*a^2*b^3*c + 1168*a^3*b*c^2)*x^15 + 8*(21*a^2*b^4 - 124*a^
3*b^2*c + 128*a^4*c^2)*x^12 - 16*(9*a^3*b^3 - 44*a^4*b*c)*x^9 + 6
400*a^5*b*x^3 + 128*(a^4*b^2 + 64*a^5*c)*x^6 + 5120*a^6)*sqrt(c*x
^6 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*a^5*x^21)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{22}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{22}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

3.214 $\int x^3 \left(a + bx^3 + cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=141

$$\frac{ax^4\sqrt{a + bx^3 + cx^6}F_1\left(\frac{4}{3}; -\frac{3}{2}, -\frac{3}{2}; \frac{7}{3}; -\frac{2cx^3}{b-\sqrt{b^2-4ac}}, -\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}} + 1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b} + 1}}$$

```
[Out] (a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.50433, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{ax^4\sqrt{a + bx^3 + cx^6}F_1\left(\frac{4}{3}; -\frac{3}{2}, -\frac{3}{2}; \frac{7}{3}; -\frac{2cx^3}{b-\sqrt{b^2-4ac}}, -\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}} + 1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b} + 1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 39.4235, size = 126, normalized size = 0.89

$$\frac{ax^4\sqrt{a + bx^3 + cx^6}\operatorname{appellf}_1\left(\frac{4}{3}, -\frac{3}{2}, -\frac{3}{2}, \frac{7}{3}, -\frac{2cx^3}{b-\sqrt{-4ac+b^2}}, -\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}} + 1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] a*x**4*sqrt(a + b*x**3 + c*x**6)*appellf1(4/3, -3/2, -3/2, 7/3, -
2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c +
```

```
b**2)))/(4*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*
x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 5.15932, size = 1746, normalized size = 12.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*c*(a + b*x^3 + c*x^6)^2*(-297*b^3 + 216*b^2*c*x^3 + 320*c^2
*x^3*(16*a + 7*c*x^6) + 4*b*c*(459*a + 812*c*x^6)) + (9504*a^2*b^
3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(16*a*AppellF1[1/3, 1/2
, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + S
qrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3,
 1/2, 3/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/
2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
Sqrt[b^2 - 4*a*c])])) - (58752*a^3*b*c*(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2,
 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^
2 - 4*a*c])])/(16*a*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*(
(b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b
- Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) + (103
95*a*b^4*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(28*a*Appell
F1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Ap
pellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
llF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) + (120960*a^3*c^2*x^3*(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*Appel
lF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(28*a*AppellF1[4/3, 1/2, 1/2, 7/3
, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 1/2, 3/2,
 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 3/2, 1/2, 1
0/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])])) + (76356*a^2*b^2*c*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
```

```
- 4*a*c])])/(-28*a*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((
b + Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b
- Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(23
2960*c^3*(a + b*x^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int x^{3} \left(c x^{6}+b x^{3}+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^{6}+b x^{3}+a\right)^{\frac{3}{2}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c x^{9}+b x^{6}+a x^{3}\right) \sqrt{c x^{6}+b x^{3}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral(x**3*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

3.215 $\int x \left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=141

$$\frac{ax^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.35464, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{ax^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 27.5055, size = 126, normalized size = 0.89

$$\frac{ax^2\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] a*x**2*sqrt(a + b*x**3 + c*x**6)*appellf1(2/3, -3/2, -3/2, 5/3, -
2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c +
```

```
b**2)))/(2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*
x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.56667, size = 1391, normalized size = 9.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(5*c*(a + b*x^3 + c*x^6)^2*(27*b^2 + 250*b*c*x^3 + 32*c*(14*
a + 5*c*x^6)) - (675*a^2*b^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c]
)])/(20*a*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[
b^2 - 4*a*c])*AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) + (10800*a^3*c*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*AppellF1[2/3, 1/2, 1
/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt
[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1/
2, 3/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 3/2,
1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqr
t[b^2 - 4*a*c])])) + (5616*a^2*b*c*x^3*(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[5/3, 1/2, 1/2,
 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^
2 - 4*a*c])])/(32*a*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*(
(b + Sqrt[b^2 - 4*a*c])*AppellF1[8/3, 1/2, 3/2, 11/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[8/3, 3/2, 1/2, 11/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) + (7
56*a*b^3*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(-32*a*Appel
lF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*A
ppellF1[8/3, 1/2, 3/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*App
ellF1[8/3, 3/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(8800*c^2*(a + b*x^3 + c*x^
6)^(3/2))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int x \left(c x^{6}+b x^{3}+a \right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^7 + bx^4 + ax)\sqrt{cx^6 + bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] integral((c*x^7 + b*x^4 + a*x)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

3.216 $\int \left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{ax\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.200282, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{ax\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 35.5643, size = 122, normalized size = 0.9

$$\frac{ax\sqrt{a+bx^3+cx^6}\,\mathrm{appellf}_1\left(\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] a*x*sqrt(a + b*x**3 + c*x**6)*appellf1(1/3, -3/2, -3/2, 4/3, -2*c
*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**
```

```
2)))/(sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**3/
(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.41176, size = 1389, normalized size = 10.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*c*(a + b*x^3 + c*x^6)^2*(27*b^2 + 184*b*c*x^3 + 28*c*(13*a
+ 4*c*x^6)) - (864*a^2*b^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
)/(16*a*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^
2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) + (24192*a^3*c*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*
AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(16*a*AppellF1[1/3, 1/2, 1/2
, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2,
 3/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sq
rt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/
2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[
b^2 - 4*a*c])])) + (8316*a^2*b*c*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])])/(28*a*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b
 + Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
 Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])) + (945
*a*b^3*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(-28*a*AppellF
1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*App
ellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appel
lF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(8960*c^2*(a + b*x^3 + c*x^6)
^(3/2))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \left(c x^{6}+b x^{3}+a\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2),x)

[Out] int((c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^6 + bx^3 + a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

3.217 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)])/(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi [A] time = 0.420774, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)])/(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi in Sympy [A] time = 36.849, size = 126, normalized size = 0.91

$$-\frac{a\sqrt{a+bx^3+cx^6}\,\mathrm{appellf}_1\left(-\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
```

```
[Out] -a*sqrt(a + b*x**3 + c*x**6)*appellf1(-1/3, -3/2, -3/2, 2/3, -2*c
*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**
```

```
2)))/(x*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**
3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.85329, size = 1058, normalized size = 7.61

$$\frac{540a^2\left(2cx^3+b-\sqrt{b^2-4ac}\right)\left(2cx^3+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)x^5}{32aF_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{8}{3};\frac{1}{2},\frac{3}{2};\frac{11}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{8}{3};\frac{3}{2},\frac{1}{2};\frac{11}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] ((5*(a + b*x^3 + c*x^6)^2*(-80*a + 19*b*x^3 + 10*c*x^6))/(4*x) +
(2025*a^2*b*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(20*
a*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*
a*c])*AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c
])*AppellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (540*a^2*x^5*(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*Appel
lF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(32*a*AppellF1[5/3, 1/2, 1/2, 8/3
, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[8/3, 1/2, 3/2,
 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[8/3, 3/2, 1/2, 1
1/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])])) + (27*a*b^2*x^5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c
])])/(c*(32*a*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + S
qrt[b^2 - 4*a*c])*AppellF1[8/3, 1/2, 3/2, 11/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqr
t[b^2 - 4*a*c])*AppellF1[8/3, 3/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(100*(a
+ b*x^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

3.218 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=141

$$-\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(2*x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.416652, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] -(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])/(2*x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 36.4707, size = 129, normalized size = 0.91

$$-\frac{a\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(-\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
```

```
[Out] -a*sqrt(a + b*x**3 + c*x**6)*appellf1(-2/3, -3/2, -3/2, 1/3, -2*c
*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**
```

```
2)))/(2*x**2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*
c*x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.86669, size = 1054, normalized size = 7.48

$$\frac{378a^2\left(2cx^3+b-\sqrt{b^2-4ac}\right)\left(2cx^3+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)x^4}{28aF_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{3};\frac{1}{2},\frac{3}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{3};\frac{3}{2},\frac{1}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] ((2*(a + b*x^3 + c*x^6)^2*(-28*a + 17*b*x^3 + 8*c*x^6))/x^2 + (64
8*a^2*b*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]])/(c*(16*a*Appell
F1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Ap
pellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appel
lF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (378*a^2*x^4*(b - Sqrt[b^2 -
4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[4/3,
 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])])/(28*a*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 1/2, 3/2, 10/3, (
-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 3/2, 1/2, 10/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c
])])) + (189*a*b^2*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4
*c*(28*a*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b
^2 - 4*a*c])*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(112*(a + b*x
^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

3.219 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=171

$$\frac{(48a^2c^2-120ab^2c+35b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}} - \frac{(5b(21b^2-44ac)-2cx^3(35b^2-36ac))\sqrt{a+bx^3+cx^6}}{576c^4} - \frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c}$$

```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3
 + c*x^6])/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*
c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2
*c + 48*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(384*c^(9/2))
```

Rubi [A] time = 0.497091, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{(48a^2c^2-120ab^2c+35b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}} - \frac{(5b(21b^2-44ac)-2cx^3(35b^2-36ac))\sqrt{a+bx^3+cx^6}}{576c^4} - \frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3
 + c*x^6])/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*
c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2
*c + 48*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(384*c^(9/2))
```

Rubi in Sympy [A] time = 42.0434, size = 162, normalized size = 0.95

$$-\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c}-\frac{\left(\frac{5b(-44ac+21b^2)}{8}-\frac{cx^3(-36ac+35b^2)}{4}\right)\sqrt{a+bx^3+cx^6}}{72c^4}+\frac{(48a^2c^2-120ab^2c+35b^4)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -7*b*x**6*sqrt(a + b*x**3 + c*x**6)/(72*c**2) + x**9*sqrt(a + b*x
**3 + c*x**6)/(12*c) - (5*b*(-44*a*c + 21*b**2)/8 - c*x**3*(-36*a
*c + 35*b**2)/4)*sqrt(a + b*x**3 + c*x**6)/(72*c**4) + (48*a**2*c
**2 - 120*a*b**2*c + 35*b**4)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqr
t(a + b*x**3 + c*x**6)))/(384*c**(9/2))
```

Mathematica [A] time = 0.185408, size = 135, normalized size = 0.79

$$\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)+2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(4bc\left(55a-14cx^6\right)+24c^2x^3\left(2cx^6\right.\right.}{1152c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 70*b^2*c*x^3 + 4*b
*c*(55*a - 14*c*x^6) + 24*c^2*x^3*(-3*a + 2*c*x^6)) + 3*(35*b^4 -
 120*a*b^2*c + 48*a^2*c^2)*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6]])/(1152*c^(9/2))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int x^{14}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290877, size = 1, normalized size = 0.01

$$\left[\frac{4\left(48\,c^{3}x^{9}-56\,bc^{2}x^{6}+2\left(35\,b^{2}c-36\,ac^{2}\right)x^{3}-105\,b^{3}+220\,abc\right)\sqrt{cx^{6}+bx^{3}+a}\sqrt{c}+3\left(35\,b^{4}-120\,ab^{2}c+48\,a^{2}c^{2}\right)\log\left(-\right.}{2304\,c^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/2304*(4*(48*c^3*x^9 - 56*b*c^2*x^6 + 2*(35*b^2*c - 36*a*c^2)*x
^3 - 105*b^3 + 220*a*b*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) + 3*(35
*b^4 - 120*a*b^2*c + 48*a^2*c^2)*log(-4*sqrt(c*x^6 + b*x^3 + a)*(
2*c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c))
)/c^(9/2), 1/1152*(2*(48*c^3*x^9 - 56*b*c^2*x^6 + 2*(35*b^2*c - 3
6*a*c^2)*x^3 - 105*b^3 + 220*a*b*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(
-c) + 3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*arctan(1/2*(2*c*x^3 +
 b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqrt(-c)*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\sqrt{a+bx^{3}+cx^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

3.220 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=121

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*
x^3)*Sqrt[a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/
2))
```

Rubi [A] time = 0.253063, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```


```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*
x^3)*Sqrt[a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/
2))
```

Rubi in Sympy [A] time = 26.0097, size = 114, normalized size = 0.94

$$-\frac{b\left(-12ac+5b^2\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{\frac{7}{2}}}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}+\frac{\sqrt{a+bx^3+cx^6}\left(-4ac+\frac{15b^2}{4}-\frac{5bcx^3}{2}\right)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -b*(-12*a*c + 5*b**2)*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*
x**3 + c*x**6)))/(48*c**(7/2)) + x**6*sqrt(a + b*x**3 + c*x**6)/(
9*c) + sqrt(a + b*x**3 + c*x**6)*(-4*a*c + 15*b**2/4 - 5*b*c*x**3
```

```
/2)/(18*c**3)
```

Mathematica [A] time = 0.100495, size = 102, normalized size = 0.84

$$\frac{\left(36abc-15b^3\right)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)+2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(8c\left(cx^6-2a\right)+15b^2-10bcx^3\right)}{144c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 10*b*c*x^3 + 8*c*(-2
*a + c*x^6)) + (-15*b^3 + 36*a*b*c)*Log[b + 2*c*x^3 + 2*Sqrt[c]*S
qrt[a + b*x^3 + c*x^6]])/(144*c^(7/2))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int x^{11}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28028, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8\,c^2x^6-10\,bcx^3+15\,b^2-16\,ac\right)\sqrt{cx^6+bx^3+a}\sqrt{c}-3\left(5\,b^3-12\,abc\right)\log\left(-4\,\sqrt{cx^6+bx^3+a}\left(2\,c^2x^3+bc\right)-\left(8\,c^2x^6+\right.\right.}{288\,c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/288*(4*(8*c^2*x^6 - 10*b*c*x^3 + 15*b^2 - 16*a*c)*sqrt(c*x^6 +
 b*x^3 + a)*sqrt(c) - 3*(5*b^3 - 12*a*b*c)*log(-4*sqrt(c*x^6 + b*
x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c
)*sqrt(c)))/c^(7/2), 1/144*(2*(8*c^2*x^6 - 10*b*c*x^3 + 15*b^2 -
16*a*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c) - 3*(5*b^3 - 12*a*b*c)*a
rctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(s
qrt(-c)*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{11}}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)
```

3.221 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=104

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

```
[Out] -(b*Sqrt[a + b*x^3 + c*x^6])/(4*c^2) + (x^3*Sqrt[a + b*x^3 + c*x^
6])/(6*c) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqr
t[a + b*x^3 + c*x^6])])/(24*c^(5/2))
```

Rubi [A] time = 0.195274, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*Sqrt[a + b*x^3 + c*x^6])/(4*c^2) + (x^3*Sqrt[a + b*x^3 + c*x^
6])/(6*c) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqr
t[a + b*x^3 + c*x^6])])/(24*c^(5/2))
```

Rubi in Sympy [A] time = 24.3176, size = 92, normalized size = 0.88

$$-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}+\frac{(-4ac+3b^2)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -b*sqrt(a + b*x**3 + c*x**6)/(4*c**2) + x**3*sqrt(a + b*x**3 + c*
x**6)/(6*c) + (-4*a*c + 3*b**2)*atanh((b + 2*c*x**3)/(2*sqrt(c)*s
qrt(a + b*x**3 + c*x**6)))/(24*c**(5/2))
```

Mathematica [A] time = 0.0686677, size = 86, normalized size = 0.83

$$\frac{(3b^2-4ac)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)+2\sqrt{c}\,(2cx^3-3b)\,\sqrt{a+bx^3+cx^6}}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*(-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (3*b^2 - 4*
a*c)*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(24*c^
(5/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x^8\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277303, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^6+bx^3+a}(2\,cx^3-3\,b)\sqrt{c}-\left(3\,b^2-4\,ac\right)\log\left(4\sqrt{cx^6+bx^3+a}(2\,c^2x^3+bc)-\left(8\,c^2x^6+8\,bcx^3+b^2+4\,ac\right)\sqrt{c}\right)}{48\,c^{\frac{5}{2}}},\ 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/48*(4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 - 3*b)*sqrt(c) - (3*b^2
 - 4*a*c)*log(4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^
2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/c^(5/2), 1/24*(2*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 - 3*b)*sqrt(-c) + (3*b^2 - 4*a*c)*arc
tan(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(sqr
t(-c)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{cx^6 + bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

3.222 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[
c]*Sqrt[a + b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rubi [A] time = 0.114341, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[
c]*Sqrt[a + b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rubi in Sympy [A] time = 12.9748, size = 58, normalized size = 0.85

$$-\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{\frac{3}{2}}}+\frac{\sqrt{a+bx^3+cx^6}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -b*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(6
*c**(3/2)) + sqrt(a + b*x**3 + c*x**6)/(3*c)
```

Mathematica [A] time = 0.0381865, size = 66, normalized size = 0.97

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sq
rt[a + b*x^3 + c*x^6]])/(6*c^(3/2))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int x^5 \frac{1}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290517, size = 1, normalized size = 0.01

$$\left[\frac{b \log\left(4\sqrt{cx^6 + bx^3 + a}\left(2\,c^2x^3 + bc\right) - \left(8\,c^2x^6 + 8\,bcx^3 + b^2 + 4\,ac\right)\sqrt{c}\right) + 4\sqrt{cx^6 + bx^3 + a}\sqrt{c}}{12\,c^{\frac{3}{2}}}, -\frac{b \arctan\left(\frac{\left(2\,cx^3 + b\right)\sqrt{-c}}{2\sqrt{cx^6 + bx^3 + a}c}\right) - 2\sqrt{cx^6 + bx^3 + a}\sqrt{-c}}{6\sqrt{-c}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/12*(b*log(4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2
*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)) + 4*sqrt(c*x^6 + b*x^3 +
 a)*sqrt(c))/c^(3/2), -1/6*(b*arctan(1/2*(2*c*x^3 + b)*sqrt(-c)/(
sqrt(c*x^6 + b*x^3 + a)*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c))
/(sqrt(-c)*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [A] time = 0.291017, size = 82, normalized size = 1.21

$$\frac{b\ln\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{6\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^6 + bx^3 + a}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/6*b*ln(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) -
 b))/c^(3/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

3.223 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqr
t[c])
```

Rubi [A] time = 0.0727881, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqr
t[c])
```

Rubi in Sympy [A] time = 7.5233, size = 37, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(3*sq
rt(c))
```

Mathematica [A] time = 0.0245715, size = 41, normalized size = 0.95

$$\frac{\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]

[Out] Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]]/(3*Sqrt[c])
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266205, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-4\sqrt{cx^6+bx^3+a}\left(2c^2x^3+bc\right)-\left(8c^2x^6+8bcx^3+b^2+4ac\right)\sqrt{c}\right)}{6\sqrt{c}}, \frac{\arctan\left(\frac{\left(2cx^3+b\right)\sqrt{-c}}{2\sqrt{cx^6+bx^3+ac}}\right)}{3\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/6*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2*x^
6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c))/sqrt(c), 1/3*arctan(1/2*(2*
c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c))/sqrt(-c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [A] time = 0.292777, size = 54, normalized size = 1.26

$$-\frac{\ln\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*ln(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) -
b))/sqrt(c)
```

3.224 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sq
rt[a])
```

Rubi [A] time = 0.0893834, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sq
rt[a])
```

Rubi in Sympy [A] time = 12.2494, size = 39, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(3*s
qrt(a))
```

Mathematica [A] time = 0.121684, size = 50, normalized size = 1.14

$$\frac{\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+x^3\left(b+cx^3\right)}+2a+bx^3\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + x^3*(b + c*x^3)]
])/(3*Sqrt[a])
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282031, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{cx^6+bx^3+a}\left(abx^3+2a^2\right)-\left(\left(b^2+4ac\right)x^6+8abx^3+8a^2\right)\sqrt{a}}{x^6}\right)}{6\sqrt{a}},-\frac{\arctan\left(\frac{\left(bx^3+2a\right)\sqrt{-a}}{2\sqrt{cx^6+bx^3+aa}}\right)}{3\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/6*log((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4
*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6)/sqrt(a), -1/3*arctan
(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a))/sqrt(-a)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)
```

3.225 $\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*
Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rubi [A] time = 0.135255, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*
Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rubi in Sympy [A] time = 16.8378, size = 61, normalized size = 0.85

$$-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}+\frac{b\,\mathrm{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)/(3*a*x**3) + b*atanh((2*a + b*x**3)/(2
*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(6*a**(3/2))
```

Mathematica [A] time = 0.133368, size = 78, normalized size = 1.08

$$-\frac{b\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+x^3\left(b+cx^3\right)}+2a+bx^3\right)\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) - (b*(Log[x^3] - Log[2*a + b*x
^3 + 2*Sqrt[a]*Sqrt[a + x^3*(b + c*x^3)]]))/(6*a^(3/2))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286317, size = 1, normalized size = 0.01

$$\left[\frac{bx^3\log\left(-\frac{4\sqrt{cx^6+bx^3+a}\left(abx^3+2a^2\right)+\left(\left(b^2+4ac\right)x^6+8abx^3+8a^2\right)\sqrt{a}}{x^6}\right)-4\sqrt{cx^6+bx^3+a}\sqrt{a}}{12a^{\frac{3}{2}}x^3}, \frac{bx^3\arctan\left(\frac{\left(bx^3+2a\right)\sqrt{-a}}{2\sqrt{cx^6+bx^3+a}a}\right)-2\sqrt{cx^6+\cdots}}{6\sqrt{-a}ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(b*x^3*log(-(4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) +
((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6) - 4*sqrt(c*
x^6 + b*x^3 + a)*sqrt(a))/(a^(3/2)*x^3), 1/6*(b*x^3*arctan(1/2*(b
*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)) - 2*sqrt(c*x^6
+ b*x^3 + a)*sqrt(-a))/(sqrt(-a)*a*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

3.226 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=108

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(6*a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/
(4*a^2*x^3) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(24*a^(5/2))
```

Rubi [A] time = 0.24279, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(6*a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/
(4*a^2*x^3) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(24*a^(5/2))
```

Rubi in Sympy [A] time = 27.3505, size = 95, normalized size = 0.88

$$-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{(-4ac+3b^2)\,\mathrm{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)/(6*a*x**6) + b*sqrt(a + b*x**3 + c*x**
6)/(4*a**2*x**3) - (-4*a*c + 3*b**2)*atanh((2*a + b*x**3)/(2*sqrt
(a)*sqrt(a + b*x**3 + c*x**6)))/(24*a**(5/2))
```

Mathematica [A] time = 0.305705, size = 97, normalized size = 0.9

$$\frac{(3b^2-4ac)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)-\frac{2\sqrt{a}(2a-3bx^3)\sqrt{a+bx^3+cx^6}}{x^6}}{24a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] ((-2*Sqrt[a]*(2*a - 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (3*b^
2 - 4*a*c)*(Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3
 + c*x^6]]))/(24*a^(5/2))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^7}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280626, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,b^2-4\,ac\right)x^6\log\left(-\frac{4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)+\left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}\left(3\,bx^3-2\,a\right)\sqrt{a}}{48\,a^{\frac{5}{2}}x^6},\right.$$
$$\left.-\frac{\left(3\,b^2-4\,ac\right)x^6\arctan\left(\frac{\left(bx^3+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^6+bx^3+a}a}\right)-2\,\sqrt{cx^6+bx^3+a}\left(3\,bx^3-2\,a\right)\sqrt{-a}}{24\,\sqrt{-a}a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*x^6*log(-(4*sqrt(c*x^6 + b*x^3 + a)*(a*b*
x^3 + 2*a^2) + ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x
^6) - 4*sqrt(c*x^6 + b*x^3 + a)*(3*b*x^3 - 2*a)*sqrt(a))/(a^(5/2)
*x^6), -1/24*((3*b^2 - 4*a*c)*x^6*arctan(1/2*(b*x^3 + 2*a)*sqrt(-
a)/(sqrt(c*x^6 + b*x^3 + a)*a)) - 2*sqrt(c*x^6 + b*x^3 + a)*(3*b*
x^3 - 2*a)*sqrt(-a))/(sqrt(-a)*a^2*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

3.227 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=145

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(9*a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6]
)/(36*a^2*x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*
a^3*x^3) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(48*a^(7/2))
```

Rubi [A] time = 0.394019, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(9*a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6]
)/(36*a^2*x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*
a^3*x^3) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(48*a^(7/2))
```

Rubi in Sympy [A] time = 44.584, size = 133, normalized size = 0.92

$$-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\left(-16ac+15b^2\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b\left(-12ac+5b^2\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)/(9*a*x**9) + 5*b*sqrt(a + b*x**3 + c*x
**6)/(36*a**2*x**6) - (-16*a*c + 15*b**2)*sqrt(a + b*x**3 + c*x**
6)/(72*a**3*x**3) + b*(-12*a*c + 5*b**2)*atanh((2*a + b*x**3)/(2*
sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(48*a**(7/2))
```

Mathematica [A] time = 0.488732, size = 117, normalized size = 0.81

$$\frac{b\left(12ac-5b^2\right)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{48a^{7/2}}-\frac{\sqrt{a+bx^3+cx^6}\left(8a^2-2a\left(5bx^3+8cx^6\right)+15b^2x^6\right)}{72a^3x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 15*b^2*x^6 - 2*a*(5*b*x^3 + 8*
c*x^6)))/(72*a^3*x^9) + (b*(-5*b^2 + 12*a*c)*(Log[x^3] - Log[2*a
+ b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(48*a^(7/2))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291736, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,b^3-12\,abc\right)x^9\log\left(\frac{4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right)+4\left(\left(15\,b^2-16\,ac\right)x^6-10\,abx^3+8\,a^2\right)\sqrt{cx^6+\cdots}}{288\,a^{\frac{7}{2}}x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*x^9*log((4*sqrt(c*x^6 + b*x^3 + a)*
(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(
a))/x^6) + 4*((15*b^2 - 16*a*c)*x^6 - 10*a*b*x^3 + 8*a^2)*sqrt(c*
x^6 + b*x^3 + a)*sqrt(a))/(a^(7/2)*x^9), 1/144*(3*(5*b^3 - 12*a*b
*c)*x^9*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a
)*a)) - 2*((15*b^2 - 16*a*c)*x^6 - 10*a*b*x^3 + 8*a^2)*sqrt(c*x^6
 + b*x^3 + a)*sqrt(-a))/(sqrt(-a)*a^3*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

3.228 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=192

$$\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}-\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}$$

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(12*a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^
6])/(72*a^2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(2
88*a^3*x^6) + (5*b*(21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(57
6*a^4*x^3) - ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a +
b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(9/2))
```

Rubi [A] time = 0.56324, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}-\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(12*a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^
6])/(72*a^2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(2
88*a^3*x^6) + (5*b*(21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(57
6*a^4*x^3) - ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a +
b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(9/2))
```

Rubi in Sympy [A] time = 63.0642, size = 180, normalized size = 0.94

$$-\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{\left(-36ac+35b^2\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{5b\left(-44ac+21b^2\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)/(12*a*x**12) + 7*b*sqrt(a + b*x**3 + c
*x**6)/(72*a**2*x**9) - (-36*a*c + 35*b**2)*sqrt(a + b*x**3 + c*x
**6)/(288*a**3*x**6) + 5*b*(-44*a*c + 21*b**2)*sqrt(a + b*x**3 +
c*x**6)/(576*a**4*x**3) - (48*a**2*c**2 - 120*a*b**2*c + 35*b**4)
*atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(384
*a**(9/2))
```

Mathematica [A] time = 0.241286, size = 146, normalized size = 0.76

$$\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{384a^{9/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-48a^3+8a^2\left(7bx^3+9cx^6\right)-10abx^6\left(7b+22cx^3\right)+105b^3x^9\right)}{576a^4x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 105*b^3*x^9 - 10*a*b*x^6*(7*b
 + 22*c*x^3) + 8*a^2*(7*b*x^3 + 9*c*x^6)))/(576*a^4*x^12) + ((35*
b^4 - 120*a*b^2*c + 48*a^2*c^2)*(Log[x^3] - Log[2*a + b*x^3 + 2*S
qrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(384*a^(9/2))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299178, size = 1, normalized size = 0.01

$$\left[\frac{3\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)x^{12}\log\left(\frac{4\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^6+8\,abx^3+8\,a^2\right)\sqrt{a}}{x^6}\right)+4\left(5\left(21\,b^3-44\,abc\right)x^9-2\left(35\,a\right.\right.}{2304\,a^{\frac{9}{2}}x^{12}}\right.$$

$$-\frac{3\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)x^{12}\arctan\left(\frac{\left(bx^3+2\,a\right)\sqrt{-a}}{2\sqrt{cx^6+bx^3+a}a}\right)-2\left(5\left(21\,b^3-44\,abc\right)x^9-2\left(35\,ab^2-36\,a^2c\right)x^6+56\,a^2bx^3-4\right.}{1152\sqrt{-a}a^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*x^12*log((4*sqrt(c
*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*x^6 + 8*a*b*
x^3 + 8*a^2)*sqrt(a))/x^6) + 4*(5*(21*b^3 - 44*a*b*c)*x^9 - 2*(35
*a*b^2 - 36*a^2*c)*x^6 + 56*a^2*b*x^3 - 48*a^3)*sqrt(c*x^6 + b*x^
3 + a)*sqrt(a))/(a^(9/2)*x^12), -1/1152*(3*(35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*x^12*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6
+ b*x^3 + a)*a)) - 2*(5*(21*b^3 - 44*a*b*c)*x^9 - 2*(35*a*b^2 - 3
6*a^2*c)*x^6 + 56*a^2*b*x^3 - 48*a^3)*sqrt(c*x^6 + b*x^3 + a)*sqr
t(-a))/(sqrt(-a)*a^4*x^12)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

3.229 $\int \frac{x^3}{\sqrt{a+bx^3+cx^6}} dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(4*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.475987, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(4*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 39.9591, size = 122, normalized size = 0.87

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,\mathrm{appellf_1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{4a\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] x**4*sqrt(a + b*x**3 + c*x**6)*appellf1(4/3, 1/2, 1/2, 7/3, -2*c*
x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2
```

```
)))/(4*a*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.276455, size = 380, normalized size = 2.71

$$\frac{7a^2x^4\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^3+cx^6\right)^{3/2}\left(28aF_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (7*a^2*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2
 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^3 + c*x^6)^(3/2)*(28*
a*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*
a*c])*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^3\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\sqrt{cx^6+bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

3.230 $\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.355891, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 27.5719, size = 122, normalized size = 0.87

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,\operatorname{appellf_1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2a\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] x**2*sqrt(a + b*x**3 + c*x**6)*appellf1(2/3, 1/2, 1/2, 5/3, -2*c*
x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2
```

```
)))/(2*a*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.277715, size = 380, normalized size = 2.71

$$\frac{10a^2x^2\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^3+cx^6\right)^{3/2}\left(20aF_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (10*a^2*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^
2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^3 + c*x^6)^(3/2)*(20
*a*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4
*a*c])*AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{cx^6+bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^6+bx^3+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

3.231 $\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sq
rt[a + b*x^3 + c*x^6]
```

Rubi [A] time = 0.202704, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sq
rt[a + b*x^3 + c*x^6]
```

Rubi in Sympy [A] time = 40.2347, size = 119, normalized size = 0.88

$$\frac{x\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{a\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] x*sqrt(a + b*x**3 + c*x**6)*appellf1(1/3, 1/2, 1/2, 4/3, -2*c*x**
3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)))
```

```
/(a*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**3/(b
 + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.265816, size = 378, normalized size = 2.8

$$\frac{16a^2x\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^3+cx^6\right)^{3/2}\left(16aF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (16*a^2*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2
- 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^3 + c*x^6)^(3/2)*(16*a
*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a
*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c]
)*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")

[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^6 + bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")

[Out] integral(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(1/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")

[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

3.232 $\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
x*Sqrt[a + b*x^3 + c*x^6]))
```

Rubi [A] time = 0.432274, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
x*Sqrt[a + b*x^3 + c*x^6]))
```

Rubi in Sympy [A] time = 32.749, size = 122, normalized size = 0.88

$$-\frac{\sqrt{a+bx^3+cx^6}\,\operatorname{appellf}_1\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{ax\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)*appellf1(-1/3, 1/2, 1/2, 2/3, -2*c*x**
3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)))
```

```
/(a*x*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**3/
(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.16312, size = 705, normalized size = 5.11

$$\frac{25abx^3\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{4c\left(20aF_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{5}{3};\frac{1}{2},\frac{3}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{3};\frac{3}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-5*(a + b*x^3 + c*x^6)^2 + (25*a*b*x^3*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2
, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])])/(4*c*(20*a*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3
*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
 + (16*a*x^6*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(32*a*Appell
F1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Ap
pellF1[8/3, 1/2, 3/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
llF1[8/3, 3/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))/(5*a*x*(a + b*x^3 + c*x^6)^(3
/2))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^6 + bx^3 + ax^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**3 + c*x**6)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

3.233 $\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2
*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.431053, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2
*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 32.7759, size = 126, normalized size = 0.9

$$-\frac{\sqrt{a+bx^3+cx^6}\,\operatorname{appellf_1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2ax^2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)*appellf1(-2/3, 1/2, 1/2, 1/3, -2*c*x**
3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)))
```

```
/(2*a*x**2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*
x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.15922, size = 705, normalized size = 5.04

$$-\frac{2abx^3\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{c\left(16aF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-(a + b*x^3 + c*x^6)^2 - (2*a*b*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2, 4
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])])/(c*(16*a*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*
((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (7
*a*x^6*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*(28*a*AppellF1[
4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Appel
lF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF
1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(2*a*x^2*(a + b*x^3 + c*x^6)^(3
/2))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{cx^6 + bx^3 + a}x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**3 + c*x**6)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

3.234 $\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}-\frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) -
 (2*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*
b^2 - 52*a*c) - 2*c*(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6]
)/(12*c^3*(b^2 - 4*a*c)) + ((5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(7/2))
```

Rubi [A] time = 0.546425, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}-\frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) -
 (2*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*
b^2 - 52*a*c) - 2*c*(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6]
)/(12*c^3*(b^2 - 4*a*c)) + ((5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(7/2))
```

Rubi in Sympy [A] time = 47.4993, size = 182, normalized size = 0.93

$$-\frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(-4ac+b^2\right)}+\frac{2x^9\left(2a+bx^3\right)}{3\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}-\frac{\left(b\left(-39ac+\frac{45b^2}{4}\right)-\frac{3cx^3\left(-12ac+5b^2\right)}{2}\right)\sqrt{a+bx^3+cx^6}}{9c^3\left(-4ac+b^2\right)}+\frac{\left(-4ac+5b^2\right)\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -2*b*x**6*sqrt(a + b*x**3 + c*x**6)/(3*c*(-4*a*c + b**2)) + 2*x**
9*(2*a + b*x**3)/(3*(-4*a*c + b**2)*sqrt(a + b*x**3 + c*x**6)) -
(b*(-39*a*c + 45*b**2/4) - 3*c*x**3*(-12*a*c + 5*b**2)/2)*sqrt(a
+ b*x**3 + c*x**6)/(9*c**3*(-4*a*c + b**2)) + (-4*a*c + 5*b**2)*a
tanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(8*c**
(7/2))
```

Mathematica [A] time = 0.268702, size = 147, normalized size = 0.75

$$\frac{\sqrt{a+bx^3+cx^6}\left(-\frac{8\left(a^2c\left(2cx^3-3b\right)+ab^2\left(b-4cx^3\right)+b^4x^3\right)}{\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}-7b+2cx^3\right)}{12c^3}+\frac{\left(5b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-7*b + 2*c*x^3 - (8*(b^4*x^3 + a*b^2*(b
 - 4*c*x^3) + a^2*c*(-3*b + 2*c*x^3)))/((b^2 - 4*a*c)*(a + b*x^3
+ c*x^6))))/(12*c^3) + ((5*b^2 - 4*a*c)*Log[b + 2*c*x^3 + 2*Sqrt[
c]*Sqrt[a + b*x^3 + c*x^6]])/(8*c^(7/2))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int x^{14}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.351939, size = 1, normalized size = 0.01

$$\left[\frac{4\left(2\left(b^2c^2-4ac^3\right)x^9-5\left(b^3c-4abc^2\right)x^6-15ab^3+52a^2bc-\left(15b^4-62ab^2c+24a^2c^2\right)x^3\right)\sqrt{cx^6+bx^3+a}\sqrt{c}-3\left(\left(5b\right.\right.}{48\left(\left(b^2c^4-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(4*(2*(b^2*c^2 - 4*a*c^3)*x^9 - 5*(b^3*c - 4*a*b*c^2)*x^6 -
 15*a*b^3 + 52*a^2*b*c - (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)*x^3)*
sqrt(c*x^6 + b*x^3 + a)*sqrt(c) - 3*((5*b^4*c - 24*a*b^2*c^2 + 16
*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24
*a*b^3*c + 16*a^2*b*c^2)*x^3)*log(4*sqrt(c*x^6 + b*x^3 + a)*(2*c^
2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/((
(b^2*c^4 - 4*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*
b*c^4)*x^3)*sqrt(c)), 1/24*(2*(2*(b^2*c^2 - 4*a*c^3)*x^9 - 5*(b^3
*c - 4*a*b*c^2)*x^6 - 15*a*b^3 + 52*a^2*b*c - (15*b^4 - 62*a*b^2*
c + 24*a^2*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-c) + 3*((5*b^4
*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2*c + 16
*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*arctan(1/2*(2
*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(((b^2*c^4 - 4
*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)*
sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.235 $\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)}+\frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) +
 ((3*b^2 - 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^
2 - 4*a*c)) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(2*c^(5/2))
```

Rubi [A] time = 0.256959, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)}+\frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) +
 ((3*b^2 - 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^
2 - 4*a*c)) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(2*c^(5/2))
```

Rubi in Sympy [A] time = 29.1089, size = 126, normalized size = 0.92

$$-\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{\frac{5}{2}}}+\frac{2x^6(2a+bx^3)}{3(-4ac+b^2)\sqrt{a+bx^3+cx^6}}+\frac{\sqrt{a+bx^3+cx^6}\,(-8ac+3b^2-2bcx^3)}{3c^2(-4ac+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -b*atanh((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(2
*c**(5/2)) + 2*x**6*(2*a + b*x**3)/(3*(-4*a*c + b**2)*sqrt(a + b*
```

```
x**3 + c*x**6)) + sqrt(a + b*x**3 + c*x**6)*(-8*a*c + 3*b**2 - 2*
b*c*x**3)/(3*c**2*(-4*a*c + b**2))
```

Mathematica [A] time = 0.19916, size = 127, normalized size = 0.93

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}\left(\frac{2\left(2a^2c-ab^2+3abcx^3-b^3x^3\right)}{c^2\left(4ac-b^2\right)\left(a+bx^3+cx^6\right)}+\frac{1}{c^2}\right)-\frac{b\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(c^(-2) + (2*(-(a*b^2) + 2*a^2*c - b^3*x
^3 + 3*a*b*c*x^3))/(c^2*(-b^2 + 4*a*c)*(a + b*x^3 + c*x^6))))/3 -
 (b*Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(2*c^(5
/2))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^{11}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302761, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(b^2c-4\,ac^2\right)x^6+\left(3\,b^3-10\,abc\right)x^3+3\,ab^2-8\,a^2c\right)\sqrt{cx^6+bx^3+a}\sqrt{c}+3\left(\left(b^3c-4\,abc^2\right)x^6+ab^3-4\,a^2bc+\left(b^4-4\,ab\right.\right.}{12\left(\left(b^2c^3-4\,ac^4\right)x^6+ab^2c^2-4\,a^2c^3+\left(b^3c^2-4\,abc\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*((b^2*c - 4*a*c^2)*x^6 + (3*b^3 - 10*a*b*c)*x^3 + 3*a*b^
2 - 8*a^2*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(c) + 3*((b^3*c - 4*a*b*
c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*log(4*sqrt(
c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2*x^6 + 8*b*c*x^3 + b
^2 + 4*a*c)*sqrt(c)))/(((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a
^2*c^3 + (b^3*c^2 - 4*a*b*c^3)*x^3)*sqrt(c)), 1/6*(2*((b^2*c - 4*
a*c^2)*x^6 + (3*b^3 - 10*a*b*c)*x^3 + 3*a*b^2 - 8*a^2*c)*sqrt(c*x
^6 + b*x^3 + a)*sqrt(-c) - 3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4
*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*arctan(1/2*(2*c*x^3 + b)*sqrt(-
c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(((b^2*c^3 - 4*a*c^4)*x^6 + a*b^
2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c^3)*x^3)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.236 $\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

```
[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) -
 (2*b*Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b +
 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Rubi [A] time = 0.196579, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) -
 (2*b*Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b +
 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Rubi in Sympy [A] time = 27.6022, size = 107, normalized size = 0.89

$$-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(-4ac+b^2\right)}+\frac{2x^3\left(2a+bx^3\right)}{3\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}+\frac{\operatorname{atanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -2*b*sqrt(a + b*x**3 + c*x**6)/(3*c*(-4*a*c + b**2)) + 2*x**3*(2*
a + b*x**3)/(3*(-4*a*c + b**2)*sqrt(a + b*x**3 + c*x**6)) + atanh
((b + 2*c*x**3)/(2*sqrt(c)*sqrt(a + b*x**3 + c*x**6)))/(3*c**(3/2
))
```

Mathematica [A] time = 0.179836, size = 95, normalized size = 0.79

$$\frac{2\left(ab-2acx^3+b^2x^3\right)}{3c\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}+\frac{\log\left(2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(a*b + b^2*x^3 - 2*a*c*x^3))/(3*c*(-b^2 + 4*a*c)*Sqrt[a + b*x^
3 + c*x^6]) + Log[b + 2*c*x^3 + 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
]/(3*c^(3/2))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^8\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29223, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{cx^6+bx^3+a}\left(\left(b^2-2\,ac\right)x^3+ab\right)\sqrt{c}-\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)\log\left(-4\sqrt{cx^6+bx^3+a}\left(2\,c^2\right.\right.}{6\left((b^2c^2-4\,ac^3)x^6+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^3\right)\sqrt{c}}\right.$$

$$\left.-\frac{2\sqrt{cx^6+bx^3+a}\left(\left(b^2-2\,ac\right)x^3+ab\right)\sqrt{-c}-\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)\arctan\left(\frac{(2\,cx^3+b)\sqrt{-c}}{2\sqrt{cx^6+bx^3+a}c}\right)}{3\left((b^2c^2-4\,ac^3)x^6+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^3\right)\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*x^6 + b*x^3 + a)*((b^2 - 2*a*c)*x^3 + a*b)*sqrt(c
) - ((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*
c)*log(-4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c) - (8*c^2*x^6
+ 8*b*c*x^3 + b^2 + 4*a*c)*sqrt(c)))/(((b^2*c^2 - 4*a*c^3)*x^6 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^3)*sqrt(c)), -1/3*(2*
sqrt(c*x^6 + b*x^3 + a)*((b^2 - 2*a*c)*x^3 + a*b)*sqrt(-c) - ((b^
2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*arcta
n(1/2*(2*c*x^3 + b)*sqrt(-c)/(sqrt(c*x^6 + b*x^3 + a)*c)))/(((b^2
*c^2 - 4*a*c^3)*x^6 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x
^3)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")

[Out] integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.237 $\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2\,(2a+bx^3)}{3\,(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.0716919, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{2\,(2a+bx^3)}{3\,(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 10.3204, size = 34, normalized size = 0.87

$$\frac{4a+2bx^3}{3\,(-4ac+b^2)\sqrt{a+bx^3+cx^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)

[Out] (4*a + 2*b*x**3)/(3*(-4*a*c + b**2)*sqrt(a + b*x**3 + c*x**6))
```

Mathematica [A] time = 0.0374483, size = 41, normalized size = 1.05

$$-\frac{2\,(2a+bx^3)}{3\,(4ac-b^2)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (-2*(2*a + b*x^3))/(3*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [A] time = 0.009, size = 38, normalized size = 1.

$$-\frac{2\,bx^3+4\,a}{12\,ac-3\,b^2}\frac{1}{\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^6+b*x^3+a)^(3/2),x)

[Out] -2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27668, size = 92, normalized size = 2.36

$$\frac{2\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)}{3\left((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")

[Out] 2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6
+ (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [A] time = 0.273861, size = 61, normalized size = 1.56

$$\frac{2\left(\frac{bx^3}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3
+ a)
```

3.238 $\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}} \, dx$

Optimal. Leaf size=38

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.0593965, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 7.0245, size = 36, normalized size = 0.95

$$-\frac{2b+4cx^3}{3\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -(2*b + 4*c*x**3)/(3*(-4*a*c + b**2)*sqrt(a + b*x**3 + c*x**6))
```

Mathematica [A] time = 0.0269077, size = 38, normalized size = 1.

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [A] time = 0.009, size = 37, normalized size = 1.

$$\frac{4\,c x^3+2\,b}{12\,ac-3\,b^2}\frac{1}{\sqrt{c x^6+b x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277464, size = 90, normalized size = 2.37

$$-\frac{2\,\sqrt{c x^6+b x^3+a}\left(2\,c x^3+b\right)}{3\left((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6
 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [A] time = 0.300747, size = 61, normalized size = 1.61

$$-\frac{2\left(\frac{2\,cx^3}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3
 + a)
```

3.239 $\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c
*x^6]) - ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]
)]/(3*a^(3/2))
```

Rubi [A] time = 0.171916, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c
*x^6]) - ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]
)]/(3*a^(3/2))
```

Rubi in Sympy [A] time = 22.4225, size = 83, normalized size = 0.9

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}-\frac{\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x**3)/(3*a*(-4*a*c + b**2)*sqrt(a + b*x**3
 + c*x**6)) - atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c
*x**6)))/(3*a**(3/2))
```

Mathematica [A] time = 0.347083, size = 96, normalized size = 1.04

$$\frac{\frac{2\sqrt{a}(-2ac+b^2+bcx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)+\log\left(x^3\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(b^2 - 2*a*c + b*c*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^
3 + c*x^6]) + Log[x^3] - Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x
^3 + c*x^6]])/(3*a^(3/2))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293785, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^6+bx^3+a}\left(bcx^3+b^2-2\,ac\right)\sqrt{a}+\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)\log\left(\frac{4\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)-\left((b^2+\right.}{x^6}\right.}{6\left(\left(ab^2c-4\,a^2c^2\right)x^6+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^3\right)\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [1/6*(4*sqrt(c*x^6 + b*x^3 + a)*(b*c*x^3 + b^2 - 2*a*c)*sqrt(a) +
 ((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*
log((4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)
*x^6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6))/(((a*b^2*c - 4*a^2*c^2)*
x^6 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(a)), 1/3*
(2*sqrt(c*x^6 + b*x^3 + a)*(b*c*x^3 + b^2 - 2*a*c)*sqrt(-a) - ((b
^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*arct
an(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)))/(((a*
b^2*c - 4*a^2*c^2)*x^6 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
x^3)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

3.240 $\int \frac{1}{x^4(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3
 + c*x^6]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^
2 - 4*a*c)*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*
x^3 + c*x^6])])/(2*a^(5/2))
```

Rubi [A] time = 0.306353, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3
 + c*x^6]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^
2 - 4*a*c)*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*
x^3 + c*x^6])])/(2*a^(5/2))
```

Rubi in Sympy [A] time = 37.5344, size = 131, normalized size = 0.92

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}-\frac{\left(-8ac+3b^2\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(-4ac+b^2\right)}+\frac{b\,\mathrm{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x**3)/(3*a*x**3*(-4*a*c + b**2)*sqrt(a + b
*x**3 + c*x**6)) - (-8*a*c + 3*b**2)*sqrt(a + b*x**3 + c*x**6)/(3
*a**2*x**3*(-4*a*c + b**2)) + b*atanh((2*a + b*x**3)/(2*sqrt(a)*s
```

```
qrt(a + b*x**3 + c*x**6)))/(2*a**(5/2))
```

Mathematica [A] time = 0.220457, size = 132, normalized size = 0.93

$$\frac{b\left(\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)-\log\left(x^3\right)\right)}{2a^{5/2}}+\frac{-4a^2c+a\left(b^2-10bcx^3-8c^2x^6\right)+3b^2x^3\left(b+cx^3\right)}{3a^2x^3\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2*x
^6))/(3*a^2*(-b^2 + 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6]) + (b*(-Lo
g[x^3] + Log[2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(
2*a^(5/2))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302067, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(\left(3\,b^2c-8\,ac^2\right)x^6+\left(3\,b^3-10\,abc\right)x^3+ab^2-4\,a^2c\right)\sqrt{cx^6+bx^3+a}\sqrt{a}-3\left(\left(b^3c-4\,abc^2\right)x^9+\left(b^4-4\,ab^2c\right)x^6+\left(ab^3\right.\right.}{12\left((a^2b^2c-4\,a^3c^2)x^9+(a^2b^3-4\,a^3bc)x^6+(a^3b^2-4\,a^4c\right.}\right.$$

$$-\frac{2\left(\left(3\,b^2c-8\,ac^2\right)x^6+\left(3\,b^3-10\,abc\right)x^3+ab^2-4\,a^2c\right)\sqrt{cx^6+bx^3+a}\sqrt{-a}-3\left(\left(b^3c-4\,abc^2\right)x^9+\left(b^4-4\,ab^2c\right)x^6+\left(ab\right.\right.}{6\left((a^2b^2c-4\,a^3c^2)x^9+(a^2b^3-4\,a^3bc)x^6+(a^3b^2-4\,a^4c)x^3\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*((3*b^2*c - 8*a*c^2)*x^6 + (3*b^3 - 10*a*b*c)*x^3 + a*b
^2 - 4*a^2*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(a) - 3*((b^3*c - 4*a*b
*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*b*c)*x^3)*log(
-(4*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) + ((b^2 + 4*a*c)*x^
6 + 8*a*b*x^3 + 8*a^2)*sqrt(a))/x^6))/(((a^2*b^2*c - 4*a^3*c^2)*x
^9 + (a^2*b^3 - 4*a^3*b*c)*x^6 + (a^3*b^2 - 4*a^4*c)*x^3)*sqrt(a)
), -1/6*(2*((3*b^2*c - 8*a*c^2)*x^6 + (3*b^3 - 10*a*b*c)*x^3 + a*
b^2 - 4*a^2*c)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a) - 3*((b^3*c - 4*a
*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*b*c)*x^3)*ar
ctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)))/(((
a^2*b^2*c - 4*a^3*c^2)*x^9 + (a^2*b^3 - 4*a^3*b*c)*x^6 + (a^3*b^2
 - 4*a^4*c)*x^3)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

3.241 $\int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6(b^2-4ac)}+\frac{2(-2ac+b^2+bcx^3)}{3ax^6(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3
 + c*x^6]) - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b
^2 - 4*a*c)*x^6) + (b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/
(12*a^3*(b^2 - 4*a*c)*x^3) - ((5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^
3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(7/2))
```

Rubi [A] time = 0.495767, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6(b^2-4ac)}+\frac{2(-2ac+b^2+bcx^3)}{3ax^6(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3
 + c*x^6]) - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b
^2 - 4*a*c)*x^6) + (b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/
(12*a^3*(b^2 - 4*a*c)*x^3) - ((5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^
3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(7/2))
```

Rubi in Sympy [A] time = 57.0319, size = 184, normalized size = 0.93

$$\frac{2(-2ac+b^2+bcx^3)}{3ax^6(-4ac+b^2)\sqrt{a+bx^3+cx^6}}-\frac{(-12ac+5b^2)\sqrt{a+bx^3+cx^6}}{6a^2x^6(-4ac+b^2)}+\frac{b(-52ac+15b^2)\sqrt{a+bx^3+cx^6}}{12a^3x^3(-4ac+b^2)}-\frac{(-4ac+5b^2)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x**3)/(3*a*x**6*(-4*a*c + b**2)*sqrt(a + b
*x**3 + c*x**6)) - (-12*a*c + 5*b**2)*sqrt(a + b*x**3 + c*x**6)/(
6*a**2*x**6*(-4*a*c + b**2)) + b*(-52*a*c + 15*b**2)*sqrt(a + b*x
**3 + c*x**6)/(12*a**3*x**3*(-4*a*c + b**2)) - (-4*a*c + 5*b**2)*
atanh((2*a + b*x**3)/(2*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(8*a*
*(7/2))
```

Mathematica [A] time = 0.350352, size = 159, normalized size = 0.8

$$\frac{(5b^2-4ac)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{8a^{7/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(\frac{8(2a^2c^2-4ab^2c-3abc^2x^3+b^4+b^3cx^3)}{(b^2-4ac)(a+bx^3+cx^6)}-\frac{2a}{x^6}+\frac{7b}{x^3}\right)}{12a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*((-2*a)/x^6 + (7*b)/x^3 + (8*(b^4 - 4*a*
b^2*c + 2*a^2*c^2 + b^3*c*x^3 - 3*a*b*c^2*x^3))/((b^2 - 4*a*c)*(a
 + b*x^3 + c*x^6))))/(12*a^3) + ((5*b^2 - 4*a*c)*(Log[x^3] - Log[
2*a + b*x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(8*a^(7/2))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{x^7}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.33167, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(15\,b^3c-52\,abc^2\right)x^9+\left(15\,b^4-62\,ab^2c+24\,a^2c^2\right)x^6-2\,a^2b^2+8\,a^3c+5\left(ab^3-4\,a^2bc\right)x^3\right)\sqrt{cx^6+bx^3+a}\sqrt{a}-3\left(\left(5\,b\right.\right.}{48\left(\left(a^3b^2c-4\,a^4c^2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7),x, algorithm="fricas")
```

```
[Out] [1/48*(4*((15*b^3*c - 52*a*b*c^2)*x^9 + (15*b^4 - 62*a*b^2*c + 24
*a^2*c^2)*x^6 - 2*a^2*b^2 + 8*a^3*c + 5*(a*b^3 - 4*a^2*b*c)*x^3)*
sqrt(c*x^6 + b*x^3 + a)*sqrt(a) - 3*((5*b^4*c - 24*a*b^2*c^2 + 16
*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (5*a*b
^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*log(-(4*sqrt(c*x^6 + b*x^3 +
 a)*(a*b*x^3 + 2*a^2) + ((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 8*a^2)*s
qrt(a))/x^6))/(((a^3*b^2*c - 4*a^4*c^2)*x^12 + (a^3*b^3 - 4*a^4*b
*c)*x^9 + (a^4*b^2 - 4*a^5*c)*x^6)*sqrt(a)), 1/24*(2*((15*b^3*c -
 52*a*b*c^2)*x^9 + (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)*x^6 - 2*a^2
*b^2 + 8*a^3*c + 5*(a*b^3 - 4*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 +
a)*sqrt(-a) - 3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*
b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c +
16*a^3*c^2)*x^6)*arctan(1/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 +
b*x^3 + a)*a)))/(((a^3*b^2*c - 4*a^4*c^2)*x^12 + (a^3*b^3 - 4*a^4
*b*c)*x^9 + (a^4*b^2 - 4*a^5*c)*x^6)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

3.242 $\int \frac{1}{x^{10}(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3
 + c*x^6]) - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b
^2 - 4*a*c)*x^9) + (b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])
/(36*a^3*(b^2 - 4*a*c)*x^6) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c
^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^2 - 4*a*c)*x^3) + (5*b*(7
*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/(48*a^(9/2))
```

Rubi [A] time = 0.683328, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3
 + c*x^6]) - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b
^2 - 4*a*c)*x^9) + (b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])
/(36*a^3*(b^2 - 4*a*c)*x^6) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c
^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^2 - 4*a*c)*x^3) + (5*b*(7
*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/(48*a^(9/2))
```

Rubi in Sympy [A] time = 94.4524, size = 241, normalized size = 0.94

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^9\left(-4ac+b^2\right)\sqrt{a+bx^3+cx^6}}-\frac{\left(-16ac+7b^2\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(-4ac+b^2\right)}+\frac{b\left(-116ac+35b^2\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(-4ac+b^2\right)}-\frac{\sqrt{a+bx^3+cx^6}\left(256a^2c^2-460ab^2c+105b^4\right)}{72a^4x^3\left(-4ac+b^2\right)}+\frac{5b\left(-12ac+7b^2\right)\operatorname{atanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x**3)/(3*a*x**9*(-4*a*c + b**2)*sqrt(a + b
*x**3 + c*x**6)) - (-16*a*c + 7*b**2)*sqrt(a + b*x**3 + c*x**6)/(
9*a**2*x**9*(-4*a*c + b**2)) + b*(-116*a*c + 35*b**2)*sqrt(a + b*
x**3 + c*x**6)/(36*a**3*x**6*(-4*a*c + b**2)) - sqrt(a + b*x**3 +
 c*x**6)*(256*a**2*c**2 - 460*a*b**2*c + 105*b**4)/(72*a**4*x**3*
(-4*a*c + b**2)) + 5*b*(-12*a*c + 7*b**2)*atanh((2*a + b*x**3)/(2
*sqrt(a)*sqrt(a + b*x**3 + c*x**6)))/(48*a**(9/2))
```

Mathematica [A] time = 0.522135, size = 191, normalized size = 0.75

$$\frac{5b\left(12ac-7b^2\right)\left(\log\left(x^3\right)-\log\left(2\sqrt{a}\sqrt{a+bx^3+cx^6}+2a+bx^3\right)\right)}{48a^{9/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-\frac{48\left(5a^2bc^2+2a^2c^3x^3-5ab^3c-4ab^2c^2x^3+b^5+b^4cx^3\right)}{\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}-\frac{8a^2}{x^9}+\frac{40ac-57b^2}{x^3}+\frac{22ab}{x^6}\right)}{72a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*((-8*a^2)/x^9 + (22*a*b)/x^6 + (-57*b^2
+ 40*a*c)/x^3 - (48*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + b^4*c*x^3 -
4*a*b^2*c^2*x^3 + 2*a^2*c^3*x^3))/((b^2 - 4*a*c)*(a + b*x^3 + c*x
^6))))/(72*a^4) + (5*b*(-7*b^2 + 12*a*c)*(Log[x^3] - Log[2*a + b*
x^3 + 2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]))/(48*a^(9/2))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}}\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.388079, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10),x, algorithm="fricas")
```

```
[Out] [-1/288*(4*((105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*x^12 + (105
*b^5 - 530*a*b^3*c + 488*a^2*b*c^2)*x^9 + (35*a*b^4 - 172*a^2*b^2
*c + 128*a^3*c^2)*x^6 + 8*a^3*b^2 - 32*a^4*c - 14*(a^2*b^3 - 4*a^
3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(a) + 15*((7*b^5*c - 40*a
*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*
c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*log((4*s
qrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2) - ((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 + 8*a^2)*sqrt(a))/x^6))/(((a^4*b^2*c - 4*a^5*c^2)*x^15 +
 (a^4*b^3 - 4*a^5*b*c)*x^12 + (a^5*b^2 - 4*a^6*c)*x^9)*sqrt(a)),
-1/144*(2*((105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*x^12 + (105*
b^5 - 530*a*b^3*c + 488*a^2*b*c^2)*x^9 + (35*a*b^4 - 172*a^2*b^2*
c + 128*a^3*c^2)*x^6 + 8*a^3*b^2 - 32*a^4*c - 14*(a^2*b^3 - 4*a^3
*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a)*sqrt(-a) - 15*((7*b^5*c - 40*a
*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*
c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*arctan(1
/2*(b*x^3 + 2*a)*sqrt(-a)/(sqrt(c*x^6 + b*x^3 + a)*a)))/(((a^4*b^
2*c - 4*a^5*c^2)*x^15 + (a^4*b^3 - 4*a^5*b*c)*x^12 + (a^5*b^2 - 4
*a^6*c)*x^9)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{10} \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^{10}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)
```

3.243 $\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}} dx$

Optimal. Leaf size=143

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(4*a*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.498967, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(4*a*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 44.7924, size = 124, normalized size = 0.87

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{4}{3},\frac{3}{2},\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{4a^2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] x**4*sqrt(a + b*x**3 + c*x**6)*appellf1(4/3, 3/2, 3/2, 7/3, -2*c*
x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2
```

```
)))/(4*a**2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c
*x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.71025, size = 711, normalized size = 4.97

$$2x\left(\frac{7ax^3\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{56aF_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-6x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{3};\frac{1}{2},\frac{3}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{3};\frac{3}{2},\frac{1}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac-}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x*(-((b + 2*c*x^3)*(a + b*x^3 + c*x^6)) + (4*a*b*(b - Sqrt[b^2
 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1
/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/
(-b + Sqrt[b^2 - 4*a*c])])/(c*(16*a*AppellF1[1/3, 1/2, 1/2, 4/3,
(-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*
a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4
*a*c])]))) + (7*a*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(56
*a*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 6*x^3*((b + Sqrt[b^2 - 4
*a*c])*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a
*c])*AppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))))/(3*(b^2 - 4*a*c)*(a
 + b*x^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^3\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.244 $\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(2*a*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.351741, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/
(2*a*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 28.2386, size = 124, normalized size = 0.87

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] x**2*sqrt(a + b*x**3 + c*x**6)*appellf1(2/3, 3/2, 3/2, 5/3, -2*c*
x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2
```

```
)))/(2*a**2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c
*x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.46759, size = 1054, normalized size = 7.37

$$x^2\left(\frac{64ab\left(2cx^3+b-\sqrt{b^2-4ac}\right)\left(2cx^3+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)x^3}{32aF_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{8}{3};\frac{1}{2},\frac{3}{2};\frac{11}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{8}{3};\frac{3}{2},\frac{1}{2};\frac{11}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac-}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(-20*(b^2 - 2*a*c + b*c*x^3)*(a + b*x^3 + c*x^6) + (100*a^2*
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*AppellF1[2/3, 1/2,
1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqr
t[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1
/2, 3/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 3/2,
 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sq
rt[b^2 - 4*a*c])])) + (25*a*b^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2, 5/3, (
-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])])/(c*(20*a*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b +
 Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sq
rt[b^2 - 4*a*c])*AppellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (64*a*b
*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(32*a*AppellF1[5/3,
1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
+ Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[8
/3, 1/2, 3/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[8/3
, 3/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(
-b + Sqrt[b^2 - 4*a*c])]))))/(30*a*(-b^2 + 4*a*c)*(a + b*x^3 + c*
x^6)^(3/2))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x\left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.245 $\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a
*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.209417, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a
*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 36.7774, size = 121, normalized size = 0.88

$$\frac{x\sqrt{a+bx^3+cx^6}\,\operatorname{appellf}_1\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] x*sqrt(a + b*x**3 + c*x**6)*appellf1(1/3, 3/2, 3/2, 4/3, -2*c*x**
3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)))
```

```
/(a**2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**3
/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.8096, size = 1056, normalized size = 7.65

$$2\left(\frac{7ab\left(2cx^3+b-\sqrt{b^2-4ac}\right)\left(2cx^3+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)x^4}{4\left(28aF_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{3};\frac{1}{2},\frac{3}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{3};\frac{3}{2},\frac{1}{2};\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}\cdots}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (2*(-(x*(b^2 - 2*a*c + b*c*x^3)*(a + b*x^3 + c*x^6)) + (16*a^2*x*
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(16*a*AppellF1[1/3, 1/2,
1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqr
t[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1
/2, 3/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2,
 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sq
rt[b^2 - 4*a*c])])) - (2*a*b^2*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2, 4/3,
(-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*
a*c])])/(c*(16*a*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b
+ Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - S
qrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (7*a*b
*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*(28*a*AppellF1[4/
3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(
-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF
1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[
7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3
)/(-b + Sqrt[b^2 - 4*a*c])])))))/(3*a*(-b^2 + 4*a*c)*(a + b*x^3 +
 c*x^6)^(3/2))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

3.246 $\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
a*x*Sqrt[a + b*x^3 + c*x^6]))
```

Rubi [A] time = 0.425583, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
a*x*Sqrt[a + b*x^3 + c*x^6]))
```

Rubi in Sympy [A] time = 44.2013, size = 124, normalized size = 0.88

$$-\frac{\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(-\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{a^2x\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)*appellf1(-1/3, 3/2, 3/2, 2/3, -2*c*x**
3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)))
```

```
/(a**2*x*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.26451, size = 1599, normalized size = 11.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((10*x^2*(b^3 - 3*a*b*c + b^2*c*x^3 - 2*a*c^2*x^3)*(a + b*x^3 + c
*x^6))/(a^2*(-b^2 + 4*a*c)) - (15*(a + b*x^3 + c*x^6)^2)/(a^2*x)
+ (125*b^3*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*
a*c)*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-20*a*Appe
llF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*
AppellF1[5/3, 1/2, 3/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*App
ellF1[5/3, 3/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) - (300*a*b*c*x^2*(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[
2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c
])*(b + Sqrt[b^2 - 4*a*c])*(-20*a*AppellF1[2/3, 1/2, 1/2, 5/3, (-
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*
c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1/2, 3/2, 8/3
, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 3/2, 1/2, 8/3, (
-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])]))) + (320*b^2*c*x^5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])
/((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*
(-32*a*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2
 - 4*a*c])*AppellF1[8/3, 1/2, 3/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
 4*a*c])*AppellF1[8/3, 3/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) - (1024*a*c^2*x^
5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c)*(-b + Sq
rt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-32*a*AppellF1[5/3, 1/2
, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + S
qrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[8/3,
 1/2, 3/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-
b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[8/3, 3
```

```
/2, 1/2, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
+ Sqrt[b^2 - 4*a*c])]))))/(15*(a + b*x^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left(cx^6 + bx^3 + a\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^8 + bx^5 + ax^2)\sqrt{cx^6 + bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((c*x^8 + b*x^5 + a*x^2)*sqrt(c*x^6 + b*x^3 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3 + c*x**6)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

3.247 $\int \frac{1}{x^3(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2
*a*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.423321, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2
*a*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 44.8635, size = 128, normalized size = 0.9

$$-\frac{\sqrt{a+bx^3+cx^6}\,\mathrm{appellf}_1\left(-\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2a^2x^2\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] -sqrt(a + b*x**3 + c*x**6)*appellf1(-2/3, 3/2, 3/2, 1/3, -2*c*x**
3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)))
```

```
/(2*a**2*x**2*sqrt(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2
*c*x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.52696, size = 1593, normalized size = 11.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((4*x*(b^3 - 3*a*b*c + b^2*c*x^3 - 2*a*c^2*x^3)*(a + b*x^3 + c*x^
6))/(a^2*(-b^2 + 4*a*c)) - (3*(a + b*x^3 + c*x^6)^2)/(a^2*x^2) -
(56*b^3*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c)*
(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-16*a*AppellF1[
1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Appel
lF1[4/3, 1/2, 3/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1
[4/3, 3/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3
)/(-b + Sqrt[b^2 - 4*a*c])]))) + (288*a*b*c*x*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[1/3, 1/
2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])*(b +
 Sqrt[b^2 - 4*a*c])*(-16*a*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] +
3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1/2, 3/2, 7/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])
] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 3/2, 1/2, 7/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))
) + (49*b^2*c*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b^2 -
 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-28*a*A
ppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c
])*AppellF1[7/3, 1/2, 3/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])
*AppellF1[7/3, 3/2, 1/2, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))) - (140*a*c^2*x^4*(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*Appe
llF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c)*(-b + Sqrt[b^2 -
4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-28*a*AppellF1[4/3, 1/2, 1/2, 7/
3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 1/2, 3/2
, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[
b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 3/2, 1/2,
```

```
10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^
2 - 4*a*c])])))))/(6*(a + b*x^3 + c*x^6)^(3/2))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \left(cx^6 + bx^3 + a\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3),x, algorithm="maxima")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^9 + bx^6 + ax^3)\sqrt{cx^6 + bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3),x, algorithm="fricas")

[Out] integral(1/((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral(1/(x**3*(a + b*x**3 + c*x**6)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

3.248 $\int (dx)^m \left(a+bx^3+cx^6\right)^2 dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 +
 m)) + ((b^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)
^(10 + m))/(d^10*(10 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rubi [A] time = 0.128446, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 +
 m)) + ((b^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)
^(10 + m))/(d^10*(10 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rubi in Sympy [A] time = 26.3899, size = 90, normalized size = 0.89

$$\frac{a^2\,(dx)^{m+1}}{d\,(m+1)}+\frac{2ab\,(dx)^{m+4}}{d^4\,(m+4)}+\frac{2bc\,(dx)^{m+10}}{d^{10}\,(m+10)}+\frac{c^2\,(dx)^{m+13}}{d^{13}\,(m+13)}+\frac{(dx)^{m+7}\left(2ac+b^2\right)}{d^7\,(m+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

```
[Out] a**2*(d*x)**(m + 1)/(d*(m + 1)) + 2*a*b*(d*x)**(m + 4)/(d**4*(m +
 4)) + 2*b*c*(d*x)**(m + 10)/(d**10*(m + 10)) + c**2*(d*x)**(m +
13)/(d**13*(m + 13)) + (d*x)**(m + 7)*(2*a*c + b**2)/(d**7*(m + 7
))
```

Mathematica [A] time = 0.0842854, size = 70, normalized size = 0.69

$$(dx)^m\left(\frac{a^2x}{m+1}+\frac{x^7\left(2ac+b^2\right)}{m+7}+\frac{2abx^4}{m+4}+\frac{2bcx^{10}}{m+10}+\frac{c^2x^{13}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (d*x)^m*((a^2*x)/(1 + m) + (2*a*b*x^4)/(4 + m) + ((b^2 + 2*a*c)*x
^7)/(7 + m) + (2*b*c*x^10)/(10 + m) + (c^2*x^13)/(13 + m))
```

Maple [B] time = 0.009, size = 301, normalized size = 3.

$$\frac{\left(c^2m^4x^{12}+22\,c^2m^3x^{12}+159\,c^2m^2x^{12}+2\,bcm^4x^9+418\,c^2mx^{12}+50\,bcm^3x^9+280\,c^2x^{12}+390\,bcm^2x^9+2\,acm^4x^6+b^2m^4x^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^6+b*x^3+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+41
8*c^2*m*x^12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^
4*x^6+b^2*m^4*x^6+1070*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+72
8*b*c*x^9+498*a*c*m^2*x^6+249*b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*
m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1040*a*c*x^6+520*b^2*x^6+642*a
*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+1820*a*b*x^3+411*a^2
*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(13+m)/(10+m)/(7+m)/(4+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^2*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301979, size = 325, normalized size = 3.22

$$\frac{\left(\left(c^2m^4 + 22\,c^2m^3 + 159\,c^2m^2 + 418\,c^2m + 280\,c^2\right)x^{13} + 2\,\left(bcm^4 + 25\,bcm^3 + 195\,bcm^2 + 535\,bcm + 364\,bc\right)x^{10} + \left(\left(b^2 + 2\,ac\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^2*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13
+ 2*(b*c*m^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^
10 + ((b^2 + 2*a*c)*m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c
)*m^2 + 520*b^2 + 1040*a*c + 742*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^
4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*a*b*m + 910*a*b)*x^4 + (a^2*m
^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m + 3640*a^2)*x)*(d*x)^m
/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

Sympy [A] time = 18.5219, size = 1510, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b*
*2/(6*x**6) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), (
(-a**2/(9*x**9) - a*b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) +
 2*b*c*log(x) + c**2*x**3/3)/d**10, Eq(m, -10)), ((-a**2/(6*x**6)
 - 2*a*b/(3*x**3) + 2*a*c*log(x) + b**2*log(x) + 2*b*c*x**3/3 + c
**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3) + 2*a*b*log(x) + 2
*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/d**4, Eq(m,
 -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6 +
2*b*c*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*d**m*m**4*x*x*
*m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a
**2*d**m*m**3*x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 571
4*m + 3640) + 411*a**2*d**m*m**2*x*x**m/(m**5 + 35*m**4 + 445*m**
3 + 2485*m**2 + 5714*m + 3640) + 2074*a**2*d**m*m*x*x**m/(m**5 +
35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 3640*a**2*d**m*
x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
2*a*b*d**m*m**4*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2
+ 5714*m + 3640) + 62*a*b*d**m*m**3*x**4*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + 642*a*b*d**m*m**2*x**4*x**
m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2402*
a*b*d**m*m*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 571
4*m + 3640) + 1820*a*b*d**m*x**4*x**m/(m**5 + 35*m**4 + 445*m**3
+ 2485*m**2 + 5714*m + 3640) + 2*a*c*d**m*m**4*x**7*x**m/(m**5 +
```

```
35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*d**m*m**
3*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 364
0) + 498*a*c*d**m*m**2*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 248
5*m**2 + 5714*m + 3640) + 1484*a*c*d**m*m*x**7*x**m/(m**5 + 35*m*
*4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1040*a*c*d**m*x**7*x
**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + b**
2*d**m*m**4*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 57
14*m + 3640) + 28*b**2*d**m*m**3*x**7*x**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640) + 249*b**2*d**m*m**2*x**7*x**m/
(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 742*b**
2*d**m*m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*
m + 3640) + 520*b**2*d**m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 +
2485*m**2 + 5714*m + 3640) + 2*b*c*d**m*m**4*x**10*x**m/(m**5 + 3
5*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 50*b*c*d**m*m**3
*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 364
0) + 390*b*c*d**m*m**2*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 24
85*m**2 + 5714*m + 3640) + 1070*b*c*d**m*m*x**10*x**m/(m**5 + 35*
m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 728*b*c*d**m*x**10
*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + c
**2*d**m*m**4*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
 5714*m + 3640) + 22*c**2*d**m*m**3*x**13*x**m/(m**5 + 35*m**4 +
445*m**3 + 2485*m**2 + 5714*m + 3640) + 159*c**2*d**m*m**2*x**13*
x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 41
8*c**2*d**m*m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
 5714*m + 3640) + 280*c**2*d**m*x**13*x**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640), True))
```

GIAC/XCAS [A] time = 0.291641, size = 687, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^2*(d*x)^m,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^13*e^(m*ln(d*x)) + 22*c^2*m^3*x^13*e^(m*ln(d*x)) + 159
*c^2*m^2*x^13*e^(m*ln(d*x)) + 2*b*c*m^4*x^10*e^(m*ln(d*x)) + 418*
c^2*m*x^13*e^(m*ln(d*x)) + 50*b*c*m^3*x^10*e^(m*ln(d*x)) + 280*c^
2*x^13*e^(m*ln(d*x)) + 390*b*c*m^2*x^10*e^(m*ln(d*x)) + b^2*m^4*x
^7*e^(m*ln(d*x)) + 2*a*c*m^4*x^7*e^(m*ln(d*x)) + 1070*b*c*m*x^10*
e^(m*ln(d*x)) + 28*b^2*m^3*x^7*e^(m*ln(d*x)) + 56*a*c*m^3*x^7*e^(
m*ln(d*x)) + 728*b*c*x^10*e^(m*ln(d*x)) + 249*b^2*m^2*x^7*e^(m*ln
(d*x)) + 498*a*c*m^2*x^7*e^(m*ln(d*x)) + 2*a*b*m^4*x^4*e^(m*ln(d*
x)) + 742*b^2*m*x^7*e^(m*ln(d*x)) + 1484*a*c*m*x^7*e^(m*ln(d*x))
+ 62*a*b*m^3*x^4*e^(m*ln(d*x)) + 520*b^2*x^7*e^(m*ln(d*x)) + 1040
*a*c*x^7*e^(m*ln(d*x)) + 642*a*b*m^2*x^4*e^(m*ln(d*x)) + a^2*m^4*
x*e^(m*ln(d*x)) + 2402*a*b*m*x^4*e^(m*ln(d*x)) + 34*a^2*m^3*x*e^(
m*ln(d*x)) + 1820*a*b*x^4*e^(m*ln(d*x)) + 411*a^2*m^2*x*e^(m*ln(d
*x)) + 2074*a^2*m*x*e^(m*ln(d*x)) + 3640*a^2*x*e^(m*ln(d*x)))/(m^
```

```
5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

3.249 $\int (dx)^m \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) +
 (c*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rubi [A] time = 0.0469034, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) +
 (c*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rubi in Sympy [A] time = 11.1272, size = 42, normalized size = 0.81

$$\frac{a\left(dx\right)^{m+1}}{d\left(m+1\right)}+\frac{b\left(dx\right)^{m+4}}{d^4\left(m+4\right)}+\frac{c\left(dx\right)^{m+7}}{d^7\left(m+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

```
[Out] a*(d*x)**(m + 1)/(d*(m + 1)) + b*(d*x)**(m + 4)/(d**4*(m + 4)) +
c*(d*x)**(m + 7)/(d**7*(m + 7))
```

Mathematica [A] time = 0.0319807, size = 35, normalized size = 0.67

$$(dx)^m\left(\frac{ax}{m+1}+\frac{bx^4}{m+4}+\frac{cx^7}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] (d*x)^m*((a*x)/(1 + m) + (b*x^4)/(4 + m) + (c*x^7)/(7 + m))
```

Maple [A] time = 0.005, size = 78, normalized size = 1.5

$$\frac{\left(cm^2x^6+5\,cmx^6+4\,cx^6+bm^2x^3+8\,bmx^3+7\,bx^3+am^2+11\,am+28\,a\right)\,x\,(dx)^m}{(7+m)\,(4+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^6+b*x^3+a),x)
```

```
[Out] x*(c*m^2*x^6+5*c*m*x^6+4*c*x^6+b*m^2*x^3+8*b*m*x^3+7*b*x^3+a*m^2+
11*a*m+28*a)*(d*x)^m/(7+m)/(4+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269514, size = 96, normalized size = 1.85

$$\frac{\left(\left(cm^2+5\,cm+4\,c\right)x^7+\left(bm^2+8\,bm+7\,b\right)x^4+\left(am^2+11\,am+28\,a\right)x\right)\,(dx)^m}{m^3+12\,m^2+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 +
 11*a*m + 28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

Sympy [A] time = 4.40074, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{6x^6}-\frac{b}{3x^3}+c\log(x)}{d^7} \\ \frac{-\frac{a}{3x^3}+b\log(x)+\frac{cx^3}{3}}{d^4} \\ \frac{a\log(x)+\frac{bx^3}{3}+\frac{cx^6}{6}}{d} \\ \frac{ad^m m^2 x x^m}{m^3+12m^2+39m+28} + \frac{11ad^m m x x^m}{m^3+12m^2+39m+28} + \frac{28ad^m x x^m}{m^3+12m^2+39m+28} + \frac{bd^m m^2 x^4 x^m}{m^3+12m^2+39m+28} + \frac{8bd^m m x^4 x^m}{m^3+12m^2+39m+28} + \frac{7bd^m x^4 x^m}{m^3+12m^2+39m+28} + \frac{cd^m m^2 x^7 x^m}{m^3+12m^2+39m+2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

```
[Out] Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7))
, ((-a/(3*x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(
x) + b*x**3/3 + c*x**6/6)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**
3 + 12*m**2 + 39*m + 28) + 11*a*d**m*m*x*x**m/(m**3 + 12*m**2 + 3
9*m + 28) + 28*a*d**m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + b*d**
m*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 8*b*d**m*m*x**4*x
**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*d**m*x**4*x**m/(m**3 + 12*
m**2 + 39*m + 28) + c*d**m*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m
+ 28) + 5*c*d**m*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*d
**m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

GIAC/XCAS [A] time = 0.266169, size = 185, normalized size = 3.56

$$\frac{cm^2x^7e^{(m\ln(dx))} + 5\,cmx^7e^{(m\ln(dx))} + 4\,cx^7e^{(m\ln(dx))} + bm^2x^4e^{(m\ln(dx))} + 8\,bmx^4e^{(m\ln(dx))} + 7\,bx^4e^{(m\ln(dx))} + am^2xe^{(m\ln(dx))} +}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (c*m^2*x^7*e^(m*ln(d*x)) + 5*c*m*x^7*e^(m*ln(d*x)) + 4*c*x^7*e^(m
*ln(d*x)) + b*m^2*x^4*e^(m*ln(d*x)) + 8*b*m*x^4*e^(m*ln(d*x)) + 7
*b*x^4*e^(m*ln(d*x)) + a*m^2*x*e^(m*ln(d*x)) + 11*a*m*x*e^(m*ln(d
*x)) + 28*a*x*e^(m*ln(d*x)))/(m^3 + 12*m^2 + 39*m + 28)
```

3.250 $\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^
2 - 4*a*c])*d*(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi [A] time = 0.483241, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^
2 - 4*a*c])*d*(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi in Sympy [A] time = 31.7794, size = 148, normalized size = 0.86

$$-\frac{2c\,(dx)^{m+1}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{d\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}+\frac{2c\,(dx)^{m+1}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|-\frac{2cx^3}{b-\sqrt{-4ac+b^2}}\right)}{d\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**6+b*x**3+a),x)
```

```
[Out] -2*c*(d*x)**(m + 1)*hyper((1, m/3 + 1/3), (m/3 + 4/3,), -2*c*x**3
/(b + sqrt(-4*a*c + b**2)))/(d*(b + sqrt(-4*a*c + b**2))*(m + 1)*
sqrt(-4*a*c + b**2)) + 2*c*(d*x)**(m + 1)*hyper((1, m/3 + 1/3), (
m/3 + 4/3,), -2*c*x**3/(b - sqrt(-4*a*c + b**2)))/(d*(b - sqrt(-4
*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0940049, size = 84, normalized size = 0.49

$$\frac{(dx)^m \text{RootSum}\left[\#1^6 c+\#1^3 b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m} {}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^5 c+\#1^2 b}\&\right]}{3m}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((d*x)^m*RootSum[a + b*#1^3 + c*#1^6 & , Hypergeometric2F1[-m, -m
, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1^2 + 2*c*#1^5)) & ]
)/(3*m)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^6+bx^3+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^6+b*x^3+a),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^6+bx^3+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{cx^6+bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

3.251 $\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx$

Optimal. Leaf size=315

$$\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a +
 b*x^3 + c*x^6)) + (c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m)
- 4*a*c*(5 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4
 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])])/(3*a*(b^2 - 4*a*c)^
(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (c*(b^2*(2 - m) - b*Sq
rt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1 + m)*Hypergeome
tric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi [A] time = 1.51084, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a +
 b*x^3 + c*x^6)) + (c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m)
- 4*a*c*(5 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4
 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])])/(3*a*(b^2 - 4*a*c)^
(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (c*(b^2*(2 - m) - b*Sq
```

```
rt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1 + m)*Hypergeome
tric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi in Sympy [A] time = 114.466, size = 264, normalized size = 0.84

$$-\frac{c\,(dx)^{m+1}\left(-4ac\,(-m+5)+b^2\,(-m+2)-b\,(-m+2)\,\sqrt{-4ac+b^2}\right)\,{}_2F_1\left(\begin{matrix}1,\ \frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{3ad\left(b+\sqrt{-4ac+b^2}\right)(m+1)\,(-4ac+b^2)^{\frac{3}{2}}}+\frac{c\,(dx)^{m+1}\left(-4ac\,(-m+5)+b^2\,(-m+2)+b\,(-m+2)\,\sqrt{-4ac+b^2}\right)\,{}_2F_1\left(\begin{matrix}1,\ \frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|-\frac{2cx^3}{b-\sqrt{-4ac+b^2}}\right)}{3ad\left(b-\sqrt{-4ac+b^2}\right)(m+1)\,(-4ac+b^2)^{\frac{3}{2}}}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\,(-4ac+b^2)\,(a+bx^3+cx^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)
```

```
[Out] -c*(d*x)**(m + 1)*(-4*a*c*(-m + 5) + b**2*(-m + 2) - b*(-m + 2)*s
qrt(-4*a*c + b**2))*hyper((1, m/3 + 1/3), (m/3 + 4/3,), -2*c*x**3
/(b + sqrt(-4*a*c + b**2)))/(3*a*d*(b + sqrt(-4*a*c + b**2))*(m +
 1)*(-4*a*c + b**2)**(3/2)) + c*(d*x)**(m + 1)*(-4*a*c*(-m + 5) +
 b**2*(-m + 2) + b*(-m + 2)*sqrt(-4*a*c + b**2))*hyper((1, m/3 +
1/3), (m/3 + 4/3,), -2*c*x**3/(b - sqrt(-4*a*c + b**2)))/(3*a*d*(
b - sqrt(-4*a*c + b**2))*(m + 1)*(-4*a*c + b**2)**(3/2)) + (d*x)*
*(m + 1)*(-2*a*c + b**2 + b*c*x**3)/(3*a*d*(-4*a*c + b**2)*(a + b
*x**3 + c*x**6))
```

Mathematica [C] time = 1.6524, size = 376, normalized size = 1.19

$$\frac{a(m+4)x(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)F_1\left(\frac{m}{}\right.}{4c(m+1)\left(a+bx^3+cx^6\right)^3\left(a(m+4)F_1\left(\frac{m+1}{3};2,2;\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{m+4}{3};2,3;\frac{m+7}{3};-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (a*(4 + m)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^3)*AppellF1[(1 + m)/3, 2, 2, (4 + m)/3, (-2*
```

```
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c]
)])/(4*c*(1 + m)*(a + b*x^3 + c*x^6)^3*(a*(4 + m)*AppellF1[(1 + m
)/3, 2, 2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
3)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*App
ellF1[(4 + m)/3, 2, 3, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c]
)*AppellF1[(4 + m)/3, 3, 2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^6 + bx^3 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^6+b*x^3+a)^2,x)

[Out] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^6 + bx^3 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2,x, algorithm="maxima")

[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^2x^{12} + 2\,bcx^9 + (b^2 + 2\,ac)x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*
b*x^3 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^6 + bx^3 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

3.252 $\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{3}{2},-\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2
, -3/2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.464182, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{3}{2},-\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```


```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2
, -3/2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 41.4125, size = 139, normalized size = 0.88

$$\frac{a\,(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\mathrm{appellf_1}\left(\frac{m}{3}+\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{m}{3}+\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{d\,(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] a*(d*x)**(m + 1)*sqrt(a + b*x**3 + c*x**6)*appellf1(m/3 + 1/3, -3
/2, -3/2, m/3 + 4/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x*
```

```
*3/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1)*sqrt(2*c*x**3/(b - sqrt(
-4*a*c + b**2)) + 1)*sqrt(2*c*x**3/(b + sqrt(-4*a*c + b**2)) + 1)
)
```

Mathematica [B] time = 10.5565, size = 1083, normalized size = 6.85

$$\frac{\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)x(dx)^m\left(2cx^3+b-\sqrt{b^2-4ac}\right)\left(2cx^3+b+\sqrt{b^2-4ac}\right)\left(\frac{}{(m+7)\left(3\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+10}{3};-\frac{1}{2},\frac{1}{2};\right.\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x*(d*x)^m*(b - S
qrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*((a
*(4 + m)*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]])/((1 +
 m)*(4*a*(4 + m)*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])
] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + m)/3, -1/2, 1/2,
 (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(4 + m)/3,
1/2, -1/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
3)/(-b + Sqrt[b^2 - 4*a*c])]))) + (b*(7 + m)*x^3*AppellF1[(4 + m)
/3, -1/2, -1/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((4 + m)*(4*a*(7 + m)*AppellF1
[(4 + m)/3, -1/2, -1/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2
- 4*a*c])*AppellF1[(7 + m)/3, -1/2, 1/2, (10 + m)/3, (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b
- Sqrt[b^2 - 4*a*c])*AppellF1[(7 + m)/3, 1/2, -1/2, (10 + m)/3, (
-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])]))) + (c*(10 + m)*x^6*AppellF1[(7 + m)/3, -1/2, -1/2, (10 +
m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^
2 - 4*a*c])])/((7 + m)*(4*a*(10 + m)*AppellF1[(7 + m)/3, -1/2, -1
/2, (10 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
(10 + m)/3, -1/2, 1/2, (13 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c
])*AppellF1[(10 + m)/3, 1/2, -1/2, (13 + m)/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))))/(4*c^2
*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

3.253 $\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2,
-1/2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.449134, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2,
-1/2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 35.3433, size = 138, normalized size = 0.88

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{m}{3}+\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{m}{3}+\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{d\,(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**3 + c*x**6)*appellf1(m/3 + 1/3, -1/2
, -1/2, m/3 + 4/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3
```

```
/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1)*sqrt(2*c*x**3/(b - sqrt(-4
*a*c + b**2)) + 1)*sqrt(2*c*x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.289458, size = 424, normalized size = 2.7

$$\frac{(m+4)x\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4a\right.}{4c^2(m+1)\sqrt{a+bx^3+cx^6}\left(3x^3\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{m+4}{3};-\frac{1}{2},\frac{1}{2};\frac{m+7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{m+4}{3};\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(4 + m)*x*(d*x)^
m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c^2*(1
 + m)*Sqrt[a + b*x^3 + c*x^6]*(4*a*(4 + m)*AppellF1[(1 + m)/3, -1
/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3
)/(-b + Sqrt[b^2 - 4*a*c])] + 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Appe
llF1[(4 + m)/3, -1/2, 1/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*
a*c])*AppellF1[(4 + m)/3, 1/2, -1/2, (7 + m)/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6+bx^3+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^6 + bx^3 + a}\,(dx)^m\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m,x, algorithm="fricas")

[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\,\sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral((d*x)**m*sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^6 + bx^3 + a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

3.254 $\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/
2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.445819, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/
2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 36.1623, size = 136, normalized size = 0.87

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{m}{3}+\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{m}{3}+\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{ad\,(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**3 + c*x**6)*appellf1(m/3 + 1/3, 1/2,
 1/2, m/3 + 4/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(
```

```
b + sqrt(-4*a*c + b**2)))/(a*d*(m + 1)*sqrt(2*c*x**3/(b - sqrt(-4
*a*c + b**2)) + 1)*sqrt(2*c*x**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 2.35687, size = 426, normalized size = 2.71

$$\frac{4a^2(m+4)x(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4a\right.}{(m+1)\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^3+cx^6\right)^{3/2}\left(4a(m+4)F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-3x^3\left(\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (4*a^2*(4 + m)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 +
 m)*(a + b*x^3 + c*x^6)^(3/2)*(4*a*(4 + m)*AppellF1[(1 + m)/3, 1/
2, 1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/
(-b + Sqrt[b^2 - 4*a*c])] - 3*x^3*((b + Sqrt[b^2 - 4*a*c])*Appell
F1[(4 + m)/3, 1/2, 3/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c
])*AppellF1[(4 + m)/3, 3/2, 1/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a),x, algorithm="maxima")

[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{cx^6 + bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a),x, algorithm="fricas")

[Out] integral((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral((d*x)**m/sqrt(a + b*x**3 + c*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^6 + bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a),x, algorithm="giac")

[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

3.255 $\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}} \, dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/
2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c])])/(a*d*(1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] time = 0.445709, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/
2, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c])])/(a*d*(1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi in Sympy [A] time = 45.7743, size = 138, normalized size = 0.86

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{appellf}_1\left(\frac{m}{3}+\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{m}{3}+\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{a^2d\,(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**3 + c*x**6)*appellf1(m/3 + 1/3, 3/2,
 3/2, m/3 + 4/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/(
```

```
b + sqrt(-4*a*c + b**2)))/(a**2*d*(m + 1)*sqrt(2*c*x**3/(b - sqrt
(-4*a*c + b**2)) + 1)*sqrt(2*c*x**3/(b + sqrt(-4*a*c + b**2)) + 1
))
```

Mathematica [B] time = 2.88843, size = 426, normalized size = 2.66

$$\frac{4a^2(m+4)x(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\sqrt{b^2-4a\right.}{(m+1)\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^3+cx^6\right)^{5/2}\left(4a(m+4)F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-9x^3\left(\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (4*a^2*(4 + m)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 +
 m)*(a + b*x^3 + c*x^6)^(5/2)*(4*a*(4 + m)*AppellF1[(1 + m)/3, 3/
2, 3/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/
(-b + Sqrt[b^2 - 4*a*c])] - 9*x^3*((b + Sqrt[b^2 - 4*a*c])*Appell
F1[(4 + m)/3, 3/2, 5/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c
])*AppellF1[(4 + m)/3, 5/2, 3/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^6+bx^3+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + bx^3 + cx^6)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**3 + c*x**6)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^6 + bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.256 $\int (dx)^m \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=155

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p,
(4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])])/(d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*
c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.278161, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p,
(4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])])/(d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*
c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 33.8801, size = 129, normalized size = 0.83

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(\frac{m}{3}+\frac{1}{3},-p,-p,\frac{m}{3}+\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b}}\right.}{d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] (d*x)**(m + 1)*(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*
c*x**3/(b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)
**p*appellf1(m/3 + 1/3, -p, -p, m/3 + 4/3, -2*c*x**3/(b - sqrt(-4
```

```
*a*c + b**2)), -2*c*x**3/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1))
```

Mathematica [B] time = 4.33275, size = 501, normalized size = 3.23

$$\frac{c(m+4)2^{-p-1}x\left(\sqrt{b^2-4ac}+b\right)(dx)^m\left(x^3\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{m+4}{3};1-p,-p;\frac{m+7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(-1 - p)*c*(b + Sqrt[b^2 - 4*a*c])*(4 + m)*x*(d*x)^m*((b - Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a
*c])*x^3)^2*(a + b*x^3 + c*x^6)^(-1 + p)*AppellF1[(1 + m)/3, -p,
-p, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
+ Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*(1 + m)*((b - Sq
rt[b^2 - 4*a*c])/(2*c) + x^3)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
*(-2*a*(4 + m)*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*
p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + m)/3, 1 - p, -p, (7
 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt
[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + m)/3, -p,
 1 - p, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/
(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int (dx)^m\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^p(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

3.257 $\int x^8 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=224

$$\frac{2^p \left(2ac-b^2(p+2)\right) \left(a+bx^3+cx^6\right)^{p+1} \left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1} {}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{6c^2(p+1)(2p+3)} + \frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{3c(2p+3)}$$

```
[Out] -(b*(2 + p)*(a + b*x^3 + c*x^6)^(1 + p))/(6*c^2*(1 + p)*(3 + 2*p)
) + (x^3*(a + b*x^3 + c*x^6)^(1 + p))/(3*c*(3 + 2*p)) + (2^p*(2*a
*c - b^2*(2 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[
-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 -
 4*a*c])])/(3*c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi [A] time = 0.478169, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{2^p \left(2ac-b^2(p+2)\right) \left(a+bx^3+cx^6\right)^{p+1} \left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1} {}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{6c^2(p+1)(2p+3)} + \frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{3c(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -(b*(2 + p)*(a + b*x^3 + c*x^6)^(1 + p))/(6*c^2*(1 + p)*(3 + 2*p)
) + (x^3*(a + b*x^3 + c*x^6)^(1 + p))/(3*c*(3 + 2*p)) + (2^p*(2*a
*c - b^2*(2 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[
-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 -
 4*a*c])])/(3*c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi in Sympy [A] time = 48.2856, size = 192, normalized size = 0.86

$$-\frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{6c^2(p+1)(2p+3)}+\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{3c(2p+3)}+\frac{\left(\frac{-\frac{b}{2}-cx^3+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1}\left(2ac-b^2(p+2)\right)\left(a+bx^3+cx^6\right)^{p+1}{}_2F_1\left(\begin{matrix}-p,p+1\\p+2\end{matrix}\middle|\frac{\frac{b}{2}+cx^3+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{6c^2(p+1)(2p+3)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] -b*(p + 2)*(a + b*x**3 + c*x**6)**(p + 1)/(6*c**2*(p + 1)*(2*p +
3)) + x**3*(a + b*x**3 + c*x**6)**(p + 1)/(3*c*(2*p + 3)) + ((-b/
2 - c*x**3 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**(-p - 1
)*(2*a*c - b**2*(p + 2))*(a + b*x**3 + c*x**6)**(p + 1)*hyper((-p
, p + 1), (p + 2,), (b/2 + c*x**3 + sqrt(-4*a*c + b**2)/2)/sqrt(-
4*a*c + b**2))/(6*c**2*(p + 1)*(2*p + 3)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.613316, size = 395, normalized size = 1.76

$$\frac{2x^9\left(\sqrt{b^2-4ac}+b\right)\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(x^3\left(b-\sqrt{b^2-4ac}\right)+2a\right)^2\left(a+x\right.}{9\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(4;1-p,-p;5;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4a\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2*(b + Sqrt[b^2 - 4*a*c])*x^9*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*
(2*a + (b - Sqrt[b^2 - 4*a*c])*x^3)^2*(a + x^3*(b + c*x^3))^(-1 +
 p)*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(9*(-b + Sqrt[b^2 - 4*a*c])*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(-8*a*AppellF1[3, -p, -p, 4, (-2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c]
)] + p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[4, 1 - p, -p, 5, (-
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*
c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[4, -p, 1 - p, 5, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))
)
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int x^{8} \left(c x^{6}+b x^{3}+a \right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(c*x^6+b*x^3+a)^p,x)

[Out] int(x^8*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)^p x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^8,x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^6 + bx^3 + a)^p x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^8,x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(c*x**6+b*x**3+a)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^8,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

3.258 $\int x^5 \left(a + bx^3 + cx^6\right)^p \, dx$

Optimal. Leaf size=161

$$\frac{b2^p \left(a + bx^3 + cx^6\right)^{p+1} \left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1} {}_2F_1\left(-p, p+1; p+2; \frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}} + \frac{\left(a + bx^3 + cx^6\right)^{p+1}}{6c(p+1)}$$

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(6*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b
^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 +
c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c*Sqrt[b^2 - 4*a*c
]*(1 + p))
```

Rubi [A] time = 0.253238, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{b2^p \left(a + bx^3 + cx^6\right)^{p+1} \left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1} {}_2F_1\left(-p, p+1; p+2; \frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}} + \frac{\left(a + bx^3 + cx^6\right)^{p+1}}{6c(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^3 + c*x^6)^p,x]
```


```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(6*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b
^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 +
c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c*Sqrt[b^2 - 4*a*c
]*(1 + p))
```

Rubi in Sympy [A] time = 22.1113, size = 136, normalized size = 0.84

$$\frac{b\left(\frac{-\frac{b}{2} - cx^3 + \frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1} \left(a + bx^3 + cx^6\right)^{p+1} {}_2F_1\left(\begin{matrix} -p, p+1 \\ p+2 \end{matrix}\middle| \frac{\frac{b}{2} + cx^3 + \frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{6c\left(p+1\right)\sqrt{-4ac+b^2}} + \frac{\left(a + bx^3 + cx^6\right)^{p+1}}{6c\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] b*((-b/2 - c*x**3 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**
(-p - 1)*(a + b*x**3 + c*x**6)**(p + 1)*hyper((-p, p + 1), (p + 2
,), (b/2 + c*x**3 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))/(
6*c*(p + 1)*sqrt(-4*a*c + b**2)) + (a + b*x**3 + c*x**6)**(p + 1)
/(6*c*(p + 1))
```

Mathematica [C] time = 0.903061, size = 439, normalized size = 2.73

$$\frac{c2^{-p-2}x^6\left(\sqrt{b^2-4ac}+b\right)\left(x^3\left(b-\sqrt{b^2-4ac}\right)+2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{c}\right)}{\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(3;1-p,-p;4;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4ac}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^5*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(-2 - p)*c*(b + Sqrt[b^2 - 4*a*c])*x^6*((b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/c)^(1 + p)*(2*a + (b - Sqrt[b^2 - 4*a*c])*x^3)^2*(a +
 x^3*(b + c*x^3))^(-1 + p)*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b +
Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)*(-6*a*AppellF1[2, -p, -p, 3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] +
 p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[3, 1 - p, -p, 4, (-2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 - (b + Sqrt[b^2 - 4*a*c])*AppellF1[3, -p, 1 - p, 4, (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int x^5\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^6 + bx^3 + a)^p x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^5,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^5,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

3.259 $\int x^2 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=130

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1} {}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c
]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1
+ p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c
])])/(3*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rubi [A] time = 0.152087, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1} {}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c
]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1
+ p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c
])])/(3*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rubi in Sympy [A] time = 11.9058, size = 112, normalized size = 0.86

$$-\frac{\left(\frac{-\frac{b}{2}-cx^3+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1} {}_2F_1\left(\begin{matrix}-p,p+1\\ p+2\end{matrix}\middle|\frac{\frac{b}{2}+cx^3+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{3\left(p+1\right)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] -((-b/2 - c*x**3 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**(
-p - 1)*(a + b*x**3 + c*x**6)**(p + 1)*hyper((-p, p + 1), (p + 2,
```

```
), (b/2 + c*x**3 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))/(3
*(p + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.175943, size = 138, normalized size = 1.06

$$\frac{2^{p-1}\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\ {}_2F_1\left(-p,p+1;p+2;\frac{-2cx^3-b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)
^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] -
2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c*(1 + p)*((b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c])^p)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x^2\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

3.260 $\int x^4 \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*
(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.302218, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*
(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 30.8772, size = 116, normalized size = 0.84

$$\frac{x^5\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] x**5*(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b
+ sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appell
f1(5/3, -p, -p, 8/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x*
*3/(b + sqrt(-4*a*c + b**2)))/5
```

Mathematica [B] time = 3.20649, size = 411, normalized size = 2.98

$$\frac{4x^5\left(\sqrt{b^2-4ac}+b\right)\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(x^3\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2(a}{5\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{8}{3};1-p,-p;\frac{11}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (4*(b + Sqrt[b^2 - 4*a*c])*x^5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*
(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^3)^2*(a + b*x^3 + c*x^6)^(-1 +
 p)*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*(-b + Sqrt[b^2 - 4*a*c]
)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(-16*a*AppellF1[5/3, -p, -p,
8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])] + 3*p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[8/3, 1 -
 p, -p, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[8/3, -p,
1 - p, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int x^4\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

3.261 $\int x^3 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*
(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.282396, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*
(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 33.5669, size = 116, normalized size = 0.84

$$\frac{x^4\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] x**4*(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b
+ sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appell
f1(4/3, -p, -p, 7/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x*
*3/(b + sqrt(-4*a*c + b**2)))/4
```

Mathematica [B] time = 3.45189, size = 456, normalized size = 3.3

$$\frac{7c2^{-p-3}x^4\left(\sqrt{b^2-4ac}+b\right)\left(x^3\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2}{c}\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{7}{3};1-p,-p;\frac{10}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4a\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (7*2^(-3 - p)*c*(b + Sqrt[b^2 - 4*a*c])*x^4*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^3)^2*
(a + b*x^3 + c*x^6)^(-1 + p)*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(
(-b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(-14*a*AppellF1[4/3, -p, -p, 7/
3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])] + 3*p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[7/3, 1 - p
, -p, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + S
qrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[7/3, -p, 1
- p, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sq
rt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int x^3\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

3.262 $\int x\left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*
(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.231168, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*
(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 26.7243, size = 116, normalized size = 0.84

$$\frac{x^2\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] x**2*(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b
+ sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appell
f1(2/3, -p, -p, 5/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x*
*3/(b + sqrt(-4*a*c + b**2)))/2
```

Mathematica [B] time = 3.19844, size = 454, normalized size = 3.29

$$\frac{5c2^{-p-2}\left(\sqrt{b^2-4ac}+b\right)\left(x^4\left(\sqrt{b^2-4ac}-b\right)-2ax\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c}{c}\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{5}{3};1-p,-p;\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4a\right.\right.\right.}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (5*2^(-2 - p)*c*(b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/c)^(1 + p)*(-2*a*x + (-b + Sqrt[b^2 - 4*a*c])*x^4)^2*(a
 + b*x^3 + c*x^6)^(-1 + p)*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-
b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(-10*a*AppellF1[2/3, -p, -p, 5/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4
*a*c])] + 3*p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, 1 - p,
-p, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt
[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[5/3, -p, 1 - p
, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])])))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p*x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

3.263 $\int \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=133

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 +
 (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]))^p)
```

Rubi [A] time = 0.150893, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 +
 (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 37.5018, size = 112, normalized size = 0.84

$$x\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p,x)
```

```
[Out] x*(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b + s
qrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appellf1(
1/3, -p, -p, 4/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/
(b + sqrt(-4*a*c + b**2)))
```

Mathematica [B] time = 3.67082, size = 487, normalized size = 3.66

$$\frac{2^{1-2p} x \left(\sqrt{b^2-4ac}+b\right)\left(x^3\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{c}\right)^{p+1}\left(\frac{\sqrt{b^2-4a}}{}\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{4}{3};1-p,-p;\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{4}{3};-p,1-p;\frac{7}{3}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(1 - 2*p)*(b + Sqrt[b^2 - 4*a*c])*x*((b - Sqrt[b^2 - 4*a*c] +
2*c*x^3)/c)^(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/c)^(-1 + p
)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^3)^2*(a + b*x^3 + c*x^6)^(-1
 + p)*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c]
)*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*((b + Sqrt[b^2 - 4*a*c]
)/(2*c) + x^3)^p*(-8*a*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*p*x^
3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, 1 - p, -p, 7/3, (-2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
- (b + Sqrt[b^2 - 4*a*c])*AppellF1[4/3, -p, 1 - p, 7/3, (-2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p,x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6+bx^3+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p,x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

3.264 $\int \frac{(a+bx^3+cx^6)^p}{x}\,dx$

Optimal. Leaf size=157

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*
p, -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(
2*c*x^3)])/(3*p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi [A] time = 0.329476, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*
p, -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(
2*c*x^3)])/(3*p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi in Sympy [A] time = 27.7788, size = 128, normalized size = 0.82

$$\frac{\left(\frac{b+2cx^3-\sqrt{-4ac+b^2}}{2cx^3}\right)^{-p}\left(\frac{b+2cx^3+\sqrt{-4ac+b^2}}{2cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf_1}\left(-2p,-p,-p,-2p+1,-\frac{b-\sqrt{-4ac+b^2}}{2cx^3},-\frac{b+\sqrt{-4ac+b^2}}{2cx^3}\right)}{6p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p/x,x)
```

```
[Out] ((b + 2*c*x**3 - sqrt(-4*a*c + b**2))/(2*c*x**3))**(-p)*((b + 2*c
*x**3 + sqrt(-4*a*c + b**2))/(2*c*x**3))**(-p)*(a + b*x**3 + c*x*
*6)**p*appellf1(-2*p, -p, -p, -2*p + 1, -(b - sqrt(-4*a*c + b**2)
```

```
)/(2*c*x**3), -(b + sqrt(-4*a*c + b**2))/(2*c*x**3))/(6*p)
```

Mathematica [B] time = 2.93977, size = 500, normalized size = 3.18

$$\frac{c4^{-p-1}(2p-1)x^3\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+1\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{c}\right)^{p+1}\left(\frac{-\sqrt{b^2-4}}{}\right.}{3p\left(2c(2p-1)x^3F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)-p\left(\sqrt{b^2-4ac}+b\right)F_1\left(1-2p;1-p,-p;2-2p;-\frac{b+\sqrt{b^2-}}{2cx}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (4^(-1 - p)*c*(-1 + 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/c
)^(1 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^(-1 + p)*AppellF1[-2
*p, -p, -p, 1 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sq
rt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*(1 + (b - Sqrt[b^2 - 4*a*c])/(2
*c*x^3))^p*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*(-((b + Sqrt[b
^2 - 4*a*c])*p*AppellF1[1 - 2*p, 1 - p, -p, 2 - 2*p, -(b + Sqrt[b
^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)]) + (-
b + Sqrt[b^2 - 4*a*c])*p*AppellF1[1 - 2*p, -p, 1 - p, 2 - 2*p, -(
b + Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x
^3)] + 2*c*(-1 + 2*p)*x^3*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b + S
qrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])
)
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x,x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x,x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p/x,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^6+bx^3+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

3.265 $\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(
1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]))^p))
```

Rubi [A] time = 0.250433, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(
1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]))^p))
```

Rubi in Sympy [A] time = 30.2027, size = 116, normalized size = 0.85

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p/x**2,x)
```

```
[Out] -(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b + sq
rt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appellf1(-
1/3, -p, -p, 2/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/
```

```
(b + sqrt(-4*a*c + b**2)))/x
```

Mathematica [B] time = 3.19541, size = 408, normalized size = 3.

$$\frac{\left(\sqrt{b^2-4ac}+b\right)\left(\sqrt{b^2-4ac}-b-2cx^3\right)\left(x^3\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2(a+b}{x\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{2}{3};1-p,-p;\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] ((b + Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*(-2*a
 + (-b + Sqrt[b^2 - 4*a*c])*x^3)^2*(a + b*x^3 + c*x^6)^(-1 + p)*A
ppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*x*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(-4*a*AppellF1[-1/3, -p, -p, 2/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4
*a*c])] + 3*p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[2/3, 1 - p,
-p, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt
[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[2/3, -p, 1 - p
, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])])))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

3.266 $\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^
2*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.244858, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^
2*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 30.4946, size = 119, normalized size = 0.86

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p/x**3,x)
```

```
[Out] -(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b + sq
rt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appellf1(-
2/3, -p, -p, 1/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3/
```

```
(b + sqrt(-4*a*c + b**2)))/(2*x**2)
```

Mathematica [B] time = 3.64329, size = 474, normalized size = 3.43

$$\frac{2^{-p-2}\left(\sqrt{b^2-4ac}+b\right)\left(\sqrt{b^2-4ac}-b-2cx^3\right)\left(x^3\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\right.}{x^2\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{1}{3};1-p,-p;\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] (2^(-2 - p)*(b + Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2 - 4*a*c] - 2*c
*x^3)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/c)^p*(-2*a + (-b + Sqrt[
b^2 - 4*a*c])*x^3)^2*(a + b*x^3 + c*x^6)^(-1 + p)*AppellF1[-2/3,
-p, -p, 1/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*x^2*((b - Sqrt[b^2
 - 4*a*c])/(2*c) + x^3)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(-2*a
*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*p*x^3*((-b + Sqrt[b^2 - 4
*a*c])*AppellF1[1/3, 1 - p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a
*c])*AppellF1[1/3, -p, 1 - p, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

3.267 $\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx$

Optimal. Leaf size=164

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-2p)x^3}$$

```
[Out] -(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p),
-(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c
*x^3)])/(3*(1 - 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^
3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi [A] time = 0.323034, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-2p)x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] -(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p),
-(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c
*x^3)])/(3*(1 - 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^
3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi in Sympy [A] time = 28.9353, size = 151, normalized size = 0.92

$$-\frac{\left(\frac{b+2cx^3-\sqrt{-4ac+b^2}}{2cx^3}\right)^{-p}\left(\frac{b+2cx^3+\sqrt{-4ac+b^2}}{2cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{1}{x^3}\right)^{-2p+1}\operatorname{appellf}_1\left(-2p+1,-p,-p,-2p+2,-\frac{b-\sqrt{-4a}}{2c}\right.}{3\left(-2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p/x**4,x)
```

```
[Out] -((b + 2*c*x**3 - sqrt(-4*a*c + b**2))/(2*c*x**3))**(-p)*((b + 2*
c*x**3 + sqrt(-4*a*c + b**2))/(2*c*x**3))**(-p)*(a + b*x**3 + c*x
**6)**p*(x**(-3))**(2*p)*(x**(-3))**(-2*p + 1)*appellf1(-2*p + 1,
```

```
 -p, -p, -2*p + 2, -(b - sqrt(-4*a*c + b**2))/(2*c*x**3), -(b + s
qrt(-4*a*c + b**2))/(2*c*x**3))/(3*(-2*p + 1))
```

Mathematica [B] time = 3.31729, size = 510, normalized size = 3.11

$$\frac{(p-1)\left(\sqrt{b^2-4ac}-b-2cx^3\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(\frac{2\left(b-\sqrt{b^2-4ac}\right)}{cx^3}+4\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{c}\right.}{3(2p-1)\left(-4c(p-1)x^3F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)+p\left(\sqrt{b^2-4ac}+b\right)F_1\left(2-2p;1-p,-p;3-2p\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] ((-1 + p)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/c)^p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^(-1 + p)*A
ppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x
^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1 + 2*p)*(4 + (2*(
b - Sqrt[b^2 - 4*a*c]))/(c*x^3))^p*((b - Sqrt[b^2 - 4*a*c])/(2*c)
 + x^3)^p*(-4*c*(-1 + p)*x^3*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -
(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*
x^3)] + (b + Sqrt[b^2 - 4*a*c])*p*AppellF1[2 - 2*p, 1 - p, -p, 3
- 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c
])/(2*c*x^3)] + (b - Sqrt[b^2 - 4*a*c])*p*AppellF1[2 - 2*p, -p, 1
 - p, 3 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2
 - 4*a*c])/(2*c*x^3)]))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6 + bx^3 + a)^p}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^6 + bx^3 + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p/x**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6 + bx^3 + a)^p}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^4,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

3.268 $\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*x
^4*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.261506, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*x
^4*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 30.6235, size = 121, normalized size = 0.88

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p/x**5,x)
```

```
[Out] -(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b + sq
rt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appellf1(-
4/3, -p, -p, -1/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3
```

```
/(b + sqrt(-4*a*c + b**2)))/(4*x**4)
```

Mathematica [B] time = 3.42631, size = 455, normalized size = 3.3

$$\frac{c2^{-p-3}\left(\sqrt{b^2-4ac}+b\right)\left(x^3\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2\ldots}{c}\right.}{x^4\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(3px^3\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(-\frac{1}{3};1-p,-p;\frac{2}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2\ldots}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] (2^(-3 - p)*c*(b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c] + 2
*c*x^3)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^3)^2*(a + b
*x^3 + c*x^6)^(-1 + p)*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b
+ Sqrt[b^2 - 4*a*c])*x^4*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(2*a*AppellF1[-4/3, -p, -p, -1/
3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])] + 3*p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[-1/3, 1 -
p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + S
qrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[-1/3, -p, 1
 - p, 2/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sq
rt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p/x^5,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^5,x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6 + bx^3 + a\right)^p}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^5,x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p/x**5,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^5,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

3.269 $\int \frac{(a+bx^3+cx^6)^p}{x^6} dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*x
^5*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.252764, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*x
^5*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 31.6245, size = 121, normalized size = 0.88

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^3}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{appellf}_1\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{-4ac+b^2}},-\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p/x**6,x)
```

```
[Out] -(2*c*x**3/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**3/(b + sq
rt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**3 + c*x**6)**p*appellf1(-
5/3, -p, -p, -2/3, -2*c*x**3/(b - sqrt(-4*a*c + b**2)), -2*c*x**3
```

```
/(b + sqrt(-4*a*c + b**2)))/(5*x**5)
```

Mathematica [B] time = 2.94187, size = 411, normalized size = 2.98

$$\frac{(\sqrt{b^2-4ac}+b)(-\sqrt{b^2-4ac}+b+2cx^3)(x^3(\sqrt{b^2-4ac}-b)-2a)^2(a}{5x^5(\sqrt{b^2-4ac}-b)(\sqrt{b^2-4ac}+b+2cx^3)(3px^3((\sqrt{b^2-4ac}-b)F_1(-\frac{2}{3};1-p,-p;\frac{1}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b})-(\sqrt{b^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] ((b + Sqrt[b^2 - 4*a*c])*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(-2*a
+ (-b + Sqrt[b^2 - 4*a*c])*x^3)^2*(a + b*x^3 + c*x^6)^(-1 + p)*Ap
pellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*(-b + Sqrt[b^2 - 4*a*c])*x^
5*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(4*a*AppellF1[-5/3, -p, -p, -
2/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])] + 3*p*x^3*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[-2/3, 1
- p, -p, 1/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[-2/3, -p,
 1 - p, 1/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p/x^6,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^6,x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^6,x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p/x**6,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^6+bx^3+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^6,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

3.270 $\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx$

Optimal. Leaf size=168

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-p)x^6}$$

```
[Out] -(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2*(1 - p), -p, -p,
3 - 2*p, -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a
*c])/(2*c*x^3)])/(3*(1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi [A] time = 0.324099, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-p)x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] -(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2*(1 - p), -p, -p,
3 - 2*p, -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a
*c])/(2*c*x^3)])/(3*(1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi in Sympy [A] time = 29.5427, size = 150, normalized size = 0.89

$$-\frac{\left(\frac{b+2cx^3-\sqrt{-4ac+b^2}}{2cx^3}\right)^{-p}\left(\frac{b+2cx^3+\sqrt{-4ac+b^2}}{2cx^3}\right)^{-p}(a+bx^3+cx^6)^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{1}{x^3}\right)^{-2p+2}\operatorname{appellf}_1\left(-2p+2,-p,-p,-2p+3,-\frac{b-\sqrt{-4a}}{2cx}\right.}{6(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)**p/x**7,x)
```

```
[Out] -((b + 2*c*x**3 - sqrt(-4*a*c + b**2))/(2*c*x**3))**(-p)*((b + 2*
c*x**3 + sqrt(-4*a*c + b**2))/(2*c*x**3))**(-p)*(a + b*x**3 + c*x
**6)**p*(x**(-3))**(2*p)*(x**(-3))**(-2*p + 2)*appellf1(-2*p + 2,
```

```
 -p, -p, -2*p + 3, -(b - sqrt(-4*a*c + b**2))/(2*c*x**3), -(b + s
qrt(-4*a*c + b**2))/(2*c*x**3))/(6*(-p + 1))
```

Mathematica [B] time = 3.1528, size = 507, normalized size = 3.02

$$\frac{c4^{-p-1}(2p-3)\left(\sqrt{b^2-4ac}+b+2cx^3\right)\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+1\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{c}\right)^{p+1}\left(\frac{-\sqrt{}\cdots\right.}{3(p-1)x^3\left(2c(2p-3)x^3F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)-p\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(3-2p;1-p,-p;4-\cdots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] (4^(-1 - p)*c*(-3 + 2*p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/c)^(1
 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^(-1 + p)*AppellF1[2 - 2*
p, -p, -p, 3 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqr
t[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1 + p)*(1 + (b - Sqrt[b^2 - 4*a*
c])/(2*c*x^3))^p*x^3*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*(2*c
*(-3 + 2*p)*x^3*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -(b + Sqrt[b^2
 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)] - p*((b
 + Sqrt[b^2 - 4*a*c])*AppellF1[3 - 2*p, 1 - p, -p, 4 - 2*p, -(b +
 Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)
] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[3 - 2*p, -p, 1 - p, 4 - 2*p,
 -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*
c*x^3)])))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)^p/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^7,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^7,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)**p/x**7,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^6+bx^3+a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^p/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

3.271 $\int \frac{x^m}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=32

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};-x^4\right)}{m+1}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 +
 m)
```

Rubi [A] time = 0.0220683, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};-x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 +
 m)
```

Rubi in Sympy [A] time = 4.52388, size = 24, normalized size = 0.75

$$\frac{x^{m+1}{}_2F_1\left(\begin{matrix}2,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-x^4\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(x**8+2*x**4+1),x)
```

```
[Out] x**(m + 1)*hyper((2, m/4 + 1/4), (m/4 + 5/4,), -x**4)/(m + 1)
```

Mathematica [A] time = 0.0201887, size = 34, normalized size = 1.06

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+1}{4}+1;-x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -x^4])/
(1 + m)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + 2x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(x^8+2*x^4+1),x)
```

```
[Out] int(x^m/(x^8+2*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + 2x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8 + 2x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + 2*x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(x^4+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(x**8+2*x**4+1),x)
```

```
[Out] Integral(x**m/(x**4 + 1)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

3.272 $\int \frac{x^9}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=30

$$\frac{3x^2}{4}-\frac{3}{4}\tan^{-1}\left(x^2\right)-\frac{x^6}{4\left(x^4+1\right)}$$

```
[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rubi [A] time = 0.0359942, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{3x^2}{4}-\frac{3}{4}\tan^{-1}\left(x^2\right)-\frac{x^6}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(1 + 2*x^4 + x^8),x]

[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rubi in Sympy [A] time = 8.04813, size = 24, normalized size = 0.8

$$-\frac{x^6}{4\left(x^4+1\right)}+\frac{3x^2}{4}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**8+2*x**4+1),x)

[Out] -x**6/(4*(x**4 + 1)) + 3*x**2/4 - 3*atan(x**2)/4
```

Mathematica [A] time = 0.02205, size = 24, normalized size = 0.8

$$\frac{1}{4}\left(x^2\left(\frac{1}{x^4+1}+2\right)-3\tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(1 + 2*x^4 + x^8),x]

[Out] (x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

Maple [A] time = 0.009, size = 25, normalized size = 0.8

$$\frac{x^2}{2}+\frac{x^2}{4\,x^4+4}-\frac{3\,\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^8+2*x^4+1),x)

[Out] 1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

Maxima [A] time = 0.841102, size = 32, normalized size = 1.07

$$\frac{1}{2}\,x^2+\frac{x^2}{4\,(x^4+1)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + 2*x^4 + 1),x, algorithm="maxima")

[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Fricas [A] time = 0.249563, size = 42, normalized size = 1.4

$$\frac{2\,x^6+3\,x^2-3\,\left(x^4+1\right)\,\arctan\left(x^2\right)}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + 2*x^4 + 1),x, algorithm="fricas")

[Out] 1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] time = 0.296007, size = 22, normalized size = 0.73

$$\frac{x^2}{2} + \frac{x^2}{4x^4 + 4} - \frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**8+2*x**4+1),x)
```

```
[Out] x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

GIAC/XCAS [A] time = 0.284733, size = 32, normalized size = 1.07

$$\frac{1}{2}\,x^2 + \frac{x^2}{4\,(x^4 + 1)} - \frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

3.273 $\int \frac{x^7}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=22

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

```
[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4
```

Rubi [A] time = 0.026405, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 + 2*x^4 + x^8),x]

[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4
```

Rubi in Sympy [A] time = 4.80556, size = 15, normalized size = 0.68

$$\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**8+2*x**4+1),x)

[Out] log(x**4 + 1)/4 + 1/(4*(x**4 + 1))
```

Mathematica [A] time = 0.00798902, size = 18, normalized size = 0.82

$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log\left(x^4+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((1 + x^4)^(-1) + Log[1 + x^4])/4
```

Maple [A] time = 0.009, size = 19, normalized size = 0.9

$$\frac{1}{4\,x^{4}+4}+\frac{\ln\left(x^{4}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^8+2*x^4+1),x)
```

```
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

Maxima [A] time = 0.792124, size = 24, normalized size = 1.09

$$\frac{1}{4\left(x^{4}+1\right)}+\frac{1}{4}\,\log\left(x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Fricas [A] time = 0.245339, size = 31, normalized size = 1.41

$$\frac{\left(x^{4}+1\right)\log\left(x^{4}+1\right)+1}{4\left(x^{4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

Sympy [A] time = 0.241917, size = 15, normalized size = 0.68

$$\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**8+2*x**4+1),x)
```

```
[Out] log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

GIAC/XCAS [A] time = 0.275785, size = 24, normalized size = 1.09

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\ln\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4/(x^4 + 1) + 1/4*ln(x^4 + 1)
```

3.274 $\int \frac{x^5}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

```
[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rubi [A] time = 0.0267787, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 + 2*x^4 + x^8),x]

[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rubi in Sympy [A] time = 5.84579, size = 15, normalized size = 0.65

$$-\frac{x^2}{4\left(x^4+1\right)}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**8+2*x**4+1),x)

[Out] -x**2/(4*(x**4 + 1)) + atan(x**2)/4
```

Mathematica [A] time = 0.0138274, size = 23, normalized size = 1.

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Maple [A] time = 0.009, size = 20, normalized size = 0.9

$$-\frac{x^2}{4\,x^4+4}+\frac{\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Maxima [A] time = 0.844045, size = 26, normalized size = 1.13

$$-\frac{x^2}{4\,(x^4+1)}+\frac{1}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Fricas [A] time = 0.245134, size = 32, normalized size = 1.39

$$-\frac{x^2-\left(x^4+1\right)\arctan\left(x^2\right)}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] time = 0.322669, size = 15, normalized size = 0.65

$$-\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**8+2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 + 4) + atan(x**2)/4
```

GIAC/XCAS [A] time = 0.288737, size = 26, normalized size = 1.13

$$-\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.275 $\int \frac{x^3}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{4\left(x^4+1\right)}$$

```
[Out] -1/(4*(1 + x^4))
```

Rubi [A] time = 0.00693275, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{1}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + 2*x^4 + x^8),x]

[Out] -1/(4*(1 + x^4))
```

Rubi in Sympy [A] time = 2.85374, size = 8, normalized size = 0.73

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**8+2*x**4+1),x)

[Out] -1/(4*(x**4 + 1))
```

Mathematica [A] time = 0.00306416, size = 11, normalized size = 1.

$$-\frac{1}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/(4*(1 + x^4))
```

Maple [A] time = 0.004, size = 10, normalized size = 0.9

$$-\frac{1}{4\,x^{4}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^8+2*x^4+1),x)
```

```
[Out] -1/4/(x^4+1)
```

Maxima [A] time = 0.766338, size = 12, normalized size = 1.09

$$-\frac{1}{4\,(x^{4}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 + 1)
```

Fricas [A] time = 0.248724, size = 12, normalized size = 1.09

$$-\frac{1}{4\,(x^{4}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 + 1)
```

Sympy [A] time = 0.227761, size = 8, normalized size = 0.73

$$-\frac{1}{4x^{4}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**8+2*x**4+1),x)
```

```
[Out] -1/(4*x**4 + 4)
```

GIAC/XCAS [A] time = 0.261829, size = 12, normalized size = 1.09

$$-\frac{1}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + 1)
```

3.276 $\int \frac{x}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{x^2}{4\left(x^4+1\right)}$$

```
[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rubi [A] time = 0.0190204, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + 2*x^4 + x^8),x]

[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rubi in Sympy [A] time = 3.63516, size = 15, normalized size = 0.65

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**8+2*x**4+1),x)

[Out] x**2/(4*(x**4 + 1)) + atan(x**2)/4
```

Mathematica [A] time = 0.00789238, size = 20, normalized size = 0.87

$$\frac{1}{4}\left(\tan^{-1}\left(x^2\right)+\frac{x^2}{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + 2*x^4 + x^8),x]

[Out] (x^2/(1 + x^4) + ArcTan[x^2])/4
```

Maple [A] time = 0.005, size = 20, normalized size = 0.9

$$\frac{x^2}{4\,x^4+4}+\frac{\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^8+2*x^4+1),x)

[Out] 1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Maxima [A] time = 0.855547, size = 26, normalized size = 1.13

$$\frac{x^2}{4\,(x^4+1)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + 2*x^4 + 1),x, algorithm="maxima")

[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Fricas [A] time = 0.245911, size = 31, normalized size = 1.35

$$\frac{x^2+\left(x^4+1\right)\arctan\left(x^2\right)}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + 2*x^4 + 1),x, algorithm="fricas")

[Out] 1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] time = 0.281674, size = 15, normalized size = 0.65

$$\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**8+2*x**4+1),x)
```

```
[Out] x**2/(4*x**4 + 4) + atan(x**2)/4
```

GIAC/XCAS [A] time = 0.260709, size = 26, normalized size = 1.13

$$\frac{x^2}{4\,(x^4+1)}+\frac{1}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.277 $\int \frac{1}{x(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \log(x)$$

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rubi [A] time = 0.028266, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + 2*x^4 + x^8)),x]

[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rubi in Sympy [A] time = 5.37042, size = 22, normalized size = 0.92

$$\frac{\log\left(x^4\right)}{4} - \frac{\log\left(x^4+1\right)}{4} + \frac{1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**8+2*x**4+1),x)

[Out] log(x**4)/4 - log(x**4 + 1)/4 + 1/(4*(x**4 + 1))
```

Mathematica [A] time = 0.0133068, size = 24, normalized size = 1.

$$\frac{1}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + 2*x^4 + x^8)),x]

[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Maple [A] time = 0.019, size = 21, normalized size = 0.9

$$\frac{1}{4\,x^{4}+4}+\ln\left(x\right)-\frac{\ln\left(x^{4}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^8+2*x^4+1),x)

[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

Maxima [A] time = 0.764091, size = 32, normalized size = 1.33

$$\frac{1}{4\,(x^{4}+1)}-\frac{1}{4}\,\log\left(x^{4}+1\right)+\frac{1}{4}\,\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x),x, algorithm="maxima")

[Out] 1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.253871, size = 43, normalized size = 1.79

$$-\frac{\left(x^{4}+1\right)\,\log\left(x^{4}+1\right)-4\,\left(x^{4}+1\right)\,\log\left(x\right)-1}{4\,(x^{4}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x),x, algorithm="fricas")

[Out] -1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

Sympy [A] time = 0.301816, size = 19, normalized size = 0.79

$$\log(x) - \frac{\log(x^4 + 1)}{4} + \frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**8+2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

GIAC/XCAS [A] time = 0.303119, size = 39, normalized size = 1.62

$$\frac{x^4 + 2}{4(x^4 + 1)} - \frac{1}{4}\ln(x^4 + 1) + \frac{1}{4}\ln(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 + 2)/(x^4 + 1) - 1/4*ln(x^4 + 1) + 1/4*ln(x^4)
```

3.278 $\int \frac{1}{x^3(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=30

$$-\frac{3}{4x^2}-\frac{3}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^2\left(x^4+1\right)}$$

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rubi [A] time = 0.0331186, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{3}{4x^2}-\frac{3}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^2\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 + 2*x^4 + x^8)),x]

[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rubi in Sympy [A] time = 7.58688, size = 26, normalized size = 0.87

$$-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}-\frac{3}{4x^2}+\frac{1}{4x^2\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**8+2*x**4+1),x)

[Out] -3*atan(x**2)/4 - 3/(4*x**2) + 1/(4*x**2*(x**4 + 1))
```

Mathematica [A] time = 0.0181117, size = 30, normalized size = 1.

$$-\frac{1}{2x^2}+\frac{3}{4}\tan^{-1}\left(\frac{1}{x^2}\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]

[Out] -1/(2*x^2) - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

Maple [A] time = 0.015, size = 25, normalized size = 0.8

$$-\frac{x^2}{4x^4+4}-\frac{3\arctan\left(x^2\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^8+2*x^4+1),x)

[Out] -1/4*x^2/(x^4+1)-3/4*arctan(x^2)-1/2/x^2
```

Maxima [A] time = 0.851052, size = 34, normalized size = 1.13

$$-\frac{3x^4+2}{4\left(x^6+x^2\right)}-\frac{3}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^3),x, algorithm="maxima")

[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Fricas [A] time = 0.251319, size = 42, normalized size = 1.4

$$-\frac{3x^4+3\left(x^6+x^2\right)\arctan\left(x^2\right)+2}{4\left(x^6+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^3),x, algorithm="fricas")

[Out] -1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

Sympy [A] time = 0.394401, size = 26, normalized size = 0.87

$$-\frac{3x^4+2}{4x^6+4x^2}-\frac{3\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**8+2*x**4+1),x)
```

```
[Out] -(3*x**4 + 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

GIAC/XCAS [A] time = 0.299475, size = 34, normalized size = 1.13

$$-\frac{3\,x^4+2}{4\,(x^6+x^2)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^3),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

3.279 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{4(x^4+1)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

```
[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rubi [A] time = 0.0358058, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{1}{4(x^4+1)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 + 2*x^4 + x^8)),x]

[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rubi in Sympy [A] time = 5.74552, size = 29, normalized size = 0.88

$$-\frac{\log\left(x^4\right)}{2}+\frac{\log\left(x^4+1\right)}{2}-\frac{1}{4(x^4+1)}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**8+2*x**4+1),x)

[Out] -log(x**4)/2 + log(x**4 + 1)/2 - 1/(4*(x**4 + 1)) - 1/(4*x**4)
```

Mathematica [A] time = 0.0191513, size = 33, normalized size = 1.

$$-\frac{1}{4(x^4+1)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Maple [A] time = 0.021, size = 28, normalized size = 0.9

$$-\frac{1}{4x^4} - \frac{1}{4x^4+4} - 2\ln(x) + \frac{\ln(x^4+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^8+2*x^4+1),x)
```

```
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

Maxima [A] time = 0.759771, size = 45, normalized size = 1.36

$$-\frac{2x^4+1}{4(x^8+x^4)} + \frac{1}{2}\log(x^4+1) - \frac{1}{2}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^5),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Fricas [A] time = 0.252424, size = 59, normalized size = 1.79

$$-\frac{2x^4 - 2(x^8+x^4)\log(x^4+1) + 8(x^8+x^4)\log(x) + 1}{4(x^8+x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^5),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) +
 1)/(x^8 + x^4)
```

Sympy [A] time = 0.428095, size = 29, normalized size = 0.88

$$-\frac{2x^4+1}{4x^8+4x^4}-2\log(x)+\frac{\log(x^4+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**8+2*x**4+1),x)

[Out] -(2*x**4 + 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

GIAC/XCAS [A] time = 0.29694, size = 45, normalized size = 1.36

$$-\frac{2\,x^4+1}{4\,(x^8+x^4)}+\frac{1}{2}\ln(x^4+1)-\frac{1}{2}\ln(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^5),x, algorithm="giac")

[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*ln(x^4 + 1) - 1/2*ln(x^4)
```

3.280 $\int \frac{1}{x^7(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=37

$$-\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{5}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^6\left(x^4+1\right)}$$

```
[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rubi [A] time = 0.0417847, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{5}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^6\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rubi in Sympy [A] time = 9.52098, size = 32, normalized size = 0.86

$$\frac{5\,\text{atan}\left(x^2\right)}{4}+\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{1}{4x^6\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(x**8+2*x**4+1),x)
```

```
[Out] 5*atan(x**2)/4 + 5/(4*x**2) - 5/(12*x**6) + 1/(4*x**6*(x**4 + 1))
```

Mathematica [A] time = 0.0163911, size = 33, normalized size = 0.89

$$-\frac{1}{6x^6}+\frac{1}{x^2}-\frac{5}{4}\tan^{-1}\left(\frac{1}{x^2}\right)+\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

Maple [A] time = 0.017, size = 28, normalized size = 0.8

$$\frac{x^2}{4x^4+4}+\frac{5\arctan\left(x^2\right)}{4}-\frac{1}{6x^6}+x^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x^2/(x^4+1)+5/4*arctan(x^2)-1/6/x^6+1/x^2
```

Maxima [A] time = 0.860928, size = 41, normalized size = 1.11

$$\frac{15x^8+10x^4-2}{12\left(x^{10}+x^6\right)}+\frac{5}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^7),x, algorithm="maxima")
```

```
[Out] 1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

Fricas [A] time = 0.245721, size = 49, normalized size = 1.32

$$\frac{15x^8+10x^4+15\left(x^{10}+x^6\right)\arctan\left(x^2\right)-2}{12\left(x^{10}+x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^7),x, algorithm="fricas")
```

```
[Out] 1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 +
x^6)
```

Sympy [A] time = 0.515027, size = 29, normalized size = 0.78

$$\frac{5\operatorname{atan}\left(x^{2}\right)}{4}+\frac{15x^{8}+10x^{4}-2}{12x^{10}+12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**8+2*x**4+1),x)
```

```
[Out] 5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

GIAC/XCAS [A] time = 0.271181, size = 42, normalized size = 1.14

$$\frac{x^{2}}{4\left(x^{4}+1\right)}+\frac{6\,x^{4}-1}{6\,x^{6}}+\frac{5}{4}\,\arctan\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^7),x, algorithm="giac")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

3.281 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=104

$$\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{x^5}{4\left(x^4+1\right)}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2
]) - (5*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x
 + x^2])/(16*Sqrt[2]) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.110529, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{x^5}{4\left(x^4+1\right)}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2
]) - (5*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x
 + x^2])/(16*Sqrt[2]) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 18.1511, size = 95, normalized size = 0.91

$$-\frac{x^5}{4\left(x^4+1\right)}+\frac{5x}{4}+\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**8+2*x**4+1),x)
```

```
[Out] -x**5/(4*(x**4 + 1)) + 5*x/4 + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1
)/32 - 5*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sq
```

```
rt(2)*x - 1)/16 - 5*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Mathematica [A] time = 0.140497, size = 94, normalized size = 0.9

$$\frac{1}{32}\left(\frac{8x}{x^4+1}+5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+32x+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*S
qrt[2]*ArcTan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
 - 5*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.012, size = 69, normalized size = 0.7

$$x+\frac{x}{4x^4+4}-\frac{5\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}-\frac{5\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}-\frac{5\sqrt{2}}{32}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^8+2*x^4+1),x)
```

```
[Out] x+1/4*x/(x^4+1)-5/16*arctan(1+2^(1/2)*x)*2^(1/2)-5/16*arctan(2^(1
/2)*x-1)*2^(1/2)-5/32*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)
*x))
```

Maxima [A] time = 0.849475, size = 112, normalized size = 1.08

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+x+\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 5/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x
^4 + 1)
```

Fricas [A] time = 0.266356, size = 181, normalized size = 1.74

$$\frac{32\,x^5 + 20\,\sqrt{2}\left(x^4+1\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right) + 20\,\sqrt{2}\left(x^4+1\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right) - 5\,\sqrt{2}\left(x^4+1\right)\log\left(x^2+\cdots\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/32*(32*x^5 + 20*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x + sqrt(2)
*sqrt(x^2 + sqrt(2)*x + 1) + 1)) + 20*sqrt(2)*(x^4 + 1)*arctan(1/
(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) - 5*sqrt(2)*
(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + 5*sqrt(2)*(x^4 + 1)*log(x^2
- sqrt(2)*x + 1) + 40*x)/(x^4 + 1)
```

Sympy [A] time = 0.514333, size = 90, normalized size = 0.87

$$x + \frac{x}{4x^4+4} + \frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**8+2*x**4+1),x)
```

```
[Out] x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*s
qrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x -
1)/16 - 5*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

GIAC/XCAS [A] time = 0.301405, size = 112, normalized size = 1.08

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{5}{32}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)+\frac{5}{32}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)+x+\frac{x}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*ln(x^2 + sqrt(
2)*x + 1) + 5/32*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4
 + 1)
```

3.282 $\int \frac{x^6}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{x^3}{4\left(x^4+1\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -x^3/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/
(16*Sqrt[2]) - (3*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.10567, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{x^3}{4\left(x^4+1\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^3/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/
(16*Sqrt[2]) - (3*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 17.5987, size = 90, normalized size = 0.91

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**8+2*x**4+1),x)
```

```
[Out] -x**3/(4*(x**4 + 1)) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3
*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x
```

```
- 1)/16 + 3*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Mathematica [A] time = 0.0992917, size = 93, normalized size = 0.94

$$\frac{1}{32}\left(3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{8x^3}{x^4+1}-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*S
qrt[2]*Log[1 + Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.01, size = 70, normalized size = 0.7

$$-\frac{x^3}{4x^4+4}+\frac{3\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}+\frac{3\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}+\frac{3\sqrt{2}}{32}\ln\left(\frac{1+x^2-\sqrt{2}x}{1+x^2+\sqrt{2}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x^3/(x^4+1)+3/16*arctan(1+2^(1/2)*x)*2^(1/2)+3/16*arctan(2^(
1/2)*x-1)*2^(1/2)+3/32*2^(1/2)*ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2
)*x))
```

Maxima [A] time = 0.847641, size = 113, normalized size = 1.14

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2))) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sq
rt(2)*log(x^2 + sqrt(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x
 + 1)
```

Fricas [A] time = 0.279266, size = 177, normalized size = 1.79

$$-\frac{8\,x^3+12\,\sqrt{2}\left(x^4+1\right)\,\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)+12\,\sqrt{2}\left(x^4+1\right)\,\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)+3\,\sqrt{2}\left(x^4+1\right)\,\log\left(x^2+\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*x^3 + 12*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x + sqrt(2)
*sqrt(x^2 + sqrt(2)*x + 1) + 1)) + 12*sqrt(2)*(x^4 + 1)*arctan(1/
(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) + 3*sqrt(2)*
(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) - 3*sqrt(2)*(x^4 + 1)*log(x^2
- sqrt(2)*x + 1))/(x^4 + 1)
```

Sympy [A] time = 0.525326, size = 90, normalized size = 0.91

$$-\frac{x^3}{4x^4+4}+\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**8+2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*s
qrt(2)*log(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x -
1)/16 + 3*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

GIAC/XCAS [A] time = 0.267445, size = 113, normalized size = 1.14

$$-\frac{x^3}{4\,(x^4+1)}+\frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,x+\sqrt{2}\Big)\right)+\frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,x-\sqrt{2}\Big)\right)-\frac{3}{32}\,\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)+\frac{3}{32}\,\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2))) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sq
rt(2)*ln(x^2 + sqrt(2)*x + 1) + 3/32*sqrt(2)*ln(x^2 - sqrt(2)*x +
 1)
```

3.283 $\int \frac{x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -x/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*x]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) +
 Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2])
```

Rubi [A] time = 0.0992792, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*x]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) +
 Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 16.5481, size = 82, normalized size = 0.85

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**8+2*x**4+1),x)
```

```
[Out] -x/(4*(x**4 + 1)) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2
)*log(x**2 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 +
 sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Mathematica [A] time = 0.15934, size = 90, normalized size = 0.93

$$\frac{1}{32}\left(-\frac{8x}{x^4+1}-\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*A
rcTan[1 + Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]
*Log[1 + Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.009, size = 68, normalized size = 0.7

$$-\frac{x}{4\,x^4+4}+\frac{\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}+\frac{\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}+\frac{\sqrt{2}}{32}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x/(x^4+1)+1/16*arctan(1+2^(1/2)*x)*2^(1/2)+1/16*arctan(2^(1/
2)*x-1)*2^(1/2)+1/32*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*
x))
```

Maxima [A] time = 0.853659, size = 111, normalized size = 1.14

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(
```

```
2)*x + 1) - 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 +
1)
```

Fricas [A] time = 0.271101, size = 173, normalized size = 1.78

$$-\frac{4\sqrt{2}(x^4+1)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)+4\sqrt{2}(x^4+1)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)-\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{2}x+1\right)+}{32\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2
 + sqrt(2)*x + 1) + 1)) + 4*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x
 + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) - sqrt(2)*(x^4 + 1)*lo
g(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x +
1) + 8*x)/(x^4 + 1)
```

Sympy [A] time = 0.509865, size = 82, normalized size = 0.85

$$-\frac{x}{4x^4+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**8+2*x**4+1),x)
```

```
[Out] -x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*
log(x**2 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + s
qrt(2)*atan(sqrt(2)*x + 1)/16
```

GIAC/XCAS [A] time = 0.296154, size = 111, normalized size = 1.14

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*ln(x^2 + sqrt(2
)*x + 1) - 1/32*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

3.284 $\int \frac{x^2}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=99

$$\frac{\log \left(x^2-\sqrt{2} x+1\right)}{16 \sqrt{2}}-\frac{\log \left(x^2+\sqrt{2} x+1\right)}{16 \sqrt{2}}+\frac{x^3}{4 \left(x^4+1\right)}-\frac{\tan^{-1}\left(1-\sqrt{2} x\right)}{8 \sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2} x+1\right)}{8 \sqrt{2}}$$

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1
+ Sqrt[2]*x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2])
- Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2])
```

Rubi [A] time = 0.104604, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{\log \left(x^2-\sqrt{2} x+1\right)}{16 \sqrt{2}}-\frac{\log \left(x^2+\sqrt{2} x+1\right)}{16 \sqrt{2}}+\frac{x^3}{4 \left(x^4+1\right)}-\frac{\tan^{-1}\left(1-\sqrt{2} x\right)}{8 \sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2} x+1\right)}{8 \sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1
+ Sqrt[2]*x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2])
- Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 17.3987, size = 83, normalized size = 0.84

$$\frac{x^3}{4 \left(x^4+1\right)}+\frac{\sqrt{2} \log \left(x^2-\sqrt{2} x+1\right)}{32}-\frac{\sqrt{2} \log \left(x^2+\sqrt{2} x+1\right)}{32}+\frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2} x-1\right)}{16}+\frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2} x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**8+2*x**4+1),x)
```

```
[Out] x**3/(4*(x**4 + 1)) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt
(2)*log(x**2 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16
 + sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Mathematica [A] time = 0.103118, size = 92, normalized size = 0.93

$$\frac{1}{32}\left(\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{8x^3}{x^4+1}-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*
ArcTan[1 + Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2
]*Log[1 + Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.006, size = 70, normalized size = 0.7

$$\frac{x^3}{4\,x^4+4}+\frac{\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}+\frac{\sqrt{2}}{32}\ln\left(\frac{1+x^2-\sqrt{2}x}{1+x^2+\sqrt{2}x}\right)+\frac{\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x^3/(x^4+1)+1/16*arctan(2^(1/2)*x-1)*2^(1/2)+1/32*2^(1/2)*ln(
(1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+1/16*arctan(1+2^(1/2)*x)*2^(
1/2)
```

Maxima [A] time = 0.844844, size = 113, normalized size = 1.14

$$\frac{x^3}{4\,(x^4+1)}+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
))) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqr
```

```
t(2)*log(x^2 + sqrt(2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x
+ 1)
```

Fricas [A] time = 0.260577, size = 176, normalized size = 1.78

$$\frac{8\,x^3 - 4\,\sqrt{2}\left(x^4+1\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right) - 4\,\sqrt{2}\left(x^4+1\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right) - \sqrt{2}\left(x^4+1\right)\log\left(x^2+\sqrt{2}x\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 4*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x + sqrt(2)*s
qrt(x^2 + sqrt(2)*x + 1) + 1)) - 4*sqrt(2)*(x^4 + 1)*arctan(1/(sq
rt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) - sqrt(2)*(x^4
+ 1)*log(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(
2)*x + 1))/(x^4 + 1)
```

Sympy [A] time = 0.532567, size = 83, normalized size = 0.84

$$\frac{x^3}{4x^4+4} + \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**8+2*x**4+1),x)
```

```
[Out] x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2
)*log(x**2 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 +
 sqrt(2)*atan(sqrt(2)*x + 1)/16
```

GIAC/XCAS [A] time = 0.277819, size = 113, normalized size = 1.14

$$\frac{x^3}{4\left(x^4+1\right)} + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
))) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqr
t(2)*ln(x^2 + sqrt(2)*x + 1) + 1/32*sqrt(2)*ln(x^2 - sqrt(2)*x +
1)
```

3.285 $\int \frac{1}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$\frac{x}{4(x^4+1)}-\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcT
an[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16
*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.0930661, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$

$$\frac{x}{4(x^4+1)}-\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcT
an[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16
*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 14.3421, size = 88, normalized size = 0.91

$$\frac{x}{4(x^4+1)}-\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8+2*x**4+1),x)
```

```
[Out] x/(4*(x**4 + 1)) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqr
t(2)*log(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)
/16 + 3*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Mathematica [A] time = 0.0848057, size = 91, normalized size = 0.94

$$\frac{1}{32}\left(\frac{8x}{x^4+1}-3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*Ar
cTan[1 + Sqrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt
[2]*Log[1 + Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.005, size = 68, normalized size = 0.7

$$\frac{x}{4\,x^4+4}+\frac{3\,\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}+\frac{3\,\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}+\frac{3\,\sqrt{2}}{32}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x/(x^4+1)+3/16*arctan(1+2^(1/2)*x)*2^(1/2)+3/16*arctan(2^(1/2
)*x-1)*2^(1/2)+3/32*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x
))
```

Maxima [A] time = 0.86165, size = 111, normalized size = 1.14

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(
```

```
2)*x + 1) - 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 +
1)
```

Fricas [A] time = 0.281308, size = 174, normalized size = 1.79

$$-\frac{12\sqrt{2}\left(x^4+1\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)+12\sqrt{2}\left(x^4+1\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)-3\sqrt{2}\left(x^4+1\right)\log\left(x^2+\sqrt{2}x+\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/32*(12*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^
2 + sqrt(2)*x + 1) + 1)) + 12*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)
*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) - 3*sqrt(2)*(x^4 + 1
)*log(x^2 + sqrt(2)*x + 1) + 3*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2
)*x + 1) - 8*x)/(x^4 + 1)
```

Sympy [A] time = 0.511355, size = 88, normalized size = 0.91

$$\frac{x}{4x^4+4}-\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8+2*x**4+1),x)
```

```
[Out] x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(
2)*log(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/1
6 + 3*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

GIAC/XCAS [A] time = 0.276658, size = 111, normalized size = 1.14

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*ln(x^2 + sqrt(2
)*x + 1) - 3/32*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

3.286 $\int \frac{1}{x^2(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$\frac{1}{4x(x^4+1)} - \frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}} + \frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}} - \frac{5}{4x} + \frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[
2]) - (5*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*
x + x^2])/(16*Sqrt[2]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2]
)
```

Rubi [A] time = 0.108605, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$

$$\frac{1}{4x(x^4+1)} - \frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}} + \frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}} - \frac{5}{4x} + \frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[
2]) - (5*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*
x + x^2])/(16*Sqrt[2]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2]
)
```

Rubi in Sympy [A] time = 18.6997, size = 95, normalized size = 0.9

$$-\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16} - \frac{5}{4x} + \frac{1}{4x(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**8+2*x**4+1),x)
```

```
[Out] -5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*sqrt(2)*log(x**2 + sq
rt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*at
an(sqrt(2)*x + 1)/16 - 5/(4*x) + 1/(4*x*(x**4 + 1))
```

Mathematica [A] time = 0.125364, size = 98, normalized size = 0.92

$$\frac{1}{32}\left(-5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{8x^3}{x^4+1}-\frac{32}{x}+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 1
0*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x
^2] + 5*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32
```

Maple [A] time = 0.012, size = 75, normalized size = 0.7

$$-\frac{x^3}{4x^4+4}-\frac{5\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}-\frac{5\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}-\frac{5\sqrt{2}}{32}\ln\left(\frac{1+x^2-\sqrt{2}x}{1+x^2+\sqrt{2}x}\right)-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x^3/(x^4+1)-5/16*arctan(1+2^(1/2)*x)*2^(1/2)-5/16*arctan(2^(
1/2)*x-1)*2^(1/2)-5/32*2^(1/2)*ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2
)*x))-1/x
```

Maxima [A] time = 0.847049, size = 119, normalized size = 1.12

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{5x^4+4}{4\left(x^5+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) - 5/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 +
 4)/(x^5 + x)
```

Fricas [A] time = 0.271719, size = 178, normalized size = 1.68

$$-\frac{40\,x^4 - 20\,\sqrt{2}\left(x^5 + x\right)\arctan\left(\frac{1}{\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} + 1}\right) - 20\,\sqrt{2}\left(x^5 + x\right)\arctan\left(\frac{1}{\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} - 1}\right) - 5\,\sqrt{2}\left(x^5 + x\right)\log\left(x^2\right.}{32\left(x^5 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] -1/32*(40*x^4 - 20*sqrt(2)*(x^5 + x)*arctan(1/(sqrt(2)*x + sqrt(2
)*sqrt(x^2 + sqrt(2)*x + 1) + 1)) - 20*sqrt(2)*(x^5 + x)*arctan(1
/(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) - 5*sqrt(2)
*(x^5 + x)*log(x^2 + sqrt(2)*x + 1) + 5*sqrt(2)*(x^5 + x)*log(x^2
 - sqrt(2)*x + 1) + 32)/(x^5 + x)
```

Sympy [A] time = 0.607307, size = 95, normalized size = 0.9

$$-\frac{5x^4 + 4}{4x^5 + 4x} - \frac{5\sqrt{2}\log\left(x^2 - \sqrt{2}x + 1\right)}{32} + \frac{5\sqrt{2}\log\left(x^2 + \sqrt{2}x + 1\right)}{32} - \frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x - 1\right)}{16} - \frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x + 1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**8+2*x**4+1),x)
```

```
[Out] -(5*x**4 + 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1
)/32 + 5*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sq
rt(2)*x - 1)/16 - 5*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

GIAC/XCAS [A] time = 0.293438, size = 119, normalized size = 1.12

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)-\frac{5x^4+4}{4(x^5+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^2),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*ln(x^2 + sqrt(
2)*x + 1) - 5/32*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4
)/(x^5 + x)
```

3.287 $\int \frac{1}{x^4(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^3(x^4+1)}+\frac{7\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqr
t[2]*x + x^2])/(16*Sqrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sq
rt[2])
```

Rubi [A] time = 0.104168, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^3(x^4+1)}+\frac{7\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```


```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqr
t[2]*x + x^2])/(16*Sqrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sq
rt[2])
```

Rubi in Sympy [A] time = 17.6087, size = 99, normalized size = 0.93

$$\frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}-\frac{7}{12x^3}+\frac{1}{4x^3(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**8+2*x**4+1),x)
```

```
[Out] 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 7*sqrt(2)*log(x**2 + sqr
t(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 7*sqrt(2)*ata
n(sqrt(2)*x + 1)/16 - 7/(12*x**3) + 1/(4*x**3*(x**4 + 1))
```

Mathematica [A] time = 0.132524, size = 96, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{24x}{x^4+1}-\frac{32}{x^3}+21\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-21\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+42\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-42\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] -
42*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x +
 x^2] - 21*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/96
```

Maple [A] time = 0.012, size = 73, normalized size = 0.7

$$-\frac{x}{4x^4+4}-\frac{7\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}-\frac{7\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}-\frac{7\sqrt{2}}{32}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x/(x^4+1)-7/16*arctan(1+2^(1/2)*x)*2^(1/2)-7/16*arctan(2^(1/
2)*x-1)*2^(1/2)-7/32*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*
x))-1/3/x^3
```

Maxima [A] time = 0.852348, size = 122, normalized size = 1.15

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{7x^4+4}{12(x^7+x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 7/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4
+ 4)/(x^7 + x^3)
```

Fricas [A] time = 0.274103, size = 192, normalized size = 1.81

$$-\frac{56x^4-84\sqrt{2}(x^7+x^3)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-84\sqrt{2}(x^7+x^3)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)+21\sqrt{2}(x^7+x^3)\log(}{96(x^7+x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] -1/96*(56*x^4 - 84*sqrt(2)*(x^7 + x^3)*arctan(1/(sqrt(2)*x + sqrt
(2)*sqrt(x^2 + sqrt(2)*x + 1) + 1)) - 84*sqrt(2)*(x^7 + x^3)*arct
an(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) + 21*sq
rt(2)*(x^7 + x^3)*log(x^2 + sqrt(2)*x + 1) - 21*sqrt(2)*(x^7 + x^
3)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^7 + x^3)
```

Sympy [A] time = 0.642443, size = 97, normalized size = 0.92

$$-\frac{7x^4+4}{12x^7+12x^3}+\frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**8+2*x**4+1),x)
```

```
[Out] -(7*x**4 + 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*
x + 1)/32 - 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*at
an(sqrt(2)*x - 1)/16 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

GIAC/XCAS [A] time = 0.274178, size = 117, normalized size = 1.1

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{7}{32}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)+\frac{7}{32}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\left(x^4+1\right)}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^4),x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*ln(x^2 + sqrt(
2)*x + 1) + 7/32*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1
) - 1/3/x^3
```

3.288 $\int \frac{1}{x^6(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^5\left(x^4+1\right)}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[
2]*x])/(8*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*L
og[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^
2])/(16*Sqrt[2])
```

Rubi [A] time = 0.115369, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^5\left(x^4+1\right)}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[
2]*x])/(8*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*L
og[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^
2])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 20.4032, size = 104, normalized size = 0.92

$$\frac{9\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{9\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{9}{4x}-\frac{9}{20x^5}+\frac{1}{4x^5\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**8+2*x**4+1),x)
```

```
[Out] 9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqr
t(2)*x + 1)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*ata
n(sqrt(2)*x + 1)/16 + 9/(4*x) - 9/(20*x**5) + 1/(4*x**5*(x**4 + 1
))
```

Mathematica [A] time = 0.132757, size = 103, normalized size = 0.91

$$\frac{1}{160}\left(-\frac{32}{x^5}+45\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-45\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{40x^3}{x^4+1}+\frac{320}{x}-90\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+90\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqr
t[2]*x] + 90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - S
qrt[2]*x + x^2] - 45*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160
```

Maple [A] time = 0.015, size = 80, normalized size = 0.7

$$\frac{x^3}{4x^4+4}+\frac{9\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}+\frac{9\sqrt{2}}{32}\ln\left(\frac{1+x^2-\sqrt{2}x}{1+x^2+\sqrt{2}x}\right)+\frac{9\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}-\frac{1}{5x^5}+2x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x^3/(x^4+1)+9/16*arctan(2^(1/2)*x-1)*2^(1/2)+9/32*2^(1/2)*ln(
(1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+9/16*arctan(1+2^(1/2)*x)*2^(
1/2)-1/5/x^5+2/x
```

Maxima [A] time = 0.873783, size = 128, normalized size = 1.13

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{9}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{9}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{45x^8+36x^4-4}{20(x^9+x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^6),x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/20*(45*x^8
+ 36*x^4 - 4)/(x^9 + x^5)
```

Fricas [A] time = 0.276045, size = 198, normalized size = 1.75

$$\frac{360x^8+288x^4-180\sqrt{2}(x^9+x^5)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-180\sqrt{2}(x^9+x^5)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)-45\sqrt{2}(x^9}{160(x^9+x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^6),x, algorithm="fricas")
```

```
[Out] 1/160*(360*x^8 + 288*x^4 - 180*sqrt(2)*(x^9 + x^5)*arctan(1/(sqrt
(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) + 1)) - 180*sqrt(2)*(x^
9 + x^5)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1)
- 1)) - 45*sqrt(2)*(x^9 + x^5)*log(x^2 + sqrt(2)*x + 1) + 45*sqrt
(2)*(x^9 + x^5)*log(x^2 - sqrt(2)*x + 1) - 32)/(x^9 + x^5)
```

Sympy [A] time = 0.738045, size = 102, normalized size = 0.9

$$\frac{9\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{9\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{9\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{9\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{45x^8+36x^4-4}{20x^9+20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**8+2*x**4+1),x)
```

```
[Out] 9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqr
t(2)*x + 1)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*ata
n(sqrt(2)*x + 1)/16 + (45*x**8 + 36*x**4 - 4)/(20*x**9 + 20*x**5)
```

GIAC/XCAS [A] time = 0.263094, size = 130, normalized size = 1.15

$$\frac{x^3}{4\,(x^4+1)} + \frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{9}{32}\,\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right) + \frac{9}{32}\,\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right) + \frac{10\,x^4-1}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^6),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
))) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqr
t(2)*ln(x^2 + sqrt(2)*x + 1) + 9/32*sqrt(2)*ln(x^2 - sqrt(2)*x +
1) + 1/5*(10*x^4 - 1)/x^5
```

3.289 $\int \frac{1}{x^8(1+2x^4+x^8)} \, dx$

Optimal. Leaf size=113

$$-\frac{11}{28x^7}+\frac{11}{12x^3}-\frac{11\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^7\left(x^4+1\right)}-\frac{11\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 -
 Sqrt[2]*x])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2])
 - (11*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[
2]*x + x^2])/(16*Sqrt[2])
```

Rubi [A] time = 0.112744, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$-\frac{11}{28x^7}+\frac{11}{12x^3}-\frac{11\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^7\left(x^4+1\right)}-\frac{11\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 -
 Sqrt[2]*x])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2])
 - (11*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[
2]*x + x^2])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 18.9727, size = 105, normalized size = 0.93

$$-\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{11}{12x^3}-\frac{11}{28x^7}+\frac{1}{4x^7\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**8/(x**8+2*x**4+1),x)
```

```
[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 +
sqrt(2)*x + 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2
)*atan(sqrt(2)*x + 1)/16 + 11/(12*x**3) - 11/(28*x**7) + 1/(4*x**
7*(x**4 + 1))
```

Mathematica [A] time = 0.140038, size = 101, normalized size = 0.89

$$\frac{1}{672}\left(-\frac{96}{x^7}+\frac{168x}{x^4+1}+\frac{448}{x^3}-231\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+231\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-462\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+462\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - S
qrt[2]*x] + 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1
 - Sqrt[2]*x + x^2] + 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672
```

Maple [A] time = 0.015, size = 78, normalized size = 0.7

$$\frac{x}{4x^4+4}+\frac{11\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{16}+\frac{11\sqrt{2}}{32}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)+\frac{11\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{16}-\frac{1}{7x^7}+\frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^8/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*x/(x^4+1)+11/16*arctan(2^(1/2)*x-1)*2^(1/2)+11/32*2^(1/2)*ln(
(1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+11/16*arctan(1+2^(1/2)*x)*2^
(1/2)-1/7/x^7+2/3/x^3
```

Maxima [A] time = 0.854048, size = 128, normalized size = 1.13

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{77x^8+44x^4-12}{84\left(x^{11}+x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^8),x, algorithm="maxima")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sq
rt(2)*x + 1) - 11/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*
x^8 + 44*x^4 - 12)/(x^11 + x^7)
```

Fricas [A] time = 0.260469, size = 198, normalized size = 1.75

$$\frac{616x^8+352x^4-924\sqrt{2}\left(x^{11}+x^7\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-924\sqrt{2}\left(x^{11}+x^7\right)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)+231\sqrt{2}}{672\left(x^{11}+x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^8),x, algorithm="fricas")
```

```
[Out] 1/672*(616*x^8 + 352*x^4 - 924*sqrt(2)*(x^11 + x^7)*arctan(1/(sqr
t(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) + 1)) - 924*sqrt(2)*(x
^11 + x^7)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1
) - 1)) + 231*sqrt(2)*(x^11 + x^7)*log(x^2 + sqrt(2)*x + 1) - 231
*sqrt(2)*(x^11 + x^7)*log(x^2 - sqrt(2)*x + 1) - 96)/(x^11 + x^7)
```

Sympy [A] time = 0.777807, size = 102, normalized size = 0.9

$$-\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{77x^8+44x^4-12}{84x^{11}+84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**8/(x**8+2*x**4+1),x)
```

```
[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 +
sqrt(2)*x + 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2
)*atan(sqrt(2)*x + 1)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 8
4*x**7)
```

GIAC/XCAS [A] time = 0.309526, size = 127, normalized size = 1.12

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{11}{32}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4(x^4+1)}+\frac{14x^4-3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 2*x^4 + 1)*x^8),x, algorithm="giac")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*ln(x^2 + sqr
t(2)*x + 1) - 11/32*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4
+ 1) + 1/21*(14*x^4 - 3)/x^7
```

3.290 $\int \frac{x^m}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=30

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4\right)}{m+1}$$

[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)

Rubi [A] time = 0.0180218, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[x^m/(1 - 2*x^4 + x^8),x]

[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)

Rubi in Sympy [A] time = 4.47393, size = 22, normalized size = 0.73

$$\frac{x^{m+1}{}_2F_1\left(\begin{matrix}2,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle| x^4\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x**m/(x**8-2*x**4+1),x)

[Out] x**(m + 1)*hyper((2, m/4 + 1/4), (m/4 + 5/4,), x**4)/(m + 1)

Mathematica [A] time = 0.0212722, size = 32, normalized size = 1.07

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+1}{4}+1;x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, x^4])/(
1 + m)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{x^{8}-2\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(x^8-2*x^4+1),x)
```

```
[Out] int(x^m/(x^8-2*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{x^{8}-2\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m}}{x^{8}-2\,x^{4}+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - 2*x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(x-1)^2 (x+1)^2 (x^2+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(x**8-2*x**4+1),x)

[Out] Integral(x**m/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - 2\,x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - 2*x^4 + 1),x, algorithm="giac")

[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

3.291 $\int \frac{x^9}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=32

$$\frac{3x^2}{4} - \frac{3}{4}\tanh^{-1}\left(x^2\right) + \frac{x^6}{4\left(1-x^4\right)}$$

```
[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rubi [A] time = 0.0377132, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{3x^2}{4} - \frac{3}{4}\tanh^{-1}\left(x^2\right) + \frac{x^6}{4\left(1-x^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(1 - 2*x^4 + x^8),x]

[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rubi in Sympy [A] time = 8.15939, size = 24, normalized size = 0.75

$$\frac{x^6}{4\left(-x^4+1\right)} + \frac{3x^2}{4} - \frac{3\operatorname{atanh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**8-2*x**4+1),x)

[Out] x**6/(4*(-x**4 + 1)) + 3*x**2/4 - 3*atanh(x**2)/4
```

Mathematica [A] time = 0.0440937, size = 39, normalized size = 1.22

$$\frac{1}{8}\left(3\log\left(1-x^2\right) - 3\log\left(x^2+1\right) + 2\left(\frac{1}{1-x^4}+2\right)x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(1 - 2*x^4 + x^8),x]

[Out] (2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

Maple [A] time = 0.015, size = 41, normalized size = 1.3

$$\frac{x^2}{2}-\frac{1}{8x^2-8}+\frac{3\ln\left(x^2-1\right)}{8}-\frac{1}{8x^2+8}-\frac{3\ln\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^8-2*x^4+1),x)

[Out] 1/2*x^2-1/8/(x^2-1)+3/8*ln(x^2-1)-1/8/(x^2+1)-3/8*ln(x^2+1)
```

Maxima [A] time = 0.769749, size = 46, normalized size = 1.44

$$\frac{1}{2}x^2-\frac{x^2}{4\left(x^4-1\right)}-\frac{3}{8}\log\left(x^2+1\right)+\frac{3}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - 2*x^4 + 1),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

Fricas [A] time = 0.249455, size = 62, normalized size = 1.94

$$\frac{4x^6-6x^2-3\left(x^4-1\right)\log\left(x^2+1\right)+3\left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - 2*x^4 + 1),x, algorithm="fricas")

[Out] 1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x
^2 - 1))/(x^4 - 1)
```

Sympy [A] time = 0.279651, size = 34, normalized size = 1.06

$$\frac{x^2}{2} - \frac{x^2}{4x^4 - 4} + \frac{3\log\left(x^2 - 1\right)}{8} - \frac{3\log\left(x^2 + 1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**8-2*x**4+1),x)
```

```
[Out] x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/
8
```

GIAC/XCAS [A] time = 0.280268, size = 47, normalized size = 1.47

$$\frac{1}{2}x^2 - \frac{x^2}{4\left(x^4 - 1\right)} - \frac{3}{8}\ln\left(x^2 + 1\right) + \frac{3}{8}\ln\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*ln(x^2 + 1) + 3/8*ln(abs(x^2 -
1))
```

3.292 $\int \frac{x^7}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=26

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

```
[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rubi [A] time = 0.031726, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 - 2*x^4 + x^8),x]

[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rubi in Sympy [A] time = 4.82471, size = 15, normalized size = 0.58

$$\frac{\log\left(-x^4+1\right)}{4}+\frac{1}{4\left(-x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**8-2*x**4+1),x)

[Out] log(-x**4 + 1)/4 + 1/(4*(-x**4 + 1))
```

Mathematica [A] time = 0.0101543, size = 22, normalized size = 0.85

$$\frac{1}{4}\log\left(x^4-1\right)-\frac{1}{4\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 - 2*x^4 + x^8),x]

[Out] -1/(4*(-1 + x^4)) + Log[-1 + x^4]/4
```

Maple [A] time = 0.009, size = 19, normalized size = 0.7

$$-\frac{1}{4x^4-4}+\frac{\ln\left(x^4-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^8-2*x^4+1),x)

[Out] -1/4/(x^4-1)+1/4*ln(x^4-1)
```

Maxima [A] time = 0.766091, size = 24, normalized size = 0.92

$$-\frac{1}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - 2*x^4 + 1),x, algorithm="maxima")

[Out] -1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

Fricas [A] time = 0.249237, size = 31, normalized size = 1.19

$$\frac{\left(x^4-1\right)\log\left(x^4-1\right)-1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - 2*x^4 + 1),x, algorithm="fricas")

[Out] 1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

Sympy [A] time = 0.243043, size = 15, normalized size = 0.58

$$\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**8-2*x**4+1),x)
```

```
[Out] log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

GIAC/XCAS [A] time = 0.262449, size = 26, normalized size = 1.

$$-\frac{1}{4\left(x^4-1\right)}+\frac{1}{4}\ln\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 - 1) + 1/4*ln(abs(x^4 - 1))
```

3.293 $\int \frac{x^5}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rubi [A] time = 0.0289943, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 - 2*x^4 + x^8),x]

[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rubi in Sympy [A] time = 5.9401, size = 15, normalized size = 0.6

$$\frac{x^2}{4\left(-x^4+1\right)}-\frac{\operatorname{atanh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**8-2*x**4+1),x)

[Out] x**2/(4*(-x**4 + 1)) - atanh(x**2)/4
```

Mathematica [A] time = 0.0177655, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(\log\left(1-x^2\right)-\log\left(x^2+1\right)-\frac{2x^2}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(1 - 2*x^4 + x^8),x]

[Out] ((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

Maple [A] time = 0.013, size = 36, normalized size = 1.4

$$-\frac{1}{8x^2-8}+\frac{\ln\left(x^2-1\right)}{8}-\frac{1}{8x^2+8}-\frac{\ln\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^8-2*x^4+1),x)

[Out] -1/8/(x^2-1)+1/8*ln(x^2-1)-1/8/(x^2+1)-1/8*ln(x^2+1)
```

Maxima [A] time = 0.778889, size = 39, normalized size = 1.56

$$-\frac{x^2}{4\,(x^4-1)}-\frac{1}{8}\,\log\left(x^2+1\right)+\frac{1}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - 2*x^4 + 1),x, algorithm="maxima")

[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

Fricas [A] time = 0.251481, size = 54, normalized size = 2.16

$$-\frac{2x^2+\left(x^4-1\right)\log\left(x^2+1\right)-\left(x^4-1\right)\log\left(x^2-1\right)}{8\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - 2*x^4 + 1),x, algorithm="fricas")

[Out] -1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x
^4 - 1)
```

Sympy [A] time = 0.273902, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}+\frac{\log\left(x^2-1\right)}{8}-\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

GIAC/XCAS [A] time = 0.282924, size = 41, normalized size = 1.64

$$-\frac{x^2}{4\left(x^4-1\right)}-\frac{1}{8}\ln\left(x^2+1\right)+\frac{1}{8}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/8*ln(x^2 + 1) + 1/8*ln(abs(x^2 - 1))
```

3.294 $\int \frac{x^3}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=13

$$\frac{1}{4(1-x^4)}$$

```
[Out] 1/(4*(1 - x^4))
```

Rubi [A] time = 0.00732921, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{1}{4(1-x^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 - 2*x^4 + x^8),x]

[Out] 1/(4*(1 - x^4))
```

Rubi in Sympy [A] time = 2.85154, size = 7, normalized size = 0.54

$$\frac{1}{4(-x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**8-2*x**4+1),x)

[Out] 1/(4*(-x**4 + 1))
```

Mathematica [A] time = 0.00414282, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4-1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] -1/(4*(-1 + x^4))
```

Maple [A] time = 0.005, size = 10, normalized size = 0.8

$$-\frac{1}{4\,x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-2*x^4+1),x)
```

```
[Out] -1/4/(x^4-1)
```

Maxima [A] time = 0.770972, size = 12, normalized size = 0.92

$$-\frac{1}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1)
```

Fricas [A] time = 0.241493, size = 12, normalized size = 0.92

$$-\frac{1}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 - 1)
```

Sympy [A] time = 0.219153, size = 8, normalized size = 0.62

$$-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**8-2*x**4+1),x)
```

```
[Out] -1/(4*x**4 - 4)
```

GIAC/XCAS [A] time = 0.274711, size = 12, normalized size = 0.92

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 - 1)
```

3.295 $\int \frac{x}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=25

$$\frac{1}{4}\tanh^{-1}\left(x^2\right)+\frac{x^2}{4\left(1-x^4\right)}$$

```
[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rubi [A] time = 0.0213285, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{1}{4}\tanh^{-1}\left(x^2\right)+\frac{x^2}{4\left(1-x^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 - 2*x^4 + x^8),x]

[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rubi in Sympy [A] time = 3.72043, size = 15, normalized size = 0.6

$$\frac{x^2}{4\left(-x^4+1\right)}+\frac{\operatorname{atanh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**8-2*x**4+1),x)

[Out] x**2/(4*(-x**4 + 1)) + atanh(x**2)/4
```

Mathematica [A] time = 0.0129628, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(-\log\left(1-x^2\right)+\log\left(x^2+1\right)-\frac{2x^2}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

Maple [A] time = 0.013, size = 36, normalized size = 1.4

$$-\frac{1}{8\,x^{2}-8}-\frac{\ln\left(x^{2}-1\right)}{8}-\frac{1}{8\,x^{2}+8}+\frac{\ln\left(x^{2}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(x^2-1)-1/8*ln(x^2-1)-1/8/(x^2+1)+1/8*ln(x^2+1)
```

Maxima [A] time = 0.779083, size = 39, normalized size = 1.56

$$-\frac{x^{2}}{4\,(x^{4}-1)}+\frac{1}{8}\,\log\left(x^{2}+1\right)-\frac{1}{8}\,\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

Fricas [A] time = 0.25306, size = 54, normalized size = 2.16

$$-\frac{2\,x^{2}-\left(x^{4}-1\right)\,\log\left(x^{2}+1\right)+\left(x^{4}-1\right)\,\log\left(x^{2}-1\right)}{8\,(x^{4}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x
^4 - 1)
```

Sympy [A] time = 0.276536, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

GIAC/XCAS [A] time = 0.262642, size = 41, normalized size = 1.64

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\ln\left(x^2+1\right)-\frac{1}{8}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*ln(x^2 + 1) - 1/8*ln(abs(x^2 - 1))
```

3.296 $\int \frac{1}{x(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{4(1-x^4)}-\frac{1}{4}\log\left(1-x^4\right)+\log(x)$$

```
[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4
```

Rubi [A] time = 0.0327314, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{4(1-x^4)}-\frac{1}{4}\log\left(1-x^4\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - 2*x^4 + x^8)),x]

[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4
```

Rubi in Sympy [A] time = 5.35112, size = 22, normalized size = 0.79

$$\frac{\log\left(x^4\right)}{4}-\frac{\log\left(-x^4+1\right)}{4}+\frac{1}{4\left(-x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**8-2*x**4+1),x)

[Out] log(x**4)/4 - log(-x**4 + 1)/4 + 1/(4*(-x**4 + 1))
```

Mathematica [A] time = 0.013591, size = 26, normalized size = 0.93

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4}\log\left(1-x^4\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*(-1 + x^4)) + Log[x] - Log[1 - x^4]/4
```

Maple [A] time = 0.021, size = 47, normalized size = 1.7

$$-\frac{1}{-16+16\,x}-\frac{\ln\left(-1+x\right)}{4}+\frac{1}{16+16\,x}-\frac{\ln\left(1+x\right)}{4}+\ln\left(x\right)+\frac{1}{8\,x^{2}+8}-\frac{\ln\left(x^{2}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-1/4*ln(-1+x)+1/16/(1+x)-1/4*ln(1+x)+ln(x)+1/8/(x^2+1
)-1/4*ln(x^2+1)
```

Maxima [A] time = 0.780673, size = 32, normalized size = 1.14

$$-\frac{1}{4\,(x^{4}-1)}-\frac{1}{4}\,\log\left(x^{4}-1\right)+\frac{1}{4}\,\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.248568, size = 43, normalized size = 1.54

$$-\frac{\left(x^{4}-1\right)\log\left(x^{4}-1\right)-4\left(x^{4}-1\right)\log\left(x\right)+1}{4\,(x^{4}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

Sympy [A] time = 0.298482, size = 19, normalized size = 0.68

$$\log(x) - \frac{\log(x^4 - 1)}{4} - \frac{1}{4x^4 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**8-2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

GIAC/XCAS [A] time = 0.288857, size = 41, normalized size = 1.46

$$\frac{x^4 - 2}{4(x^4 - 1)} + \frac{1}{4}\ln(x^4) - \frac{1}{4}\ln(|x^4 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 2)/(x^4 - 1) + 1/4*ln(x^4) - 1/4*ln(abs(x^4 - 1))
```

3.297 $\int \frac{1}{x^3(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=32

$$-\frac{3}{4x^2}+\frac{3}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^2\left(1-x^4\right)}$$

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rubi [A] time = 0.0345444, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{3}{4x^2}+\frac{3}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^2\left(1-x^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rubi in Sympy [A] time = 7.86682, size = 26, normalized size = 0.81

$$\frac{3\,\mathrm{atanh}\left(x^2\right)}{4}-\frac{3}{4x^2}+\frac{1}{4x^2\left(-x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**8-2*x**4+1),x)
```

```
[Out] 3*atanh(x**2)/4 - 3/(4*x**2) + 1/(4*x**2*(-x**4 + 1))
```

Mathematica [A] time = 0.0327269, size = 41, normalized size = 1.28

$$\frac{1}{8}\left(-3\log\left(1-x^2\right)+3\log\left(x^2+1\right)+\frac{4-6x^4}{x^2\left(x^4-1\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] ((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/
8
```

Maple [A] time = 0.024, size = 50, normalized size = 1.6

$$-\frac{1}{-16+16x}-\frac{3\ln(-1+x)}{8}+\frac{1}{16+16x}-\frac{3\ln(1+x)}{8}-\frac{1}{2x^2}-\frac{1}{8x^2+8}+\frac{3\ln(x^2+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-3/8*ln(-1+x)+1/16/(1+x)-3/8*ln(1+x)-1/2/x^2-1/8/(x^2
+1)+3/8*ln(x^2+1)
```

Maxima [A] time = 0.770033, size = 50, normalized size = 1.56

$$-\frac{3x^4-2}{4(x^6-x^2)}+\frac{3}{8}\log(x^2+1)-\frac{3}{8}\log(x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1
)
```

Fricas [A] time = 0.284432, size = 73, normalized size = 2.28

$$-\frac{6x^4-3(x^6-x^2)\log(x^2+1)+3(x^6-x^2)\log(x^2-1)-4}{8(x^6-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] -1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2
- 1) - 4)/(x^6 - x^2)
```

Sympy [A] time = 0.399905, size = 36, normalized size = 1.12

$$-\frac{3x^4-2}{4x^6-4x^2}-\frac{3\log\left(x^2-1\right)}{8}+\frac{3\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**8-2*x**4+1),x)
```

```
[Out] -(3*x**4 - 2)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2
+ 1)/8
```

GIAC/XCAS [A] time = 0.287366, size = 51, normalized size = 1.59

$$-\frac{3\,x^4-2}{4\,(x^6-x^2)}+\frac{3}{8}\ln\left(x^2+1\right)-\frac{3}{8}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^3),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*ln(x^2 + 1) - 3/8*ln(abs(x^2 -
 1))
```

3.298 $\int \frac{1}{x^5(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4(1-x^4)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

```
[Out] -1/(4*x^4) + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rubi [A] time = 0.0394494, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{4(1-x^4)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 - 2*x^4 + x^8)),x]

[Out] -1/(4*x^4) + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rubi in Sympy [A] time = 6.1395, size = 29, normalized size = 0.78

$$\frac{\log\left(x^4\right)}{2}-\frac{\log\left(-x^4+1\right)}{2}+\frac{1}{4(-x^4+1)}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**8-2*x**4+1),x)

[Out] log(x**4)/2 - log(-x**4 + 1)/2 + 1/(4*(-x**4 + 1)) - 1/(4*x**4)
```

Mathematica [A] time = 0.0207327, size = 35, normalized size = 0.95

$$-\frac{1}{4(x^4-1)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]

[Out] -1/(4*x^4) - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Maple [A] time = 0.022, size = 54, normalized size = 1.5

$$-\frac{1}{-16+16\,x}-\frac{\ln\left(-1+x\right)}{2}+\frac{1}{16+16\,x}-\frac{\ln\left(1+x\right)}{2}-\frac{1}{4\,x^{4}}+2\,\ln\left(x\right)+\frac{1}{8\,x^{2}+8}-\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^8-2*x^4+1),x)

[Out] -1/16/(-1+x)-1/2*ln(-1+x)+1/16/(1+x)-1/2*ln(1+x)-1/4/x^4+2*ln(x)+
1/8/(x^2+1)-1/2*ln(x^2+1)
```

Maxima [A] time = 0.765369, size = 47, normalized size = 1.27

$$-\frac{2\,x^{4}-1}{4\,(x^{8}-x^{4})}-\frac{1}{2}\,\log\left(x^{4}-1\right)+\frac{1}{2}\,\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^5),x, algorithm="maxima")

[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

Fricas [A] time = 0.250569, size = 68, normalized size = 1.84

$$-\frac{2\,x^{4}+2\,\left(x^{8}-x^{4}\right)\,\log\left(x^{4}-1\right)-8\,\left(x^{8}-x^{4}\right)\,\log\left(x\right)-1}{4\,(x^{8}-x^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^5),x, algorithm="fricas")

[Out] -1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) -
 1)/(x^8 - x^4)
```

Sympy [A] time = 0.426497, size = 29, normalized size = 0.78

$$-\frac{2x^4-1}{4x^8-4x^4}+2\log(x)-\frac{\log(x^4-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**8-2*x**4+1),x)
```

```
[Out] -(2*x**4 - 1)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

GIAC/XCAS [A] time = 0.268617, size = 49, normalized size = 1.32

$$-\frac{2x^4-1}{4(x^8-x^4)}+\frac{1}{2}\ln(x^4)-\frac{1}{2}\ln(|x^4-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^5),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*ln(x^4) - 1/2*ln(abs(x^4 - 1))
```

3.299 $\int \frac{1}{x^7(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$-\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{5}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^6\left(1-x^4\right)}$$

```
[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/
4
```

Rubi [A] time = 0.0427798, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{5}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^6\left(1-x^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/
4
```

Rubi in Sympy [A] time = 9.80445, size = 32, normalized size = 0.82

$$\frac{5\,\operatorname{atanh}\left(x^2\right)}{4}-\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{1}{4x^6\left(-x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(x**8-2*x**4+1),x)
```

```
[Out] 5*atanh(x**2)/4 - 5/(4*x**2) - 5/(12*x**6) + 1/(4*x**6*(-x**4 + 1
))
```

Mathematica [A] time = 0.0256367, size = 49, normalized size = 1.26

$$-\frac{1}{6x^6}-\frac{1}{x^2}-\frac{5}{8}\log\left(1-x^2\right)+\frac{5}{8}\log\left(x^2+1\right)-\frac{x^2}{4\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (
5*Log[1 + x^2])/8
```

Maple [A] time = 0.022, size = 55, normalized size = 1.4

$$-\frac{1}{-16+16\,x}-\frac{5\,\ln\left(-1+x\right)}{8}+\frac{1}{16+16\,x}-\frac{5\,\ln\left(1+x\right)}{8}-\frac{1}{6\,x^{6}}-x^{-2}-\frac{1}{8\,x^{2}+8}+\frac{5\,\ln\left(x^{2}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-5/8*ln(-1+x)+1/16/(1+x)-5/8*ln(1+x)-1/6/x^6-1/x^2-1/
8/(x^2+1)+5/8*ln(x^2+1)
```

Maxima [A] time = 0.759393, size = 57, normalized size = 1.46

$$-\frac{15\,x^{8}-10\,x^{4}-2}{12\,(x^{10}-x^{6})}+\frac{5}{8}\,\log\left(x^{2}+1\right)-\frac{5}{8}\,\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^7),x, algorithm="maxima")
```

```
[Out] -1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8
*log(x^2 - 1)
```

Fricas [A] time = 0.27519, size = 80, normalized size = 2.05

$$-\frac{30\,x^{8}-20\,x^{4}-15\,\left(x^{10}-x^{6}\right)\log\left(x^{2}+1\right)+15\,\left(x^{10}-x^{6}\right)\log\left(x^{2}-1\right)-4}{24\,(x^{10}-x^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^7),x, algorithm="fricas")
```

```
[Out] -1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10
- x^6)*log(x^2 - 1) - 4)/(x^10 - x^6)
```

Sympy [A] time = 0.542635, size = 41, normalized size = 1.05

$$-\frac{5\log\left(x^2-1\right)}{8}+\frac{5\log\left(x^2+1\right)}{8}-\frac{15x^8-10x^4-2}{12x^{10}-12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**8-2*x**4+1),x)
```

```
[Out] -5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 - (15*x**8 - 10*x**4 - 2)/
(12*x**10 - 12*x**6)
```

GIAC/XCAS [A] time = 0.304913, size = 57, normalized size = 1.46

$$-\frac{x^2}{4\,(x^4-1)}-\frac{6\,x^4+1}{6\,x^6}+\frac{5}{8}\ln\left(x^2+1\right)-\frac{5}{8}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^7),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*ln(x^2 + 1) - 5/8*
ln(abs(x^2 - 1))
```

3.300 $\int \frac{x^8}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=34

$$\frac{x^5}{4(1-x^4)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

```
[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8
```

Rubi [A] time = 0.0264783, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{x^5}{4(1-x^4)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(1 - 2*x^4 + x^8),x]

[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8
```

Rubi in Sympy [A] time = 6.27079, size = 27, normalized size = 0.79

$$\frac{x^5}{4(-x^4+1)}+\frac{5x}{4}-\frac{5\,\mathrm{atan}\,(x)}{8}-\frac{5\,\mathrm{atanh}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**8-2*x**4+1),x)

[Out] x**5/(4*(-x**4 + 1)) + 5*x/4 - 5*atan(x)/8 - 5*atanh(x)/8
```

Mathematica [A] time = 0.0253772, size = 38, normalized size = 1.12

$$-\frac{x}{4(x^4-1)}+x+\frac{5}{16}\log(1-x)-\frac{5}{16}\log(x+1)-\frac{5}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(1 - 2*x^4 + x^8),x]
```

```
[Out] x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*L
og[1 + x])/16
```

Maple [A] time = 0.02, size = 43, normalized size = 1.3

$$x-\frac{1}{-16+16\,x}+\frac{5\,\ln\left(-1+x\right)}{16}-\frac{1}{16+16\,x}-\frac{5\,\ln\left(1+x\right)}{16}+\frac{x}{8\,x^{2}+8}-\frac{5\,\arctan\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^8-2*x^4+1),x)
```

```
[Out] x-1/16/(-1+x)+5/16*ln(-1+x)-1/16/(1+x)-5/16*ln(1+x)+1/8*x/(x^2+1)
-5/8*arctan(x)
```

Maxima [A] time = 0.845837, size = 38, normalized size = 1.12

$$x-\frac{x}{4\,(x^{4}-1)}-\frac{5}{8}\,\arctan\left(x\right)-\frac{5}{16}\,\log\left(x+1\right)+\frac{5}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(
x - 1)
```

Fricas [A] time = 0.264062, size = 66, normalized size = 1.94

$$\frac{16\,x^{5}-10\,\left(x^{4}-1\right)\,\arctan\left(x\right)-5\,\left(x^{4}-1\right)\,\log\left(x+1\right)+5\,\left(x^{4}-1\right)\,\log\left(x-1\right)-20\,x}{16\,(x^{4}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) +
5*(x^4 - 1)*log(x - 1) - 20*x)/(x^4 - 1)
```

Sympy [A] time = 0.45463, size = 32, normalized size = 0.94

$$x - \frac{x}{4x^4 - 4} + \frac{5\log(x-1)}{16} - \frac{5\log(x+1)}{16} - \frac{5\,\mathrm{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**8-2*x**4+1),x)
```

```
[Out] x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x
)/8
```

GIAC/XCAS [A] time = 0.304627, size = 41, normalized size = 1.21

$$x - \frac{x}{4(x^4-1)} - \frac{5}{8}\arctan(x) - \frac{5}{16}\ln(|x+1|) + \frac{5}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*ln(abs(x + 1)) + 5/16*
ln(abs(x - 1))
```

3.301 $\int \frac{x^6}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)} + \frac{3}{8}\tan^{-1}(x) - \frac{3}{8}\tanh^{-1}(x)$$

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rubi [A] time = 0.025095, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{x^3}{4(1-x^4)} + \frac{3}{8}\tan^{-1}(x) - \frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rubi in Sympy [A] time = 5.69815, size = 22, normalized size = 0.76

$$\frac{x^3}{4(-x^4+1)} + \frac{3\,\mathrm{atan}(x)}{8} - \frac{3\,\mathrm{atanh}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**8-2*x**4+1),x)
```

```
[Out] x**3/(4*(-x**4 + 1)) + 3*atan(x)/8 - 3*atanh(x)/8
```

Mathematica [A] time = 0.0228906, size = 35, normalized size = 1.21

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1} + 3\log(1-x) - 3\log(x+1) + 6\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])
/16
```

Maple [A] time = 0.02, size = 42, normalized size = 1.5

$$-\frac{1}{-16+16x}+\frac{3\ln(-1+x)}{16}-\frac{1}{16+16x}-\frac{3\ln(1+x)}{16}-\frac{x}{8x^2+8}+\frac{3\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)+3/16*ln(-1+x)-1/16/(1+x)-3/16*ln(1+x)-1/8*x/(x^2+1)+
3/8*arctan(x)
```

Maxima [A] time = 0.838854, size = 39, normalized size = 1.34

$$-\frac{x^3}{4(x^4-1)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\log(x+1)+\frac{3}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x
 - 1)
```

Fricas [A] time = 0.256162, size = 62, normalized size = 2.14

$$-\frac{4x^3-6\left(x^4-1\right)\arctan(x)+3\left(x^4-1\right)\log(x+1)-3\left(x^4-1\right)\log(x-1)}{16(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3
*(x^4 - 1)*log(x - 1))/(x^4 - 1)
```

Sympy [A] time = 0.448197, size = 32, normalized size = 1.1

$$-\frac{x^3}{4x^4-4}+\frac{3\log(x-1)}{16}-\frac{3\log(x+1)}{16}+\frac{3\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x
)/8
```

GIAC/XCAS [A] time = 0.299349, size = 42, normalized size = 1.45

$$-\frac{x^3}{4(x^4-1)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\ln(|x+1|)+\frac{3}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*ln(abs(x + 1)) + 3/16*l
n(abs(x - 1))
```

3.302 $\int \frac{x^4}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)$$

```
[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rubi [A] time = 0.0197442, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{x}{4(1-x^4)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - 2*x^4 + x^8),x]

[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rubi in Sympy [A] time = 4.34331, size = 17, normalized size = 0.63

$$\frac{x}{4(-x^4+1)}-\frac{\operatorname{atan}(x)}{8}-\frac{\operatorname{atanh}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**8-2*x**4+1),x)

[Out] x/(4*(-x**4 + 1)) - atan(x)/8 - atanh(x)/8
```

Mathematica [A] time = 0.0214261, size = 31, normalized size = 1.15

$$\frac{1}{16}\left(-\frac{4x}{x^4-1}+\log(1-x)-\log(x+1)-2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

Maple [A] time = 0.018, size = 42, normalized size = 1.6

$$-\frac{1}{-16+16x}+\frac{\ln(-1+x)}{16}-\frac{1}{16+16x}-\frac{\ln(1+x)}{16}+\frac{x}{8x^{2}+8}-\frac{\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)+1/16*ln(-1+x)-1/16/(1+x)-1/16*ln(1+x)+1/8*x/(x^2+1)-
1/8*arctan(x)
```

Maxima [A] time = 0.860397, size = 36, normalized size = 1.33

$$-\frac{x}{4\left(x^{4}-1\right)}-\frac{1}{8}\arctan(x)-\frac{1}{16}\log(x+1)+\frac{1}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x -
 1)
```

Fricas [A] time = 0.282568, size = 58, normalized size = 2.15

$$-\frac{2\left(x^{4}-1\right)\arctan(x)+\left(x^{4}-1\right)\log(x+1)-\left(x^{4}-1\right)\log(x-1)+4x}{16\left(x^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*l
og(x - 1) + 4*x)/(x^4 - 1)
```

Sympy [A] time = 0.435699, size = 26, normalized size = 0.96

$$-\frac{x}{4x^4-4}+\frac{\log(x-1)}{16}-\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

GIAC/XCAS [A] time = 0.282249, size = 39, normalized size = 1.44

$$-\frac{x}{4(x^4-1)}-\frac{1}{8}\arctan(x)-\frac{1}{16}\ln(|x+1|)+\frac{1}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*ln(abs(x + 1)) + 1/16*ln(
abs(x - 1))
```

3.303 $\int \frac{x^2}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

```
[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

Rubi [A] time = 0.0249913, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{x^3}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 - 2*x^4 + x^8),x]

[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

Rubi in Sympy [A] time = 5.77625, size = 19, normalized size = 0.66

$$\frac{x^3}{4(-x^4+1)} - \frac{\operatorname{atan}(x)}{8} + \frac{\operatorname{atanh}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**8-2*x**4+1),x)

[Out] x**3/(4*(-x**4 + 1)) - atan(x)/8 + atanh(x)/8
```

Mathematica [A] time = 0.019038, size = 33, normalized size = 1.14

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1} - \log(1-x) + \log(x+1) - 2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

Maple [A] time = 0.016, size = 42, normalized size = 1.5

$$-\frac{1}{-16+16\,x}-\frac{\ln(-1+x)}{16}-\frac{1}{16+16\,x}+\frac{\ln(1+x)}{16}-\frac{x}{8\,x^2+8}-\frac{\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-1/16*ln(-1+x)-1/16/(1+x)+1/16*ln(1+x)-1/8*x/(x^2+1)-
1/8*arctan(x)
```

Maxima [A] time = 0.851383, size = 39, normalized size = 1.34

$$-\frac{x^3}{4\,(x^4-1)}-\frac{1}{8}\arctan(x)+\frac{1}{16}\log(x+1)-\frac{1}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x
 - 1)
```

Fricas [A] time = 0.257829, size = 61, normalized size = 2.1

$$-\frac{4\,x^3+2\left(x^4-1\right)\arctan(x)-\left(x^4-1\right)\log(x+1)+\left(x^4-1\right)\log(x-1)}{16\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^
4 - 1)*log(x - 1))/(x^4 - 1)
```

Sympy [A] time = 0.437867, size = 27, normalized size = 0.93

$$-\frac{x^3}{4x^4-4}-\frac{\log(x-1)}{16}+\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

GIAC/XCAS [A] time = 0.274123, size = 42, normalized size = 1.45

$$-\frac{x^3}{4\,(x^4-1)}-\frac{1}{8}\arctan(x)+\frac{1}{16}\ln(|x+1|)-\frac{1}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*ln(abs(x + 1)) - 1/16*l
n(abs(x - 1))
```

3.304 $\int \frac{1}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rubi [A] time = 0.0139036, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{x}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2*x^4 + x^8)^(-1),x]

[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rubi in Sympy [A] time = 2.00162, size = 20, normalized size = 0.74

$$\frac{x}{4(-x^4+1)}+\frac{3\,\mathrm{atan}\,(x)}{8}+\frac{3\,\mathrm{atanh}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8-2*x**4+1),x)

[Out] x/(4*(-x**4 + 1)) + 3*atan(x)/8 + 3*atanh(x)/8
```

Mathematica [A] time = 0.0149598, size = 33, normalized size = 1.22

$$\frac{1}{16}\left(-\frac{4x}{x^4-1}-3\log(1-x)+3\log(x+1)+6\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/1
6
```

Maple [A] time = 0.019, size = 42, normalized size = 1.6

$$-\frac{1}{-16+16x}-\frac{3\ln(-1+x)}{16}-\frac{1}{16+16x}+\frac{3\ln(1+x)}{16}+\frac{x}{8x^2+8}+\frac{3\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-3/16*ln(-1+x)-1/16/(1+x)+3/16*ln(1+x)+1/8*x/(x^2+1)+
3/8*arctan(x)
```

Maxima [A] time = 0.85635, size = 36, normalized size = 1.33

$$-\frac{x}{4(x^4-1)}+\frac{3}{8}\arctan(x)+\frac{3}{16}\log(x+1)-\frac{3}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x -
 1)
```

Fricas [A] time = 0.258219, size = 59, normalized size = 2.19

$$\frac{6(x^4-1)\arctan(x)+3(x^4-1)\log(x+1)-3(x^4-1)\log(x-1)-4x}{16(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1
)*log(x - 1) - 4*x)/(x^4 - 1)
```

Sympy [A] time = 0.456182, size = 31, normalized size = 1.15

$$-\frac{x}{4x^4-4}-\frac{3\log(x-1)}{16}+\frac{3\log(x+1)}{16}+\frac{3\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

GIAC/XCAS [A] time = 0.28432, size = 39, normalized size = 1.44

$$-\frac{x}{4(x^4-1)}+\frac{3}{8}\arctan(x)+\frac{3}{16}\ln(|x+1|)-\frac{3}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*ln(abs(x + 1)) - 3/16*ln(
abs(x - 1))
```

3.305 $\int \frac{1}{x^2(1-2x^4+x^8)} dx$

Optimal. Leaf size=36

$$\frac{1}{4x(1-x^4)} - \frac{5}{4x} - \frac{5}{8}\tan^{-1}(x) + \frac{5}{8}\tanh^{-1}(x)$$

```
[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rubi [A] time = 0.0308835, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{1}{4x(1-x^4)} - \frac{5}{4x} - \frac{5}{8}\tan^{-1}(x) + \frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(1 - 2*x^4 + x^8)),x]

[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rubi in Sympy [A] time = 6.95372, size = 27, normalized size = 0.75

$$-\frac{5\,\mathrm{atan}\,(x)}{8} + \frac{5\,\mathrm{atanh}\,(x)}{8} - \frac{5}{4x} + \frac{1}{4x(-x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**8-2*x**4+1),x)

[Out] -5*atan(x)/8 + 5*atanh(x)/8 - 5/(4*x) + 1/(4*x*(-x**4 + 1))
```

Mathematica [A] time = 0.0256668, size = 40, normalized size = 1.11

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1} - \frac{16}{x} - 5\log(1-x) + 5\log(x+1) - 10\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log
[1 + x])/16
```

Maple [A] time = 0.022, size = 47, normalized size = 1.3

$$-\frac{1}{-16+16\,x}-\frac{5\,\ln(-1+x)}{16}-\frac{1}{16+16\,x}+\frac{5\,\ln(1+x)}{16}-x^{-1}-\frac{x}{8\,x^{2}+8}-\frac{5\,\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-5/16*ln(-1+x)-1/16/(1+x)+5/16*ln(1+x)-1/x-1/8*x/(x^2
+1)-5/8*arctan(x)
```

Maxima [A] time = 0.846868, size = 47, normalized size = 1.31

$$-\frac{5\,x^{4}-4}{4\,(x^{5}-x)}-\frac{5}{8}\arctan(x)+\frac{5}{16}\log(x+1)-\frac{5}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/
16*log(x - 1)
```

Fricas [A] time = 0.258011, size = 74, normalized size = 2.06

$$-\frac{20\,x^{4}+10\,\left(x^{5}-x\right)\arctan(x)-5\,\left(x^{5}-x\right)\log(x+1)+5\,\left(x^{5}-x\right)\log(x-1)-16}{16\,(x^{5}-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] -1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) +
 5*(x^5 - x)*log(x - 1) - 16)/(x^5 - x)
```

Sympy [A] time = 0.545148, size = 37, normalized size = 1.03

$$-\frac{5x^4-4}{4x^5-4x}-\frac{5\log(x-1)}{16}+\frac{5\log(x+1)}{16}-\frac{5\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**8-2*x**4+1),x)
```

```
[Out] -(5*x**4 - 4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16
- 5*atan(x)/8
```

GIAC/XCAS [A] time = 0.278636, size = 50, normalized size = 1.39

$$-\frac{5x^4-4}{4(x^5-x)}-\frac{5}{8}\arctan(x)+\frac{5}{16}\ln(|x+1|)-\frac{5}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^2),x, algorithm="giac")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*ln(abs(x + 1))
- 5/16*ln(abs(x - 1))
```

3.306 $\int \frac{1}{x^4(1-2x^4+x^8)} dx$

Optimal. Leaf size=36

$$-\frac{7}{12x^3}+\frac{1}{4x^3(1-x^4)}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)$$

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[
x])/8
```

Rubi [A] time = 0.0259804, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{7}{12x^3}+\frac{1}{4x^3(1-x^4)}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[
x])/8
```

Rubi in Sympy [A] time = 6.30421, size = 31, normalized size = 0.86

$$\frac{7\,\mathrm{atan}\,(x)}{8}+\frac{7\,\mathrm{atanh}\,(x)}{8}-\frac{7}{12x^3}+\frac{1}{4x^3(-x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**8-2*x**4+1),x)
```

```
[Out] 7*atan(x)/8 + 7*atanh(x)/8 - 7/(12*x**3) + 1/(4*x**3*(-x**4 + 1))
```

Mathematica [A] time = 0.0310668, size = 38, normalized size = 1.06

$$\frac{1}{48}\left(-\frac{12x}{x^4-1}-\frac{16}{x^3}-21\log(1-x)+21\log(x+1)+42\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*
Log[1 + x])/48
```

Maple [A] time = 0.021, size = 47, normalized size = 1.3

$$-\frac{1}{-16+16\,x}-\frac{7\,\ln\left(-1+x\right)}{16}-\frac{1}{16+16\,x}+\frac{7\,\ln\left(1+x\right)}{16}-\frac{1}{3\,x^{3}}+\frac{x}{8\,x^{2}+8}+\frac{7\,\arctan\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-7/16*ln(-1+x)-1/16/(1+x)+7/16*ln(1+x)-1/3/x^3+1/8*x/
(x^2+1)+7/8*arctan(x)
```

Maxima [A] time = 0.86121, size = 50, normalized size = 1.39

$$-\frac{7\,x^{4}-4}{12\,(x^{7}-x^{3})}+\frac{7}{8}\,\arctan\left(x\right)+\frac{7}{16}\,\log\left(x+1\right)-\frac{7}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) -
 7/16*log(x - 1)
```

Fricas [A] time = 0.269199, size = 85, normalized size = 2.36

$$-\frac{28\,x^{4}-42\,\left(x^{7}-x^{3}\right)\arctan\left(x\right)-21\,\left(x^{7}-x^{3}\right)\log\left(x+1\right)+21\,\left(x^{7}-x^{3}\right)\log\left(x-1\right)-16}{48\,(x^{7}-x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] -1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x +
 1) + 21*(x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

Sympy [A] time = 0.583853, size = 39, normalized size = 1.08

$$-\frac{7x^4-4}{12x^7-12x^3}-\frac{7\log(x-1)}{16}+\frac{7\log(x+1)}{16}+\frac{7\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**8-2*x**4+1),x)
```

```
[Out] -(7*x**4 - 4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1
)/16 + 7*atan(x)/8
```

GIAC/XCAS [A] time = 0.28787, size = 46, normalized size = 1.28

$$-\frac{x}{4(x^4-1)}-\frac{1}{3x^3}+\frac{7}{8}\arctan(x)+\frac{7}{16}\ln(|x+1|)-\frac{7}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^4),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*ln(abs(x + 1))
- 7/16*ln(abs(x - 1))
```

3.307 $\int \frac{1}{x^6(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$-\frac{9}{20x^5}+\frac{1}{4x^5(1-x^4)}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

```
[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (
9*ArcTanh[x])/8
```

Rubi [A] time = 0.0376882, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{9}{20x^5}+\frac{1}{4x^5(1-x^4)}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (
9*ArcTanh[x])/8
```

Rubi in Sympy [A] time = 8.70021, size = 36, normalized size = 0.84

$$-\frac{9\,\mathrm{atan}\,(x)}{8}+\frac{9\,\mathrm{atanh}\,(x)}{8}-\frac{9}{4x}-\frac{9}{20x^5}+\frac{1}{4x^5(-x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**8-2*x**4+1),x)
```

```
[Out] -9*atan(x)/8 + 9*atanh(x)/8 - 9/(4*x) - 9/(20*x**5) + 1/(4*x**5*(
-x**4 + 1))
```

Mathematica [A] time = 0.0343185, size = 51, normalized size = 1.19

$$-\frac{1}{5x^5}-\frac{x^3}{4(x^4-1)}-\frac{2}{x}-\frac{9}{16}\log(1-x)+\frac{9}{16}\log(x+1)-\frac{9}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[
1 - x])/16 + (9*Log[1 + x])/16
```

Maple [A] time = 0.024, size = 52, normalized size = 1.2

$$-\frac{1}{-16+16\,x}-\frac{9\,\ln\left(-1+x\right)}{16}-\frac{1}{16+16\,x}+\frac{9\,\ln\left(1+x\right)}{16}-\frac{1}{5\,x^{5}}-2\,x^{-1}-\frac{x}{8\,x^{2}+8}-\frac{9\,\arctan\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-9/16*ln(-1+x)-1/16/(1+x)+9/16*ln(1+x)-1/5/x^5-2/x-1/
8*x/(x^2+1)-9/8*arctan(x)
```

Maxima [A] time = 0.850441, size = 57, normalized size = 1.33

$$-\frac{45\,x^{8}-36\,x^{4}-4}{20\,(x^{9}-x^{5})}-\frac{9}{8}\,\arctan\left(x\right)+\frac{9}{16}\,\log\left(x+1\right)-\frac{9}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^6),x, algorithm="maxima")
```

```
[Out] -1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*arctan(x) + 9/16*lo
g(x + 1) - 9/16*log(x - 1)
```

Fricas [A] time = 0.256877, size = 92, normalized size = 2.14

$$-\frac{180\,x^{8}-144\,x^{4}+90\,\left(x^{9}-x^{5}\right)\,\arctan\left(x\right)-45\,\left(x^{9}-x^{5}\right)\,\log\left(x+1\right)+45\,\left(x^{9}-x^{5}\right)\,\log\left(x-1\right)-16}{80\,(x^{9}-x^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^6),x, algorithm="fricas")
```

```
[Out] -1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x
^5)*log(x + 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)
```

Sympy [A] time = 0.713242, size = 44, normalized size = 1.02

$$-\frac{9\log(x-1)}{16}+\frac{9\log(x+1)}{16}-\frac{9\,\mathrm{atan}(x)}{8}-\frac{45x^8-36x^4-4}{20x^9-20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**8-2*x**4+1),x)
```

```
[Out] -9*log(x - 1)/16 + 9*log(x + 1)/16 - 9*atan(x)/8 - (45*x**8 - 36*
x**4 - 4)/(20*x**9 - 20*x**5)
```

GIAC/XCAS [A] time = 0.273789, size = 58, normalized size = 1.35

$$-\frac{x^3}{4\,(x^4-1)}-\frac{10\,x^4+1}{5\,x^5}-\frac{9}{8}\,\arctan(x)+\frac{9}{16}\,\ln(|x+1|)-\frac{9}{16}\,\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^6),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*
ln(abs(x + 1)) - 9/16*ln(abs(x - 1))
```

3.308 $\int \frac{1}{x^8(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7(1-x^4)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)$$

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])
/8 + (11*ArcTanh[x])/8
```

Rubi [A] time = 0.0332619, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7(1-x^4)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])
/8 + (11*ArcTanh[x])/8
```

Rubi in Sympy [A] time = 7.28232, size = 37, normalized size = 0.86

$$\frac{11\operatorname{atan}(x)}{8}+\frac{11\operatorname{atanh}(x)}{8}-\frac{11}{12x^3}-\frac{11}{28x^7}+\frac{1}{4x^7(-x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**8/(x**8-2*x**4+1),x)
```

```
[Out] 11*atan(x)/8 + 11*atanh(x)/8 - 11/(12*x**3) - 11/(28*x**7) + 1/(4
*x**7*(-x**4 + 1))
```

Mathematica [A] time = 0.0342513, size = 43, normalized size = 1.

$$\frac{1}{336}\left(-\frac{48}{x^7}-\frac{84x}{x^4-1}-\frac{224}{x^3}-231\log(1-x)+231\log(x+1)+462\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[
1 - x] + 231*Log[1 + x])/336
```

Maple [A] time = 0.024, size = 52, normalized size = 1.2

$$-\frac{1}{-16+16x}-\frac{11\ln(-1+x)}{16}-\frac{1}{16+16x}+\frac{11\ln(1+x)}{16}-\frac{1}{7x^7}-\frac{2}{3x^3}+\frac{x}{8x^2+8}+\frac{11\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^8/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(-1+x)-11/16*ln(-1+x)-1/16/(1+x)+11/16*ln(1+x)-1/7/x^7-2/3/
x^3+1/8*x/(x^2+1)+11/8*arctan(x)
```

Maxima [A] time = 0.854868, size = 57, normalized size = 1.33

$$-\frac{77x^8-44x^4-12}{84(x^{11}-x^7)}+\frac{11}{8}\arctan(x)+\frac{11}{16}\log(x+1)-\frac{11}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^8),x, algorithm="maxima")
```

```
[Out] -1/84*(77*x^8 - 44*x^4 - 12)/(x^11 - x^7) + 11/8*arctan(x) + 11/1
6*log(x + 1) - 11/16*log(x - 1)
```

Fricas [A] time = 0.280728, size = 92, normalized size = 2.14

$$-\frac{308x^8-176x^4-462\left(x^{11}-x^7\right)\arctan(x)-231\left(x^{11}-x^7\right)\log(x+1)+231\left(x^{11}-x^7\right)\log(x-1)-48}{336(x^{11}-x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 2*x^4 + 1)*x^8),x, algorithm="fricas")
```

```
[Out] -1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^1
1 - x^7)*log(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x
^7)
```

Sympy [A] time = 0.730099, size = 44, normalized size = 1.02

$$-\frac{11\log(x-1)}{16}+\frac{11\log(x+1)}{16}+\frac{11\,\mathrm{atan}(x)}{8}-\frac{77x^8-44x^4-12}{84x^{11}-84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-2*x**4+1),x)
```

```
[Out] -11*log(x - 1)/16 + 11*log(x + 1)/16 + 11*atan(x)/8 - (77*x**8 -
44*x**4 - 12)/(84*x**11 - 84*x**7)
```

GIAC/XCAS [A] time = 0.280882, size = 55, normalized size = 1.28

$$-\frac{x}{4\,(x^4-1)}-\frac{14\,x^4+3}{21\,x^7}+\frac{11}{8}\,\arctan(x)+\frac{11}{16}\ln(|x+1|)-\frac{11}{16}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^8 - 2*x^4 + 1)*x^8),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16
*ln(abs(x + 1)) - 11/16*ln(abs(x - 1))
```

3.309 $\int \frac{x^m}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=163

$$\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x
^4)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 -
4*a*c])*(1 + m)) - (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4,
 (5 + m)/4, (-2*c*x^4)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Rubi [A] time = 0.324964, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x
^4)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 -
4*a*c])*(1 + m)) - (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4,
 (5 + m)/4, (-2*c*x^4)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Rubi in Sympy [A] time = 32.074, size = 141, normalized size = 0.87

$$-\frac{2cx^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\\frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-\frac{2cx^4}{b+\sqrt{-4ac+b^2}}\right)}{\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}+\frac{2cx^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\\frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-\frac{2cx^4}{b-\sqrt{-4ac+b^2}}\right)}{\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(c*x**8+b*x**4+a),x)
```

```
[Out] -2*c*x**(m + 1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), -2*c*x**4/(b
+ sqrt(-4*a*c + b**2)))/((b + sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-
4*a*c + b**2)) + 2*c*x**(m + 1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,
), -2*c*x**4/(b - sqrt(-4*a*c + b**2)))/((b - sqrt(-4*a*c + b**2)
)*(m + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0835056, size = 82, normalized size = 0.5

$$\frac{x^m\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^7c+\#1^3b}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (x^m*RootSum[a + b*#1^4 + c*#1^8 & , Hypergeometric2F1[-m, -m, 1
- m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1^3 + 2*c*#1^7)) & ])/(4
*m)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(c*x^8+b*x^4+a),x)
```

```
[Out] int(x^m/(c*x^8+b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{cx^8 + bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(c*x^8 + b*x^4 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

3.310 $\int \frac{x^{11}}{a+bx^4+cx^8} dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c
]])/(4*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2
)
```

Rubi [A] time = 0.17379, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c
]])/(4*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2
)
```

Rubi in Sympy [A] time = 31.1966, size = 73, normalized size = 0.9

$$-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^4}{\sqrt{-4ac+b^2}}\right)}{4c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(c*x**8+b*x**4+a),x)
```

```
[Out] -b*log(a + b*x**4 + c*x**8)/(8*c**2) + x**4/(4*c) - (-2*a*c + b**
2)*atanh((b + 2*c*x**4)/sqrt(-4*a*c + b**2))/(4*c**2*sqrt(-4*a*c
+ b**2))
```

Mathematica [A] time = 0.0851206, size = 78, normalized size = 0.96

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^4+cx^8\right)+2cx^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Maple [A] time = 0.006, size = 111, normalized size = 1.4

$$\frac{x^4}{4c}-\frac{b\ln\left(cx^8+bx^4+a\right)}{8c^2}-\frac{a}{2c}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2}{4c^2}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/2/c/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*a+1/4/c^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.306908, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)\log\left(\frac{2\left(b^2c-4\,ac^2\right)x^4+b^3-4\,abc+\left(2\,c^2x^8+2\,bcx^4+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(2\,cx^4-b\log\left(cx^8+bx^4+a\right)\right)\sqrt{b^2-4\,ac}}{8\,\sqrt{b^2-4\,ac}c^2},\ \frac{2\,(b^2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 2*a*c)*log((2*(b^2*c - 4*a*c^2)*x^4 + b^3 - 4*a*b*c
 + (2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^
8 + b*x^4 + a)) - (2*c*x^4 - b*log(c*x^8 + b*x^4 + a))*sqrt(b^2 -
 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), 1/8*(2*(b^2 - 2*a*c)*arctan(-(2
*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*x^4 - b*log(
c*x^8 + b*x^4 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 11.1557, size = 316, normalized size = 3.9

$$\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{8c^2\left(4ac-b^2\right)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{8c^2(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{8c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{8c^2\left(4ac-b^2\right)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{8c^2(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{8c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c
- b**2)))*log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c
 + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(
```

```
8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**
2))))/(2*a*c - b**2)) + (-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 + (-a*b - 16*a*c**2*(-
b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c -
b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

GIAC/XCAS [A] time = 0.286349, size = 101, normalized size = 1.25

$$\frac{x^4}{4\,c} - \frac{b\ln\left(cx^8 + bx^4 + a\right)}{8\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^4 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{4\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*x^4/c - 1/8*b*ln(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.311 $\int \frac{x^9}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=192

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.708604, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 72.5695, size = 196, normalized size = 1.02

$$\frac{x^2}{2c}-\frac{\sqrt{2}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**8+b*x**4+a),x)
```

```
[Out] x**2/(2*c) - sqrt(2)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan
(sqrt(2)*sqrt(c)*x**2/sqrt(b + sqrt(-4*a*c + b**2)))/(4*c**(3/2)*
sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(-2*
a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x**2/sqr
t(b - sqrt(-4*a*c + b**2)))/(4*c**(3/2)*sqrt(b - sqrt(-4*a*c + b*
*2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.221771, size = 210, normalized size = 1.09

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+2\sqrt{c}x^2}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c +
 b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(4*c^(3/2))
```

Maple [B] time = 0.029, size = 360, normalized size = 1.9

$$\frac{x^2}{2c} - \frac{b\sqrt{2}}{4c}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}a}{2}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b^2}{4c}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}}{4c}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}a}{2}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b^2}{4c}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^9/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/2*x^2/c-1/4/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/c/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+1/4/c*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*a-1/4/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2\,c} - \frac{\int \frac{(bx^4+a)\,x}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [A] time = 0.294613, size = 1446, normalized size = 7.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))*log(-(a*b^2 - a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c
+ 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b
^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2
*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c -
(b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 + 1/2*sqr
t(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*
a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2
*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x
^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*
b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqr
t(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x^2)/c
```

Sympy [A] time = 12.3801, size = 134, normalized size = 0.7

$$\operatorname{RootSum}\left(t^4\left(4096a^2c^5-2048ab^2c^4+256b^4c^3\right)+t^2\left(192a^2bc^2-112ab^3c+16b^5\right)+a^3,\left(t\mapsto t\log\left(x^2+\frac{256t^3abc^4-64t^3}{}\right.\right.\right.$$
$$+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3)
 + _t**2*(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambd
a(_t, _t*log(x**2 + (256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*
_t*a**2*c**2 + 16*_t*a*b**2*c - 4*_t*b**4)/(a**2*c - a*b**2)))) +
 x**2/(2*c)
```

GIAC/XCAS [A] time = 0.440106, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.312 $\int \frac{x^7}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^4 + c*x^8]/(8*c)
```

Rubi [A] time = 0.119409, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^4 + c*x^8]/(8*c)
```

Rubi in Sympy [A] time = 19.5677, size = 54, normalized size = 0.86

$$\frac{b \operatorname{atanh}\left(\frac{b+2cx^4}{\sqrt{-4ac+b^2}}\right)}{4c\sqrt{-4ac+b^2}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**7/(c*x**8+b*x**4+a),x)
```

```
[Out] b*atanh((b + 2*c*x**4)/sqrt(-4*a*c + b**2))/(4*c*sqrt(-4*a*c + b*
*2)) + log(a + b*x**4 + c*x**8)/(8*c)
```

Mathematica [A] time = 0.0387938, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^4+cx^8\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c] + Log[a + b*x^4 + c*x^8])/(8*c)
```

Maple [A] time = 0.003, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^8+bx^4+a\right)}{8\,c}-\frac{b}{4\,c}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4
+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277341, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{2\left(b^2c-4\,ac^2\right)x^4+b^3-4\,abc+\left(2\,c^2x^8+2\,bcx^4+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+\sqrt{b^2-4\,ac}\log\left(cx^8+bx^4+a\right)}{8\,\sqrt{b^2-4\,ac}c},\right.$$
$$\left.-\frac{2\,b\arctan\left(-\frac{\left(2\,cx^4+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\log\left(cx^8+bx^4+a\right)}{8\,\sqrt{-b^2+4\,ac}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] [1/8*(b*log((2*(b^2*c - 4*a*c^2)*x^4 + b^3 - 4*a*b*c + (2*c^2*x^8
 + 2*b*c*x^4 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a
)) + sqrt(b^2 - 4*a*c)*log(c*x^8 + b*x^4 + a))/(sqrt(b^2 - 4*a*c)
*c), -1/8*(2*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)) - sqrt(-b^2 + 4*a*c)*log(c*x^8 + b*x^4 + a))/(sqrt(-b^2 + 4
*a*c)*c)]
```

Sympy [A] time = 6.10944, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4
+ (-16*a*c*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)
) + 2*a + 4*b**2*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1
/(8*c)))/b) + (b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*
c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2
)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c -
 b**2)) + 1/(8*c)))/b)
```

<u>GIAC/XCAS</u> [A] time = 0.277033, size = 80, normalized size = 1.27

$$-\frac{b \arctan\left(\frac{2 \, c x^{4} + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{4 \, \sqrt{-b^{2} + 4 \, a c} c} + \frac{\ln\left(c x^{8} + b x^{4} + a\right)}{8 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*c) + 1/8*ln(c*x^8 + b*x^4 + a)/c
```

3.313 $\int \frac{x^5}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.273555, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] -(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 38.7665, size = 144, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt{b-\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x**2/
sqrt(b - sqrt(-4*a*c + b**2)))/(4*sqrt(c)*sqrt(-4*a*c + b**2)) +
```

```
sqrt(2)*sqrt(b + sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x**2/s
qrt(b + sqrt(-4*a*c + b**2)))/(4*sqrt(c)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.148516, size = 171, normalized size = 1.08

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - S
qrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*
a*c]])
```

Maple [A] time = 0.021, size = 216, normalized size = 1.4

$$\frac{\sqrt{2}}{4}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}}{4}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}}{4}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}}{4}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b-1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1
/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 0.274801, size = 765, normalized size = 4.81

$$\begin{aligned}
&\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3
))/(b^2*c - 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(
-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2
))/sqrt(b^2*c^2 - 4*a*c^3)) - 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4
*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(x^2 - sqr
t(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^
2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) - 1/4*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b
^2*c - 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b -
 (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sq
rt(b^2*c^2 - 4*a*c^3)) + 1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^
2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(x^2 - sqrt(1/2
)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4
*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
```

Sympy [A] time = 7.94046, size = 76, normalized size = 0.48

$$\operatorname{RootSum}\left(t^4\left(4096a^2c^3-2048ab^2c^2+256b^4c\right)+t^2\left(-64abc+16b^3\right)+a,\left(t\mapsto t\log\left(512t^3ac^2-128t^3b^2c-4tb+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) +
_t**2*(-64*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c*
*2 - 128*_t**3*b**2*c - 4*_t*b + x**2)))
```

GIAC/XCAS [A] time = 0.395299, size = 1, normalized size = 0.01

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.314 $\int \frac{x^3}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

```
[Out] -ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.0724077, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^4 + c*x^8),x]

[Out] -ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 11.4714, size = 36, normalized size = 0.95

$$-\frac{\operatorname{atanh}\left(\frac{b+2cx^4}{\sqrt{-4ac+b^2}}\right)}{2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**8+b*x**4+a),x)

[Out] -atanh((b + 2*c*x**4)/sqrt(-4*a*c + b**2))/(2*sqrt(-4*a*c + b**2)
)
```

Mathematica [A] time = 0.0149854, size = 42, normalized size = 1.11

$$\frac{\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*x^4 + c*x^8),x]

[Out] ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

Maple [A] time = 0.002, size = 37, normalized size = 1.

$$\frac{1}{2}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^8+b*x^4+a),x)

[Out] 1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^8 + b*x^4 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263406, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{2\,(b^2c-4\,ac^2)\,x^4+b^3-4\,abc-(2\,c^2x^8+2\,bcx^4+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{4\,\sqrt{b^2-4\,ac}},\ \frac{\arctan\left(-\frac{(2\,cx^4+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(2*(b^2*c - 4*a*c^2)*x^4 + b^3 - 4*a*b*c - (2*c^2*x^8 +
 2*b*c*x^4 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a))
/sqrt(b^2 - 4*a*c), 1/2*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/
(b^2 - 4*a*c))/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 2.33962, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c
- b**2))*log(x**4 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1
/(4*a*c - b**2)) + b)/(2*c))/4
```

GIAC/XCAS [A] time = 0.259801, size = 49, normalized size = 1.29

$$\frac{\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.315 $\int \frac{x}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.228119, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 29.9423, size = 144, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x**2/sqrt(b + sqrt(-4*a*c +
 b**2)))/(2*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) +
```

```
sqrt(2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x**2/sqrt(b - sqrt(-4*a*c +
b**2)))/(2*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.137201, size = 133, normalized size = 0.86

$$\frac{\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*
Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.016, size = 120, normalized size = 0.8

$$-\frac{c\sqrt{2}}{2}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{c\sqrt{2}}{2}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2*c/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 0.279663, size = 836, normalized size = 5.43

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2-\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2-\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*
c))/(a*b^2 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (
a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1
/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
- 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a
^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqr
t(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(-(
b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))
*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqr
t(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b - (a*b^2
- 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2
- 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 -
```

```
 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c)))
```

Sympy [A] time = 8.74151, size = 88, normalized size = 0.57

$$\mathrm{RootSum}\left(t^4\left(4096a^3c^2-2048a^2b^2c+256ab^4\right)+t^2\left(-64abc+16b^3\right)+c,\left(t\mapsto t\log\left(x^2+\frac{256t^3a^2bc-64t^3ab^3+8tac-4\right.\right.\right.$$
$$c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) +
_t**2*(-64*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t
**3*a**2*b*c - 64*_t**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

GIAC/XCAS [A] time = 0.409605, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.316 $\int \frac{1}{x(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rubi [A] time = 0.140544, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rubi in Sympy [A] time = 27.7409, size = 63, normalized size = 0.91

$$\frac{b\,\operatorname{atanh}\left(\frac{b+2cx^4}{\sqrt{-4ac+b^2}}\right)}{4a\sqrt{-4ac+b^2}}+\frac{\log\left(x^4\right)}{4a}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**8+b*x**4+a),x)
```

```
[Out] b*atanh((b + 2*c*x**4)/sqrt(-4*a*c + b**2))/(4*a*sqrt(-4*a*c + b*
*2)) + log(x**4)/(4*a) - log(a + b*x**4 + c*x**8)/(8*a)
```

Mathematica [C] time = 0.0376716, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log
[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Maple [A] time = 0.008, size = 66, normalized size = 1.

$$-\frac{\ln\left(cx^8+bx^4+a\right)}{8\,a}-\frac{b}{4\,a}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/8*ln(c*x^8+b*x^4+a)/a-1/4/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
4+b)/(4*a*c-b^2)^(1/2))+ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.341948, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(\frac{2\,(b^2c-4\,ac^2)\,x^4+b^3-4\,abc+(2\,c^2x^8+2\,bcx^4+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\sqrt{b^2-4\,ac}\left(\log\left(cx^8+bx^4+a\right)-8\,\log\left(x\right)\right)}{8\,\sqrt{b^2-4\,ac}a},\right.$$
$$\left.-\frac{2\,b\arctan\left(-\frac{(2\,cx^4+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(\log\left(cx^8+bx^4+a\right)-8\,\log\left(x\right)\right)}{8\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/8*(b*log((2*(b^2*c - 4*a*c^2)*x^4 + b^3 - 4*a*b*c + (2*c^2*x^8
 + 2*b*c*x^4 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a
)) - sqrt(b^2 - 4*a*c)*(log(c*x^8 + b*x^4 + a) - 8*log(x)))/(sqrt
(b^2 - 4*a*c)*a), -1/8*(2*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*x^8 + b*x^4 + a) -
 8*log(x)))/(sqrt(-b^2 + 4*a*c)*a)]
```

Sympy [A] time = 18.4928, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)$$
$$+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4
+ (-16*a**2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8
*a)) + 4*a*b**2*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/
(8*a)) - 2*a*c + b**2)/(b*c)) + (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*
```

```
c - b**2)) - 1/(8*a))*log(x**4 + (-16*a**2*c*(b*sqrt(-4*a*c + b**
2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(b*sqrt(-4*a*c + b*
*2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c)) + log(
x)/a
```

GIAC/XCAS [A] time = 0.307605, size = 92, normalized size = 1.33

$$-\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}-\frac{\ln\left(cx^8+bx^4+a\right)}{8\,a}+\frac{\ln\left(x^4\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - 1/8*ln(c*x^8 + b*x^4 + a)/a + 1/4*ln(x^4)/a
```

3.317 $\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.471957, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(2*a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 62.193, size = 184, normalized size = 1.

$$\frac{\sqrt{2}\sqrt{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x*
*2/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(b + sqrt(-4*a*c + b**
2))*sqrt(-4*a*c + b**2)) - sqrt(2)*sqrt(c)*(b + sqrt(-4*a*c + b**
2))*atan(sqrt(2)*sqrt(c)*x**2/sqrt(b - sqrt(-4*a*c + b**2)))/(4*a
*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - 1/(2*a*x**2
)
```

Mathematica [C] time = 0.048984, size = 75, normalized size = 0.41

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^6c+\#1^2b}\&\right]}{4a}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(2*a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c
*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Maple [A] time = 0.024, size = 240, normalized size = 1.3

$$-\frac{c\sqrt{2}}{4\,a}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}b}{4\,a}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}}{4\,a}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}b}{4\,a}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*c/a/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/4*c/a*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))+1/4*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b-1/2/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{(cx^4+b)x}{cx^8+bx^4+a}\,dx}{a} - \frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

Fricas [A] time = 0.283147, size = 1531, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*
b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*
b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(b^
3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^
3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4
- 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^
4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*
c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5
```

```
*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3
*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-
(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a
*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b
^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a
^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))) + 2)/(a*x^2)
```

Sympy [A] time = 13.5555, size = 153, normalized size = 0.83

$$\operatorname{RootSum}\left(t^4\left(4096a^5c^2-2048a^4b^2c+256a^3b^4\right)+t^2\left(192a^2bc^2-112ab^3c+16b^5\right)+c^3,\left(t\mapsto t\log\left(x^2+\frac{-512t^3a^5c^2+384\right.\right.\right.$$

$$-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4)
 + _t**2*(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambd
a(_t, _t*log(x**2 + (-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c
 - 64*_t**3*a**3*b**4 - 20*_t*a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t
*b**5)/(a*c**3 - b**2*c**2)))) - 1/(2*a*x**2)
```

GIAC/XCAS [A] time = 0.457251, size = 1, normalized size = 0.01

$$\textit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^3),x, algorithm="giac")
```

```
[Out] Done
```

3.318 $\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

```
[Out] -1/(4*a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*
a*c]])/(4*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^4 + c*x^8])/(8*a^2)
```

Rubi [A] time = 0.265326, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(4*a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*
a*c]])/(4*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^4 + c*x^8])/(8*a^2)
```

Rubi in Sympy [A] time = 44.0345, size = 87, normalized size = 0.98

$$-\frac{1}{4ax^4}-\frac{b\log\left(x^4\right)}{4a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^4}{\sqrt{-4ac+b^2}}\right)}{4a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] -1/(4*a*x**4) - b*log(x**4)/(4*a**2) + b*log(a + b*x**4 + c*x**8)
/(8*a**2) - (-2*a*c + b**2)*atanh((b + 2*c*x**4)/sqrt(-4*a*c + b*
*2))/(4*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0480938, size = 92, normalized size = 1.03

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(4*a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (
b^2*Log[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*
c*#1^4) & ]/(4*a^2)
```

Maple [A] time = 0.011, size = 119, normalized size = 1.3

$$\frac{b\ln\left(cx^8+bx^4+a\right)}{8\,a^2}-\frac{c}{2\,a}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{4\,a^2}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{1}{4\,ax^4}-\frac{b\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^4+b)/(4*a*c-b^2)^(1/2))*c+1/4/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^4+b)/(4*a*c-b^2)^(1/2))*b^2-1/4/a/x^4-b*ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.418975, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)x^4\log\left(\frac{2\,\left(b^2c-4\,ac^2\right)x^4+b^3-4\,abc+\left(2\,c^2x^8+2\,bcx^4+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(bx^4\log\left(cx^8+bx^4+a\right)-8\,bx^4\log\left(x\right)-2\,a\right)\cdots}{8\,\sqrt{b^2-4\,ac}a^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 2*a*c)*x^4*log((2*(b^2*c - 4*a*c^2)*x^4 + b^3 - 4*a
*b*c + (2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(
c*x^8 + b*x^4 + a)) - (b*x^4*log(c*x^8 + b*x^4 + a) - 8*b*x^4*log
(x) - 2*a)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x^4), 1/8*(2
*(b^2 - 2*a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)) + (b*x^4*log(c*x^8 + b*x^4 + a) - 8*b*x^4*log(x) - 2*a)
*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291142, size = 127, normalized size = 1.43

$$\frac{b\ln\left(cx^8+bx^4+a\right)}{8\,a^2}-\frac{b\ln\left(x^4\right)}{4\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^4-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^5),x, algorithm="giac")
```

```
[Out] 1/8*b*ln(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*ln(x^4)/a^2 + 1/4*(b^2 -
2*a*c)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a^2) + 1/4*(b*x^4 - a)/(a^2*x^4)
```

3.319 $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=381

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^3}{3c}$$

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + ((b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/
4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.36315, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + ((b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/
4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 149.894, size = 386, normalized size = 1.01

$$\frac{x^3}{3c} + \frac{\sqrt[4]{2}\left(-2ac + b^2 - b\sqrt{-4ac + b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{7}{4}}\sqrt[4]{-b + \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} - \frac{\sqrt[4]{2}\left(-2ac + b^2 - b\sqrt{-4ac + b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{7}{4}}\sqrt[4]{-b + \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} - \frac{\sqrt[4]{2}\left(-2ac + b^2 + b\sqrt{-4ac + b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{7}{4}}\sqrt[4]{-b - \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} + \frac{\sqrt[4]{2}\left(-2ac + b^2 + b\sqrt{-4ac + b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{7}{4}}\sqrt[4]{-b - \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(c*x**8+b*x**4+a),x)
```

```
[Out] x**3/(3*c) + 2**(1/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*ata
n(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(7
/4)*(-b + sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - 2**(
1/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1
/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(7/4)*(-b + sqrt(-
4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - 2**(1/4)*(-2*a*c + b
**2 + b*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(
```

```
-4*a*c + b**2))**(1/4))/(4*c**(7/4)*(-b - sqrt(-4*a*c + b**2))**(
1/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*(-2*a*c + b**2 + b*sqrt(-4*a
*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))*
*(1/4))/(4*c**(7/4)*(-b - sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c
 + b**2))
```

Mathematica [C] time = 0.057724, size = 70, normalized size = 0.18

$$\frac{4x^3 - 3\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{12c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] (4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log
[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

Maple [C] time = 0.003, size = 63, normalized size = 0.2

$$\frac{x^3}{3c} - \frac{1}{4c}\sum_{\_R=RootOf(c\_Z^8+\_Z^4b+a)}\frac{(\_R^6b + \_R^2a)\ln(x - \_R)}{2\_R^7c + \_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/3*x^3/c-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3c} - \frac{\int \frac{bx^6+ax^2}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/c - integrate((b*x^6 + a*x^2)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [A] time = 1.18994, size = 8504, normalized size = 22.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^3 - 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2
*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqr
t((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 4
8*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c
^9)))*arctan(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^
2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6
*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^
9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b
^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^
2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c
 + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 +
 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*
c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10
*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^
2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64
*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))/((a^5*b^6 - 5*
a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x + sqrt(1/2)*(a^5*b^6 - 5*a
^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt((2*(a^3*b^6 - 5*a^4*b^4*
c + 6*a^5*b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12*a*b^9*c +
 53*a^2*b^7*c^2 - 103*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5
 - (b^8*c^7 - 13*a*b^6*c^8 + 60*a^2*b^4*c^9 - 112*a^3*b^2*c^10 +
64*a^4*c^11)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 1
2*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*
b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 1
6*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6
*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*
a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*
c^8 + 16*a^2*c^9)))/(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*
c^3)))) + 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b
^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^
2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))
)*arctan(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 3
```

28*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*
c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 3
64*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2
*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14
*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)
*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15
+ 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a
^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 -
(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*
c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))/((a^5*b^6 - 5*a^6*b
^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x + sqrt(1/2)*(a^5*b^6 - 5*a^6*b^
4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt((2*(a^3*b^6 - 5*a^4*b^4*c + 6
*a^5*b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12*a*b^9*c + 53*a
^2*b^7*c^2 - 103*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5 + (b
^8*c^7 - 13*a*b^6*c^8 + 60*a^2*b^4*c^9 - 112*a^3*b^2*c^10 + 64*a^
4*c^11)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^
3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b
^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c
+ 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 +
16*a^2*c^9)))/(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3))
)) - 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*
c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(
1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^
8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*
a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^
5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 6
4*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*
c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^
12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2
*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))
*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7
- 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(
b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6
*a^7*b^2*c^2 - a^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*
b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 1
6*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6
*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*
a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*

c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*
a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^
5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113
*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^
12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c
^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c
^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14
 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*
a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2
 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^
2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (
a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x) - 3*c*sqrt(sq
rt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (
b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37
*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^
17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*sqrt(1/2)*(b
^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*
b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*
c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4
*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sq
rt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^
3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a
^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*
a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 +
 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 1
2*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a
^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b
^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt(
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*
a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9
)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 3
28*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*
c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 3
64*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 -
 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2
*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14
*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)
*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15
 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a

```
^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 -
 (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*
c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*
b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x))/c
```

Sympy [A] time = 139.482, size = 360, normalized size = 0.94

$$\text{RootSum}\left(t^8\left(16777216a^4c^{11} - 16777216a^3b^2c^{10} + 6291456a^2b^4c^9 - 1048576ab^6c^8 + 65536b^8c^7\right) + t^4\left(-28672a^5bc^5 + 7168\right.\right.$$
$$+ \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**11 - 16777216*a**3*b**2*c**10 + 6
291456*a**2*b**4*c**9 - 1048576*a*b**6*c**8 + 65536*b**8*c**7) +
_t**4*(-28672*a**5*b*c**5 + 71680*a**4*b**3*c**4 - 59136*a**3*b**
5*c**3 + 22016*a**2*b**7*c**2 - 3840*a*b**9*c + 256*b**11) + a**7
, Lambda(_t, _t*log(x + (5242880*_t**7*a**5*b*c**12 - 9175040*_t*
*7*a**4*b**3*c**11 + 5963776*_t**7*a**3*b**5*c**10 - 1851392*_t**
7*a**2*b**7*c**9 + 278528*_t**7*a*b**9*c**8 - 16384*_t**7*b**11*c
**7 + 512*_t**3*a**7*c**7 - 9344*_t**3*a**6*b**2*c**6 + 29184*_t*
*3*a**5*b**4*c**5 - 35392*_t**3*a**4*b**6*c**4 + 20992*_t**3*a**3
*b**8*c**3 - 6528*_t**3*a**2*b**10*c**2 + 1024*_t**3*a*b**12*c -
64*_t**3*b**14)/(a**8*c**3 - 6*a**7*b**2*c**2 + 5*a**6*b**4*c - a
**5*b**6)))) + x**3/(3*c)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^10/(c*x^8 + b*x^4 + a), x)
```

3.320 $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b
 + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.28809, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b
 + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 142.909, size = 382, normalized size = 1.02

$$\frac{x}{c} - \frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}} - \frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}} + \frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}} + \frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**8+b*x**4+a),x)
```

```
[Out] x/c - 2**(3/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(2**(1
/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(5/4)*(-b
 + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) - 2**(3/4)*(-
2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(
-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(5/4)*(-b + sqrt(-4*a*c +
 b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*(-2*a*c + b**2 + b
```

```
*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c
+ b**2))**(1/4))/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*sq
rt(-4*a*c + b**2)) + 2**(3/4)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b*
*2))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))
/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2
))
```

Mathematica [C] time = 0.0529236, size = 70, normalized size = 0.19

$$\frac{x}{c}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x -
#1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] time = 0.002, size = 59, normalized size = 0.2

$$\frac{x}{c}+\frac{1}{4c}\sum_{\_R=RootOf\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^8+b*x^4+a),x)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c}-\frac{\int\frac{bx^4+a}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [A] time = 0.450383, size = 5154, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 4
8*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)))*arctan(-1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 -
 (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/((a*b^4 - 3*a^2*b^2*c + a^3*c^2
)*x + sqrt(1/2)*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt((2*(a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^
2*b^4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6
+ 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^
2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16
*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))) - 4*c*sqrt(sqrt(
1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/2*(b
^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*
c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*
c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))/((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + sqrt(1/2)*(a*b
^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt((2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^3*b
^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 6
4*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b
```

```
^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^2*b^4
 - 3*a^3*b^2*c + a^4*c^2)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5
- 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)
*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5
 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^
2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^1
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x
- 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 -
8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b
^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 1
2*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^
2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1
/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a
*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a
^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c
^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12
*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*
(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^
3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6
+ 16*a^2*c^7)))) + 4*x)/c
```

Sympy [A] time = 52.6812, size = 218, normalized size = 0.58

$$\text{RootSum}\Big(t^8\left(16777216a^4c^9-16777216a^3b^2c^8+6291456a^2b^4c^7-1048576ab^6c^6+65536b^8c^5\right)+t^4\left(20480a^4bc^4-30720a^3\right.$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**9 - 16777216*a**3*b**2*c**8 + 629
1456*a**2*b**4*c**7 - 1048576*a*b**6*c**6 + 65536*b**8*c**5) + _t
**4*(20480*a**4*b*c**4 - 30720*a**3*b**3*c**3 + 15616*a**2*b**5*c
**2 - 3328*a*b**7*c + 256*b**9) + a**5, Lambda(_t, _t*log(x + (16
384*_t**5*a**2*b*c**7 - 8192*_t**5*a*b**3*c**6 + 1024*_t**5*b**5*
c**5 - 8*_t*a**3*c**3 + 36*_t*a**2*b**2*c**2 - 24*_t*a*b**4*c + 4
*_t*b**6)/(a**3*c**2 - 3*a**2*b**2*c + a*b**4)))) + x/c
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^8 + b*x^4 + a), x)
```

3.321 $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

```
[Out] -((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c])
 + ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c
]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*
a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*
x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 -
 4*a*c])
```

Rubi [A] time = 0.700114, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c])
 + ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c
]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*
a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*
x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 -
 4*a*c])
```

Rubi in Sympy [A] time = 92.2893, size = 291, normalized size = 0.9

$$-\frac{\sqrt[4]{2}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**8+b*x**4+a),x)
```

```
[Out] -2**(1/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*atan(2**(1/4)*c**(1/4
)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*c**(3/4)*sqrt(-4*a*c +
b**2)) + 2**(1/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*atanh(2**(1/4
)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*c**(3/4)*sqrt(
-4*a*c + b**2)) + 2**(1/4)*(-b + sqrt(-4*a*c + b**2))**(3/4)*atan
(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(3/
4)*sqrt(-4*a*c + b**2)) - 2**(1/4)*(-b + sqrt(-4*a*c + b**2))**(3
```

```
/4)*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/
(4*c**(3/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0349959, size = 44, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^3\log(x-\#1)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4)
 & ]/4
```

Maple [C] time = 0.002, size = 43, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 0.364862, size = 5292, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 1
6*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5)))*arctan(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2
- 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a
^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(
-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))/((a^2*b^2 -
a^3*c)*x + sqrt(1/2)*(a^2*b^2 - a^3*c)*sqrt((2*(a*b^2 - a^2*c)*x^
2 - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (b^6*c^3 - 12*a*b^
4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqr
t(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a*b^2 -
a^2*c)))) + sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2
*c^4 + 16*a^2*c^5)))*arctan(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*
a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4
*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sq
rt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*
c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))/
((a^2*b^2 - a^3*c)*x + sqrt(1/2)*(a^2*b^2 - a^3*c)*sqrt((2*(a*b^2
 - a^2*c)*x^2 - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (b^6*c
^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^
3*c^9)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48
*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
))/(a*b^2 - a^2*c)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c +
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a
```

```
*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4
+ 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a
^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^
2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 1
2*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4
+ 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 1
6*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^
3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqrt(1
/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 -
 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^
4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c
^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(sqrt(
1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2
*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*l
og(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3
 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 1
28*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*
c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 -
3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*
c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1
/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2
- 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a
^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(
-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 -
 a^3*c)*x)
```

Sympy [A] time = 23.9215, size = 230, normalized size = 0.71

$$\mathrm{RootSum}\left(t^8\left(16777216a^4c^7-16777216a^3b^2c^6+6291456a^2b^4c^5-1048576ab^6c^4+65536b^8c^3\right)+t^4\left(-12288a^3bc^3+10240a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**7 - 16777216*a**3*b**2*c**6 + 629
1456*a**2*b**4*c**5 - 1048576*a*b**6*c**4 + 65536*b**8*c**3) + _t
**4*(-12288*a**3*b*c**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c +
256*b**7) + a**3, Lambda(_t, _t*log(x + (2097152*_t**7*a**4*c**7
- 2621440*_t**7*a**3*b**2*c**6 + 1179648*_t**7*a**2*b**4*c**5 - 2
29376*_t**7*a*b**6*c**4 + 16384*_t**7*b**8*c**3 - 1280*_t**3*a**3
*b*c**3 + 1600*_t**3*a**2*b**3*c**2 - 576*_t**3*a*b**5*c + 64*_t*
*3*b**7)/(a**3*c - a**2*b**2))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

3.322 $\int \frac{x^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b -
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c])
- ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]
) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a
*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 -
4*a*c])
```

Rubi [A] time = 0.595679, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b -
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c])
- ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
```

```
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]
) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a
*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 -
4*a*c])
```

Rubi in Sympy [A] time = 89.7584, size = 291, normalized size = 0.9

$$\frac{2^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\,\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\,\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\,\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\,\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] 2**(3/4)*(-b - sqrt(-4*a*c + b**2))**(1/4)*atan(2**(1/4)*c**(1/4)
*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*c**(1/4)*sqrt(-4*a*c + b
**2)) + 2**(3/4)*(-b - sqrt(-4*a*c + b**2))**(1/4)*atanh(2**(1/4)
*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*c**(1/4)*sqrt(-
4*a*c + b**2)) - 2**(3/4)*(-b + sqrt(-4*a*c + b**2))**(1/4)*atan(
2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(1/4
)*sqrt(-4*a*c + b**2)) - 2**(3/4)*(-b + sqrt(-4*a*c + b**2))**(1/
4)*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(
4*c**(1/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0304384, size = 42, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1\log(x-\#1)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) &
 ]/4
```

Maple [C] time = 0.002, size = 43, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 0.302566, size = 2464, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)))*arctan((b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*(x + sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a
*c)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)))))) + sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*arctan((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*(x + sqrt(x^2 +
 sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))))) + 1/4*sqrt(sqrt(1/2)*sqr
t(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/
2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2
)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sq
rt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
```

Sympy [A] time = 16.0307, size = 126, normalized size = 0.39

RootSum $\left(t^{8}\left(16777216a^{4}c^{5}-16777216a^{3}b^{2}c^{4}+6291456a^{2}b^{4}c^{3}-1048576ab^{6}c^{2}+65536b^{8}c\right)+t^{4}\left(4096a^{2}bc^{2}-2048ab^{3}c\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 629
1456*a**2*b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4
*(4096*a**2*b*c**2 - 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t
*log(-32768*_t**5*a**2*c**3 + 16384*_t**5*a*b**2*c**2 - 2048*_t**
5*b**4*c - 4*_t*b + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

3.323 $\int \frac{x^2}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)))
 + (c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
 + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)
) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4
))
```

Rubi [A] time = 0.600244, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)))
 + (c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
 + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)
) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4
))
```

Rubi in Sympy [A] time = 90.8588, size = 291, normalized size = 0.92

$$\frac{\sqrt[4]{2}\sqrt[4]{c}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] 2**(1/4)*c**(1/4)*atan(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b*
*2))**(1/4))/(2*(-b + sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b
**2)) - 2**(1/4)*c**(1/4)*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4
*a*c + b**2))**(1/4))/(2*(-b + sqrt(-4*a*c + b**2))**(1/4)*sqrt(-
4*a*c + b**2)) - 2**(1/4)*c**(1/4)*atan(2**(1/4)*c**(1/4)*x/(-b -
 sqrt(-4*a*c + b**2))**(1/4))/(2*(-b - sqrt(-4*a*c + b**2))**(1/4
)*sqrt(-4*a*c + b**2)) + 2**(1/4)*c**(1/4)*atanh(2**(1/4)*c**(1/4
)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*(-b - sqrt(-4*a*c + b**
2))**(1/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0320882, size = 43, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log(x-\#1)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]
/4
```

Maple [C] time = 0.001, size = 43, normalized size = 0.1

$$\frac{1}{4} \sum_{\_R = RootOf\left(c\_Z^8 + \_Z^4 b + a\right)} \frac{\_R^2 \ln\left(x - \_R\right)}{2\_R^7 c + \_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 0.336273, size = 3767, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
```

8*a^2*b^2*c + 16*a^3*c^2)))*arctan(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*
c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*
b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sq
rt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))/(c*x + sqrt(1/2)*c*sqrt
((2*c*x^2 - sqrt(1/2)*(b^3 - 4*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48
*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/c))) + sqrt(sqrt(1/2)*sqrt(-(b
- (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)))*arctan(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-
(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^
2*c + 16*a^3*c^2))/(c*x + sqrt(1/2)*c*sqrt((2*c*x^2 - sqrt(1/2)*(
b^3 - 4*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*s
qrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)))/c))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1
/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^
3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*
x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/
(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*
a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 6
4*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c
^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) - 1/4*sqrt
(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a
^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/s
qrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(s
qrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*

```
b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*sqr
t(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt
(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b
^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)) + c*x)
```

Sympy [A] time = 12.0883, size = 172, normalized size = 0.55

$$\operatorname{RootSum}\left(t^8\left(16777216a^5c^4 - 16777216a^4b^2c^3 + 6291456a^3b^4c^2 - 1048576a^2b^6c + 65536ab^8\right) + t^4\left(4096a^2bc^2 - 2048ab^3c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 629
1456*a**3*b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4
*(4096*a**2*b*c**2 - 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t
*log(x + (1048576*_t**7*a**4*b*c**3 - 786432*_t**7*a**3*b**3*c**2
 + 196608*_t**7*a**2*b**5*c - 16384*_t**7*a*b**7 - 512*_t**3*a**2
*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

3.324 $\int \frac{1}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=315

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
(c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) +
(c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/
4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
 (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.626329, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
```

```
(c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) +
(c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/
4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
 (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 83.9957, size = 291, normalized size = 0.92

$$-\frac{2^{\frac{3}{4}} c^{\frac{3}{4}} \operatorname{atan}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b+\sqrt{-4 a c+b^{2}}}}\right)}{2\left(-b+\sqrt{-4 a c+b^{2}}\right)^{\frac{3}{4}} \sqrt{-4 a c+b^{2}}}-\frac{2^{\frac{3}{4}} c^{\frac{3}{4}} \operatorname{atanh}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b+\sqrt{-4 a c+b^{2}}}}\right)}{2\left(-b+\sqrt{-4 a c+b^{2}}\right)^{\frac{3}{4}} \sqrt{-4 a c+b^{2}}}+\frac{2^{\frac{3}{4}} c^{\frac{3}{4}} \operatorname{atan}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b-\sqrt{-4 a c+b^{2}}}}\right)}{2\left(-b-\sqrt{-4 a c+b^{2}}\right)^{\frac{3}{4}} \sqrt{-4 a c+b^{2}}}+\frac{2^{\frac{3}{4}} c^{\frac{3}{4}} \operatorname{atanh}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b-\sqrt{-4 a c+b^{2}}}}\right)}{2\left(-b-\sqrt{-4 a c+b^{2}}\right)^{\frac{3}{4}} \sqrt{-4 a c+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**8+b*x**4+a),x)
```

```
[Out] -2**(3/4)*c**(3/4)*atan(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b
**2))**(1/4))/(2*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c +
b**2)) - 2**(3/4)*c**(3/4)*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-
4*a*c + b**2))**(1/4))/(2*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(
-4*a*c + b**2)) + 2**(3/4)*c**(3/4)*atan(2**(1/4)*c**(1/4)*x/(-b
- sqrt(-4*a*c + b**2))**(1/4))/(2*(-b - sqrt(-4*a*c + b**2))**(3/
4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*c**(3/4)*atanh(2**(1/4)*c**(1/
4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*(-b - sqrt(-4*a*c + b*
*2))**(3/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0352672, size = 45, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8 c+\#1^4 b+a\&,\frac{\log (x-\#1)}{2 \#1^7 c+\#1^3 b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) &
 ]/4
```

Maple [C] time = 0.002, size = 40, normalized size = 0.1

$$\frac{1}{4} \sum_{\_R = RootOf\left(c\_Z^8+\_Z^4b+a\right)} \frac{\ln\left(x - \_R\right)}{2\_R^7c + \_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 0.374602, size = 4000, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)))*arctan(1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a
^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*
```

```
sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/((b^2*
c - a*c^2)*x + sqrt(1/2)*(b^2*c - a*c^2)*sqrt((2*(b^2*c^2 - a*c^3
)*x^2 + sqrt(1/2)*(b^6 - 7*a*b^4*c + 14*a^2*b^2*c^2 - 8*a^3*c^3 -
 (a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*a^6*b*c^3)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)))/(b^2*c^2 - a*c^3)))) - sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(-1/2*(b^4 - 5*
a*b^2*c + 4*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)))/((b^2*c - a*c^2)*x + sqrt(1/2)*(b^2
*c - a*c^2)*sqrt((2*(b^2*c^2 - a*c^3)*x^2 + sqrt(1/2)*(b^6 - 7*a*
b^4*c + 14*a^2*b^2*c^2 - 8*a^3*c^3 + (a^3*b^7 - 12*a^4*b^5*c + 48
*a^5*b^3*c^2 - 64*a^6*b*c^3)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3
- 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 - a*c^3)
))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c
+ 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 6
4*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
+ (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/
2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
 a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) + 1/4*sqrt(sqrt(1/2)*sq
rt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-(b^
2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (a^3*b^5 - 8*
a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
```

```
*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/
4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^
4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*
c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2))))
```

Sympy [A] time = 22.4199, size = 177, normalized size = 0.56

$$\mathrm{RootSum}\left(t^8\left(16777216a^7c^4 - 16777216a^6b^2c^3 + 6291456a^5b^4c^2 - 1048576a^4b^6c + 65536a^3b^8\right) + t^4\left(-12288a^3bc^3 + 10240\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**7*c**4 - 16777216*a**6*b**2*c**3 + 629
1456*a**5*b**4*c**2 - 1048576*a**4*b**6*c + 65536*a**3*b**8) + _t
**4*(-12288*a**3*b*c**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c +
256*b**7) + c**3, Lambda(_t, _t*log(x + (16384*_t**5*a**5*b*c**2
- 8192*_t**5*a**4*b**3*c + 1024*_t**5*a**3*b**5 + 8*_t*a**2*c**2
- 16*_t*a*b**2*c + 4*_t*b**4)/(a*c**2 - b**2*c))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

3.325 $\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c
^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqr
t[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTa
n[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)
*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4
*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)
])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 + b
/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.896001, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c
^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqr
t[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTa
n[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)
*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4
*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)
])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 + b
/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 129.01, size = 357, normalized size = 0.98

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] -2**(1/4)*c**(1/4)*(b + sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/
4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b + sqrt(-4*a*c +
b**2))**(1/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*c**(1/4)*(b + sqrt(
-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**
2))**(1/4))/(4*a*(-b + sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c +
b**2)) + 2**(1/4)*c**(1/4)*(b - sqrt(-4*a*c + b**2))*atan(2**(1/4
)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b - sqrt(-
4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - 2**(1/4)*c**(1/4)*(b
```

```
 - sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a
*c + b**2))**(1/4))/(4*a*(-b - sqrt(-4*a*c + b**2))**(1/4)*sqrt(-
4*a*c + b**2)) - 1/(a*x)
```

Mathematica [C] time = 0.0527092, size = 71, normalized size = 0.2

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*L
og[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Maple [C] time = 0.002, size = 63, normalized size = 0.2

$$-\frac{1}{4\,a}\sum_{\_R=RootOf\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6c+\_R^2b\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_
Z^8*c+_Z^4*b+a))-1/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{cx^6+bx^2}{cx^8+bx^4+a}\,dx}{a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^6 + b*x^2)/(c*x^8 + b*x^4 + a), x)/a - 1/(a*x)
```

Fricas [A] time = 0.754121, size = 6971, normalized size = 19.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 +
(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7
*c^2)))*arctan(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2
- 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 -
16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4
 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))/((b^4*c^
4 - 3*a*b^2*c^5 + a^2*c^6)*x + sqrt(1/2)*(b^4*c^4 - 3*a*b^2*c^5 +
 a^2*c^6)*sqrt((2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x^2 - sqrt(1/
2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 43*a^3*b^3*c^3 + 12*a^4*b
*c^4 - (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3
 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c
^3 - 3*a*b^2*c^4 + a^2*c^5)))) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5
- 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*sqrt(1/2)*(b^11 -
 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4
- 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*
a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^1
1*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*
c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
```

```
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2))/((b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x + sqrt(
1/2)*(b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt((2*(b^4*c^3 - 3*a*b^2
*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^
2 - 43*a^3*b^3*c^3 + 12*a^4*b*c^4 + (a^5*b^8 - 13*a^6*b^6*c + 60*
a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)))) + a*x
*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*lo
g(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5
*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c
+ 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c
^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*
c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*
c^5 + a^2*c^6)*x) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12
*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b
^2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a
^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (
a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*
a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*sqrt(sqrt(1/2)*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b
^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3
*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 -
```

```
 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8
*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x)
- a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)
))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a
^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*
b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*
a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*
a*b^2*c^5 + a^2*c^6)*x) - 4)/(a*x)
```

Sympy [A] time = 56.7829, size = 304, normalized size = 0.84

$$\operatorname{RootSum}\left(t^8\left(16777216a^9c^4 - 16777216a^8b^2c^3 + 6291456a^7b^4c^2 - 1048576a^6b^6c + 65536a^5b^8\right) + t^4\left(20480a^4bc^4 - 30720a^3\right.\right.$$

$$-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**9*c**4 - 16777216*a**8*b**2*c**3 + 629
1456*a**7*b**4*c**2 - 1048576*a**6*b**6*c + 65536*a**5*b**8) + _t
**4*(20480*a**4*b*c**4 - 30720*a**3*b**3*c**3 + 15616*a**2*b**5*c
**2 - 3328*a*b**7*c + 256*b**9) + c**5, Lambda(_t, _t*log(x + (-2
097152*_t**7*a**10*c**5 + 5767168*_t**7*a**9*b**2*c**4 - 4587520*
_t**7*a**8*b**4*c**3 + 1605632*_t**7*a**7*b**6*c**2 - 262144*_t**
7*a**6*b**8*c + 16384*_t**7*a**5*b**10 - 2304*_t**3*a**5*b*c**5 +
 8256*_t**3*a**4*b**3*c**4 - 8832*_t**3*a**3*b**5*c**3 + 4032*_t*
```

```
*3*a**2*b**7*c**2 - 832*_t**3*a*b**9*c + 64*_t**3*b**11)/(a**2*c*
*6 - 3*a*b**2*c**5 + b**4*c**4)))) - 1/(a*x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^8 + bx^4 + a)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^2), x)
```

3.326 $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=365

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/
4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 - b/Sqrt[b^2 -
 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/
4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 +
 b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.841326, antiderivative size = 365, normalized size of antiderivative = 1., number

of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(3*a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/
4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 - b/Sqrt[b^2 -
 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/
4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 +
 b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 123.403, size = 360, normalized size = 0.99

$$\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] 2**(3/4)*c**(3/4)*(b + sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4
)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b + sqrt(-4*a*c + b
**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*c**(3/4)*(b + sqrt(-
4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2
))**(1/4))/(4*a*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b
**2)) - 2**(3/4)*c**(3/4)*(b - sqrt(-4*a*c + b**2))*atan(2**(1/4)
*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b - sqrt(-4
*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) - 2**(3/4)*c**(3/4)*(b
- sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*
c + b**2))**(1/4))/(4*a*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4
*a*c + b**2)) - 1/(3*a*x**3)
```

Mathematica [C] time = 0.0584215, size = 75, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4a}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(3*a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c
*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

Maple [C] time = 0.002, size = 62, normalized size = 0.2

$$-\frac{1}{3\,a x^{3}}+\frac{1}{4\,a}\sum_{\_R=RootOf\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}c-b\right)\ln\left(x-\_R\right)}{2\,\_R^{7}c+\_R^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/3/a/x^3+1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{cx^{4}+b}{cx^{8}+bx^{4}+a}\,dx}{a}-\frac{1}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)/(c*x^8 + b*x^4 + a), x)/a - 1/3/(a*x^3)
```

Fricas [A] time = 0.727835, size = 6533, normalized size = 17.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b
^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))
)*arctan(-1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3
+ 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^1
0*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b
^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*
b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*
b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)))/((b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*
c^4 - a^3*c^5)*x + sqrt(1/2)*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c
^4 - a^3*c^5)*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^
3*c^7)*x^2 + sqrt(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c^2 - 147
*a^3*b^6*c^3 + 156*a^4*b^4*c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 - (a^
7*b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2 - 160*a^10*b^3*c^3 + 128*a^
11*b*c^4)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*
c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a
^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5
*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^
3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c
 + 16*a^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7
)))) - 12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^
3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4
*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)
))*arctan(1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3
+ 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^1
0*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b
^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*
b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*
b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)))/((b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*
c^4 - a^3*c^5)*x + sqrt(1/2)*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c
^4 - a^3*c^5)*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^
3*c^7)*x^2 + sqrt(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c^2 - 147
*a^3*b^6*c^3 + 156*a^4*b^4*c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 + (a^
7*b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2 - 160*a^10*b^3*c^3 + 128*a^
11*b*c^4)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*
c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a
^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5
*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^
3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^1
```

5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)))) - 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x - 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x - 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^

```
2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a
^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/
(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^1
7*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 4)/(a*x^3)
```

Sympy [A] time = 123.503, size = 277, normalized size = 0.76

$$\mathrm{RootSum}\left(t^8\left(16777216a^{11}c^4 - 16777216a^{10}b^2c^3 + 6291456a^9b^4c^2 - 1048576a^8b^6c + 65536a^7b^8\right) + t^4\left(-28672a^5bc^5 + 7168\right.\right.$$

$$-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**11*c**4 - 16777216*a**10*b**2*c**3 + 6
291456*a**9*b**4*c**2 - 1048576*a**8*b**6*c + 65536*a**7*b**8) +
_t**4*(-28672*a**5*b*c**5 + 71680*a**4*b**3*c**4 - 59136*a**3*b**
5*c**3 + 22016*a**2*b**7*c**2 - 3840*a*b**9*c + 256*b**11) + c**7
, Lambda(_t, _t*log(x + (32768*_t**5*a**10*c**3 - 32768*_t**5*a**
9*b**2*c**2 + 10240*_t**5*a**8*b**4*c - 1024*_t**5*a**7*b**6 - 36
*_t*a**4*b*c**4 + 120*_t*a**3*b**3*c**3 - 108*_t*a**2*b**5*c**2 +
 36*_t*a*b**7*c - 4*_t*b**9)/(a**3*c**5 - 6*a**2*b**2*c**4 + 5*a*
b**4*c**3 - b**6*c**2)))) - 1/(3*a*x**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^8 + bx^4 + a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^8 + b*x^4 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^4), x)
```

3.327 $\int \frac{x^m}{1+x^4+x^8}\,dx$

Optimal. Leaf size=127

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/
(1 - I*Sqrt[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*
Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3
])])/(Sqrt[3]*(I - Sqrt[3])*(1 + m))
```

Rubi [A] time = 0.151875, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/
(1 - I*Sqrt[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*
Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3
])])/(Sqrt[3]*(I - Sqrt[3])*(1 + m))
```

Rubi in Sympy [A] time = 17.576, size = 102, normalized size = 0.8

$$\frac{2\sqrt{3}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|x^4\left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\right)}{3\left(\sqrt{3}+i\right)(m+1)}+\frac{2\sqrt{3}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|x^4\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\right)}{3\left(\sqrt{3}-i\right)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(x**8+x**4+1),x)
```

```
[Out] 2*sqrt(3)*x**(m + 1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), x**4*(-1
/2 - sqrt(3)*I/2))/(3*(sqrt(3) + I)*(m + 1)) + 2*sqrt(3)*x**(m +
1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), x**4*(-1/2 + sqrt(3)*I/2))
/(3*(sqrt(3) - I)*(m + 1))
```

Mathematica [C] time = 2.42724, size = 488, normalized size = 3.84

$$x^m\left(-\frac{\text{RootSum}\left[\#1^4-\#1^2+1\&,\frac{\#1^2m^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+3\#1^2m\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+2\#1^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\right.}{2\#1^3-\#1}\right.}{m^2+3m+2}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (x^m*(((-I)*(Hypergeometric2F1[-m, -m, 1 - m, (-1)^(1/3)/((-1)^(1
/3) - x)]/(x/(-(-1)^(1/3) + x))^m + Hypergeometric2F1[-m, -m, 1 -
 m, (-1)^(2/3)/((-1)^(2/3) - x)]/(x/(-(-1)^(2/3) + x))^m - Hyperg
eometric2F1[-m, -m, 1 - m, (-1)^(1/3)/((-1)^(1/3) + x)]/(x/((-1)^
(1/3) + x))^m - Hypergeometric2F1[-m, -m, 1 - m, (-1)^(2/3)/((-1)
^(2/3) + x)]/(x/((-1)^(2/3) + x))^m))/Sqrt[3] + RootSum[1 - #1^2
+ #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(
x - #1))^m*(-#1 + 2*#1^3)) & ] - RootSum[1 - #1^2 + #1^4 & , (m*x
^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m,
 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometri
c2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*H
ypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1)
)^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2*#1^3) & ]/(2 + 3*m + m^2)))/(4*
m)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(x^8+x^4+1),x)
```

```
[Out] int(x^m/(x^8+x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8 + x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(x^2 - x + 1)(x^2 + x + 1)(x^4 - x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(x**8+x**4+1),x)
```

```
[Out] Integral(x**m/((x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

3.328 $\int \frac{x^{11}}{1+x^4+x^8}\,dx$

Optimal. Leaf size=44

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x
^8]/8
```

Rubi [A] time = 0.0778663, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x
^8]/8
```

Rubi in Sympy [A] time = 11.4976, size = 39, normalized size = 0.89

$$\frac{x^4}{4}-\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}+\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(x**8+x**4+1),x)
```

```
[Out] x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(sqrt(3)*(2*x**4/3
+ 1/3))/12
```

Mathematica [A] time = 0.0182896, size = 44, normalized size = 1.

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x
^8]/8
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$\frac{x^{4}}{4}-\frac{\ln\left(x^{8}+x^{4}+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(x^8+x^4+1),x)
```

```
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1
/2)
```

Maxima [A] time = 0.822668, size = 47, normalized size = 1.07

$$\frac{1}{4}x^{4}-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}+1\right)\right)-\frac{1}{8}\log\left(x^{8}+x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(
x^8 + x^4 + 1)
```

Fricas [A] time = 0.25262, size = 58, normalized size = 1.32

$$\frac{1}{24}\sqrt{3}\left(2\sqrt{3}x^{4}-\sqrt{3}\log\left(x^{8}+x^{4}+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(2*sqrt(3)*x^4 - sqrt(3)*log(x^8 + x^4 + 1) - 2*arct
an(1/3*sqrt(3)*(2*x^4 + 1)))
```

Sympy [A] time = 0.332669, size = 42, normalized size = 0.95

$$\frac{x^4}{4} - \frac{\log\left(x^8 + x^4 + 1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(x**8+x**4+1),x)
```

```
[Out] x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 +
 sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.265038, size = 47, normalized size = 1.07

$$\frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4 + 1\right)\right) - \frac{1}{8}\ln\left(x^8 + x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*ln(x
^8 + x^4 + 1)
```

3.329 $\int \frac{x^9}{1+x^4+x^8} \, dx$

Optimal. Leaf size=54

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x
^2)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.108533, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x
^2)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 15.7214, size = 51, normalized size = 0.94

$$\frac{x^2}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}-\frac{1}{3}\right)\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**8+x**4+1),x)
```

```
[Out] x**2/2 - sqrt(3)*atan(sqrt(3)*(2*x**2/3 - 1/3))/6 - sqrt(3)*atan(
sqrt(3)*(2*x**2/3 + 1/3))/6
```

Mathematica [C] time = 0.292973, size = 98, normalized size = 1.81

$$\frac{x^2}{2}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)}{2\sqrt{6+6i\sqrt{3}}}-\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6
+ (6*I)*Sqrt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2]
)/(2*Sqrt[6 - (6*I)*Sqrt[3]])
```

Maple [A] time = 0.008, size = 43, normalized size = 0.8

$$\frac{x^2}{2} - \frac{\sqrt{3}}{6}\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right) - \frac{\sqrt{3}}{6}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^8+x^4+1),x)
```

```
[Out] 1/2*x^2-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/6*3^(1/2)*arc
tan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 0.820385, size = 57, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
```

Fricas [A] time = 0.248687, size = 55, normalized size = 1.02

$$\frac{1}{6}\sqrt{3}\left(\sqrt{3}x^2 - \arctan\left(\frac{1}{3}\sqrt{3}x^2\right) - \arctan\left(\frac{1}{3}\sqrt{3}(x^6+2x^2)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(sqrt(3)*x^2 - arctan(1/3*sqrt(3)*x^2) - arctan(1/3*s
qrt(3)*(x^6 + 2*x^2)))
```

Sympy [A] time = 0.343318, size = 51, normalized size = 0.94

$$\frac{x^2}{2} + \frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right) - 2\operatorname{atan}\left(\frac{\sqrt{3}x^6}{3} + \frac{2\sqrt{3}x^2}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**8+x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3
 + 2*sqrt(3)*x**2/3))/12
```

GIAC/XCAS [A] time = 0.29242, size = 57, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
```

3.330 $\int \frac{x^7}{1+x^4+x^8} \, dx$

Optimal. Leaf size=37

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Rubi [A] time = 0.0619881, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Rubi in Sympy [A] time = 8.2776, size = 34, normalized size = 0.92

$$\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}+\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(sqrt(3)*(2*x**4/3 + 1/3))/1
2
```

Mathematica [A] time = 0.0168237, size = 37, normalized size = 1.

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Maple [A] time = 0.004, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^{8}+x^{4}+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2\,x^{4}+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 0.821607, size = 41, normalized size = 1.11

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{4}+1\right)\right)+\frac{1}{8}\log\left(x^{8}+x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4
 + 1)
```

Fricas [A] time = 0.250371, size = 46, normalized size = 1.24

$$\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(x^{8}+x^{4}+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{4}+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(sqrt(3)*log(x^8 + x^4 + 1) - 2*arctan(1/3*sqrt(3)*(
2*x^4 + 1)))
```

Sympy [A] time = 0.313318, size = 37, normalized size = 1.

$$\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12
```

GIAC/XCAS [A] time = 0.269861, size = 41, normalized size = 1.11

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\ln\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*ln(x^8 + x^4
+ 1)
```

3.331 $\int \frac{x^5}{1+x^4+x^8}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqr
t[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rubi [A] time = 0.142103, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqr
t[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rubi in Sympy [A] time = 25.2466, size = 70, normalized size = 0.93

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}-\frac{1}{3}\right)\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(sq
rt(3)*(2*x**2/3 - 1/3))/12 + sqrt(3)*atan(sqrt(3)*(2*x**2/3 + 1/3
))/12
```

Mathematica [C] time = 0.209666, size = 94, normalized size = 1.25

$$\frac{\sqrt{1-i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)+\sqrt{1+i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^5/(1 + x^4 + x^8),x]
```

```
[Out] (Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2
] + Sqrt[1 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/
2])/(4*Sqrt[6])
```

Maple [A] time = 0.005, size = 62, normalized size = 0.8

$$-\frac{\ln\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)+\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/8
*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 0.818125, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4
 - x^2 + 1)
```

Fricas [A] time = 0.261774, size = 88, normalized size = 1.17

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(x^4+x^2+1\right)-\sqrt{3}\log\left(x^4-x^2+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(x^4 + x^2 + 1) - sqrt(3)*log(x^4 - x^2
 + 1) - 2*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 2*arctan(1/3*sqrt(3)*
(2*x^2 - 1)))
```

Sympy [A] time = 0.597719, size = 76, normalized size = 1.01

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*
sqrt(3)*x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 +
sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.316075, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\ln\left(x^4+x^2+1\right)+\frac{1}{8}\ln\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*ln(x^4 + x^2 + 1) + 1/8*ln(x^4 -
 x^2 + 1)
```

3.332 $\int \frac{x^3}{1+x^4+x^8} \, dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0438793, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + x^4 + x^8),x]

[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 5.26444, size = 22, normalized size = 0.96

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**8+x**4+1),x)

[Out] sqrt(3)*atan(sqrt(3)*(2*x**4/3 + 1/3))/6
```

Mathematica [A] time = 0.010338, size = 23, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + x^4 + x^8),x]

[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.002, size = 19, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2\,x^{4}+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^8+x^4+1),x)

[Out] 1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 0.826147, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{4}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + x^4 + 1),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Fricas [A] time = 0.250017, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{4}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + x^4 + 1),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Sympy [A] time = 0.287448, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

GIAC/XCAS [A] time = 0.282584, size = 24, normalized size = 1.04

$$\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

3.333 $\int \frac{x}{1+x^4+x^8} \, dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqr
t[3]]/(4*Sqrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rubi [A] time = 0.113985, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqr
t[3]]/(4*Sqrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rubi in Sympy [A] time = 14.9639, size = 70, normalized size = 0.93

$$-\frac{\log\left(x^4-x^2+1\right)}{8}+\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}-\frac{1}{3}\right)\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**8+x**4+1),x)
```

```
[Out] -log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(s
qrt(3)*(2*x**2/3 - 1/3))/12 + sqrt(3)*atan(sqrt(3)*(2*x**2/3 + 1/
3))/12
```

Mathematica [C] time = 0.0929768, size = 79, normalized size = 1.05

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)-\sqrt{1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(1 + x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt
[1 + I*Sqrt[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [A] time = 0.005, size = 62, normalized size = 0.8

$$\frac{\ln\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)-\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^4+x^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/8*
ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 0.821219, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4
 - x^2 + 1)
```

Fricas [A] time = 0.259487, size = 88, normalized size = 1.17

$$\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(x^4+x^2+1\right)-\sqrt{3}\log\left(x^4-x^2+1\right)+2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(sqrt(3)*log(x^4 + x^2 + 1) - sqrt(3)*log(x^4 - x^2
+ 1) + 2*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*arctan(1/3*sqrt(3)*(
2*x^2 - 1)))
```

Sympy [A] time = 0.60481, size = 76, normalized size = 1.01

$$-\frac{\log\left(x^4-x^2+1\right)}{8}+\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**8+x**4+1),x)
```

```
[Out] -log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2
*sqrt(3)*x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 +
 sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.290922, size = 82, normalized size = 1.09

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\ln\left(x^4+x^2+1\right)-\frac{1}{8}\ln\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) + 1/8*ln(x^4 + x^2 + 1) - 1/8*ln(x^4 -
 x^2 + 1)
```

3.334 $\int \frac{1}{x(1+x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 + x^4 +
 x^8]/8
```

Rubi [A] time = 0.0657754, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 + x^4 +
 x^8]/8
```

Rubi in Sympy [A] time = 11.5548, size = 41, normalized size = 1.05

$$\frac{\log\left(x^4\right)}{4}-\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}+\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**8+x**4+1),x)
```

```
[Out] log(x**4)/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(sqrt(3)*(2*x*
*4/3 + 1/3))/12
```

Mathematica [C] time = 0.157781, size = 138, normalized size = 3.54

$$\frac{1}{24}\left(-\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)-\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)-3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+24\log(x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x
)/Sqrt[3]] + 24*Log[x] - Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*
Sqrt[3] + x^2] - Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) +
x^2] - 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

Maple [B] time = 0.013, size = 87, normalized size = 2.2

$$-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(x^4-x^2+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)+\ln(x)-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/8*ln(
x^4-x^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+ln(x)-1/8*l
n(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.823128, size = 49, normalized size = 1.26

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.249995, size = 55, normalized size = 1.41

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(x^8+x^4+1\right)-8\sqrt{3}\log\left(x\right)+2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(x^8 + x^4 + 1) - 8*sqrt(3)*log(x) + 2*
arctan(1/3*sqrt(3)*(2*x^4 + 1)))
```

Sympy [A] time = 0.369719, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**8+x**4+1),x)
```

```
[Out] log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 +
 sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.288732, size = 49, normalized size = 1.26

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\ln\left(x^8+x^4+1\right)+\frac{1}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*ln(x^8 + x^4
+ 1) + 1/4*ln(x^4)
```

3.335 $\int \frac{1}{x^3(1+x^4+x^8)}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1
+ 2*x^2)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0995506, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1
+ 2*x^2)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 15.7505, size = 53, normalized size = 0.98

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{6}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*(-2*x**2/3 + 1/3))/6 - sqrt(3)*atan(sqrt(3)*
(2*x**2/3 + 1/3))/6 - 1/(2*x**2)
```

Mathematica [C] time = 0.0849388, size = 100, normalized size = 1.85

$$\frac{1}{12}\left(-\frac{6}{x^2}+i\sqrt{3}\log\left(2x^2-i\sqrt{3}-1\right)-i\sqrt{3}\log\left(2x^2+i\sqrt{3}-1\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] + I*Sqrt[3]*Log[-1 - I*Sqrt[3] + 2*x^2] - I*S
qrt[3]*Log[-1 + I*Sqrt[3] + 2*x^2])/12
```

Maple [A]   time = 0.009, size = 57, normalized size = 1.1

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)-\frac{1}{2x^2}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*3^(1/2)*arctan(1/3*(2
*x^2-1)*3^(1/2))-1/2/x^2-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]   time = 0.819949, size = 57, normalized size = 1.06

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2/x^2
```

Fricas [A]   time = 0.247279, size = 59, normalized size = 1.09

$$-\frac{\sqrt{3}\left(x^2\arctan\left(\frac{1}{3}\sqrt{3}x^2\right)+x^2\arctan\left(\frac{1}{3}\sqrt{3}(x^6+2x^2)\right)+\sqrt{3}\right)}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*(x^2*arctan(1/3*sqrt(3)*x^2) + x^2*arctan(1/3*sqrt(3
)*(x^6 + 2*x^2)) + sqrt(3))/x^2
```

Sympy [A] time = 0.440587, size = 53, normalized size = 0.98

$$\frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^2}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt
(3)*x**2/3))/12 - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.311122, size = 57, normalized size = 1.06

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^3),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2/x^2
```

3.336 $\int \frac{1}{x^5(1+x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\log(x)$$

```
[Out] -1/(4*x^4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + L
og[1 + x^4 + x^8]/8
```

Rubi [A] time = 0.10025, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + L
og[1 + x^4 + x^8]/8
```

Rubi in Sympy [A] time = 14.3619, size = 48, normalized size = 1.

$$-\frac{\log\left(x^4\right)}{4}+\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}+\frac{1}{3}\right)\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**8+x**4+1),x)
```

```
[Out] -log(x**4)/4 + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(sqrt(3)*(2*x
**4/3 + 1/3))/12 - 1/(4*x**4)
```

Mathematica [C] time = 0.253038, size = 141, normalized size = 2.94

$$\frac{1}{24}\left(-\frac{6}{x^4}+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)+3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-24\log(x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(I + Sqrt[3])*Log[-1/2
- (I/2)*Sqrt[3] + x^2] + Sqrt[3]*(-I + Sqrt[3])*Log[(I/2)*(I + Sq
rt[3]) + x^2] + 3*Log[1 - x + x^2] + 3*Log[1 + x + x^2])/24
```

Maple [B] time = 0.012, size = 94, normalized size = 2.

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^4-x^2+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)-\frac{1}{4x^4}-\ln(x)+\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(x
^4-x^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/4/x^4-ln(x
)+1/8*ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.822466, size = 55, normalized size = 1.15

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^5),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log
(x^8 + x^4 + 1) - 1/4*log(x^4)
```

Fricas [A] time = 0.254229, size = 78, normalized size = 1.62

$$\frac{\sqrt{3}\left(\sqrt{3}x^4 \log\left(x^8 + x^4 + 1\right) - 8\sqrt{3}x^4 \log\left(x\right) - 2\,x^4 \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4 + 1\right)\right) - 2\sqrt{3}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^5),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(sqrt(3)*x^4*log(x^8 + x^4 + 1) - 8*sqrt(3)*x^4*log(
x) - 2*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 2*sqrt(3))/x^4
```

Sympy [A] time = 0.522131, size = 48, normalized size = 1.

$$-\log\left(x\right) + \frac{\log\left(x^8 + x^4 + 1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**8+x**4+1),x)
```

```
[Out] -log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3
+ sqrt(3)/3)/12 - 1/(4*x**4)
```

GIAC/XCAS [A] time = 0.309429, size = 62, normalized size = 1.29

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4 + 1\right)\right) + \frac{x^4 - 1}{4\,x^4} + \frac{1}{8}\ln\left(x^8 + x^4 + 1\right) - \frac{1}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^5),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4
 + 1/8*ln(x^8 + x^4 + 1) - 1/4*ln(x^4)
```

3.337 $\int \frac{1}{x^7(1+x^4+x^8)} dx$

Optimal. Leaf size=89

$$-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/(6*x^6) + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3])
+ ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8
- Log[1 + x^2 + x^4]/8
```

Rubi [A] time = 0.189117, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714

$$-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3])
+ ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8
- Log[1 + x^2 + x^4]/8
```

Rubi in Sympy [A] time = 37.3575, size = 83, normalized size = 0.93

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}-\frac{1}{3}\right)\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{12}+\frac{1}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(sq
rt(3)*(2*x**2/3 - 1/3))/12 + sqrt(3)*atan(sqrt(3)*(2*x**2/3 + 1/3
))/12 + 1/(2*x**2) - 1/(6*x**6)
```

Mathematica [C] time = 0.233839, size = 142, normalized size = 1.6

$$\frac{1}{24}\left(-\frac{4}{x^6}+\frac{12}{x^2}+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)\right.$$
$$\left.-3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] (-4/x^6 + 12/x^2 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[
3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 -
(I/2)*Sqrt[3] + x^2] + Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[
3]) + x^2] - 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.012, size = 95, normalized size = 1.1

$$-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^4-x^2+1\right)}{8}$$
$$+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(
x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/6/x^6+1/2
/x^2-1/8*ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.821896, size = 99, normalized size = 1.11

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)$$
$$+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^7),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 +
x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Fricas [A] time = 0.25767, size = 126, normalized size = 1.42

$$-\frac{\sqrt{3}\left(3\sqrt{3}x^6\log\left(x^4+x^2+1\right)-3\sqrt{3}x^6\log\left(x^4-x^2+1\right)-6\,x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2+1\right)\right)-6\,x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right)-\right.}{72\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^7),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(3)*(3*sqrt(3)*x^6*log(x^4 + x^2 + 1) - 3*sqrt(3)*x^6*l
og(x^4 - x^2 + 1) - 6*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 6*x^6
*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 4*sqrt(3)*(3*x^4 - 1))/x^6
```

Sympy [A] time = 0.800666, size = 88, normalized size = 0.99

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*
sqrt(3)*x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 +
sqrt(3)/3)/12 + (3*x**4 - 1)/(6*x**6)
```

GIAC/XCAS [A] time = 0.298674, size = 99, normalized size = 1.11

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right)+\frac{3\,x^4-1}{6\,x^6}-\frac{1}{8}\ln\left(x^4+x^2+1\right)+\frac{1}{8}\ln\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^7),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*ln(x^4 + x
^2 + 1) + 1/8*ln(x^4 - x^2 + 1)
```

3.338 $\int \frac{x^8}{1+x^4+x^8}\,dx$

Optimal. Leaf size=141

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]
/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x
]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x
 + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.182506, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(1 + x^4 + x^8),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]
/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x
]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x
 + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 33.4815, size = 129, normalized size = 0.91

$$x+\frac{\log\left(x^2-x+1\right)}{8}-\frac{\log\left(x^2+x+1\right)}{8}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}-\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{4}-\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**8+x**4+1),x)
```

```
[Out] x + log(x**2 - x + 1)/8 - log(x**2 + x + 1)/8 + sqrt(3)*log(x**2
- sqrt(3)*x + 1)/24 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/24 - sqrt
(3)*atan(sqrt(3)*(2*x/3 - 1/3))/12 - sqrt(3)*atan(sqrt(3)*(2*x/3
+ 1/3))/12 - atan(2*x - sqrt(3))/4 - atan(2*x + sqrt(3))/4
```

Mathematica [C] time = 0.447036, size = 139, normalized size = 0.99

$$\frac{1}{24}\left(3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+24x-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}+\frac{i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^8/(1 + x^4 + x^8),x]
```

```
[Out] ((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (
I*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x
 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2
*x)/Sqrt[3]] + 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.03, size = 110, normalized size = 0.8

$$x-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^8+x^4+1),x)
```

```
[Out] x-1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*l
n(1+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))-1/24*ln(1+x^2+
x*3^(1/2))*3^(1/2)-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3
```

```
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x)
 - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 0.278275, size = 220, normalized size = 1.56

$$\frac{1}{24}\sqrt{3}\left(8\sqrt{3}x+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2+\sqrt{3}x+1}+3}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2-\sqrt{3}x+1}-3}\right)-\sqrt{3}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(8*sqrt(3)*x + 4*sqrt(3)*arctan(sqrt(3)/(2*sqrt(3)*x
 + 2*sqrt(3)*sqrt(x^2 + sqrt(3)*x + 1) + 3)) + 4*sqrt(3)*arctan(s
qrt(3)/(2*sqrt(3)*x + 2*sqrt(3)*sqrt(x^2 - sqrt(3)*x + 1) - 3)) -
 sqrt(3)*log(x^2 + x + 1) + sqrt(3)*log(x^2 - x + 1) - 2*arctan(1
/3*sqrt(3)*(2*x + 1)) - 2*arctan(1/3*sqrt(3)*(2*x - 1)) - log(x^2
 + sqrt(3)*x + 1) + log(x^2 - sqrt(3)*x + 1))
```

Sympy [A] time = 2.88825, size = 192, normalized size = 1.36

$$x+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**8+x**4+1),x)
```

```
[Out] x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sq
rt(3)*I/24)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sq
rt(3)*I/24)**5 + sqrt(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 -
 sqrt(3)*I/3 - 9216*(-1/8 + sqrt(3)*I/24)**5) + (-1/8 - sqrt(3)*I
/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)**5 + sqrt(3)*I/3) + R
ootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9216*_t**5 -
 8*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^8 + x^4 + 1), x)
```

3.339 $\int \frac{x^6}{1+x^4+x^8}\,dx$

Optimal. Leaf size=88

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sq
rt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.112379, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sq
rt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 24.3977, size = 83, normalized size = 0.94

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)
*x + 1)/12 + sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/6 + sqrt(3)*atan
(sqrt(3)*(2*x/3 + 1/3))/6
```

Mathematica [A] time = 0.0380885, size = 68, normalized size = 0.77

$$\frac{\log\left(-x^2+\sqrt{3}x-1\right)-\log\left(x^2+\sqrt{3}x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 + x^4 + x^8),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log
[-1 + Sqrt[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A] time = 0.018, size = 67, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*
3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/6*3^(1/2)*arctan(1/3*(
2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x
)
```

Fricas [A] time = 0.258608, size = 93, normalized size = 1.06

$$\frac{1}{12}\sqrt{3}\left(2\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+2\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\log\left(-\frac{6x^3-\sqrt{3}\left(x^4+5x^2+1\right)+6x}{x^4-x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(2*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 2*arctan(1/3*sq
rt(3)*x) + log(-(6*x^3 - sqrt(3)*(x^4 + 5*x^2 + 1) + 6*x)/(x^4 -
x^2 + 1)))
```

Sympy [A] time = 0.504953, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*
x/3))/12 + sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**
2 + sqrt(3)*x + 1)/12
```

GIAC/XCAS [A] time = 0.284739, size = 97, normalized size = 1.1

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{12}\sqrt{3}\ln\left(\frac{\left|2x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/12*sqrt(3)*ln(abs(2*x - 2*sqrt(3) + 2/x)
/abs(2*x + 2*sqrt(3) + 2/x))
```

3.340 $\int \frac{x^4}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4
- Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x
^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.204631, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4
- Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x
^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 48.7079, size = 128, normalized size = 0.91

$$-\frac{\log\left(x^2-x+1\right)}{8}+\frac{\log\left(x^2+x+1\right)}{8}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}+\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{4}+\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**8+x**4+1),x)
```

```
[Out] -log(x**2 - x + 1)/8 + log(x**2 + x + 1)/8 + sqrt(3)*log(x**2 - s
qrt(3)*x + 1)/24 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/24 - sqrt(3)
*atan(sqrt(3)*(2*x/3 - 1/3))/12 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1
/3))/12 + atan(2*x - sqrt(3))/4 + atan(2*x + sqrt(3))/4
```

Mathematica [C] time = 0.305339, size = 135, normalized size = 0.96

$$\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-2i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+2i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] +
(2*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*
Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/S
qrt[3]] - 3*Log[1 - x + x^2] + 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.017, size = 109, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln(
1+x^2-x*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(1/2))-1/24*ln(1+x^2+x*
3^(1/2))*3^(1/2)+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)-1/12*3^(
1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) +
 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 0.288243, size = 193, normalized size = 1.38

$$-\frac{1}{24}\sqrt{3}\left(4\sqrt{3}\arctan\left(\frac{1}{2x+\sqrt{3}+2\sqrt{x^2+\sqrt{3}x+1}}\right)+4\sqrt{3}\arctan\left(\frac{1}{2x-\sqrt{3}+2\sqrt{x^2-\sqrt{3}x+1}}\right)-\sqrt{3}\log\left(x^2+x+1\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(4*sqrt(3)*arctan(1/(2*x + sqrt(3) + 2*sqrt(x^2 + s
qrt(3)*x + 1))) + 4*sqrt(3)*arctan(1/(2*x - sqrt(3) + 2*sqrt(x^2
- sqrt(3)*x + 1))) - sqrt(3)*log(x^2 + x + 1) + sqrt(3)*log(x^2 -
 x + 1) + 2*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*arctan(1/3*sqrt(3)*
(2*x - 1)) + log(x^2 + sqrt(3)*x + 1) - log(x^2 - sqrt(3)*x + 1))
```

Sympy [A] time = 2.88616, size = 197, normalized size = 1.41

$$\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\log\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{6} - 18432\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\log\left(x - \frac{1}{2} - 18432\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^5 - \frac{\sqrt{3}i}{6}\right) + \left(-\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\log\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{6} - 18432\left(-\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\log\left(x + \frac{1}{2} - 18432\left(-\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^5 - \frac{\sqrt{3}i}{6}\right) + \operatorname{RootSum}\left(2304t^4 + 48t^2 + 1, \left(t \mapsto t\log\left(-18432t^5 - 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqr
t(3)*I/24)**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 +
sqrt(3)*I/24)**5 - sqrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1
/2 + sqrt(3)*I/6 - 18432*(-1/8 - sqrt(3)*I/24)**5) + (-1/8 + sqrt
(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sqrt(3)*I/24)**5 - sqrt(3)*
I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-1843
2*_t**5 - 4*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^8 + x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^8 + x^4 + 1), x)
```

3.341 $\int \frac{x^2}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4
+ Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x
^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.164832, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4
+ Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x
^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 29.8471, size = 128, normalized size = 0.91

$$\frac{\log\left(x^2-x+1\right)}{8}-\frac{\log\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{24}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{12}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**8+x**4+1),x)
```

```
[Out] log(x**2 - x + 1)/8 - log(x**2 + x + 1)/8 - sqrt(3)*log(x**2 - sq
rt(3)*x + 1)/24 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/24 - sqrt(3)*
atan(sqrt(3)*(2*x/3 - 1/3))/12 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/
3))/12 + atan(2*x - sqrt(3))/4 + atan(2*x + sqrt(3))/4
```

Mathematica [C] time = 0.296594, size = 135, normalized size = 0.96

$$\frac{1}{48}\left(6\log\left(x^2-x+1\right)-6\log\left(x^2+x+1\right)+4i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-4i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-4\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-4\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (
4*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*S
qrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 6*Log[1 - x + x^2] - 6*Log[1 + x + x^2])/48
```

Maple [A] time = 0.012, size = 109, normalized size = 0.8

$$-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/24*ln
(1+x^2-x*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(1/2))+1/24*ln(1+x^2+x
*3^(1/2))*3^(1/2)+1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^
(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1
/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 0.270786, size = 212, normalized size = 1.51

$$-\frac{1}{24}\sqrt{3}\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2+\sqrt{3}x+1}+3}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2-\sqrt{3}x+1}-3}\right)+\sqrt{3}\log\left(x^2+x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(4*sqrt(3)*arctan(sqrt(3)/(2*sqrt(3)*x + 2*sqrt(3)*
sqrt(x^2 + sqrt(3)*x + 1) + 3)) + 4*sqrt(3)*arctan(sqrt(3)/(2*sqr
t(3)*x + 2*sqrt(3)*sqrt(x^2 - sqrt(3)*x + 1) - 3)) + sqrt(3)*log(
x^2 + x + 1) - sqrt(3)*log(x^2 - x + 1) + 2*arctan(1/3*sqrt(3)*(2
*x + 1)) + 2*arctan(1/3*sqrt(3)*(2*x - 1)) - log(x^2 + sqrt(3)*x
+ 1) + log(x^2 - sqrt(3)*x + 1))
```

Sympy [A] time = 2.89033, size = 214, normalized size = 1.53

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(442368t^7-192t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 1
92*(-1/8 - sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*
(-1/8 + sqrt(3)*I/24)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/
8 - sqrt(3)*I/24)*log(x + 442368*(1/8 - sqrt(3)*I/24)**7 - 192*(1
/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*I/24)*log(x - 192*(1/8 +
sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24)**7) + RootSum(2304
*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7 - 192*_t**3
 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^8 + x^4 + 1), x)
```

3.342 $\int \frac{1}{1+x^4+x^8}\,dx$

Optimal. Leaf size=88

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sq
rt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.0986079, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]
]/(2*Sqrt[3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sq
rt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 21.1603, size = 83, normalized size = 0.94

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8+x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3
)*x + 1)/12 + sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/6 + sqrt(3)*ata
n(sqrt(3)*(2*x/3 + 1/3))/6
```

Mathematica [A] time = 0.0350676, size = 68, normalized size = 0.77

$$\frac{-\log\left(-x^2+\sqrt{3}x-1\right)+\log\left(x^2+\sqrt{3}x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log
[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A] time = 0.015, size = 67, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/12*ln(1+x^2-x*3^(1/2))*
3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/6*3^(1/2)*arctan(1/3*(
2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x
)
```

Fricas [A] time = 0.269096, size = 90, normalized size = 1.02

$$\frac{1}{12}\sqrt{3}\left(2\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+2\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\log\left(\frac{6x^3+\sqrt{3}\left(x^4+5x^2+1\right)+6x}{x^4-x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(2*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 2*arctan(1/3*sq
rt(3)*x) + log((6*x^3 + sqrt(3)*(x^4 + 5*x^2 + 1) + 6*x)/(x^4 - x
^2 + 1)))
```

Sympy [A] time = 0.515138, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*
x/3))/12 - sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**
2 + sqrt(3)*x + 1)/12
```

GIAC/XCAS [A] time = 0.300524, size = 97, normalized size = 1.1

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\sqrt{3}\ln\left(\frac{\left|2x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) - 1/12*sqrt(3)*ln(abs(2*x - 2*sqrt(3) + 2/x)
/abs(2*x + 2*sqrt(3) + 2/x))
```

3.343 $\int \frac{1}{x^2(1+x^4+x^8)}\,dx$

Optimal. Leaf size=145

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3]
- 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3]
 + 2*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqr
t[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.226359, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3]
- 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3]
 + 2*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqr
t[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 55.7349, size = 131, normalized size = 0.9

$$-\frac{\log\left(x^2-x+1\right)}{8}+\frac{\log\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{24}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}-\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{4}-\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{4}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**8+x**4+1),x)
```

```
[Out] -log(x**2 - x + 1)/8 + log(x**2 + x + 1)/8 - sqrt(3)*log(x**2 - s
qrt(3)*x + 1)/24 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/24 - sqrt(3)
*atan(sqrt(3)*(2*x/3 - 1/3))/12 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1
/3))/12 - atan(2*x - sqrt(3))/4 - atan(2*x + sqrt(3))/4 - 1/x
```

Mathematica [C] time = 0.386753, size = 140, normalized size = 0.97

$$\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-\frac{24}{x}+2i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] (-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x
)/2] - (2*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/
2] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 +
 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2] + 3*Log[1 + x + x^2])/24
```

Maple [A] time = 0.016, size = 114, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-x^{-1}-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/24*ln(
1+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))+1/24*ln(1+x^2+x*
3^(1/2))*3^(1/2)-1/4*arctan(2*x+3^(1/2))-1/x-1/8*ln(x^2-x+1)-1/12
*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}-\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1)
, x) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 0.280712, size = 235, normalized size = 1.62

$$\frac{\sqrt{3}\left(4\sqrt{3}x\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2+\sqrt{3}x+1}+3}\right)+4\sqrt{3}x\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2-\sqrt{3}x+1}-3}\right)+\sqrt{3}x\log\left(x^2+x+1\right)-\sqrt{3}x\log\left(x^2-x\right.\right.}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(4*sqrt(3)*x*arctan(sqrt(3)/(2*sqrt(3)*x + 2*sqrt(3)
*sqrt(x^2 + sqrt(3)*x + 1) + 3)) + 4*sqrt(3)*x*arctan(sqrt(3)/(2*
sqrt(3)*x + 2*sqrt(3)*sqrt(x^2 - sqrt(3)*x + 1) - 3)) + sqrt(3)*x
*log(x^2 + x + 1) - sqrt(3)*x*log(x^2 - x + 1) - 2*x*arctan(1/3*s
qrt(3)*(2*x + 1)) - 2*x*arctan(1/3*sqrt(3)*(2*x - 1)) + x*log(x^2
 + sqrt(3)*x + 1) - x*log(x^2 - sqrt(3)*x + 1) - 8*sqrt(3))/x
```

Sympy [A] time = 2.9611, size = 218, normalized size = 1.5

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{7}-384\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{3}-442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{7}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{7}-384\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{3}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{3}-442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{7}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(-442368t^{7}-384t^{3}+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 3
84*(-1/8 - sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*
(-1/8 + sqrt(3)*I/24)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/
8 - sqrt(3)*I/24)*log(x - 442368*(1/8 - sqrt(3)*I/24)**7 - 384*(1
/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*I/24)*log(x - 384*(1/8 +
sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24)**7) + RootSum(2304
*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7 - 384*_t**
3 + x))) - 1/x
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^{8}+x^{4}+1)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + x^4 + 1)*x^2), x)
```

3.344 $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

Optimal. Leaf size=147

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[
3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt
[3] + 2*x]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 -
Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3
])
```

Rubi [A] time = 0.188172, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[
3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt
[3] + 2*x]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 -
Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3
])
```

Rubi in Sympy [A] time = 34.9141, size = 134, normalized size = 0.91

$$\frac{\log\left(x^2-x+1\right)}{8}-\frac{\log\left(x^2+x+1\right)}{8}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}-\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{4}-\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{4}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**8+x**4+1),x)
```

```
[Out] log(x**2 - x + 1)/8 - log(x**2 + x + 1)/8 + sqrt(3)*log(x**2 - sq
rt(3)*x + 1)/24 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/24 - sqrt(3)*
atan(sqrt(3)*(2*x/3 - 1/3))/12 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/
3))/12 - atan(2*x - sqrt(3))/4 - atan(2*x + sqrt(3))/4 - 1/(3*x**
3)
```

Mathematica [C] time = 0.554714, size = 148, normalized size = 1.01

$$\frac{1}{24}\left(-\frac{8}{x^3}+3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)-\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}i\left(\sqrt{3}+i\right)}}+\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{1}{6}i\left(\sqrt{3}-i\right)}}-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] (-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + S
qrt[3])] + ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-I/6)*(-I
+ Sqrt[3])] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*Ar
cTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2]
)/24
```

Maple [A] time = 0.009, size = 114, normalized size = 0.8

$$-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{1}{3x^3}+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln
(1+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))-1/24*ln(1+x^2+x
*3^(1/2))*3^(1/2)-1/4*arctan(2*x+3^(1/2))-1/3/x^3+1/8*ln(x^2-x+1)
-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{3x^3}-\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 +
1), x) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 0.275027, size = 259, normalized size = 1.76

$$\sqrt{3}\left(12\sqrt{3}x^3\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2+\sqrt{3}x+1}+3}\right)+12\sqrt{3}x^3\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2-\sqrt{3}x+1}-3}\right)-3\sqrt{3}x^3\log\left(x^2+x+1\right)+3\sqrt{3}x^3\,\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(12*sqrt(3)*x^3*arctan(sqrt(3)/(2*sqrt(3)*x + 2*sqrt
(3)*sqrt(x^2 + sqrt(3)*x + 1) + 3)) + 12*sqrt(3)*x^3*arctan(sqrt(
3)/(2*sqrt(3)*x + 2*sqrt(3)*sqrt(x^2 - sqrt(3)*x + 1) - 3)) - 3*s
qrt(3)*x^3*log(x^2 + x + 1) + 3*sqrt(3)*x^3*log(x^2 - x + 1) - 6*
x^3*arctan(1/3*sqrt(3)*(2*x + 1)) - 6*x^3*arctan(1/3*sqrt(3)*(2*x
 - 1)) - 3*x^3*log(x^2 + sqrt(3)*x + 1) + 3*x^3*log(x^2 - sqrt(3)
*x + 1) - 8*sqrt(3))/x^3
```

Sympy [A] time = 3.00226, size = 197, normalized size = 1.34

$$\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3
)*I/24)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3
)*I/24)**5 + sqrt(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqr
t(3)*I/3 - 9216*(-1/8 + sqrt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)
*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)**5 + sqrt(3)*I/3) + RootS
um(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9216*_t**5 - 8*_
t + x))) - 1/(3*x**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^8+x^4+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + x^4 + 1)*x^4), x)
```

3.345 $\int \frac{1}{x^6(1+x^4+x^8)}\,dx$

Optimal. Leaf size=98

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/(5*x^5) + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + Arc
Tan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*
Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.166289, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + Arc
Tan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*
Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 34.8661, size = 94, normalized size = 0.96

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}+\frac{1}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)
*x + 1)/12 + sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/6 + sqrt(3)*atan
```

```
(sqrt(3)*(2*x/3 + 1/3))/6 + 1/x - 1/(5*x**5)
```

Mathematica [A] time = 0.0637317, size = 95, normalized size = 0.97

$$\frac{1}{60}\left(-\frac{12}{x^5}+5\sqrt{3}\log\left(-x^2+\sqrt{3}x-1\right)-5\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{60}{x}+10\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+10\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] (-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt
[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^
2] - 5*Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/60
```

Maple [A] time = 0.011, size = 75, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}-\frac{1}{5\,x^5}+x^{-1}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^8+x^4+1),x)
```

```
[Out] 1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*
3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)-1/5/x^5+1/x+1/6*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{5\,x^4-1}{5\,x^5}+\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 -
 1)/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 0.264504, size = 128, normalized size = 1.31

$$\frac{\sqrt{3}\left(10\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2\,x\right)\right)+10\,x^5\arctan\left(\frac{1}{3}\sqrt{3}x\right)+5\,x^5\log\left(-\frac{6\,x^3-\sqrt{3}\left(x^4+5\,x^2+1\right)+6\,x}{x^4-x^2+1}\right)+4\,\sqrt{3}\left(5\,x^4-1\right)\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^6),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(3)*(10*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*x^5*arc
tan(1/3*sqrt(3)*x) + 5*x^5*log(-(6*x^3 - sqrt(3)*(x^4 + 5*x^2 + 1
) + 6*x)/(x^4 - x^2 + 1)) + 4*sqrt(3)*(5*x^4 - 1))/x^5
```

Sympy [A] time = 0.696406, size = 94, normalized size = 0.96

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{5x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*
x/3))/12 + sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**
2 + sqrt(3)*x + 1)/12 + (5*x**4 - 1)/(5*x**5)
```

GIAC/XCAS [A] time = 0.277816, size = 113, normalized size = 1.15

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{12}\sqrt{3}\ln\left(\frac{\left|2\,x-2\,\sqrt{3}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{3}+\frac{2}{x}\right|}\right)+\frac{5\,x^4-1}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/12*sqrt(3)*ln(abs(2*x - 2*sqrt(3) + 2/x)
/abs(2*x + 2*sqrt(3) + 2/x)) + 1/5*(5*x^4 - 1)/x^5
```

3.346 $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -1/(7*x^7) + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) -
ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) +
 ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/
8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^
2]/(8*Sqrt[3])
```

Rubi [A] time = 0.284647, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$

$$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) -
ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) +
 ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/
8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^
2]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 62.7571, size = 141, normalized size = 0.92

$$-\frac{\log\left(x^2-x+1\right)}{8}+\frac{\log\left(x^2+x+1\right)}{8}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}+\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{4}+\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{4}+\frac{1}{3x^3}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**8/(x**8+x**4+1),x)
```

```
[Out] -log(x**2 - x + 1)/8 + log(x**2 + x + 1)/8 + sqrt(3)*log(x**2 - s
qrt(3)*x + 1)/24 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/24 - sqrt(3)
*atan(sqrt(3)*(2*x/3 - 1/3))/12 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1
/3))/12 + atan(2*x - sqrt(3))/4 + atan(2*x + sqrt(3))/4 + 1/(3*x*
*3) - 1/(7*x**7)
```

Mathematica [C] time = 0.693319, size = 171, normalized size = 1.11

$$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{2\sqrt{-6+6i\sqrt{3}}}+\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{-6-6i\sqrt{3}}}-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x
)/2])/(2*Sqrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 +
 I*Sqrt[3])*x)/2])/(2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*
x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) -
 Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8
```

Maple [A] time = 0.012, size = 119, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^8/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln(
1+x^2-x*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(1/2))-1/24*ln(1+x^2+x*
3^(1/2))*3^(1/2)+1/4*arctan(2*x+3^(1/2))-1/7/x^7+1/3/x^3-1/8*ln(x
^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{7x^4-3}{21x^7}+\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^8),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^
2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x +
1)
```

Fricas [A] time = 0.274457, size = 250, normalized size = 1.62

$$-\frac{\sqrt{3}\left(84\sqrt{3}x^7\arctan\left(\frac{1}{2x+\sqrt{3}+2\sqrt{x^2+\sqrt{3}x+1}}\right)+84\sqrt{3}x^7\arctan\left(\frac{1}{2x-\sqrt{3}+2\sqrt{x^2-\sqrt{3}x+1}}\right)-21\sqrt{3}x^7\log\left(x^2+x+1\right)+21\sqrt{3}x^7\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^8),x, algorithm="fricas")
```

```
[Out] -1/504*sqrt(3)*(84*sqrt(3)*x^7*arctan(1/(2*x + sqrt(3) + 2*sqrt(x
^2 + sqrt(3)*x + 1))) + 84*sqrt(3)*x^7*arctan(1/(2*x - sqrt(3) +
2*sqrt(x^2 - sqrt(3)*x + 1))) - 21*sqrt(3)*x^7*log(x^2 + x + 1) +
 21*sqrt(3)*x^7*log(x^2 - x + 1) + 42*x^7*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 42*x^7*arctan(1/3*sqrt(3)*(2*x - 1)) + 21*x^7*log(x^2 +
sqrt(3)*x + 1) - 21*x^7*log(x^2 - sqrt(3)*x + 1) - 8*sqrt(3)*(7*x
^4 - 3))/x^7
```

Sympy [A] time = 3.12639, size = 209, normalized size = 1.36

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{5}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{5}-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{5}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{5}-\frac{\sqrt{3}i}{6}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(-18432t^{5}-4t+x\right)\right)\right)+\frac{7x^{4}-3}{21x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**8/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqr
t(3)*I/24)**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 +
sqrt(3)*I/24)**5 - sqrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1
/2 + sqrt(3)*I/6 - 18432*(-1/8 - sqrt(3)*I/24)**5) + (-1/8 + sqrt
(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sqrt(3)*I/24)**5 - sqrt(3)*
I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-1843
2*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^8+x^4+1)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + x^4 + 1)*x^8),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + x^4 + 1)*x^8), x)
```

3.347 $\int \frac{x^m}{1-x^4+x^8}\,dx$

Optimal. Leaf size=127

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(
1 - I*Sqrt[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*H
ypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])
])/(Sqrt[3]*(I - Sqrt[3])*(1 + m))
```

Rubi [A] time = 0.114895, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(
1 - I*Sqrt[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*H
ypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])
])/(Sqrt[3]*(I - Sqrt[3])*(1 + m))
```

Rubi in Sympy [A] time = 18.287, size = 100, normalized size = 0.79

$$\frac{2\sqrt{3}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\\frac{m}{4}+\frac{5}{4}\end{matrix}\middle|x^4\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\right)}{3\left(\sqrt{3}-i\right)(m+1)}+\frac{2\sqrt{3}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\\frac{m}{4}+\frac{5}{4}\end{matrix}\middle|x^4\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\right)}{3\left(\sqrt{3}+i\right)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(x**8-x**4+1),x)
```

```
[Out] 2*sqrt(3)*x**(m + 1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), x**4*(1/
2 - sqrt(3)*I/2))/(3*(sqrt(3) - I)*(m + 1)) + 2*sqrt(3)*x**(m + 1
)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), x**4*(1/2 + sqrt(3)*I/2))/(
3*(sqrt(3) + I)*(m + 1))
```

Mathematica [C] time = 0.0770746, size = 79, normalized size = 0.62

$$\frac{x^m \text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}\,_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^7-\#1^3}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (x^m*RootSum[1 - #1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m,
 -(#1/(x - #1))]/((x/(x - #1))^m*(-#1^3 + 2*#1^7)) & ])/(4*m)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(x^8-x^4+1),x)
```

```
[Out] int(x^m/(x^8-x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8 - x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - x^4 + 1),x, algorithm="fricas")

[Out] integral(x^m/(x^8 - x^4 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(x**8-x**4+1),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - x^4 + 1),x, algorithm="giac")

[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

3.348 $\int \frac{x^{11}}{1-x^4+x^8}\,dx$

Optimal. Leaf size=46

$$\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

```
[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x
^8]/8
```

Rubi [A] time = 0.0779255, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x
^8]/8
```

Rubi in Sympy [A] time = 12.4994, size = 39, normalized size = 0.85

$$\frac{x^4}{4}+\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(x**8-x**4+1),x)
```

```
[Out] x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(sqrt(3)*(2*x**4/3
- 1/3))/12
```

Mathematica [A] time = 0.0186173, size = 46, normalized size = 1.

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 +
x^8]/8
```

Maple [A] time = 0.006, size = 38, normalized size = 0.8

$$\frac{x^{4}}{4}+\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(x^8-x^4+1),x)
```

```
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/
2))
```

Maxima [A] time = 0.823322, size = 50, normalized size = 1.09

$$\frac{1}{4}x^{4}-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)+\frac{1}{8}\log\left(x^{8}-x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(
x^8 - x^4 + 1)
```

Fricas [A] time = 0.266189, size = 59, normalized size = 1.28

$$\frac{1}{24}\sqrt{3}\left(2\sqrt{3}x^{4}+\sqrt{3}\log\left(x^{8}-x^{4}+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(2*sqrt(3)*x^4 + sqrt(3)*log(x^8 - x^4 + 1) - 2*arct
an(1/3*sqrt(3)*(2*x^4 - 1)))
```

Sympy [A] time = 0.325583, size = 42, normalized size = 0.91

$$\frac{x^4}{4} + \frac{\log\left(x^8 - x^4 + 1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(x**8-x**4+1),x)
```

```
[Out] x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 -
 sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.278669, size = 50, normalized size = 1.09

$$\frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4 - 1\right)\right) + \frac{1}{8}\,\ln\left(x^8 - x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*ln(x
^8 - x^4 + 1)
```

3.349 $\int \frac{x^9}{1-x^4+x^8} \, dx$

Optimal. Leaf size=57

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*
x^2 + x^4]/(4*Sqrt[3])
```

Rubi [A] time = 0.0865429, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(1 - x^4 + x^8),x]
```

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*
x^2 + x^4]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 22.4176, size = 48, normalized size = 0.84

$$\frac{x^2}{2}+\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**8-x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x*
*4 + sqrt(3)*x**2 + 1)/12
```

Mathematica [A] time = 0.0247852, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(6x^2+\sqrt{3}\log\left(-x^4+\sqrt{3}x^2-1\right)-\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(1 - x^4 + x^8),x]
```

```
[Out] (6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sq
rt[3]*x^2 + x^4])/12
```

Maple [A] time = 0.01, size = 44, normalized size = 0.8

$$\frac{x^2}{2}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^8-x^4+1),x)
```

```
[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1
/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2+\int\frac{(x^4-1)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.261312, size = 74, normalized size = 1.3

$$\frac{1}{12}\sqrt{3}\left(2\sqrt{3}x^2+\log\left(-\frac{6x^6+6x^2-\sqrt{3}\left(x^8+5x^4+1\right)}{x^8-x^4+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(2*sqrt(3)*x^2 + log(-(6*x^6 + 6*x^2 - sqrt(3)*(x^8
+ 5*x^4 + 1))/(x^8 - x^4 + 1)))
```

Sympy [A] time = 0.28893, size = 48, normalized size = 0.84

$$\frac{x^2}{2} + \frac{\sqrt{3}\log\left(x^4 - \sqrt{3}x^2 + 1\right)}{12} - \frac{\sqrt{3}\log\left(x^4 + \sqrt{3}x^2 + 1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**8-x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x*
*4 + sqrt(3)*x**2 + 1)/12
```

GIAC/XCAS [A] time = 0.343373, size = 348, normalized size = 6.11

$$\frac{1}{2}x^2 - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt
(2))/(sqrt(6) + sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*
x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/48*(sqrt(6) - 3*s
qrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1
/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(
6) - sqrt(2))) - 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(
6) + sqrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(s
qrt(6) + sqrt(2)) + 1) - 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*
x*(sqrt(6) - sqrt(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 -
1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.350 $\int \frac{x^7}{1-x^4+x^8}\,dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.0668157, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 - x^4 + x^8),x]

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rubi in Sympy [A] time = 9.42995, size = 34, normalized size = 0.87

$$\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**8-x**4+1),x)

[Out] log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(sqrt(3)*(2*x**4/3 - 1/3))/1
2
```

Mathematica [A] time = 0.0119846, size = 39, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Maple [A] time = 0.002, size = 33, normalized size = 0.9

$$\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^8-x^4+1),x)
```

```
[Out] 1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 0.826273, size = 43, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)+\frac{1}{8}\log\left(x^{8}-x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Fricas [A] time = 0.27005, size = 49, normalized size = 1.26

$$\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(x^{8}-x^{4}+1\right)+2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(sqrt(3)*log(x^8 - x^4 + 1) + 2*arctan(1/3*sqrt(3)*(
2*x^4 - 1)))
```

Sympy [A] time = 0.314063, size = 37, normalized size = 0.95

$$\frac{\log\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^{4}}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**8-x**4+1),x)
```

```
[Out] log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12
```

GIAC/XCAS [A] time = 0.292154, size = 43, normalized size = 1.1

$$\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^{4}-1\right)\right)+\frac{1}{8}\,\ln\left(x^{8}-x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*ln(x^8 - x^4 +
 1)
```

3.351 $\int \frac{x^5}{1-x^4+x^8}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 -
Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sq
rt[3])
```

Rubi [A] time = 0.132671, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 -
Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sq
rt[3])
```

Rubi in Sympy [A] time = 29.1467, size = 70, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt
(3)*x**2 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(
3))/4
```

Mathematica [C] time = 0.227787, size = 98, normalized size = 1.2

$$\frac{\sqrt{-1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)+\sqrt{-1+i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^5/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/
2] + Sqrt[-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*
x^2)/2])/(4*Sqrt[6])
```

Maple [A] time = 0.01, size = 65, normalized size = 0.8

$$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*ln(1+x^4
-x^2*3^(1/2))*3^(1/2)-1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.275634, size = 159, normalized size = 1.94

$$-\frac{1}{24}\sqrt{3}\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4+\sqrt{3}x^2+1}+3}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4-\sqrt{3}x^2+1}-3}\right)+\log\left(x^4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(4*sqrt(3)*arctan(sqrt(3)/(2*sqrt(3)*x^2 + 2*sqrt(3
)*sqrt(x^4 + sqrt(3)*x^2 + 1) + 3)) + 4*sqrt(3)*arctan(sqrt(3)/(2
*sqrt(3)*x^2 + 2*sqrt(3)*sqrt(x^4 - sqrt(3)*x^2 + 1) - 3)) + log(
x^4 + sqrt(3)*x^2 + 1) - log(x^4 - sqrt(3)*x^2 + 1))
```

Sympy [A] time = 0.593273, size = 70, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt
(3)*x**2 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(
3))/4
```

GIAC/XCAS [A] time = 0.33499, size = 342, normalized size = 4.17

$$\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$
$$+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$
$$+\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)$$
$$+\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*ar
```

```
ctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(
6) + 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(
2)) + 1) - 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + s
qrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6)
 - sqrt(2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqr
t(6) - sqrt(2)) + 1)
```

3.352 $\int \frac{x^3}{1-x^4+x^8} \, dx$

Optimal. Leaf size=23

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.045978, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 - x^4 + x^8),x]

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 5.98158, size = 22, normalized size = 0.96

$$\frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**8-x**4+1),x)

[Out] sqrt(3)*atan(sqrt(3)*(2*x**4/3 - 1/3))/6
```

Mathematica [A] time = 0.0109242, size = 23, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.001, size = 19, normalized size = 0.8

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^8-x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 0.824623, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Fricas [A] time = 0.256167, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Sympy [A] time = 0.292467, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**8-x**4+1),x)

[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

GIAC/XCAS [A] time = 0.274977, size = 24, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 - x^4 + 1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

3.353 $\int \frac{x}{1-x^4+x^8} dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 -
Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sq
rt[3])
```

Rubi [A] time = 0.108086, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 -
Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sq
rt[3])
```

Rubi in Sympy [A] time = 18.366, size = 70, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt
(3))/4
```

Mathematica [C] time = 0.104553, size = 83, normalized size = 1.01

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)-\sqrt{-1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(1 - x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sq
rt[-1 + I*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [A] time = 0.009, size = 65, normalized size = 0.8

$$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4
-x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.265392, size = 159, normalized size = 1.94

$$-\frac{1}{24}\sqrt{3}\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4+\sqrt{3}x^2+1}+3}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4-\sqrt{3}x^2+1}-3}\right)-\log\left(x^4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(4*sqrt(3)*arctan(sqrt(3)/(2*sqrt(3)*x^2 + 2*sqrt(3
)*sqrt(x^4 + sqrt(3)*x^2 + 1) + 3)) + 4*sqrt(3)*arctan(sqrt(3)/(2
*sqrt(3)*x^2 + 2*sqrt(3)*sqrt(x^4 - sqrt(3)*x^2 + 1) - 3)) - log(
x^4 + sqrt(3)*x^2 + 1) + log(x^4 - sqrt(3)*x^2 + 1))
```

Sympy [A] time = 0.617644, size = 70, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt
(3))/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

3.354 $\int \frac{1}{x(1-x^4+x^8)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 +
 x^8]/8
```

Rubi [A] time = 0.0718375, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 +
 x^8]/8
```

Rubi in Sympy [A] time = 13.4691, size = 41, normalized size = 1.

$$\frac{\log\left(x^4\right)}{4}-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**8-x**4+1),x)
```

```
[Out] log(x**4)/4 - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(sqrt(3)*(2*x*
*4/3 - 1/3))/12
```

Mathematica [C] time = 0.0209464, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*
#1^4)/(-1 + 2*#1^4) & ]/4
```

Maple [A] time = 0.011, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3}}{12}\arctan\left(\frac{(2x^4 - 1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2)
)
```

Maxima [A] time = 0.827508, size = 51, normalized size = 1.24

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4 - 1)\right) - \frac{1}{8}\log(x^8 - x^4 + 1) + \frac{1}{4}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4
+ 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.256825, size = 58, normalized size = 1.41

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log(x^8 - x^4 + 1) - 8\sqrt{3}\log(x) - 2\arctan\left(\frac{1}{3}\sqrt{3}(2x^4 - 1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(x^8 - x^4 + 1) - 8*sqrt(3)*log(x) - 2*
arctan(1/3*sqrt(3)*(2*x^4 - 1)))
```

Sympy [A] time = 0.383117, size = 41, normalized size = 1.

$$\log(x) - \frac{\log(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 -
 sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.275213, size = 51, normalized size = 1.24

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{1}{8}\ln\left(x^8 - x^4 + 1\right) + \frac{1}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*ln(x^8 - x^4 +
 1) + 1/4*ln(x^4)
```

3.355 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[Out] -1/(2*x^2) - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqr
t[3]*x^2 + x^4]/(4*Sqrt[3])
```

Rubi [A] time = 0.0798921, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqr
t[3]*x^2 + x^4]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 22.1231, size = 49, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/12 - 1/(2*x**2)
```

Mathematica [A] time = 0.0287988, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{6}{x^2}-\sqrt{3}\log\left(-x^4+\sqrt{3}x^2-1\right)+\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + S
qrt[3]*x^2 + x^4])/12
```

Maple [A] time = 0.008, size = 44, normalized size = 0.8

$$-\frac{1}{2\,x^{2}}-\frac{\ln\left(1+x^{4}-x^{2}\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^{4}+x^{2}\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^8-x^4+1),x)
```

```
[Out] -1/2/x^2-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(
1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\,x^{2}}-\int\frac{\left(x^{4}-1\right)x}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.275455, size = 77, normalized size = 1.35

$$\frac{\sqrt{3}\left(x^{2}\log\left(\frac{6\,x^{6}+6\,x^{2}+\sqrt{3}\left(x^{8}+5\,x^{4}+1\right)}{x^{8}-x^{4}+1}\right)-2\,\sqrt{3}\right)}{12\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(x^2*log((6*x^6 + 6*x^2 + sqrt(3)*(x^8 + 5*x^4 + 1))
/(x^8 - x^4 + 1)) - 2*sqrt(3))/x^2
```

Sympy [A] time = 0.40514, size = 49, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/12 - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.336939, size = 348, normalized size = 6.11

$$\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$
$$+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$
$$+\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)$$
$$+\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*ar
ctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(
6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(
2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + s
qrt(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6)
 - sqrt(2)) + 1) - 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqr
t(6) - sqrt(2)) + 1) - 1/2/x^2
```

3.356 $\int \frac{1}{x^5(1-x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - L
og[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.107153, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - L
og[1 - x^4 + x^8]/8
```

Rubi in Sympy [A] time = 15.7455, size = 48, normalized size = 1.

$$\frac{\log\left(x^4\right)}{4}-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**8-x**4+1),x)
```

```
[Out] log(x**4)/4 - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(sqrt(3)*(2*x*
*4/3 - 1/3))/12 - 1/(4*x**4)
```

Mathematica [C] time = 0.0219268, size = 51, normalized size = 1.06

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{4x^4}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1
^4)/(-1 + 2*#1^4) & ]/4
```

Maple [A] time = 0.01, size = 40, normalized size = 0.8

$$-\frac{1}{4x^4}+\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^8-x^4+1),x)
```

```
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1
)*3^(1/2))
```

Maxima [A] time = 0.825723, size = 58, normalized size = 1.21

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)-\frac{1}{4x^4}-\frac{1}{8}\log(x^8-x^4+1)+\frac{1}{4}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^5),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log
(x^8 - x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.252381, size = 81, normalized size = 1.69

$$-\frac{\sqrt{3}\left(\sqrt{3}x^4\log(x^8-x^4+1)-8\sqrt{3}x^4\log(x)+2x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)+2\sqrt{3}\right)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^5),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*x^4*log(x^8 - x^4 + 1) - 8*sqrt(3)*x^4*log
(x) + 2*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 2*sqrt(3))/x^4
```

Sympy [A] time = 0.540033, size = 48, normalized size = 1.

$$\log(x) - \frac{\log(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 -
 sqrt(3)/3)/12 - 1/(4*x**4)
```

GIAC/XCAS [A] time = 0.275807, size = 65, normalized size = 1.35

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{x^4 + 1}{4x^4} - \frac{1}{8}\ln\left(x^8 - x^4 + 1\right) + \frac{1}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^5),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4
 - 1/8*ln(x^8 - x^4 + 1) + 1/4*ln(x^4)
```

3.357 $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[Out] -1/(6*x^6) - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[
3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 +
Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rubi [A] time = 0.185119, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625

$$-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[
3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 +
Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 40.9123, size = 83, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt
(3))/4 - 1/(2*x**2) - 1/(6*x**6)
```

Mathematica [C] time = 0.0271525, size = 56, normalized size = 0.58

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{6x^6}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]
*#1^2)/(-1 + 2*#1^4) & ]/4
```

Maple [A] time = 0.009, size = 75, normalized size = 0.8

$$-\frac{1}{6\,x^6}-\frac{1}{2\,x^2}-\frac{\arctan\left(2\,x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2\,x^2+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^8-x^4+1),x)
```

```
[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/
2))-1/24*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))
*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\,x^4+1}{6\,x^6}-\int\frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^7),x, algorithm="maxima")
```

```
[Out] -1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.269092, size = 198, normalized size = 2.06

$$\frac{\sqrt{3}\left(12\sqrt{3}x^6\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4+\sqrt{3}x^2+1}+3}\right)+12\sqrt{3}x^6\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4-\sqrt{3}x^2+1}-3}\right)+3\,x^6\log\left(x^4+\sqrt{3}x^2+1\right)-3\,x^6\right.}{72\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^7),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(12*sqrt(3)*x^6*arctan(sqrt(3)/(2*sqrt(3)*x^2 + 2*sq
rt(3)*sqrt(x^4 + sqrt(3)*x^2 + 1) + 3)) + 12*sqrt(3)*x^6*arctan(s
qrt(3)/(2*sqrt(3)*x^2 + 2*sqrt(3)*sqrt(x^4 - sqrt(3)*x^2 + 1) - 3
)) + 3*x^6*log(x^4 + sqrt(3)*x^2 + 1) - 3*x^6*log(x^4 - sqrt(3)*x
^2 + 1) - 4*sqrt(3)*(3*x^4 + 1))/x^6
```

Sympy [A] time = 0.861041, size = 82, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{3x^4+1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt
(3))/4 - (3*x**4 + 1)/(6*x**6)
```

GIAC/XCAS [A] time = 0.344287, size = 358, normalized size = 3.73

$$\begin{aligned}&-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\&-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\&-\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\&+\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\&+\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{3x^4+1}{6x^6}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^7),x, algorithm="giac")
```

```
[Out] -1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*a
rctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt
(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt
(2))) - 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt
(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) +
sqrt(2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6
) - sqrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/6*(3*x^4 + 1)/x^6
```

3.358 $\int \frac{x^8}{1-x^4+x^8}\,dx$

Optimal. Leaf size=356

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)+x+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt
[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x
)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 +
 Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - (
Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 + (Sqrt
[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(
2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.692935, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)+x+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt
[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x
)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 +
```

```
 Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - (
Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 + (Sqrt
[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(
2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi in Sympy [A] time = 78.869, size = 496, normalized size = 1.39

$$x - \frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2 - x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2 + x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2 - x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2 + x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2} + \sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2} + \sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2} + \sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2} + \sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**8-x**4+1),x)
```

```
[Out] x - sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)
/(12*sqrt(-sqrt(3) + 2)) + sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 + x*
sqrt(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) + sqrt(3)*(sqrt(3
)/2 + 1)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) + 2
)) - sqrt(3)*(sqrt(3)/2 + 1)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/
(12*sqrt(sqrt(3) + 2)) - sqrt(3)*(-(sqrt(3) + 2)**(3/2)/2 + sqrt(
3)*sqrt(sqrt(3) + 2))*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-sqrt(3
) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt(3)*(-(sqr
t(3) + 2)**(3/2)/2 + sqrt(3)*sqrt(sqrt(3) + 2))*atan((2*x + sqrt(
sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt
```

```
(3) + 2)) - sqrt(3)*((-sqrt(3) + 2)**(3/2)/2 + sqrt(3)*sqrt(-sqrt
(3) + 2))*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*s
qrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt(3)*((-sqrt(3) + 2)**(
3/2)/2 + sqrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x + sqrt(-sqrt(3) +
2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0214334, size = 59, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)
/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.012, size = 44, normalized size = 0.1

$$x+\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^4-1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^8-x^4+1),x)
```

```
[Out] x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x+\int\frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.296449, size = 1284, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*(4*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arc
tan(1/(2*(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) + (20
9*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(5
6*sqrt(3) - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)
*x - 2*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - sqrt(3) + 2)) + 4
*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2
*(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3
)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3
) - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x
)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(3) - 2)) + (2*sqrt(3
) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(194*x^2 + 112*sqrt
(3)*(x^2 + 1) + 2*(209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*s
qrt(3) + 7)) + 194) - (2*sqrt(3) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(
3) - 7))*log(194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x +
 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 194) - (2*sqrt(3) +
 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(-194*x^2 + 112*sqrt(3
)*(x^2 + 1) + 2*(209*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqr
t(3) - 7)) - 194) + (2*sqrt(3) + 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3)
 + 7))*log(-194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x -
362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - 194) - 24*(sqrt(3)*x
 + 2*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/(4
*sqrt(3) - 7)) + 4*sqrt(3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*ar
ctan(1/(2*(sqrt(3) + 2)*sqrt((97*x^2 + 56*sqrt(3)*(x^2 + 1) + (20
9*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 97)/(5
6*sqrt(3) + 97))*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)
*x + 2*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + sqrt(3) + 2)) + 4
*sqrt(3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/(2*(sqrt(3)
 + 2)*sqrt((97*x^2 + 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3)*x + 362*
x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(3) + 97))*s
qrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sq
rt(3) + 2)/(4*sqrt(3) + 7)) - sqrt(3) - 2)))/((sqrt(3) + 2)*sqrt(
(sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)
))
```

Sympy [A] time = 4.97091, size = 26, normalized size = 0.07

$$x + \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**8-x**4+1),x)
```

```
[Out] x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(921
6*_t**5 - 8*_t + x)))
```

GIAC/XCAS [A] time = 0.300472, size = 343, normalized size = 0.96

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(
6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*a
rctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt
(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt
(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt
(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) +
sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6
) - sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1) + x
```

3.359 $\int \frac{x^6}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6])
- ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6])
 + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]
) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6
]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt[2 + Sqrt[3]]*x
 + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[
6])
```

Rubi [A] time = 0.515849, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6])
- ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6])
 + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]
) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6
```

```
]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt[2 + Sqrt[3]]*x
 + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[
6])
```

Rubi in Sympy [A] time = 73.7589, size = 508, normalized size = 1.85

$$-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\sqrt{\sqrt{3}+2}+\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\sqrt{\sqrt{3}+2}+\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/
(12*sqrt(-sqrt(3) + 2)) + sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 + x
*sqrt(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) + sqrt(3)*(1/2 +
 sqrt(3)/2)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3)
+ 2)) - sqrt(3)*(1/2 + sqrt(3)/2)*log(x**2 + x*sqrt(sqrt(3) + 2)
+ 1)/(12*sqrt(sqrt(3) + 2)) + sqrt(3)*(-sqrt(sqrt(3) + 2) + (1 +
sqrt(3))*sqrt(sqrt(3) + 2)/2)*atan((2*x - sqrt(sqrt(3) + 2))/sqrt
(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3
)*(-sqrt(sqrt(3) + 2) + (1 + sqrt(3))*sqrt(sqrt(3) + 2)/2)*atan((
2*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2
)*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2
)/2 + sqrt(-sqrt(3) + 2))*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sq
rt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*(-
```

```
(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)/2 + sqrt(-sqrt(3) + 2))*atan((2
*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)
*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.017545, size = 41, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/
4
```

Maple [C] time = 0.028, size = 32, normalized size = 0.1

$$\frac{\sum_{\_R=RootOf\left(9\_Z^4+1\right)}\_R\ln\left(9\_R^3x-3\_R^2+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.266756, size = 271, normalized size = 0.99

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}+3\,x}{\sqrt{3}\sqrt{2}x^2+\sqrt{3}\sqrt{2}\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1}+3\,x}\right)+\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}-3\,x}{\sqrt{3}\sqrt{2}x^2+\sqrt{3}\sqrt{2}\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1}-3\,x}\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3\,x^2+1\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan((sqrt(3)*sqrt(2) + 3*x)/(sqrt(3)*sqrt
(2)*x^2 + sqrt(3)*sqrt(2)*sqrt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) +
3*x^2 + 1) + 3*x)) + 1/6*sqrt(3)*sqrt(2)*arctan((sqrt(3)*sqrt(2)
- 3*x)/(sqrt(3)*sqrt(2)*x^2 + sqrt(3)*sqrt(2)*sqrt(x^4 - sqrt(3)*
sqrt(2)*(x^3 + x) + 3*x^2 + 1) - 3*x)) - 1/24*sqrt(3)*sqrt(2)*log
(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) + 1/24*sqrt(3)*sqrt
(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

Sympy [A] time = 0.633401, size = 165, normalized size = 0.6

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}+\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}-\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2
 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*
atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x
**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 - sqrt(6)*log(x**
4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24
```

GIAC/XCAS [A] time = 0.302906, size = 277, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))
) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/12*sqrt(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) -
sqrt(2))) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6
) - sqrt(2))) - 1/24*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) +
 1) + 1/24*sqrt(6)*ln(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24
*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/24*sqrt(6)*l
n(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.360 $\int \frac{x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sq
rt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqr
t[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - Log[1
 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[1 - S
qrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqr
t[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.456919, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sq
rt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqr
t[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - Log[1
 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[1 - S
qrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqr
t[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rubi in Sympy [A] time = 64.0809, size = 311, normalized size = 0.9

$$-\frac{\sqrt{3}\log\left(x^2 - x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\log\left(x^2 + x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\log\left(x^2 - x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\log\left(x^2 + x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqrt(3) +
 2)) + sqrt(3)*log(x**2 + x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqr
t(3) + 2)) + sqrt(3)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(24*sqrt
(sqrt(3) + 2)) - sqrt(3)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(24*
sqrt(sqrt(3) + 2)) + sqrt(3)*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(
-sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) + sqrt(3)*atan((2*x + sqrt
(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) - sqrt
(3)*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(12*sqrt(s
qrt(3) + 2)) - sqrt(3)*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(
3) + 2))/(12*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0156203, size = 39, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

Maple [C] time = 0.01, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{4}}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.286281, size = 1067, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*(4*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arc
tan((sqrt(3)*sqrt(2) - 2*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqrt(
```

```
3) - 2)/(4*sqrt(3) - 7)) + 2)*(sqrt(3) - 2)*sqrt((sqrt(3) - 2)/(4
*sqrt(3) - 7)) + 2*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt((sqrt(3
) - 2)/(4*sqrt(3) - 7)) - sqrt(2))) + 4*(7*sqrt(3) + 12)*sqrt((sq
rt(3) + 2)/(4*sqrt(3) + 7))*arctan((sqrt(3)*sqrt(2) - 2*sqrt(2))/
(2*sqrt(2*x^2 - 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2)*(sqr
t(3) - 2)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*sqrt(2
)*x - 2*sqrt(2)*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(2))
) - (2*sqrt(3) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(2*x^2
 + 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2) + (2*sqrt(3) + 3)
*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(2*x^2 - 2*x*sqrt((sqrt(3
) + 2)/(4*sqrt(3) + 7)) + 2) + (2*sqrt(3) + 3)*sqrt((sqrt(3) + 2)
/(4*sqrt(3) + 7))*log(2*x^2 + 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) -
 7)) + 2) - (2*sqrt(3) + 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*l
og(2*x^2 - 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2) - 4*sqrt(
3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan((sqrt(3)*sqrt(2) +
2*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)
) + 2)*(sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqr
t(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))
 + sqrt(2))) - 4*sqrt(3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arct
an((sqrt(3)*sqrt(2) + 2*sqrt(2))/(2*sqrt(2*x^2 - 2*x*sqrt((sqrt(3
) + 2)/(4*sqrt(3) + 7)) + 2)*(sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*
sqrt(3) + 7)) + 2*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt((sqrt(3)
 + 2)/(4*sqrt(3) + 7)) - sqrt(2))))/((sqrt(3) + 2)*sqrt((sqrt(3)
+ 2)/(4*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.8585, size = 24, normalized size = 0.07

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-18432t^5 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*
_t**5 + 4*_t + x)))
```

GIAC/XCAS [A] time = 0.288111, size = 342, normalized size = 0.99

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*ar
ctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(
6) + 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(
2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + s
qrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6)
 - sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqr
t(6) - sqrt(2)) + 1)
```

3.361 $\int \frac{x^2}{1-x^4+x^8} \, dx$

Optimal. Leaf size=355

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} -\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} +\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) -\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 +
Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] -
 2*x)/Sqrt[2 - Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt
[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/
3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + Log[1
 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[1 - S
qrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqr
t[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.435571, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} -\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} +\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) -\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 +
Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] -
 2*x)/Sqrt[2 - Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt
[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/
3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + Log[1
 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[1 - S
qrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqr
t[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rubi in Sympy [A] time = 43.0205, size = 311, normalized size = 0.88

$$\frac{\sqrt{3}\log\left(x^2 - x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\log\left(x^2 + x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\log\left(x^2 - x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} + \frac{\sqrt{3}\log\left(x^2 + x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqrt(3) +
2)) - sqrt(3)*log(x**2 + x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqrt
(3) + 2)) - sqrt(3)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(24*sqrt(
sqrt(3) + 2)) + sqrt(3)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(24*s
qrt(sqrt(3) + 2)) + sqrt(3)*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-
sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) + sqrt(3)*atan((2*x + sqrt(
sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) - sqrt(
3)*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(12*sqrt(sq
rt(3) + 2)) - sqrt(3)*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3
) + 2))/(12*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.016386, size = 40, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

Maple [C]   time = 0.01, size = 40, normalized size = 0.1

$$\frac{1}{4} \sum_{\_R = RootOf(\_Z^8 - \_Z^4 + 1)} \frac{\_R^2 \ln (x - \_R)}{2\_R^7 - \_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - x^4 + 1), x)
```

Fricas [A]   time = 0.284676, size = 1107, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*(4*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arc
tan((2*sqrt(3)*sqrt(2) - 3*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqr
```

```
t(3) - 2)/(4*sqrt(3) - 7)) + 2)*(2*sqrt(3) - 3)*sqrt((sqrt(3) - 2
)/(4*sqrt(3) - 7)) + 2*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt((
sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(3)*sqrt(2))) + 4*(7*sqrt(3)
+ 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan((2*sqrt(3)*sqrt(
2) - 3*sqrt(2))/(2*sqrt(2*x^2 - 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3)
 - 7)) + 2)*(2*sqrt(3) - 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) +
 2*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt((sqrt(3) - 2)/(4*sqrt
(3) - 7)) - sqrt(3)*sqrt(2))) + (2*sqrt(3) + 3)*sqrt((sqrt(3) - 2
)/(4*sqrt(3) - 7))*log(2*x^2 + 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3)
+ 7)) + 2) - (2*sqrt(3) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*
log(2*x^2 - 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2) - (2*sqr
t(3) + 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(2*x^2 + 2*x*sqr
t((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2) + (2*sqrt(3) + 3)*sqrt((sqr
t(3) + 2)/(4*sqrt(3) + 7))*log(2*x^2 - 2*x*sqrt((sqrt(3) - 2)/(4*
sqrt(3) - 7)) + 2) - 4*sqrt(3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)
)*arctan((2*sqrt(3)*sqrt(2) + 3*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt
((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2)*(2*sqrt(3) + 3)*sqrt((sqrt(3
) + 2)/(4*sqrt(3) + 7)) + 2*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*s
qrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + sqrt(3)*sqrt(2))) - 4*sqrt(3
)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan((2*sqrt(3)*sqrt(2) +
 3*sqrt(2))/(2*sqrt(2*x^2 - 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7
)) + 2)*(2*sqrt(3) + 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(
2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3)
+ 7)) - sqrt(3)*sqrt(2))))/((sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*s
qrt(3) + 7))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.89047, size = 26, normalized size = 0.07

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-442368t^7 - 192t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368
*_t**7 - 192*_t**3 + x)))
```

GIAC/XCAS [A] time = 0.2881, size = 342, normalized size = 0.96

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*ar
ctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(
6) + 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(
2))) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(
2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + s
qrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6)
 - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqr
t(6) - sqrt(2)) + 1)
```

3.362 $\int \frac{1}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6])
- ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6])
 + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]
) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6
]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x
 + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[
6])
```

Rubi [A] time = 0.596381, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6])
- ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6])
 + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]
) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6
```

```
]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x
 + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[
6])
```

Rubi in Sympy [A] time = 65.9423, size = 529, normalized size = 1.92

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{(1+\sqrt{3})\sqrt{\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{(1+\sqrt{3})\sqrt{\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{(-\sqrt{3}+1)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{(-\sqrt{3}+1)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(
12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 + x*
sqrt(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(1/2 +
sqrt(3)/2)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) +
 2)) + sqrt(3)*(1/2 + sqrt(3)/2)*log(x**2 + x*sqrt(sqrt(3) + 2) +
 1)/(12*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(1 + sqrt(3))*sqrt(sqrt(3)
 + 2)/2 + sqrt(3)*sqrt(sqrt(3) + 2))*atan((2*x - sqrt(sqrt(3) + 2
))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) +
 sqrt(3)*(-(1 + sqrt(3))*sqrt(sqrt(3) + 2)/2 + sqrt(3)*sqrt(sqrt(
3) + 2))*atan((2*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sq
rt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*((-sqrt(3) + 1)*sqr
t(-sqrt(3) + 2)/2 + sqrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x - sqrt(
-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt
```

```
(3) + 2)) + sqrt(3)*((-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)/2 + sqrt(3
)*sqrt(-sqrt(3) + 2))*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3
) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0158417, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.007, size = 30, normalized size = 0.1

$$\frac{\sum_{\_R=RootOf\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.264503, size = 257, normalized size = 0.93

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}x+2}{\sqrt{3}\sqrt{2}x+2\,x^2+2\,\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1}}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}x-2}{\sqrt{3}\sqrt{2}x-2\,x^2-2\,\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1}}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4+\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4-\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan((sqrt(3)*sqrt(2)*x + 2)/(sqrt(3)*sqrt
(2)*x + 2*x^2 + 2*sqrt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 +
1))) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*x - 2)/(sqrt(
3)*sqrt(2)*x - 2*x^2 - 2*sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3
*x^2 + 1))) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3
 + x) + 3*x^2 + 1) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(
2)*(x^3 + x) + 3*x^2 + 1)
```

Sympy [A] time = 0.643296, size = 165, normalized size = 0.6

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2
 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*
atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x
**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**
4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24
```

GIAC/XCAS [A] time = 0.294547, size = 277, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))
) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/12*sqrt(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) -
sqrt(2))) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6
) - sqrt(2))) + 1/24*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) +
 1) - 1/24*sqrt(6)*ln(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24
*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*sqrt(6)*l
n(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.363 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=360

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2
- Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]]
 + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sq
rt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8
 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 -
 (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (
Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.694282, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2
- Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]]
 + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sq
```

```
rt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8
 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 -
 (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (
Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi in Sympy [A] time = 99.6204, size = 484, normalized size = 1.34

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(12
*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 + x*sqrt
(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(sqrt(3)/2
+ 1)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) + 2)) +
 sqrt(3)*(sqrt(3)/2 + 1)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(12*
sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3) + 2)**(3/2)/2 + 2*sqrt(sq
rt(3) + 2))*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6
*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3) + 2)*
*(3/2)/2 + 2*sqrt(sqrt(3) + 2))*atan((2*x + sqrt(sqrt(3) + 2))/sq
rt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt
(3)*(-(-sqrt(3) + 2)**(3/2)/2 + 2*sqrt(-sqrt(3) + 2))*atan((2*x -
 sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqr
t(sqrt(3) + 2)) - sqrt(3)*(-(-sqrt(3) + 2)**(3/2)/2 + 2*sqrt(-sqr
t(3) + 2))*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*
```

```
sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - 1/x
```

Mathematica [C] time = 0.0218228, size = 61, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]
*#1^4)/(-#1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.015, size = 52, normalized size = 0.1

$$-x^{-1}-\frac{1}{4}\sum_{\_R=RootOf(\_Z^8-\_Z^4+1)}\frac{(\_R^6-\_R^2)\ln(x-\_R)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/x-1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z
^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\int\frac{x^6-x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.282247, size = 1311, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*sqrt(3)*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/(
2*(sqrt(3) + 2)*sqrt((97*x^2 + 56*sqrt(3)*(x^2 + 1) + (209*sqrt(3
)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(3
) + 97))*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*x
)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + sqrt(3) + 2)) + 4*sqrt(3)
*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/(2*(sqrt(3) + 2)*
sqrt((97*x^2 + 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3)*x + 362*x)*sqr
t((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt((s
qrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(3)
+ 2)/(4*sqrt(3) + 7)) - sqrt(3) - 2)) + 4*(7*sqrt(3)*x + 12*x)*sq
rt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2*(sqrt(3) - 2)*sqrt(
-(97*x^2 - 56*sqrt(3)*(x^2 + 1) + (209*sqrt(3)*x - 362*x)*sqrt((s
qrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3) - 97))*sqrt((sqrt(
3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x)*sqrt((sqrt(3) - 2)
/(4*sqrt(3) - 7)) - sqrt(3) + 2)) + 4*(7*sqrt(3)*x + 12*x)*sqrt((
sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2*(sqrt(3) - 2)*sqrt(-(97
*x^2 - 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3)*x - 362*x)*sqrt((sqrt(
3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3) - 97))*sqrt((sqrt(3) -
 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x)*sqrt((sqrt(3) - 2)/(4*
sqrt(3) - 7)) + sqrt(3) - 2)) - (2*sqrt(3)*x + 3*x)*sqrt((sqrt(3)
 - 2)/(4*sqrt(3) - 7))*log(194*x^2 + 112*sqrt(3)*(x^2 + 1) + 2*(2
09*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 194)
+ (2*sqrt(3)*x + 3*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(194
*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x + 362*x)*sqrt((sq
rt(3) + 2)/(4*sqrt(3) + 7)) + 194) + (2*sqrt(3)*x + 3*x)*sqrt((sq
rt(3) + 2)/(4*sqrt(3) + 7))*log(-194*x^2 + 112*sqrt(3)*(x^2 + 1)
+ 2*(209*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) -
 194) - (2*sqrt(3)*x + 3*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*l
og(-194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x - 362*x)*s
qrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - 194) + 24*(sqrt(3) + 2)*sqrt
((sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7
)))/((sqrt(3)*x + 2*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt((
sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.91863, size = 29, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-442368t^7 + 384t^3 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368
*_t**7 + 384*_t**3 + x))) - 1/x
```

GIAC/XCAS [A] time = 0.292224, size = 348, normalized size = 0.97

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^2),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(
6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*a
rctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt
(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt
(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt
(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) +
sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6
) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/x
```

3.364 $\int \frac{1}{x^4(1-x^4+x^8)}\,dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] -1/(3*x^3) - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2
*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2
 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]
*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(
2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]
]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/
8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8
+ (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.54314, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2
*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2
```

```
 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]
*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(
2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]
]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/
8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8
+ (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi in Sympy [A] time = 81.1481, size = 549, normalized size = 1.48

$$\frac{\sqrt{3}\left(-\frac{3\sqrt{3}}{2}+3\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{36\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{3\sqrt{3}}{2}+3\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{36\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{3\sqrt{3}}{2}+3\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{36\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{3\sqrt{3}}{2}+3\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{36\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\sqrt{\sqrt{3}+2}\left(3\sqrt{3}+6\right)}{2}+3\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{18\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\sqrt{\sqrt{3}+2}\left(3\sqrt{3}+6\right)}{2}+3\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{18\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\left(-3\sqrt{3}+6\right)\sqrt{-\sqrt{3}+2}}{2}+3\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{18\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\left(-3\sqrt{3}+6\right)\sqrt{-\sqrt{3}+2}}{2}+3\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{18\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-3*sqrt(3)/2 + 3)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(
36*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-3*sqrt(3)/2 + 3)*log(x**2 + x*
sqrt(-sqrt(3) + 2) + 1)/(36*sqrt(-sqrt(3) + 2)) - sqrt(3)*(3*sqrt
(3)/2 + 3)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(36*sqrt(sqrt(3) +
 2)) + sqrt(3)*(3*sqrt(3)/2 + 3)*log(x**2 + x*sqrt(sqrt(3) + 2) +
 1)/(36*sqrt(sqrt(3) + 2)) + sqrt(3)*(-sqrt(sqrt(3) + 2)*(3*sqrt(
3) + 6)/2 + 3*sqrt(3)*sqrt(sqrt(3) + 2))*atan((2*x - sqrt(sqrt(3)
 + 2))/sqrt(-sqrt(3) + 2))/(18*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) +
2)) + sqrt(3)*(-sqrt(sqrt(3) + 2)*(3*sqrt(3) + 6)/2 + 3*sqrt(3)*s
qrt(sqrt(3) + 2))*atan((2*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) +
```

```
2))/(18*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*((-3*sqrt
(3) + 6)*sqrt(-sqrt(3) + 2)/2 + 3*sqrt(3)*sqrt(-sqrt(3) + 2))*ata
n((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(18*sqrt(-sqrt(3)
 + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*((-3*sqrt(3) + 6)*sqrt(-sqrt(3
) + 2)/2 + 3*sqrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x + sqrt(-sqrt(3
) + 2))/sqrt(sqrt(3) + 2))/(18*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) +
2)) - 1/(3*x**3)
```

Mathematica [C] time = 0.0193475, size = 65, normalized size = 0.18

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x -
#1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.014, size = 50, normalized size = 0.1

$$-\frac{1}{3\,x^3}+\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\,\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^8-x^4+1),x)
```

```
[Out] -1/3/x^3+1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-
_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3\,x^3}-\int\frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.285668, size = 1354, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(3)*x^3*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/
(2*(sqrt(3) + 2)*sqrt((97*x^2 + 56*sqrt(3)*(x^2 + 1) + (209*sqrt(
3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(
3) + 97))*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*
x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + sqrt(3) + 2)) + 4*sqrt(3
)*x^3*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/(2*(sqrt(3) +
2)*sqrt((97*x^2 + 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3)*x + 362*x)*
sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt
((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(
3) + 2)/(4*sqrt(3) + 7)) - sqrt(3) - 2)) + 4*(7*sqrt(3)*x^3 + 12*
x^3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2*(sqrt(3) - 2
)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) + (209*sqrt(3)*x - 362*x)*
sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3) - 97))*sqrt
((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x)*sqrt((sqrt(
3) - 2)/(4*sqrt(3) - 7)) - sqrt(3) + 2)) + 4*(7*sqrt(3)*x^3 + 12*
x^3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2*(sqrt(3) - 2
)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3)*x - 362*x)*
sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3) - 97))*sqrt
((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x)*sqrt((sqrt(
3) - 2)/(4*sqrt(3) - 7)) + sqrt(3) - 2)) + (2*sqrt(3)*x^3 + 3*x^3
)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(194*x^2 + 112*sqrt(3)*(
x^2 + 1) + 2*(209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3
) + 7)) + 194) - (2*sqrt(3)*x^3 + 3*x^3)*sqrt((sqrt(3) - 2)/(4*sq
rt(3) - 7))*log(194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*
x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 194) - (2*sqrt(3
)*x^3 + 3*x^3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(-194*x^2 +
 112*sqrt(3)*(x^2 + 1) + 2*(209*sqrt(3)*x - 362*x)*sqrt((sqrt(3)
- 2)/(4*sqrt(3) - 7)) - 194) + (2*sqrt(3)*x^3 + 3*x^3)*sqrt((sqrt
(3) + 2)/(4*sqrt(3) + 7))*log(-194*x^2 + 112*sqrt(3)*(x^2 + 1) -
2*(209*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - 1
94) - 8*(sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt((s
qrt(3) - 2)/(4*sqrt(3) - 7)))/((sqrt(3)*x^3 + 2*x^3)*sqrt((sqrt(3
) + 2)/(4*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 5.13596, size = 31, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-9216t^5 + 8t + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_
t**5 + 8*_t + x))) - 1/(3*x**3)
```

GIAC/XCAS [A] time = 0.29846, size = 348, normalized size = 0.94

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^4),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*ar
ctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(
6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(
2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + s
qrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6)
 - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqr
t(6) - sqrt(2)) + 1) - 1/3/x^3
```

3.365 $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

Optimal. Leaf size=287

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -1/(5*x^5) - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 -
Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2
- Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*S
qrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1
- Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[
3]]*x + x^2]/(4*Sqrt[6])
```

Rubi [A] time = 0.572112, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 -
Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2
```

```
- Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*S
qrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1
- Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[
3]]*x + x^2]/(4*Sqrt[6])
```

Rubi in Sympy [A] time = 86.3535, size = 518, normalized size = 1.8

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\sqrt{\sqrt{3}+2}+\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\sqrt{\sqrt{3}+2}+\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{1}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(
12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 + x*
sqrt(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(1/2 +
sqrt(3)/2)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) +
 2)) + sqrt(3)*(1/2 + sqrt(3)/2)*log(x**2 + x*sqrt(sqrt(3) + 2) +
 1)/(12*sqrt(sqrt(3) + 2)) - sqrt(3)*(-sqrt(sqrt(3) + 2) + (1 + s
qrt(3))*sqrt(sqrt(3) + 2)/2)*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(
-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt(3)
*(-sqrt(sqrt(3) + 2) + (1 + sqrt(3))*sqrt(sqrt(3) + 2)/2)*atan((2
*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)
*sqrt(sqrt(3) + 2)) - sqrt(3)*(-(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)
/2 + sqrt(-sqrt(3) + 2))*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqr
t(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt(3)*(-(
```

```
-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)/2 + sqrt(-sqrt(3) + 2))*atan((2*
x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*
sqrt(sqrt(3) + 2)) - 1/x - 1/(5*x**5)
```

Mathematica [C] time = 0.0239242, size = 54, normalized size = 0.19

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{5x^5}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1
^3)/(-1 + 2*#1^4) & ]/4
```

Maple [C] time = 0.014, size = 43, normalized size = 0.2

$$-\frac{1}{5\,x^5}-x^{-1}-\frac{\sum_{\_R=RootOf\left(9\_Z^4+1\right)}\_R\ln\left(9\_R^3x-3\_R^2+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^8-x^4+1),x)
```

```
[Out] -1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+
1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{5\,x^4+1}{5\,x^5}-\int\frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^6),x, algorithm="maxima")
```

```
[Out] -1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.270996, size = 302, normalized size = 1.05

$$\frac{20\,\sqrt{3}\sqrt{2}x^{5}\arctan\left(\frac{\sqrt{3}\sqrt{2}+3\,x}{\sqrt{3}\sqrt{2}x^{2}+\sqrt{3}\sqrt{2}\sqrt{x^{4}+\sqrt{3}\sqrt{2}(x^{3}+x)+3\,x^{2}+1}+3\,x}\right)-20\,\sqrt{3}\sqrt{2}x^{5}\arctan\left(\frac{\sqrt{3}\sqrt{2}-3\,x}{\sqrt{3}\sqrt{2}x^{2}+\sqrt{3}\sqrt{2}\sqrt{x^{4}-\sqrt{3}\sqrt{2}(x^{3}+x)+3\,x^{2}+1}-3\,x}\right)+5\,\sqrt{}}{120\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^6),x, algorithm="fricas")
```

```
[Out] 1/120*(20*sqrt(3)*sqrt(2)*x^5*arctan((sqrt(3)*sqrt(2) + 3*x)/(sqr
t(3)*sqrt(2)*x^2 + sqrt(3)*sqrt(2)*sqrt(x^4 + sqrt(3)*sqrt(2)*(x^
3 + x) + 3*x^2 + 1) + 3*x)) - 20*sqrt(3)*sqrt(2)*x^5*arctan((sqrt
(3)*sqrt(2) - 3*x)/(sqrt(3)*sqrt(2)*x^2 + sqrt(3)*sqrt(2)*sqrt(x^
4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) - 3*x)) + 5*sqrt(3)*sq
rt(2)*x^5*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) - 5*sq
rt(3)*sqrt(2)*x^5*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 120*x^4 - 24)/x^5
```

Sympy [A] time = 0.866249, size = 180, normalized size = 0.63

$$\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^{3}-4x^{2}+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^{3}+4x^{2}+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^{4}-\sqrt{6}x^{3}+3x^{2}-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^{4}+\sqrt{6}x^{3}+3x^{2}+\sqrt{6}x+1\right)}{24}-\frac{5x^{4}+1}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**
2 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) -
2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log
(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x
**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24 - (5*x**4 + 1)/(5
*x**5)
```

GIAC/XCAS [A] time = 0.285729, size = 293, normalized size = 1.02

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{5x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^6),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2)
)) - 1/12*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/12*sqrt(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) -
 sqrt(2))) - 1/12*sqrt(6)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(
6) - sqrt(2))) + 1/24*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(2))
+ 1) - 1/24*sqrt(6)*ln(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/2
4*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*sqrt(6)*
ln(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x^5
```

3.366 $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

Optimal. Leaf size=377

$$-\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] -1/(7*x^7) - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 -
Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Ar
cTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 -
 Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]
]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*
x + x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x
+ x^2])/8
```

Rubi [A] time = 0.658774, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$

$$-\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 -
Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Ar
```

```
cTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 -
 Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]
]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*
x + x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x
+ x^2])/8
```

Rubi in Sympy [A] time = 76.811, size = 325, normalized size = 0.86

$$\frac{\sqrt{3}\log\left(x^2 - x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\log\left(x^2 + x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\log\left(x^2 - x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} + \frac{\sqrt{3}\log\left(x^2 + x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}} - \frac{1}{3x^3} - \frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**8/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqrt(3) +
2)) - sqrt(3)*log(x**2 + x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqrt
(3) + 2)) - sqrt(3)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(24*sqrt(
sqrt(3) + 2)) + sqrt(3)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(24*s
qrt(sqrt(3) + 2)) - sqrt(3)*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-
sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*atan((2*x + sqrt(
sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) + sqrt(
3)*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(12*sqrt(sq
rt(3) + 2)) + sqrt(3)*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3
) + 2))/(12*sqrt(sqrt(3) + 2)) - 1/(3*x**3) - 1/(7*x**7)
```

Mathematica [C] time = 0.0232919, size = 54, normalized size = 0.14

$$-\frac{1}{4}\mathrm{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{7x^7}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]
*#1)/(-1 + 2*#1^4) & ]/4
```

Maple [C] time = 0.015, size = 51, normalized size = 0.1

$$-\frac{1}{7\,x^{7}}-\frac{1}{3\,x^{3}}-\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\,\_R^{7}-\_R^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^8/(x^8-x^4+1),x)
```

```
[Out] -1/7/x^7-1/3/x^3-1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z
^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7\,x^{4}+3}{21\,x^{7}}-\int\frac{x^{4}}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^8),x, algorithm="maxima")
```

```
[Out] -1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.291739, size = 1218, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^8),x, algorithm="fricas")
```

```
[Out] -1/168*(28*sqrt(3)*x^7*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan
((sqrt(3)*sqrt(2) + 2*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqrt(3)
```

```
+ 2)/(4*sqrt(3) + 7)) + 2)*(sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sq
rt(3) + 7)) + 2*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt((sqrt(3) +
 2)/(4*sqrt(3) + 7)) + sqrt(2))) + 28*sqrt(3)*x^7*sqrt((sqrt(3) -
 2)/(4*sqrt(3) - 7))*arctan((sqrt(3)*sqrt(2) + 2*sqrt(2))/(2*sqrt
(2*x^2 - 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2)*(sqrt(3) +
2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*sqrt(2)*x + 2
*sqrt(2)*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) - sqrt(2))) - 28*
(7*sqrt(3)*x^7 + 12*x^7)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arct
an((sqrt(3)*sqrt(2) - 2*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqrt(3
) - 2)/(4*sqrt(3) - 7)) + 2)*(sqrt(3) - 2)*sqrt((sqrt(3) - 2)/(4*
sqrt(3) - 7)) + 2*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt((sqrt(3)
 - 2)/(4*sqrt(3) - 7)) - sqrt(2))) - 28*(7*sqrt(3)*x^7 + 12*x^7)*
sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan((sqrt(3)*sqrt(2) - 2*s
qrt(2))/(2*sqrt(2*x^2 - 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) +
 2)*(sqrt(3) - 2)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3
)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) +
sqrt(2))) + 7*(2*sqrt(3)*x^7 + 3*x^7)*sqrt((sqrt(3) - 2)/(4*sqrt(
3) - 7))*log(2*x^2 + 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2)
 - 7*(2*sqrt(3)*x^7 + 3*x^7)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*
log(2*x^2 - 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2) - 7*(2*s
qrt(3)*x^7 + 3*x^7)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(2*x^2
 + 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2) + 7*(2*sqrt(3)*x^
7 + 3*x^7)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(2*x^2 - 2*x*sq
rt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2) + 8*(14*x^4 + sqrt(3)*(7*x
^4 + 3) + 6)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt((sqrt(3) -
2)/(4*sqrt(3) - 7)))/((sqrt(3)*x^7 + 2*x^7)*sqrt((sqrt(3) + 2)/(4
*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 5.16786, size = 36, normalized size = 0.1

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(18432t^5 - 4t + x\right)\right)\right) - \frac{7x^4+3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**8/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_
t**5 - 4*_t + x))) - (7*x**4 + 3)/(21*x**7)
```

GIAC/XCAS [A] time = 0.305881, size = 358, normalized size = 0.95

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$
$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$
$$-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)$$
$$+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$
$$+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{7x^4+3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - x^4 + 1)*x^8),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*a
rctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt
(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt
(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt
(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) +
sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6
) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

3.367 $\int \frac{x^m}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/
(3 - Sqrt[5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hy
pergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])
/(Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rubi [A] time = 0.153278, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/
(3 - Sqrt[5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hy
pergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])
/(Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rubi in Sympy [A] time = 13.6053, size = 104, normalized size = 0.89

$$\frac{\sqrt{5}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\\frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-\frac{x^4}{-\frac{\sqrt{5}}{2}+\frac{3}{2}}\right)}{5\left(-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)(m+1)}-\frac{\sqrt{5}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\\frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-\frac{x^4}{\frac{\sqrt{5}}{2}+\frac{3}{2}}\right)}{5\left(\frac{\sqrt{5}}{2}+\frac{3}{2}\right)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(5)*x**(m + 1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), -x**4/(-sq
rt(5)/2 + 3/2))/(5*(-sqrt(5)/2 + 3/2)*(m + 1)) - sqrt(5)*x**(m +
1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), -x**4/(sqrt(5)/2 + 3/2))/(
5*(sqrt(5)/2 + 3/2)*(m + 1))
```

Mathematica [C] time = 0.0731782, size = 79, normalized size = 0.68

$$\frac{x^m\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^7+3\#1^3}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (x^m*RootSum[1 + 3*#1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 -
m, -(#1/(x - #1))]/((x/(x - #1))^m*(3*#1^3 + 2*#1^7)) & ])/(4*m)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(x^8+3*x^4+1),x)
```

```
[Out] int(x^m/(x^8+3*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8 + 3\,x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + 3*x^4 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(x**8+3*x**4+1),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

3.368 $\int \frac{x^{11}}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7
*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] time = 0.106361, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7
*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi in Sympy [A] time = 11.6429, size = 68, normalized size = 1.1

$$\frac{x^4}{4}+\frac{\sqrt{5}\left(-\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(2x^4-\sqrt{5}+3\right)}{20}-\frac{\sqrt{5}\left(\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(2x^4+\sqrt{5}+3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(x**8+3*x**4+1),x)
```

```
[Out] x**4/4 + sqrt(5)*(-3*sqrt(5)/2 + 7/2)*log(2*x**4 - sqrt(5) + 3)/2
0 - sqrt(5)*(3*sqrt(5)/2 + 7/2)*log(2*x**4 + sqrt(5) + 3)/20
```

Mathematica [A] time = 0.0554396, size = 57, normalized size = 0.92

$$\frac{1}{40}\left(10x^4+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}-3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*S
qrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.005, size = 38, normalized size = 0.6

$$\frac{x^4}{4} - \frac{3\,\ln\left(x^8+3\,x^4+1\right)}{8} - \frac{7\,\sqrt{5}}{20}\mathit{Artanh}\left(\frac{\left(2\,x^4+3\right)\,\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5
^(1/2)
```

Maxima [A] time = 0.826583, size = 68, normalized size = 1.1

$$\frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right) - \frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5)
 + 3)) - 3/8*log(x^8 + 3*x^4 + 1)
```

Fricas [A] time = 0.248572, size = 96, normalized size = 1.55

$$\frac{1}{40}\,\sqrt{5}\left(2\,\sqrt{5}x^4 - 3\,\sqrt{5}\log\left(x^8+3\,x^4+1\right) + 7\,\log\left(-\frac{10\,x^4-\sqrt{5}\left(2\,x^8+6\,x^4+7\right)+15}{x^8+3\,x^4+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*(2*sqrt(5)*x^4 - 3*sqrt(5)*log(x^8 + 3*x^4 + 1) + 7*
log(-(10*x^4 - sqrt(5)*(2*x^8 + 6*x^4 + 7) + 15)/(x^8 + 3*x^4 + 1
)))
```

Sympy [A] time = 0.330164, size = 60, normalized size = 0.97

$$\frac{x^{4}}{4} + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^{4} - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^{4} + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(x**8+3*x**4+1),x)
```

```
[Out] x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*
sqrt(5)/40 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

GIAC/XCAS [A] time = 0.271132, size = 68, normalized size = 1.1

$$\frac{1}{4}\,x^{4} + \frac{7}{40}\,\sqrt{5}\ln\left(\frac{2\,x^{4} - \sqrt{5} + 3}{2\,x^{4} + \sqrt{5} + 3}\right) - \frac{3}{8}\,\ln\left(x^{8} + 3\,x^{4} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*ln((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5)
+ 3)) - 3/8*ln(x^8 + 3*x^4 + 1)
```

3.369 $\int \frac{x^9}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2
])/2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2]
)/2
```

Rubi [A] time = 0.265382, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2
])/2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2]
)/2
```

Rubi in Sympy [A] time = 20.1815, size = 102, normalized size = 1.13

$$\frac{x^2}{2}-\frac{\sqrt{2}\left(-\frac{7\sqrt{5}}{10}+\frac{3}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{2}\left(\frac{3}{2}+\frac{7\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**8+3*x**4+1),x)
```

```
[Out] x**2/2 - sqrt(2)*(-7*sqrt(5)/10 + 3/2)*atan(sqrt(2)*x**2/sqrt(-sq
rt(5) + 3))/(2*sqrt(-sqrt(5) + 3)) - sqrt(2)*(3/2 + 7*sqrt(5)/10)
*atan(sqrt(2)*x**2/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5) + 3))
```

Mathematica [A] time = 0.264443, size = 97, normalized size = 1.08

$$\frac{1}{40}\left(20x^2-\sqrt{6-2\sqrt{5}}\left(15+7\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\sqrt{2\left(3+\sqrt{5}\right)}\left(7\sqrt{5}-15\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] (20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 +
 Sqrt[5])]*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[
Sqrt[(3 + Sqrt[5])/2]*x^2])/40
```

Maple [B] time = 0.044, size = 117, normalized size = 1.3

$$\frac{x^2}{2}-\frac{7\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x^2}{2\sqrt{5}+2}\right)-3\frac{1}{2\sqrt{5}+2}\arctan\left(4\frac{x^2}{2\sqrt{5}+2}\right)+\frac{7\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)-3\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^8+3*x^4+1),x)
```

```
[Out] 1/2*x^2-7/5*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))-3/(
2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))+7/5*5^(1/2)/(-2+2*5^(1/2
))*arctan(4*x^2/(-2+2*5^(1/2)))-3/(-2+2*5^(1/2))*arctan(4*x^2/(-2
+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2-\int\frac{\left(3x^4+1\right)x}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.265717, size = 203, normalized size = 2.26

$$\frac{1}{2}\,x^2-\frac{1}{5}\,\sqrt{\sqrt{5}\left(9\,\sqrt{5}-20\right)}\arctan\left(\frac{\sqrt{\sqrt{5}\left(9\,\sqrt{5}-20\right)}\left(\sqrt{5}+3\right)}{2\left(\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2\,x^4+3)-5\right)}\right)}\right)-\frac{1}{5}\,\sqrt{\sqrt{5}\left(9\,\sqrt{5}+20\right)}\arctan\left(\frac{\sqrt{\sqrt{5}\left(9\,\sqrt{5}+20\right)}\left(\sqrt{5}-3\right)}{2\left(\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2\,x^4+3)+5\right)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/5*sqrt(sqrt(5)*(9*sqrt(5) - 20))*arctan(1/2*sqrt(sqrt
(5)*(9*sqrt(5) - 20))*(sqrt(5) + 3)/(sqrt(5)*x^2 + sqrt(5)*sqrt(1
/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^4 + 3) - 5)))) - 1/5*sqrt(sqrt(5)
*(9*sqrt(5) + 20))*arctan(1/2*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(sqr
t(5) - 3)/(sqrt(5)*x^2 + sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)
*(2*x^4 + 3) + 5))))
```

Sympy [A] time = 0.617601, size = 54, normalized size = 0.6

$$\frac{x^2}{2}+2\left(-\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**8+3*x**4+1),x)
```

```
[Out] x**2/2 + 2*(-sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) - 2*(s
qrt(5)/10 + 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

GIAC/XCAS [A] time = 0.281519, size = 89, normalized size = 0.99

$$\frac{1}{2}\,x^2-\frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(
sqrt(5) + 1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2
*x^2/(sqrt(5) - 1))
```

3.370 $\int \frac{x^7}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] time = 0.0684024, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi in Sympy [A] time = 8.48622, size = 60, normalized size = 1.09

$$-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(2x^4-\sqrt{5}+3\right)}{20}+\frac{\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(2x^4+\sqrt{5}+3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**8+3*x**4+1),x)
```

```
[Out] -sqrt(5)*(-sqrt(5)/2 + 3/2)*log(2*x**4 - sqrt(5) + 3)/20 + sqrt(5
)*(sqrt(5)/2 + 3/2)*log(2*x**4 + sqrt(5) + 3)/20
```

Mathematica [A] time = 0.0403947, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])
*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.004, size = 33, normalized size = 0.6

$$\frac{\ln\left(x^8+3x^4+1\right)}{8}+\frac{3\sqrt{5}}{20}Artanh\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^8+3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 0.826187, size = 61, normalized size = 1.11

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) +
1/8*log(x^8 + 3*x^4 + 1)
```

Fricas [A] time = 0.262203, size = 81, normalized size = 1.47

$$\frac{1}{40}\sqrt{5}\left(\sqrt{5}\log\left(x^8+3x^4+1\right)+3\log\left(\frac{10x^4+\sqrt{5}\left(2x^8+6x^4+7\right)+15}{x^8+3x^4+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*(sqrt(5)*log(x^8 + 3*x^4 + 1) + 3*log((10*x^4 + sqrt
(5)*(2*x^8 + 6*x^4 + 7) + 15)/(x^8 + 3*x^4 + 1)))
```

Sympy [A] time = 0.30008, size = 53, normalized size = 0.96

$$\left(-\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**8+3*x**4+1),x)
```

```
[Out] (-3*sqrt(5)/40 + 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt
(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

GIAC/XCAS [A] time = 0.283719, size = 61, normalized size = 1.11

$$-\frac{3}{40}\sqrt{5}\ln\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)+\frac{1}{8}\ln\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*ln((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1
/8*ln(x^8 + 3*x^4 + 1)
```

3.371 $\int \frac{x^5}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (S
qrt[(3 - Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.155115, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (S
qrt[(3 - Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi in Sympy [A] time = 13.7169, size = 97, normalized size = 1.2

$$\frac{\sqrt{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}+\frac{\sqrt{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(2)*(-3*sqrt(5)/10 + 1/2)*atan(sqrt(2)*x**2/sqrt(-sqrt(5) + 3
))/(2*sqrt(-sqrt(5) + 3)) + sqrt(2)*(1/2 + 3*sqrt(5)/10)*atan(sqr
t(2)*x**2/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5) + 3))
```

Mathematica [A] time = 0.0926063, size = 75, normalized size = 0.93

$$\frac{2\sqrt{5}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\left(5-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*Ar
cTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

Maple [B] time = 0.029, size = 110, normalized size = 1.4

$$\frac{1}{2\sqrt{5}+2}\arctan\left(4\frac{x^2}{2\sqrt{5}+2}\right)+\frac{3\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x^2}{2\sqrt{5}+2}\right)+\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)-\frac{3\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^8+3*x^4+1),x)
```

```
[Out] 1/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))+3/5*5^(1/2)/(2*5^(1/2
)+2)*arctan(4*x^2/(2*5^(1/2)+2))+1/(-2+2*5^(1/2))*arctan(4*x^2/(-
2+2*5^(1/2)))-3/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.271149, size = 196, normalized size = 2.42

$$-\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}+5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}+5\right)}}{\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^4+3)+5\right)}}\right)+\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}-5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}-5\right)}}{\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^4+3)-5\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(1/2)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*arctan(sqrt(1/2)*sqr
t(sqrt(5)*(3*sqrt(5) + 5))/(sqrt(5)*x^2 + sqrt(5)*sqrt(1/10)*sqrt
(sqrt(5)*(sqrt(5)*(2*x^4 + 3) + 5)))) + 1/5*sqrt(1/2)*sqrt(sqrt(5
)*(3*sqrt(5) - 5))*arctan(sqrt(1/2)*sqrt(sqrt(5)*(3*sqrt(5) - 5))
/(sqrt(5)*x^2 + sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^4 +
 3) - 5))))
```

Sympy [A] time = 0.609175, size = 49, normalized size = 0.6

$$-2\left(-\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**8+3*x**4+1),x)
```

```
[Out] -2*(-sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/4
0 + 1/8)*atan(2*x**2/(1 + sqrt(5)))
```

GIAC/XCAS [A] time = 0.268334, size = 63, normalized size = 0.78

$$\frac{1}{20}x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sq
rt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1))
```

3.372 $\int \frac{x^3}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -ArcTanh[(3 + 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rubi [A] time = 0.0499458, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + 3*x^4 + x^8),x]

[Out] -ArcTanh[(3 + 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 5.65705, size = 24, normalized size = 1.04

$$-\frac{\sqrt{5}\,\operatorname{atanh}\left(\sqrt{5}\left(\frac{2x^4}{5}+\frac{3}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**8+3*x**4+1),x)

[Out] -sqrt(5)*atanh(sqrt(5)*(2*x**4/5 + 3/5))/10
```

Mathematica [A] time = 0.0155355, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}-3\right)-\log\left(2x^4+\sqrt{5}+3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + 3*x^4 + x^8),x]

[Out] (Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5]
)
```

Maple [A] time = 0.003, size = 19, normalized size = 0.8

$$-\frac{\sqrt{5}}{10}\mathit{Artanh}\left(\frac{(2\,x^4+3)\,\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^8+3*x^4+1),x)

[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 0.824023, size = 42, normalized size = 1.83

$$\frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + 3*x^4 + 1),x, algorithm="maxima")

[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Fricas [A] time = 0.257924, size = 59, normalized size = 2.57

$$\frac{1}{20}\,\sqrt{5}\log\left(-\frac{10\,x^4-\sqrt{5}(2\,x^8+6\,x^4+7)+15}{x^8+3\,x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log(-(10*x^4 - sqrt(5)*(2*x^8 + 6*x^4 + 7) + 15)/(x^
8 + 3*x^4 + 1))
```

Sympy [A] time = 0.260736, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{20} - \frac{\sqrt{5}\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(
5)/2 + 3/2)/20
```

GIAC/XCAS [A] time = 0.279537, size = 42, normalized size = 1.83

$$\frac{1}{20}\sqrt{5}\ln\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*ln((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

3.373 $\int \frac{x}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sq
rt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.103794, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sq
rt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi in Sympy [A] time = 7.37182, size = 73, normalized size = 0.97

$$\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{10\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{10\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(10)*atan(sqrt(2)*x**2/sqrt(-sqrt(5) + 3))/(10*sqrt(-sqrt(5)
+ 3)) - sqrt(10)*atan(sqrt(2)*x**2/sqrt(sqrt(5) + 3))/(10*sqrt(sq
rt(5) + 3))
```

Mathematica [A] time = 0.0621362, size = 74, normalized size = 0.99

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan
[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

Maple [A] time = 0.018, size = 60, normalized size = 0.8

$$-\frac{2\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x^2}{2\sqrt{5}+2}\right)+\frac{2\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^8+3*x^4+1),x)
```

```
[Out] -2/5*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))+2/5*5^(1/2
)/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.280995, size = 212, normalized size = 2.83

$$\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}-5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}-5\right)}\left(\sqrt{5}+3\right)}{2\left(\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^4+3)+5\right)}\right)}\right)+\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}+5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(3\sqrt{5}+5\right)}\left(\sqrt{5}-3\right)}{2\left(\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^4+3)-5\right)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(1/2)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*arctan(1/2*sqrt(1/2)*
sqrt(sqrt(5)*(3*sqrt(5) - 5))*(sqrt(5) + 3)/(sqrt(5)*x^2 + sqrt(5
)*sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^4 + 3) + 5)))) + 1/5*sqrt
(1/2)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*arctan(1/2*sqrt(1/2)*sqrt(sqr
t(5)*(3*sqrt(5) + 5))*(sqrt(5) - 3)/(sqrt(5)*x^2 + sqrt(5)*sqrt(1
/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^4 + 3) - 5))))
```

Sympy [A] time = 0.593474, size = 49, normalized size = 0.65

$$2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(-\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**8+3*x**4+1),x)
```

```
[Out] 2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(-sqrt(5)/40
 + 1/8)*atan(2*x**2/(1 + sqrt(5)))
```

GIAC/XCAS [A] time = 0.284052, size = 55, normalized size = 0.73

$$\frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) +
5)*arctan(2*x^2/(sqrt(5) - 1))
```

3.374 $\int \frac{1}{x(1+3x^4+x^8)} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*
Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] time = 0.078824, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*
Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi in Sympy [A] time = 11.2325, size = 66, normalized size = 1.16

$$\frac{\log\left(x^4\right)}{4}-\frac{\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(2x^4-\sqrt{5}+3\right)}{20}+\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(2x^4+\sqrt{5}+3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**8+3*x**4+1),x)
```

```
[Out] log(x**4)/4 - sqrt(5)*(sqrt(5)/2 + 3/2)*log(2*x**4 - sqrt(5) + 3)
/20 + sqrt(5)*(-sqrt(5)/2 + 3/2)*log(2*x**4 + sqrt(5) + 3)/20
```

Mathematica [A] time = 0.0593927, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 +
 3*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.01, size = 35, normalized size = 0.6

$$\ln(x) - \frac{\ln(x^8 + 3x^4 + 1)}{8} + \frac{3\sqrt{5}}{20} Artanh\left(\frac{(2x^4 + 3)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^8+3*x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(
1/2)
```

Maxima [A] time = 0.825035, size = 69, normalized size = 1.21

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{1}{8}\log(x^8 + 3x^4 + 1) + \frac{1}{4}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) -
1/8*log(x^8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.256837, size = 90, normalized size = 1.58

$$-\frac{1}{40}\sqrt{5}\left(\sqrt{5}\log(x^8 + 3x^4 + 1) - 8\sqrt{5}\log(x) - 3\log\left(\frac{10x^4 + \sqrt{5}(2x^8 + 6x^4 + 7) + 15}{x^8 + 3x^4 + 1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*(sqrt(5)*log(x^8 + 3*x^4 + 1) - 8*sqrt(5)*log(x) -
3*log((10*x^4 + sqrt(5)*(2*x^8 + 6*x^4 + 7) + 15)/(x^8 + 3*x^4 +
1)))
```

Sympy [A] time = 0.377179, size = 58, normalized size = 1.02

$$\log(x) + \left(-\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**8+3*x**4+1),x)
```

```
[Out] log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/
8 + 3*sqrt(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

GIAC/XCAS [A] time = 0.276181, size = 69, normalized size = 1.21

$$-\frac{3}{40}\sqrt{5}\ln\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{1}{8}\ln\left(x^8 + 3x^4 + 1\right) + \frac{1}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*ln((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1
/8*ln(x^8 + 3*x^4 + 1) + 1/4*ln(x^4)
```

3.375 $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/(2*x^2) + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])
]*x^2])/2 - ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2
])/(4*Sqrt[10])
```

Rubi [A] time = 0.167353, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])
]*x^2])/2 - ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2
])/(4*Sqrt[10])
```

Rubi in Sympy [A] time = 18.0556, size = 105, normalized size = 1.18

$$-\frac{\sqrt{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**8+3*x**4+1),x)
```

```
[Out] -sqrt(2)*(1/2 + 3*sqrt(5)/10)*atan(sqrt(2)*x**2/sqrt(-sqrt(5) + 3
))/(2*sqrt(-sqrt(5) + 3)) - sqrt(2)*(-3*sqrt(5)/10 + 1/2)*atan(sq
```

```
rt(2)*x**2/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5) + 3)) - 1/(2*x**2)
```

Mathematica [C] time = 0.0270219, size = 65, normalized size = 0.73

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x
 - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

Maple [B] time = 0.006, size = 117, normalized size = 1.3

$$-\frac{1}{2\sqrt{5}+2}\arctan\left(4\frac{x^2}{2\sqrt{5}+2}\right)+\frac{3\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x^2}{2\sqrt{5}+2}\right)-\frac{3\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)-\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x^2}{-2+2\sqrt{5}}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^8+3*x^4+1),x)
```

```
[Out] -1/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))+3/5*5^(1/2)/(2*5^(1/
2)+2)*arctan(4*x^2/(2*5^(1/2)+2))-3/5*5^(1/2)/(-2+2*5^(1/2))*arct
an(4*x^2/(-2+2*5^(1/2)))-1/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1
/2)))-1/2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2x^2}-\int\frac{(x^4+3)x}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.288127, size = 217, normalized size = 2.44

$$-\frac{2\sqrt{\sqrt{5}\left(9\sqrt{5}-20\right)}x^2\arctan\left(\frac{\sqrt{\sqrt{5}\left(9\sqrt{5}-20\right)}\left(3\sqrt{5}+7\right)}{2\left(\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^4+3)+5\right)}\right)}\right)+2\sqrt{\sqrt{5}\left(9\sqrt{5}+20\right)}x^2\arctan\left(\frac{\sqrt{\sqrt{5}\left(9\sqrt{5}+20\right)}\left(3\sqrt{5}-7\right)}{2\left(\sqrt{5}x^2+\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^4+3)-\right.}\right.}\right.}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(sqrt(5)*(9*sqrt(5) - 20))*x^2*arctan(1/2*sqrt(sqrt(
5)*(9*sqrt(5) - 20))*(3*sqrt(5) + 7)/(sqrt(5)*x^2 + sqrt(5)*sqrt(
1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^4 + 3) + 5)))) + 2*sqrt(sqrt(5)*
(9*sqrt(5) + 20))*x^2*arctan(1/2*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(
3*sqrt(5) - 7)/(sqrt(5)*x^2 + sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sq
rt(5)*(2*x^4 + 3) - 5)))) + 5)/x^2
```

Sympy [A] time = 0.723562, size = 56, normalized size = 0.63

$$-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(-\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**8+3*x**4+1),x)
```

```
[Out] -2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(-sqrt(5)/1
0 + 1/4)*atan(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.280451, size = 92, normalized size = 1.03

$$-\frac{1}{20}\left(x^4\left(\sqrt{5}-5\right)+3\sqrt{5}-15\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(x^4\left(\sqrt{5}+5\right)+3\sqrt{5}+15\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^3),x, algorithm="giac")
```

```
[Out] -1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5)
+ 1)) - 1/20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(s
qrt(5) - 1)) - 1/2/x^2
```

3.376 $\int \frac{1}{x^5(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

```
[Out] -1/(4*x^4) - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4
])/40 + ((15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] time = 0.144261, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4
])/40 + ((15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi in Sympy [A] time = 16.5915, size = 78, normalized size = 1.18

$$-\frac{3\log\left(x^4\right)}{4}+\frac{\sqrt{5}\left(\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(2x^4-\sqrt{5}+3\right)}{20}-\frac{\sqrt{5}\left(-\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(2x^4+\sqrt{5}+3\right)}{20}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**8+3*x**4+1),x)
```

```
[Out] -3*log(x**4)/4 + sqrt(5)*(3*sqrt(5)/2 + 7/2)*log(2*x**4 - sqrt(5)
 + 3)/20 - sqrt(5)*(-3*sqrt(5)/2 + 7/2)*log(2*x**4 + sqrt(5) + 3)
/20 - 1/(4*x**4)
```

Mathematica [A] time = 0.059073, size = 60, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4
] + (15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.011, size = 42, normalized size = 0.6

$$-\frac{1}{4x^{4}}-3\ln\left(x\right)+\frac{3\ln\left(x^{8}+3x^{4}+1\right)}{8}-\frac{7\sqrt{5}}{20}\mathit{Artanh}\left(\frac{\left(2x^{4}+3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^8+3*x^4+1),x)
```

```
[Out] -1/4/x^4-3*ln(x)+3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5
^(1/2))*5^(1/2)
```

Maxima [A] time = 0.823955, size = 76, normalized size = 1.15

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^{4}-\sqrt{5}+3}{2x^{4}+\sqrt{5}+3}\right)-\frac{1}{4x^{4}}+\frac{3}{8}\log\left(x^{8}+3x^{4}+1\right)-\frac{3}{4}\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^5),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1
/4/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Fricas [A] time = 0.251386, size = 117, normalized size = 1.77

$$\frac{\sqrt{5}\left(3\sqrt{5}x^{4}\log\left(x^{8}+3x^{4}+1\right)-24\sqrt{5}x^{4}\log\left(x\right)+7x^{4}\log\left(-\frac{10x^{4}-\sqrt{5}\left(2x^{8}+6x^{4}+7\right)+15}{x^{8}+3x^{4}+1}\right)-2\sqrt{5}\right)}{40x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^5),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*(3*sqrt(5)*x^4*log(x^8 + 3*x^4 + 1) - 24*sqrt(5)*x^4
*log(x) + 7*x^4*log(-(10*x^4 - sqrt(5)*(2*x^8 + 6*x^4 + 7) + 15)/
(x^8 + 3*x^4 + 1)) - 2*sqrt(5))/x^4
```

Sympy [A] time = 0.529782, size = 65, normalized size = 0.98

$$-3\log(x) + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40} + \frac{3}{8}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right) - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**8+3*x**4+1),x)
```

```
[Out] -3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-
7*sqrt(5)/40 + 3/8)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

GIAC/XCAS [A] time = 0.282319, size = 85, normalized size = 1.29

$$\frac{7}{40}\sqrt{5}\ln\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) + \frac{3x^4 - 1}{4x^4} + \frac{3}{8}\ln\left(x^8 + 3x^4 + 1\right) - \frac{3}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^5),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*ln((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/
4*(3*x^4 - 1)/x^4 + 3/8*ln(x^8 + 3*x^4 + 1) - 3/4*ln(x^4)
```

3.377 $\int \frac{1}{x^7(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/(6*x^6) + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt
[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[S
qrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.280785, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt
[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[S
qrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi in Sympy [A] time = 25.1671, size = 110, normalized size = 1.13

$$\frac{\sqrt{2}\left(\frac{9}{2}+\frac{21\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{6\sqrt{-\sqrt{5}+3}}+\frac{\sqrt{2}\left(-\frac{21\sqrt{5}}{10}+\frac{9}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{6\sqrt{\sqrt{5}+3}}+\frac{3}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(2)*(9/2 + 21*sqrt(5)/10)*atan(sqrt(2)*x**2/sqrt(-sqrt(5) + 3
))/(6*sqrt(-sqrt(5) + 3)) + sqrt(2)*(-21*sqrt(5)/10 + 9/2)*atan(s
qrt(2)*x**2/sqrt(sqrt(5) + 3))/(6*sqrt(sqrt(5) + 3)) + 3/(2*x**2)
 - 1/(6*x**6)
```

Mathematica [C] time = 0.0285652, size = 73, normalized size = 0.75

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+8\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]-\frac{1}{6x^6}+\frac{3}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x -
 #1] + 3*Log[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

Maple [B] time = 0.026, size = 122, normalized size = 1.3

$$-\frac{1}{6\,x^6}+\frac{3}{2\,x^2}-\frac{7\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\,\frac{x^2}{2\sqrt{5}+2}\right)+3\,\frac{1}{2\sqrt{5}+2}\arctan\left(4\,\frac{x^2}{2\sqrt{5}+2}\right)+\frac{7\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\,\frac{x^2}{-2+2\sqrt{5}}\right)+3\,\frac{1}{-2+2\sqrt{5}}\arctan\left(4\,\frac{x^2}{-2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^8+3*x^4+1),x)
```

```
[Out] -1/6/x^6+3/2/x^2-7/5*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2
)+2))+3/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))+7/5*5^(1/2)/(-2
+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))+3/(-2+2*5^(1/2))*arctan(
4*x^2/(-2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{9\,x^4-1}{6\,x^6}+\int\frac{(3\,x^4+8)\,x}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^7),x, algorithm="maxima")
```

```
[Out] 1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1),
x)
```

Fricas [A] time = 0.282979, size = 215, normalized size = 2.22

$$-\frac{\sqrt{10}\left(6\,x^6\sqrt{55\,\sqrt{5}+123}\arctan\left(\frac{\left(9\,\sqrt{5}\sqrt{2}-20\,\sqrt{2}\right)\sqrt{55\,\sqrt{5}+123}}{\sqrt{10}\sqrt{2}x^2+\sqrt{10}\sqrt{2\,x^4-\sqrt{5}+3}}\right)-6\,x^6\sqrt{-55\,\sqrt{5}+123}\arctan\left(\frac{\left(9\,\sqrt{5}\sqrt{2}+20\,\sqrt{2}\right)\sqrt{-55\,\sqrt{5}+123}}{\sqrt{10}\sqrt{2}x^2+\sqrt{10}\sqrt{2\,x^4+\sqrt{5}+3}}\right)-\sqrt{10}(9}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^7),x, algorithm="fricas")
```

```
[Out] -1/60*sqrt(10)*(6*x^6*sqrt(55*sqrt(5) + 123)*arctan((9*sqrt(5)*sq
rt(2) - 20*sqrt(2))*sqrt(55*sqrt(5) + 123)/(sqrt(10)*sqrt(2)*x^2
+ sqrt(10)*sqrt(2*x^4 - sqrt(5) + 3))) - 6*x^6*sqrt(-55*sqrt(5) +
 123)*arctan((9*sqrt(5)*sqrt(2) + 20*sqrt(2))*sqrt(-55*sqrt(5) +
123)/(sqrt(10)*sqrt(2)*x^2 + sqrt(10)*sqrt(2*x^4 + sqrt(5) + 3)))
 - sqrt(10)*(9*x^4 - 1))/x^6
```

Sympy [A] time = 0.869662, size = 65, normalized size = 0.67

$$2\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(-\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)+\frac{9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**8+3*x**4+1),x)
```

```
[Out] 2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(-11*sqrt
(5)/40 + 5/8)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

GIAC/XCAS [A] time = 0.278921, size = 104, normalized size = 1.07

$$\frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+8\,\sqrt{5}-40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+8\,\sqrt{5}+40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)+\frac{9\,x^4-1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^7),x, algorithm="giac")
```

```
[Out] 1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5)
 + 1)) + 1/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2
/(sqrt(5) - 1)) + 1/6*(9*x^4 - 1)/x^6
```

3.378 $\int \frac{x^8}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=458

$$-\frac{\sqrt[4]{984-440\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}+x-\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\sqrt{10}}+\frac{\sqrt[4]{123+55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] x - ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5
])^(1/4)])/(4*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 + (
2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(4*Sqrt[10]) + ((123 + 55*Sqrt[5
])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*
Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)
*Log[Sqrt[3 - Sqrt[5]] - 2^(3/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*
x^2])/(8*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt
[5]] + 2^(3/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(8*Sqrt[10])
 + ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[3 + Sqrt[5]] - 2^(3/4)*(3 +
 Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) - ((123 + 5
5*Sqrt[5])^(1/4)*Log[Sqrt[3 + Sqrt[5]] + 2^(3/4)*(3 + Sqrt[5])^(1
/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.873226, antiderivative size = 440, normalized size of antiderivative = 0.96, number

of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+x-\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - ((123 - 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5]
)^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((123 - 55*Sqrt[5])^(1/4)*ArcTan[
1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((123
 + 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])
/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3
/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 - 55*Sqr
t[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)
*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((123 - 55*Sqrt[5])^(1/4)*Log[
Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) + ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt
[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
 - ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3
+ Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rubi in Sympy [A] time = 91.0684, size = 605, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**8+3*x**4+1),x)
```

```
[Out] x + 2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-7*sqrt(5)/10 + 3/2)*log(2*x**
2 - 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(8
*(-sqrt(5) + 3)**(5/4)) - 2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-7*sqrt(
5)/10 + 3/2)*log(2*x**2 + 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sq
rt(-2*sqrt(5) + 6))/(8*(-sqrt(5) + 3)**(5/4)) + 2**(3/4)*(3/2 + 7
*sqrt(5)/10)*sqrt(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(sqrt(
5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5) + 3)**(5/4)) -
2**(3/4)*(3/2 + 7*sqrt(5)/10)*sqrt(2*sqrt(5) + 6)*log(2*x**2 + 2*
2**(1/4)*x*(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5
) + 3)**(5/4)) - 2**(3/4)*(-7*sqrt(5)/10 + 3/2)*atan(2**(3/4)*(x
- (-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(2*sqrt(-2*sq
rt(5) + 6)*(-sqrt(5) + 3)**(1/4)) - 2**(3/4)*(-7*sqrt(5)/10 + 3/2
)*atan(2**(3/4)*(x + (-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(
1/4))/(2*sqrt(-2*sqrt(5) + 6)*(-sqrt(5) + 3)**(1/4)) - 2**(3/4)*(
3/2 + 7*sqrt(5)/10)*atan(2**(3/4)*(x - (2*sqrt(5) + 6)**(1/4)/2)/
(sqrt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6)
) - 2**(3/4)*(3/2 + 7*sqrt(5)/10)*atan(2**(3/4)*(x + (2*sqrt(5) +
 6)**(1/4)/2)/(sqrt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)*sqrt(
2*sqrt(5) + 6))
```

Mathematica [C] time = 0.0222967, size = 58, normalized size = 0.13

$$x-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1
^4)/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.011, size = 46, normalized size = 0.1

$$x+\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-3\_R^4-1\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^8+3*x^4+1),x)
```

```
[Out] x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_
Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x - \int \frac{3\,x^4 + 1}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.307641, size = 1837, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*sqrt(2)*(4*sqrt(2)*sqrt(sqrt(5)*(123*sqrt(5) + 275))
*sqrt(sqrt(5)*(123*sqrt(5) - 275))*x - 4*(1/250)^(1/4)*sqrt(sqrt(
5)*(123*sqrt(5) + 275))*(sqrt(5)*(123*sqrt(5) - 275))^(3/4)*arcta
n(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) - 275))^(3/4)*
(sqrt(5) + 3)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(123*sqr
t(5) - 275))*x + 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5)
 - 275))^(3/4)*(sqrt(5) + 3) + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(
sqrt(5)*(123*sqrt(5) - 275))*sqrt((123*sqrt(5)*x^2 - 275*x^2 + 2*
sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*(9*sqrt(5) - 20) - 5
*(1/250)^(1/4)*(21*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(sqrt(5)*(12
3*sqrt(5) - 275))^(1/4))/(123*sqrt(5) - 275)))) - 4*(1/250)^(1/4)
*sqrt(sqrt(5)*(123*sqrt(5) + 275))*(sqrt(5)*(123*sqrt(5) - 275))^
(3/4)*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) - 2
75))^(3/4)*(sqrt(5) + 3)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(
5)*(123*sqrt(5) - 275))*x - 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(
123*sqrt(5) - 275))^(3/4)*(sqrt(5) + 3) + 2*sqrt(5)*sqrt(2)*sqrt(
1/10)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*sqrt((123*sqrt(5)*x^2 - 2
75*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*(9*sqrt(5
```

```
) - 20) + 5*(1/250)^(1/4)*(21*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(
sqrt(5)*(123*sqrt(5) - 275))^(1/4))/(123*sqrt(5) - 275)))) - 4*(1
/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(3/4)*sqrt(sqrt(5)*(123
*sqrt(5) - 275))*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(123*
sqrt(5) + 275))^(3/4)*(sqrt(5) - 3)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)
*sqrt(sqrt(5)*(123*sqrt(5) + 275))*x + 5*sqrt(1/10)*(1/250)^(1/4)
*(sqrt(5)*(123*sqrt(5) + 275))^(3/4)*(sqrt(5) - 3) + 2*sqrt(5)*sq
rt(2)*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*sqrt((123*sqrt
(5)*x^2 + 275*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275)
)*(9*sqrt(5) + 20) - 5*(1/250)^(1/4)*(21*sqrt(5)*sqrt(2)*x + 47*s
qrt(2)*x)*(sqrt(5)*(123*sqrt(5) + 275))^(1/4))/(123*sqrt(5) + 275
)))) - 4*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(3/4)*sqrt(s
qrt(5)*(123*sqrt(5) - 275))*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sq
rt(5)*(123*sqrt(5) + 275))^(3/4)*(sqrt(5) - 3)/(2*sqrt(5)*sqrt(2)
*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*x - 5*sqrt(1/10)*(1
/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(3/4)*(sqrt(5) - 3) + 2
*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*sqr
t((123*sqrt(5)*x^2 + 275*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(123*sqr
t(5) + 275))*(9*sqrt(5) + 20) + 5*(1/250)^(1/4)*(21*sqrt(5)*sqrt(
2)*x + 47*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5) + 275))^(1/4))/(123*sq
rt(5) + 275)))) - (1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(3/
4)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*log(123*sqrt(5)*x^2 + 275*x^
2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*(9*sqrt(5) + 2
0) + 5*(1/250)^(1/4)*(21*sqrt(5)*sqrt(2)*x + 47*sqrt(2)*x)*(sqrt(
5)*(123*sqrt(5) + 275))^(1/4)) + (1/250)^(1/4)*(sqrt(5)*(123*sqrt
(5) + 275))^(3/4)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*log(123*sqrt(
5)*x^2 + 275*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))
*(9*sqrt(5) + 20) - 5*(1/250)^(1/4)*(21*sqrt(5)*sqrt(2)*x + 47*sq
rt(2)*x)*(sqrt(5)*(123*sqrt(5) + 275))^(1/4)) - (1/250)^(1/4)*sqr
t(sqrt(5)*(123*sqrt(5) + 275))*(sqrt(5)*(123*sqrt(5) - 275))^(3/4
)*log(123*sqrt(5)*x^2 - 275*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(123*
sqrt(5) - 275))*(9*sqrt(5) - 20) + 5*(1/250)^(1/4)*(21*sqrt(5)*sq
rt(2)*x - 47*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5) - 275))^(1/4)) + (1
/250)^(1/4)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*(sqrt(5)*(123*sqrt(
5) - 275))^(3/4)*log(123*sqrt(5)*x^2 - 275*x^2 + 2*sqrt(1/10)*sqr
t(sqrt(5)*(123*sqrt(5) - 275))*(9*sqrt(5) - 20) - 5*(1/250)^(1/4)
*(21*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5) - 27
5))^(1/4)))/(sqrt(123*sqrt(5) + 275)*sqrt(123*sqrt(5) - 275))
```

Sympy [A] time = 3.86152, size = 29, normalized size = 0.06

$$x + \operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**8+3*x**4+1),x)
```

```
[Out] x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(
```

```
15360*_t**5/11 + 1288*_t/55 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{x^8 + 3x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^8 + 3*x^4 + 1), x)
```

3.379 $\int \frac{x^6}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=449

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{8\sqrt[4]{2}\sqrt{5}}+\frac{\sqrt[4]{9-4\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}-\frac{\sqrt[4]{9-4\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{4\sqrt[4]{2}\sqrt{5}}$$

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)
])/(2*Sqrt[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(
3 - Sqrt[5])^(1/4)])/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1
 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 +
Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(4*2^
(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] - 2
^(3/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*Sqrt[10]) + ((9 -
 4*Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] + 2^(3/4)*(3 - Sqrt[5])^(
1/4)*x + Sqrt[2]*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sq
rt[3 + Sqrt[5]] - 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(
8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(3/4)*Log[Sqrt[3 + Sqrt[5]] +
 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(8*2^(1/4)*Sqrt[5]
)
```

Rubi [A] time = 0.704739, antiderivative size = 431, normalized size of antiderivative = 0.96, number

of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}}+\frac{\sqrt[4]{9-4\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}-\frac{\sqrt[4]{9-4\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{4\sqrt[4]{2}\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)
])/(2*Sqrt[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(
3 - Sqrt[5])^(1/4)])/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1
 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 +
Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(4*2^
(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 - 4*S
qrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/
4)*x + 2*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3
+ Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sq
rt[5]) - ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3
 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sqrt[5])
```

Rubi in Sympy [A] time = 86.547, size = 542, normalized size = 1.21

$$\frac{2^{\frac{3}{4}}\left(-2\sqrt{5}+6\right)\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(2x^{2}-2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{16\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{2^{\frac{3}{4}}\left(-2\sqrt{5}+6\right)\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(2x^{2}+2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{16\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} + \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\left(2\sqrt{5}+6\right)\log\left(2x^{2}-2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{16\left(\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\left(2\sqrt{5}+6\right)\log\left(2x^{2}+2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{16\left(\sqrt{5}+3\right)^{\frac{5}{4}}} + \frac{2^{\frac{3}{4}}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{4\sqrt[4]{-\sqrt{5}+3}} + \frac{2^{\frac{3}{4}}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{4\sqrt[4]{-\sqrt{5}+3}} + \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{4\sqrt[4]{\sqrt{5}+3}} + \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{4\sqrt[4]{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**8+3*x**4+1),x)
```

```
[Out] 2**(3/4)*(-2*sqrt(5) + 6)*(-3*sqrt(5)/10 + 1/2)*log(2*x**2 - 2*2*
*(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(16*(-sqrt
(5) + 3)**(5/4)) - 2**(3/4)*(-2*sqrt(5) + 6)*(-3*sqrt(5)/10 + 1/2
)*log(2*x**2 + 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(
5) + 6))/(16*(-sqrt(5) + 3)**(5/4)) + 2**(3/4)*(1/2 + 3*sqrt(5)/1
0)*(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(sqrt(5) + 3)**(1/4)
 + sqrt(2*sqrt(5) + 6))/(16*(sqrt(5) + 3)**(5/4)) - 2**(3/4)*(1/2
```

```
 + 3*sqrt(5)/10)*(2*sqrt(5) + 6)*log(2*x**2 + 2*2**(1/4)*x*(sqrt(
5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(16*(sqrt(5) + 3)**(5/4)) +
 2**(3/4)*(-3*sqrt(5)/10 + 1/2)*atan(2**(3/4)*(x - (-2*sqrt(5) +
6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(4*(-sqrt(5) + 3)**(1/4)) + 2
**(3/4)*(-3*sqrt(5)/10 + 1/2)*atan(2**(3/4)*(x + (-2*sqrt(5) + 6)
**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(4*(-sqrt(5) + 3)**(1/4)) + 2**
(3/4)*(1/2 + 3*sqrt(5)/10)*atan(2**(3/4)*(x - (2*sqrt(5) + 6)**(1
/4)/2)/(sqrt(5) + 3)**(1/4))/(4*(sqrt(5) + 3)**(1/4)) + 2**(3/4)*
(1/2 + 3*sqrt(5)/10)*atan(2**(3/4)*(x + (2*sqrt(5) + 6)**(1/4)/2)
/(sqrt(5) + 3)**(1/4))/(4*(sqrt(5) + 3)**(1/4))
```

Mathematica [C] time = 0.0171233, size = 41, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]
/4
```

Maple [C] time = 0.009, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.310487, size = 1864, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*sqrt(2)*(4*(1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 2
0))*(sqrt(5)*(9*sqrt(5) - 20))^(3/4)*arctan(sqrt(5)*(1/125)^(1/4)
*(sqrt(5)*(9*sqrt(5) - 20))^(1/4)*(9*sqrt(5) - 20)*(3*sqrt(5) + 7
)/(2*sqrt(5)*sqrt(2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*x +
 2*sqrt(5)*sqrt(2)*sqrt(1/2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) -
20))*sqrt((sqrt(1/5)*(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(
2)*x)*(sqrt(5)*(9*sqrt(5) - 20))^(3/4) + 8*sqrt(5)*x^2 - 18*x^2 +
 sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*(3*sqrt(5) - 7))/(4*sqr
t(5) - 9)) + 5*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) - 20))^(1/4)*(sq
rt(5) - 3))) + 4*(1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(sq
rt(5)*(9*sqrt(5) - 20))^(3/4)*arctan(sqrt(5)*(1/125)^(1/4)*(sqrt(
5)*(9*sqrt(5) - 20))^(1/4)*(9*sqrt(5) - 20)*(3*sqrt(5) + 7)/(2*sq
rt(5)*sqrt(2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*x + 2*sqrt
(5)*sqrt(2)*sqrt(1/2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*sq
rt(-(sqrt(1/5)*(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*
(sqrt(5)*(9*sqrt(5) - 20))^(3/4) - 8*sqrt(5)*x^2 + 18*x^2 - sqrt(
1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*(3*sqrt(5) - 7))/(4*sqrt(5) -
 9)) - 5*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) - 20))^(1/4)*(sqrt(5)
- 3))) + 4*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*sqrt(sq
rt(5)*(9*sqrt(5) - 20))*arctan(sqrt(5)*(1/125)^(1/4)*(sqrt(5)*(9*
sqrt(5) + 20))^(1/4)*(9*sqrt(5) + 20)*(3*sqrt(5) - 7)/(2*sqrt(5)*
sqrt(2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*x + 2*sqrt(5)*sq
rt(2)*sqrt(1/2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*sqrt((sq
rt(1/5)*(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(sqrt(5
)*(9*sqrt(5) + 20))^(3/4) + 8*sqrt(5)*x^2 + 18*x^2 + sqrt(1/5)*sq
rt(sqrt(5)*(9*sqrt(5) + 20))*(3*sqrt(5) + 7))/(4*sqrt(5) + 9)) +
5*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(1/4)*(sqrt(5) + 3)))
+ 4*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*sqrt(sqrt(5)*(
9*sqrt(5) - 20))*arctan(sqrt(5)*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5)
 + 20))^(1/4)*(9*sqrt(5) + 20)*(3*sqrt(5) - 7)/(2*sqrt(5)*sqrt(2)
*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*x + 2*sqrt(5)*sqrt(2)*s
qrt(1/2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*sqrt(-(sqrt(1/5
)*(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(sqrt(5)*(9*s
qrt(5) + 20))^(3/4) - 8*sqrt(5)*x^2 - 18*x^2 - sqrt(1/5)*sqrt(sqr
t(5)*(9*sqrt(5) + 20))*(3*sqrt(5) + 7))/(4*sqrt(5) + 9)) - 5*(1/1
25)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(1/4)*(sqrt(5) + 3))) - (1/1
25)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*sqrt(sqrt(5)*(9*sqrt(5
```

```
) - 20))*log(1/2*sqrt(1/5)*(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x + 5
*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4) + 4*sqrt(5)*x^2 + 9*
x^2 + 1/2*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(3*sqrt(5) + 7
)) + (1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*sqrt(sqrt(5)*
(9*sqrt(5) - 20))*log(-1/2*sqrt(1/5)*(1/125)^(1/4)*(3*sqrt(5)*sqr
t(2)*x + 5*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4) + 4*sqrt(5
)*x^2 + 9*x^2 + 1/2*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(3*s
qrt(5) + 7)) - (1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(sqrt
(5)*(9*sqrt(5) - 20))^(3/4)*log(1/2*sqrt(1/5)*(1/125)^(1/4)*(3*sq
rt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) - 20))^(3/4) +
 4*sqrt(5)*x^2 - 9*x^2 + 1/2*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) -
20))*(3*sqrt(5) - 7)) + (1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 2
0))*(sqrt(5)*(9*sqrt(5) - 20))^(3/4)*log(-1/2*sqrt(1/5)*(1/125)^(
1/4)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) - 20
))^(3/4) + 4*sqrt(5)*x^2 - 9*x^2 + 1/2*sqrt(1/5)*sqrt(sqrt(5)*(9*
sqrt(5) - 20))*(3*sqrt(5) - 7)))/(sqrt(9*sqrt(5) + 20)*sqrt(9*sqr
t(5) - 20))
```

Sympy [A] time = 3.7956, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(40960000t^{8}+115200t^{4}+1,\left(t\mapsto t\log\left(-1792000t^{7}-4920t^{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-179
2000*_t**7 - 4920*_t**3 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{x^{8}+3\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

3.380 $\int \frac{x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=469

$$\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x
)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4
)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/
4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt
[5]] - 2^(3/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*S
qrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] + 2^(3/4)*(3
 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + S
qrt[5])^(1/4)*Log[Sqrt[3 + Sqrt[5]] - 2^(3/4)*(3 + Sqrt[5])^(1/4)
*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*Log
[Sqrt[3 + Sqrt[5]] + 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2]
)/(4*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.661987, antiderivative size = 451, normalized size of antiderivative = 0.96, number

of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x
)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4
)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/
4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - S
qrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[
5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 -
Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2
*x^2])/(4*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 +
 Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqr
t[5])
```

Rubi in Sympy [A] time = 84.7237, size = 604, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**8+3*x**4+1),x)
```

```
[Out] -2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-3*sqrt(5)/10 + 1/2)*log(2*x**2 -
 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(8*(-
sqrt(5) + 3)**(5/4)) + 2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-3*sqrt(5)/
10 + 1/2)*log(2*x**2 + 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(
-2*sqrt(5) + 6))/(8*(-sqrt(5) + 3)**(5/4)) - 2**(3/4)*(1/2 + 3*sq
rt(5)/10)*sqrt(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(sqrt(5)
+ 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5) + 3)**(5/4)) + 2**
(3/4)*(1/2 + 3*sqrt(5)/10)*sqrt(2*sqrt(5) + 6)*log(2*x**2 + 2*2**
(1/4)*x*(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5) +
 3)**(5/4)) + 2**(3/4)*(-3*sqrt(5)/10 + 1/2)*atan(2**(3/4)*(x - (
-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(2*sqrt(-2*sqrt(
5) + 6)*(-sqrt(5) + 3)**(1/4)) + 2**(3/4)*(-3*sqrt(5)/10 + 1/2)*a
tan(2**(3/4)*(x + (-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4
))/(2*sqrt(-2*sqrt(5) + 6)*(-sqrt(5) + 3)**(1/4)) + 2**(3/4)*(1/2
 + 3*sqrt(5)/10)*atan(2**(3/4)*(x - (2*sqrt(5) + 6)**(1/4)/2)/(sq
rt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6)) +
 2**(3/4)*(1/2 + 3*sqrt(5)/10)*atan(2**(3/4)*(x + (2*sqrt(5) + 6)
**(1/4)/2)/(sqrt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)*sqrt(2*s
qrt(5) + 6))
```

Mathematica [C] time = 0.0155784, size = 39, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

Maple [C] time = 0.01, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf(\_Z^8+3\_Z^4+1)}\frac{\_R^4\ln(x-\_R)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^8 + 3x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.310072, size = 1378, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*sqrt(2)*(4*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^
(3/4)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*arctan(5*sqrt(1/10)*(1/250)^(
1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)/(sqrt(5)*sqrt(2)*sqrt(1/10)*
sqrt(sqrt(5)*(3*sqrt(5) + 5))*x + sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt
(sqrt(5)*sqrt(2)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(1/4)*x
+ x^2 + sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5)))*sqrt(sqrt(5)*(3
*sqrt(5) + 5)) + 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) +
 5))^(3/4))) + 4*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*sq
rt(sqrt(5)*(3*sqrt(5) - 5))*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sq
rt(5)*(3*sqrt(5) + 5))^(3/4)/(sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqr
t(5)*(3*sqrt(5) + 5))*x + sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(-sqrt(5
)*sqrt(2)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(1/4)*x + x^2 +
 sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5)))*sqrt(sqrt(5)*(3*sqrt(5
) + 5)) - 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3
/4))) - 4*(1/250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*(sqrt(5)*(3
*sqrt(5) - 5))^(3/4)*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(
3*sqrt(5) - 5))^(3/4)/(sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3
*sqrt(5) - 5))*x + sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*sqrt(2
)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(1/4)*x + x^2 + sqrt(1/
```

```
10)*sqrt(sqrt(5)*(3*sqrt(5) - 5)))*sqrt(sqrt(5)*(3*sqrt(5) - 5))
+ 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4))) -
4*(1/250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*(sqrt(5)*(3*sqrt(5)
 - 5))^(3/4)*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5
) - 5))^(3/4)/(sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5)
 - 5))*x + sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(-sqrt(5)*sqrt(2)*(1/25
0)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(1/4)*x + x^2 + sqrt(1/10)*sqr
t(sqrt(5)*(3*sqrt(5) - 5)))*sqrt(sqrt(5)*(3*sqrt(5) - 5)) - 5*sqr
t(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4))) - (1/250)
^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*sqrt(sqrt(5)*(3*sqrt(5) -
5))*log(sqrt(5)*sqrt(2)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(
1/4)*x + x^2 + sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))) + (1/250
)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*sqrt(sqrt(5)*(3*sqrt(5) -
 5))*log(-sqrt(5)*sqrt(2)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))
^(1/4)*x + x^2 + sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))) + (1/2
50)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*(sqrt(5)*(3*sqrt(5) - 5))
^(3/4)*log(sqrt(5)*sqrt(2)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5
)^(1/4)*x + x^2 + sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))) - (1/
250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*(sqrt(5)*(3*sqrt(5) - 5)
)^(3/4)*log(-sqrt(5)*sqrt(2)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) -
5))^(1/4)*x + x^2 + sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))))/(s
qrt(3*sqrt(5) + 5)*sqrt(3*sqrt(5) - 5))
```

Sympy [A] time = 3.82122, size = 24, normalized size = 0.05

$$\operatorname{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t\log\left(-51200t^5 - 12t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-5120
0*_t**5 - 12*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

3.381 $\int \frac{x^2}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*
x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ArcTan[1 - (2^(3/4
)*x)/(3 + Sqrt[5])^(1/4)]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) - A
rcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*Sqrt[5]*(2*(3 + Sqr
t[5]))^(1/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] - 2^(3
/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) - (
(3 + Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] + 2^(3/4)*(3 - Sqrt[5])
^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) - Log[Sqrt[3 + Sqrt[
5]] - 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2]/(4*Sqrt[5]*(2*
(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[3 + Sqrt[5]] + 2^(3/4)*(3 + Sqrt
[5])^(1/4)*x + Sqrt[2]*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4))
```

Rubi [A] time = 0.518147, antiderivative size = 431, normalized size of antiderivative = 0.96, number

of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*
x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ArcTan[1 - (2^(3/4
)*x)/(3 + Sqrt[5])^(1/4)]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) - A
rcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*Sqrt[5]*(2*(3 + Sqr
t[5]))^(1/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] -
2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 +
 Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(
1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - Log[Sqrt[2*(3 + Sqrt[5])]
- 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]
))^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)
*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4))
```

Rubi in Sympy [A] time = 86.542, size = 488, normalized size = 1.09

$$\frac{2^{\frac{3}{4}}\sqrt{5}\left(-2\sqrt{5}+6\right)\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{80\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{2^{\frac{3}{4}}\sqrt{5}\left(-2\sqrt{5}+6\right)\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{80\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{2^{\frac{3}{4}}\sqrt{5}\left(2\sqrt{5}+6\right)\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{80\left(\sqrt{5}+3\right)^{\frac{5}{4}}} + \frac{2^{\frac{3}{4}}\sqrt{5}\left(2\sqrt{5}+6\right)\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{80\left(\sqrt{5}+3\right)^{\frac{5}{4}}} + \frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{20\sqrt[4]{-\sqrt{5}+3}} + \frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{20\sqrt[4]{-\sqrt{5}+3}} - \frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{20\sqrt[4]{\sqrt{5}+3}} - \frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{20\sqrt[4]{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**8+3*x**4+1),x)
```

```
[Out] 2**(3/4)*sqrt(5)*(-2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(-sqr
t(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(80*(-sqrt(5) + 3)**(5/4
)) - 2**(3/4)*sqrt(5)*(-2*sqrt(5) + 6)*log(2*x**2 + 2*2**(1/4)*x*
(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(80*(-sqrt(5) + 3)*
*(5/4)) - 2**(3/4)*sqrt(5)*(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4
)*x*(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(80*(sqrt(5) + 3)
**(5/4)) + 2**(3/4)*sqrt(5)*(2*sqrt(5) + 6)*log(2*x**2 + 2*2**(1/
```

```
4)*x*(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(80*(sqrt(5) + 3
)**(5/4)) + 2**(3/4)*sqrt(5)*atan(2**(3/4)*(x - (-2*sqrt(5) + 6)*
*(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(20*(-sqrt(5) + 3)**(1/4)) + 2**
(3/4)*sqrt(5)*atan(2**(3/4)*(x + (-2*sqrt(5) + 6)**(1/4)/2)/(-sqr
t(5) + 3)**(1/4))/(20*(-sqrt(5) + 3)**(1/4)) - 2**(3/4)*sqrt(5)*a
tan(2**(3/4)*(x - (2*sqrt(5) + 6)**(1/4)/2)/(sqrt(5) + 3)**(1/4))
/(20*(sqrt(5) + 3)**(1/4)) - 2**(3/4)*sqrt(5)*atan(2**(3/4)*(x +
(2*sqrt(5) + 6)**(1/4)/2)/(sqrt(5) + 3)**(1/4))/(20*(sqrt(5) + 3)
**(1/4))
```

Mathematica [C] time = 0.0154862, size = 40, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5+3\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.009, size = 40, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf(\_Z^8+3\_Z^4+1)}\frac{\_R^2\ln(x-\_R)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.311206, size = 1640, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*sqrt(2)*(4*(1/250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5
))*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*arctan(1/4*sqrt(5)*(1/250)^(1/
4)*(sqrt(5)*(3*sqrt(5) - 5))^(1/4)*(3*sqrt(5) - 5)*(sqrt(5) + 3)/
(sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*x + sqr
t(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*sqrt((10*sq
rt(2)*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*x
+ 3*sqrt(5)*x^2 - 5*x^2 + 2*sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(3*sq
rt(5) - 5)))/(3*sqrt(5) - 5)) + 5*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(
5) - 5))^(1/4))) + 4*(1/250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*
(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*arctan(1/4*sqrt(5)*(1/250)^(1/4)*
(sqrt(5)*(3*sqrt(5) - 5))^(1/4)*(3*sqrt(5) - 5)*(sqrt(5) + 3)/(sq
rt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*x + sqrt(5
)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*sqrt(-(10*sqrt
(2)*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*x -
3*sqrt(5)*x^2 + 5*x^2 - 2*sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt
(5) - 5)))/(3*sqrt(5) - 5)) - 5*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5)
 - 5))^(1/4))) + 4*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*
sqrt(sqrt(5)*(3*sqrt(5) - 5))*arctan(1/4*sqrt(5)*(1/250)^(1/4)*(s
qrt(5)*(3*sqrt(5) + 5))^(1/4)*(3*sqrt(5) + 5)*(sqrt(5) - 3)/(sqrt
(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*x + sqrt(5)*
sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*sqrt((10*sqrt(2)
*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*x + 3*s
qrt(5)*x^2 + 5*x^2 + 2*sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5)
 + 5)))/(3*sqrt(5) + 5)) + 5*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) +
5))^(1/4))) + 4*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*sqr
t(sqrt(5)*(3*sqrt(5) - 5))*arctan(1/4*sqrt(5)*(1/250)^(1/4)*(sqrt
(5)*(3*sqrt(5) + 5))^(1/4)*(3*sqrt(5) + 5)*(sqrt(5) - 3)/(sqrt(5)
*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*x + sqrt(5)*sqr
t(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*sqrt(-(10*sqrt(2)*s
qrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*x - 3*sqr
t(5)*x^2 - 5*x^2 - 2*sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) +
 5)))/(3*sqrt(5) + 5)) - 5*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5)
)^(1/4))) - (1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*sqrt(sq
rt(5)*(3*sqrt(5) - 5))*log(10*sqrt(2)*sqrt(1/10)*(1/250)^(1/4)*(s
qrt(5)*(3*sqrt(5) + 5))^(3/4)*x + 3*sqrt(5)*x^2 + 5*x^2 + 2*sqrt(
5)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))) + (1/250)^(1/4)*(sqr
t(5)*(3*sqrt(5) + 5))^(3/4)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*log(-10
*sqrt(2)*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)
*x + 3*sqrt(5)*x^2 + 5*x^2 + 2*sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(3
*sqrt(5) + 5))) + (1/250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*(sq
```

```
rt(5)*(3*sqrt(5) - 5))^(3/4)*log(10*sqrt(2)*sqrt(1/10)*(1/250)^(1
/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*x + 3*sqrt(5)*x^2 - 5*x^2 + 2
*sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))) - (1/250)^(1/4
)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*l
og(-10*sqrt(2)*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))
^(3/4)*x + 3*sqrt(5)*x^2 - 5*x^2 + 2*sqrt(5)*sqrt(1/10)*sqrt(sqrt
(5)*(3*sqrt(5) - 5))))/(sqrt(3*sqrt(5) + 5)*sqrt(3*sqrt(5) - 5))
```

Sympy [A] time = 3.75336, size = 26, normalized size = 0.06

$$\text{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t\log\left(-6144000t^7 - 2240t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144
000*_t**7 - 2240*_t**3 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

3.382 $\int \frac{1}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=311

$$-\frac{1}{40}\sqrt{20+10\sqrt{5}}\log\left(2x^2-2\sqrt{\sqrt{5}-1}x+\sqrt{5}-1\right)+\frac{1}{40}\sqrt{20+10\sqrt{5}}\log\left(2x^2+2\sqrt{\sqrt{5}-1}x+\sqrt{5}-1\right)+\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(2x^2-2\sqrt{1+\sqrt{5}}x+\sqrt{5}+1\right)-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(2x^2+2\sqrt{1+\sqrt{5}}x+\sqrt{5}+1\right)+\frac{1}{20}\sqrt{10\sqrt{5}-20}\tan^{-1}\left(1-\sqrt{\sqrt{5}-1}x\right)-\frac{1}{20}\sqrt{10\sqrt{5}-20}\tan^{-1}\left(\sqrt{\sqrt{5}-1}x+1\right)-\frac{1}{20}\sqrt{20+10\sqrt{5}}\tan^{-1}\left(1-\sqrt{1+\sqrt{5}}x\right)+\frac{1}{20}\sqrt{20+10\sqrt{5}}\tan^{-1}\left(\sqrt{1+\sqrt{5}}x+1\right)$$

```
[Out] (Sqrt[-20 + 10*Sqrt[5]]*ArcTan[1 - Sqrt[-1 + Sqrt[5]]*x])/20 - (S
qrt[-20 + 10*Sqrt[5]]*ArcTan[1 + Sqrt[-1 + Sqrt[5]]*x])/20 - (Sqr
t[20 + 10*Sqrt[5]]*ArcTan[1 - Sqrt[1 + Sqrt[5]]*x])/20 + (Sqrt[20
 + 10*Sqrt[5]]*ArcTan[1 + Sqrt[1 + Sqrt[5]]*x])/20 - (Sqrt[20 + 1
0*Sqrt[5]]*Log[-1 + Sqrt[5] - 2*Sqrt[-1 + Sqrt[5]]*x + 2*x^2])/40
 + (Sqrt[20 + 10*Sqrt[5]]*Log[-1 + Sqrt[5] + 2*Sqrt[-1 + Sqrt[5]]
*x + 2*x^2])/40 + (Sqrt[-20 + 10*Sqrt[5]]*Log[1 + Sqrt[5] - 2*Sqr
t[1 + Sqrt[5]]*x + 2*x^2])/40 - (Sqrt[-20 + 10*Sqrt[5]]*Log[1 + S
qrt[5] + 2*Sqrt[1 + Sqrt[5]]*x + 2*x^2])/40
```

Rubi [A] time = 0.592509, antiderivative size = 414, normalized size of antiderivative = 1.33, number

of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\sqrt[4]{9+4\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}}+\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] -((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4
)])/(2*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/
(3 - Sqrt[5])^(1/4)])/(2*Sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - ArcTan[1 + (2
^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4))
- ((9 + 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sq
rt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1/4)*L
og[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/
(4*Sqrt[10]) + Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - Log[Sqrt[2*(
3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*Sqrt[5]*(
2*(3 + Sqrt[5]))^(3/4))
```

Rubi in Sympy [A] time = 80.7127, size = 549, normalized size = 1.77

$$-\frac{2^{\frac{3}{4}}\sqrt{5}\sqrt{-2\sqrt{5}+6}\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{40\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{2^{\frac{3}{4}}\sqrt{5}\sqrt{-2\sqrt{5}+6}\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{40\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{2^{\frac{3}{4}}\sqrt{5}\sqrt{2\sqrt{5}+6}\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{40\left(\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{2^{\frac{3}{4}}\sqrt{5}\sqrt{2\sqrt{5}+6}\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{40\left(\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\sqrt{-2\sqrt{5}+6}\sqrt[4]{-\sqrt{5}+3}}+\frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\sqrt{-2\sqrt{5}+6}\sqrt[4]{-\sqrt{5}+3}}-\frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\sqrt[4]{\sqrt{5}+3}\sqrt{2\sqrt{5}+6}}-\frac{2^{\frac{3}{4}}\sqrt{5}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\sqrt[4]{\sqrt{5}+3}\sqrt{2\sqrt{5}+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8+3*x**4+1),x)
```

```
[Out] -2**(3/4)*sqrt(5)*sqrt(-2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*
(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(40*(-sqrt(5) + 3)*
*(5/4)) + 2**(3/4)*sqrt(5)*sqrt(-2*sqrt(5) + 6)*log(2*x**2 + 2*2*
*(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(40*(-sqrt
(5) + 3)**(5/4)) + 2**(3/4)*sqrt(5)*sqrt(2*sqrt(5) + 6)*log(2*x**
2 - 2*2**(1/4)*x*(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(40*
(sqrt(5) + 3)**(5/4)) - 2**(3/4)*sqrt(5)*sqrt(2*sqrt(5) + 6)*log(
```

```
2*x**2 + 2*2**(1/4)*x*(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))
/(40*(sqrt(5) + 3)**(5/4)) + 2**(3/4)*sqrt(5)*atan(2**(3/4)*(x -
(-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(10*sqrt(-2*sqr
t(5) + 6)*(-sqrt(5) + 3)**(1/4)) + 2**(3/4)*sqrt(5)*atan(2**(3/4)
*(x + (-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(10*sqrt(
-2*sqrt(5) + 6)*(-sqrt(5) + 3)**(1/4)) - 2**(3/4)*sqrt(5)*atan(2*
*(3/4)*(x - (2*sqrt(5) + 6)**(1/4)/2)/(sqrt(5) + 3)**(1/4))/(10*(
sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6)) - 2**(3/4)*sqrt(5)*atan(
2**(3/4)*(x + (2*sqrt(5) + 6)**(1/4)/2)/(sqrt(5) + 3)**(1/4))/(10
*(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6))
```

Mathematica [C] time = 0.0142354, size = 42, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.009, size = 37, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8+3\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(1/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 3*x^4 + 1),x, algorithm="maxima")

[Out] integrate(1/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.340465, size = 1805, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 3*x^4 + 1),x, algorithm="fricas")

[Out] 1/40*sqrt(5)*sqrt(2)*(4*(1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 2
0))*(sqrt(5)*(9*sqrt(5) - 20))^(3/4)*arctan(5*sqrt(1/5)*(1/125)^(
1/4)*(sqrt(5)*(9*sqrt(5) - 20))^(3/4)*(sqrt(5) + 3)/(2*sqrt(5)*sq
rt(2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*x + 2*sqrt(5)*sqrt
(2)*sqrt(1/2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*sqrt((18*s
qrt(5)*x^2 - 40*x^2 + sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*(3
*sqrt(5) - 5) + 5*(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*
x)*(sqrt(5)*(9*sqrt(5) - 20))^(1/4))/(9*sqrt(5) - 20)) - 5*sqrt(1
/5)*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) - 20))^(3/4)*(sqrt(5) + 3))
) + 4*(1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(sqrt(5)*(9*sq
rt(5) - 20))^(3/4)*arctan(5*sqrt(1/5)*(1/125)^(1/4)*(sqrt(5)*(9*s
qrt(5) - 20))^(3/4)*(sqrt(5) + 3)/(2*sqrt(5)*sqrt(2)*sqrt(1/5)*sq
rt(sqrt(5)*(9*sqrt(5) - 20))*x + 2*sqrt(5)*sqrt(2)*sqrt(1/2)*sqrt
(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*sqrt((18*sqrt(5)*x^2 - 40*x^
2 + sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*(3*sqrt(5) - 5) - 5*
(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)*(sqrt(5)*(9*sqr
t(5) - 20))^(1/4))/(9*sqrt(5) - 20)) + 5*sqrt(1/5)*(1/125)^(1/4)*
(sqrt(5)*(9*sqrt(5) - 20))^(3/4)*(sqrt(5) + 3))) + 4*(1/125)^(1/4
)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*sqrt(sqrt(5)*(9*sqrt(5) - 20))
*arctan(5*sqrt(1/5)*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4
)*(sqrt(5) - 3)/(2*sqrt(5)*sqrt(2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt
(5) + 20))*x + 2*sqrt(5)*sqrt(2)*sqrt(1/2)*sqrt(1/5)*sqrt(sqrt(5)
*(9*sqrt(5) + 20))*sqrt((18*sqrt(5)*x^2 + 40*x^2 + sqrt(1/5)*sqrt
(sqrt(5)*(9*sqrt(5) + 20))*(3*sqrt(5) + 5) + 5*(1/125)^(1/4)*(3*s
qrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) + 20))^(1/4))
/(9*sqrt(5) + 20)) - 5*sqrt(1/5)*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5
) + 20))^(3/4)*(sqrt(5) - 3))) + 4*(1/125)^(1/4)*(sqrt(5)*(9*sqrt
(5) + 20))^(3/4)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*arctan(5*sqrt(1/5
)*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*(sqrt(5) - 3)/(2
*sqrt(5)*sqrt(2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*x + 2*s
qrt(5)*sqrt(2)*sqrt(1/2)*sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))
*sqrt((18*sqrt(5)*x^2 + 40*x^2 + sqrt(1/5)*sqrt(sqrt(5)*(9*sqrt(5
) + 20))*(3*sqrt(5) + 5) - 5*(1/125)^(1/4)*(3*sqrt(5)*sqrt(2)*x +
 7*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) + 20))^(1/4))/(9*sqrt(5) + 20))
 + 5*sqrt(1/5)*(1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*(sq
rt(5) - 3))) + (1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*sqr
t(sqrt(5)*(9*sqrt(5) - 20))*log(9*sqrt(5)*x^2 + 20*x^2 + 1/2*sqrt
```

```
(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(3*sqrt(5) + 5) + 5/2*(1/125
)^(1/4)*(3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) +
 20))^(1/4)) - (1/125)^(1/4)*(sqrt(5)*(9*sqrt(5) + 20))^(3/4)*sqr
t(sqrt(5)*(9*sqrt(5) - 20))*log(9*sqrt(5)*x^2 + 20*x^2 + 1/2*sqrt
(1/5)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(3*sqrt(5) + 5) - 5/2*(1/125
)^(1/4)*(3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) +
 20))^(1/4)) + (1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(sqrt
(5)*(9*sqrt(5) - 20))^(3/4)*log(9*sqrt(5)*x^2 - 20*x^2 + 1/2*sqrt
(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*(3*sqrt(5) - 5) + 5/2*(1/125
)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) -
 20))^(1/4)) - (1/125)^(1/4)*sqrt(sqrt(5)*(9*sqrt(5) + 20))*(sqrt
(5)*(9*sqrt(5) - 20))^(3/4)*log(9*sqrt(5)*x^2 - 20*x^2 + 1/2*sqrt
(1/5)*sqrt(sqrt(5)*(9*sqrt(5) - 20))*(3*sqrt(5) - 5) - 5/2*(1/125
)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)*(sqrt(5)*(9*sqrt(5) -
 20))^(1/4)))/(sqrt(9*sqrt(5) + 20)*sqrt(9*sqrt(5) - 20))
```

Sympy [A] time = 3.76545, size = 26, normalized size = 0.08

$$\operatorname{RootSum}\left(40960000t^8 + 115200t^4 + 1, \left(t \mapsto t\log\left(-9600t^5 - \frac{47t}{2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-960
0*_t**5 - 47*_t/2 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(1/(x^8 + 3*x^4 + 1), x)
```

3.383 $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=434

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{8\,2^{3/4}\sqrt{5}}$$

$$+\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)-\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)-$$

```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5
])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 +
(2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((6150 -
2750*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/
20 + ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqr
t[5])^(1/4)])/20 - ((3 + Sqrt[5])^(5/4)*Log[Sqrt[3 - Sqrt[5]] - 2
^(3/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(8*2^(3/4)*Sqrt[5])
+ ((3 + Sqrt[5])^(5/4)*Log[Sqrt[3 - Sqrt[5]] + 2^(3/4)*(3 - Sqrt[
5])^(1/4)*x + Sqrt[2]*x^2])/(8*2^(3/4)*Sqrt[5]) + ((6150 - 2750*S
qrt[5])^(1/4)*Log[Sqrt[3 + Sqrt[5]] - 2^(3/4)*(3 + Sqrt[5])^(1/4)
*x + Sqrt[2]*x^2])/40 - ((6150 - 2750*Sqrt[5])^(1/4)*Log[Sqrt[3 +
 Sqrt[5]] + 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/40
```

Rubi [A] time = 0.739844, antiderivative size = 416, normalized size of antiderivative = 0.96, number

of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{1}{x}+\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5
])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 +
(2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((123 - 5
5*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*
2^(3/4)*Sqrt[5]) + ((123 - 55*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*
x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/
4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^
2])/(8*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2*(3 - Sq
rt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5
]) + ((123 - 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(
3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((123 - 55*
Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rubi in Sympy [A] time = 95.785, size = 546, normalized size = 1.26

$$-\frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\left(-2\sqrt{5}+6\right)\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{16\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\left(-2\sqrt{5}+6\right)\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{16\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{2^{\frac{3}{4}}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\left(2\sqrt{5}+6\right)\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{16\left(\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{2^{\frac{3}{4}}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\left(2\sqrt{5}+6\right)\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{16\left(\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{4\sqrt[4]{-\sqrt{5}+3}}-\frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{4\sqrt[4]{-\sqrt{5}+3}}-\frac{2^{\frac{3}{4}}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{4\sqrt[4]{\sqrt{5}+3}}-\frac{2^{\frac{3}{4}}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{4\sqrt[4]{\sqrt{5}+3}}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**8+3*x**4+1),x)
```

```
[Out] -2**(3/4)*(1/2 + 3*sqrt(5)/10)*(-2*sqrt(5) + 6)*log(2*x**2 - 2*2*
*(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(16*(-sqrt
(5) + 3)**(5/4)) + 2**(3/4)*(1/2 + 3*sqrt(5)/10)*(-2*sqrt(5) + 6)
*log(2*x**2 + 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5
) + 6))/(16*(-sqrt(5) + 3)**(5/4)) - 2**(3/4)*(-3*sqrt(5)/10 + 1/
2)*(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(sqrt(5) + 3)**(1/4)
 + sqrt(2*sqrt(5) + 6))/(16*(sqrt(5) + 3)**(5/4)) + 2**(3/4)*(-3*
```

```
sqrt(5)/10 + 1/2)*(2*sqrt(5) + 6)*log(2*x**2 + 2*2**(1/4)*x*(sqrt
(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(16*(sqrt(5) + 3)**(5/4))
- 2**(3/4)*(1/2 + 3*sqrt(5)/10)*atan(2**(3/4)*(x - (-2*sqrt(5) +
6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(4*(-sqrt(5) + 3)**(1/4)) - 2
**(3/4)*(1/2 + 3*sqrt(5)/10)*atan(2**(3/4)*(x + (-2*sqrt(5) + 6)*
*(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(4*(-sqrt(5) + 3)**(1/4)) - 2**(
3/4)*(-3*sqrt(5)/10 + 1/2)*atan(2**(3/4)*(x - (2*sqrt(5) + 6)**(1
/4)/2)/(sqrt(5) + 3)**(1/4))/(4*(sqrt(5) + 3)**(1/4)) - 2**(3/4)*
(-3*sqrt(5)/10 + 1/2)*atan(2**(3/4)*(x + (2*sqrt(5) + 6)**(1/4)/2
)/(sqrt(5) + 3)**(1/4))/(4*(sqrt(5) + 3)**(1/4)) - 1/x
```

Mathematica [C] time = 0.0229389, size = 61, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^5+3\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x -
#1]*#1^4)/(3*#1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.014, size = 52, normalized size = 0.1

$$-\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(\_R^6+3\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^8+3*x^4+1),x)
```

```
[Out] -1/4*sum((_R^6+3*_R^2)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*
_Z^4+1))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\int\frac{x^6+3\,x^2}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.320964, size = 1902, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*sqrt(2)*(4*(1/250)^(1/4)*sqrt(sqrt(5)*(123*sqrt(5)
+ 275))*(sqrt(5)*(123*sqrt(5) - 275))^(3/4)*x*arctan(1/2*sqrt(5)*
(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) - 275))^(1/4)*(123*sqrt(5) -
275)*(21*sqrt(5) + 47)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)
*(123*sqrt(5) - 275))*x + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(
5)*(123*sqrt(5) - 275))*sqrt((sqrt(1/10)*(1/250)^(1/4)*(3*sqrt(5)
*sqrt(2)*x - 5*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5) - 275))^(3/4) + 5
5*sqrt(5)*x^2 - 123*x^2 + sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) -
275))*(3*sqrt(5) - 7))/(55*sqrt(5) - 123)) + 5*(1/250)^(1/4)*(sqr
t(5)*(123*sqrt(5) - 275))^(1/4)*(sqrt(5) - 3))) + 4*(1/250)^(1/4)
*sqrt(sqrt(5)*(123*sqrt(5) + 275))*(sqrt(5)*(123*sqrt(5) - 275))^
(3/4)*x*arctan(1/2*sqrt(5)*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) -
275))^(1/4)*(123*sqrt(5) - 275)*(21*sqrt(5) + 47)/(2*sqrt(5)*sqrt
(2)*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*x + 2*sqrt(5)*sq
rt(2)*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*sqrt(-(sqrt(1/
10)*(1/250)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(sqrt(5)*(1
23*sqrt(5) - 275))^(3/4) - 55*sqrt(5)*x^2 + 123*x^2 - sqrt(1/10)*
sqrt(sqrt(5)*(123*sqrt(5) - 275))*(3*sqrt(5) - 7))/(55*sqrt(5) -
123)) - 5*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) - 275))^(1/4)*(sqrt
(5) - 3))) + 4*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(3/4)*
sqrt(sqrt(5)*(123*sqrt(5) - 275))*x*arctan(1/2*sqrt(5)*(1/250)^(1
/4)*(sqrt(5)*(123*sqrt(5) + 275))^(1/4)*(123*sqrt(5) + 275)*(21*s
qrt(5) - 47)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt
(5) + 275))*x + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(123*sq
rt(5) + 275))*sqrt((sqrt(1/10)*(1/250)^(1/4)*(3*sqrt(5)*sqrt(2)*x
 + 5*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5) + 275))^(3/4) + 55*sqrt(5)*
x^2 + 123*x^2 + sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*(3*s
qrt(5) + 7))/(55*sqrt(5) + 123)) + 5*(1/250)^(1/4)*(sqrt(5)*(123*
sqrt(5) + 275))^(1/4)*(sqrt(5) + 3))) + 4*(1/250)^(1/4)*(sqrt(5)*
(123*sqrt(5) + 275))^(3/4)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*x*ar
ctan(1/2*sqrt(5)*(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(1/4
)*(123*sqrt(5) + 275)*(21*sqrt(5) - 47)/(2*sqrt(5)*sqrt(2)*sqrt(1
/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*x + 2*sqrt(5)*sqrt(2)*sqrt
(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*sqrt(-(sqrt(1/10)*(1/250
)^(1/4)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5)
 + 275))^(3/4) - 55*sqrt(5)*x^2 - 123*x^2 - sqrt(1/10)*sqrt(sqrt(
5)*(123*sqrt(5) + 275))*(3*sqrt(5) + 7))/(55*sqrt(5) + 123)) - 5*
```

```
(1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(1/4)*(sqrt(5) + 3)))
 - (1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275))^(3/4)*sqrt(sqrt(5)
*(123*sqrt(5) - 275))*x*log(sqrt(1/10)*(1/250)^(1/4)*(3*sqrt(5)*s
qrt(2)*x + 5*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5) + 275))^(3/4) + 55*
sqrt(5)*x^2 + 123*x^2 + sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) + 27
5))*(3*sqrt(5) + 7)) + (1/250)^(1/4)*(sqrt(5)*(123*sqrt(5) + 275)
)^(3/4)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*x*log(-sqrt(1/10)*(1/25
0)^(1/4)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5
) + 275))^(3/4) + 55*sqrt(5)*x^2 + 123*x^2 + sqrt(1/10)*sqrt(sqrt
(5)*(123*sqrt(5) + 275))*(3*sqrt(5) + 7)) - (1/250)^(1/4)*sqrt(sq
rt(5)*(123*sqrt(5) + 275))*(sqrt(5)*(123*sqrt(5) - 275))^(3/4)*x*
log(sqrt(1/10)*(1/250)^(1/4)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*
(sqrt(5)*(123*sqrt(5) - 275))^(3/4) + 55*sqrt(5)*x^2 - 123*x^2 +
sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) - 275))*(3*sqrt(5) - 7)) + (
1/250)^(1/4)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*(sqrt(5)*(123*sqrt
(5) - 275))^(3/4)*x*log(-sqrt(1/10)*(1/250)^(1/4)*(3*sqrt(5)*sqrt
(2)*x - 5*sqrt(2)*x)*(sqrt(5)*(123*sqrt(5) - 275))^(3/4) + 55*sqr
t(5)*x^2 - 123*x^2 + sqrt(1/10)*sqrt(sqrt(5)*(123*sqrt(5) - 275))
*(3*sqrt(5) - 7)) + 4*sqrt(2)*sqrt(sqrt(5)*(123*sqrt(5) + 275))*s
qrt(sqrt(5)*(123*sqrt(5) - 275)))/(x*sqrt(123*sqrt(5) + 275)*sqrt
(123*sqrt(5) - 275))
```

Sympy [A] time = 3.95179, size = 32, normalized size = 0.07

$$\operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{19251200t^7}{11} + \frac{369792t^3}{11} + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(1925
1200*_t**7/11 + 369792*_t**3/11 + x))) - 1/x
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^8 + 3\,x^4 + 1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + 3*x^4 + 1)*x^2), x)
```

3.384 $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=484

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843+377\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/(3*x^3) + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3
 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1
/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[
5]) - ((843 - 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt
[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt[5])^(1/4)*Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + (
(843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] - 2^(3/4)*(3 - Sq
rt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*
Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] + 2^(3/4)*(3 - Sqrt[5])^(1/4
)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1
/4)*Log[Sqrt[3 + Sqrt[5]] - 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[
2]*x^2])/(4*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt[5])^(1/4)*Log[Sqr
t[3 + Sqrt[5]] + 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4
*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.808392, antiderivative size = 466, normalized size of antiderivative = 0.96, number

of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843+377\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3
 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1
/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[
5]) - ((843 - 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt
[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt[5])^(1/4)*Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + (
(843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - S
qrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt
[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*
x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/4)*Log[
Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqr
t[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
)
```

Rubi in Sympy [A] time = 93.5898, size = 610, normalized size = 1.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**8+3*x**4+1),x)
```

```
[Out] 2**(3/4)*(3/2 + 9*sqrt(5)/10)*sqrt(-2*sqrt(5) + 6)*log(2*x**2 - 2
*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(24*(-s
qrt(5) + 3)**(5/4)) - 2**(3/4)*(3/2 + 9*sqrt(5)/10)*sqrt(-2*sqrt(
5) + 6)*log(2*x**2 + 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2
*sqrt(5) + 6))/(24*(-sqrt(5) + 3)**(5/4)) + 2**(3/4)*(-9*sqrt(5)/
10 + 3/2)*sqrt(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(sqrt(5)
+ 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(24*(sqrt(5) + 3)**(5/4)) - 2*
*(3/4)*(-9*sqrt(5)/10 + 3/2)*sqrt(2*sqrt(5) + 6)*log(2*x**2 + 2*2
**(1/4)*x*(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(24*(sqrt(5
) + 3)**(5/4)) - 2**(3/4)*(3/2 + 9*sqrt(5)/10)*atan(2**(3/4)*(x -
 (-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(6*sqrt(-2*sqr
t(5) + 6)*(-sqrt(5) + 3)**(1/4)) - 2**(3/4)*(3/2 + 9*sqrt(5)/10)*
atan(2**(3/4)*(x + (-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/
4))/(6*sqrt(-2*sqrt(5) + 6)*(-sqrt(5) + 3)**(1/4)) - 2**(3/4)*(-9
*sqrt(5)/10 + 3/2)*atan(2**(3/4)*(x - (2*sqrt(5) + 6)**(1/4)/2)/(
sqrt(5) + 3)**(1/4))/(6*(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6))
 - 2**(3/4)*(-9*sqrt(5)/10 + 3/2)*atan(2**(3/4)*(x + (2*sqrt(5) +
 6)**(1/4)/2)/(sqrt(5) + 3)**(1/4))/(6*(sqrt(5) + 3)**(1/4)*sqrt(
2*sqrt(5) + 6)) - 1/(3*x**3)
```

Mathematica [C] time = 0.0235776, size = 65, normalized size = 0.13

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x
 - #1]*#1^4)/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.014, size = 50, normalized size = 0.1

$$-\frac{1}{3\,x^3}+\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8+3\,\_Z^4+1\right)}\frac{\left(-\_R^4-3\right)\,\ln\left(x-\_R\right)}{2\,\_R^7+3\,\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^8+3*x^4+1),x)
```

```
[Out] -1/3/x^3+1/4*sum((-_R^4-3)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^
8+3*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3\,x^3} - \int \frac{x^4+3}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.328997, size = 1897, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] -1/120*sqrt(5)*sqrt(2)*(12*(1/250)^(1/4)*sqrt(sqrt(5)*(843*sqrt(5
) + 1885))*(sqrt(5)*(843*sqrt(5) - 1885))^(3/4)*x^3*arctan(5*sqrt
(1/10)*(1/250)^(1/4)*(sqrt(5)*(843*sqrt(5) - 1885))^(3/4)*(3*sqrt
(5) + 7)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5)
- 1885))*x + 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(843*sqrt(5) - 1
885))^(3/4)*(3*sqrt(5) + 7) + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(s
qrt(5)*(843*sqrt(5) - 1885))*sqrt((843*sqrt(5)*x^2 - 1885*x^2 + 2
*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5) - 1885))*(9*sqrt(5) - 20) -
 5*(1/250)^(1/4)*(55*sqrt(5)*sqrt(2)*x - 123*sqrt(2)*x)*(sqrt(5)*
(843*sqrt(5) - 1885))^(1/4))/(843*sqrt(5) - 1885)))) + 12*(1/250)
^(1/4)*sqrt(sqrt(5)*(843*sqrt(5) + 1885))*(sqrt(5)*(843*sqrt(5) -
 1885))^(3/4)*x^3*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(843
*sqrt(5) - 1885))^(3/4)*(3*sqrt(5) + 7)/(2*sqrt(5)*sqrt(2)*sqrt(1
/10)*sqrt(sqrt(5)*(843*sqrt(5) - 1885))*x - 5*sqrt(1/10)*(1/250)^
(1/4)*(sqrt(5)*(843*sqrt(5) - 1885))^(3/4)*(3*sqrt(5) + 7) + 2*sq
rt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5) - 1885))*sqrt(
(843*sqrt(5)*x^2 - 1885*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt
(5) - 1885))*(9*sqrt(5) - 20) + 5*(1/250)^(1/4)*(55*sqrt(5)*sqrt(
```

```
2)*x - 123*sqrt(2)*x)*(sqrt(5)*(843*sqrt(5) - 1885))^(1/4))/(843*
sqrt(5) - 1885)))) + 12*(1/250)^(1/4)*(sqrt(5)*(843*sqrt(5) + 188
5))^(3/4)*sqrt(sqrt(5)*(843*sqrt(5) - 1885))*x^3*arctan(5*sqrt(1/
10)*(1/250)^(1/4)*(sqrt(5)*(843*sqrt(5) + 1885))^(3/4)*(3*sqrt(5)
 - 7)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5) + 1
885))*x + 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(843*sqrt(5) + 1885
))^(3/4)*(3*sqrt(5) - 7) + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt
(5)*(843*sqrt(5) + 1885))*sqrt((843*sqrt(5)*x^2 + 1885*x^2 + 2*sq
rt(1/10)*sqrt(sqrt(5)*(843*sqrt(5) + 1885))*(9*sqrt(5) + 20) - 5*
(1/250)^(1/4)*(55*sqrt(5)*sqrt(2)*x + 123*sqrt(2)*x)*(sqrt(5)*(84
3*sqrt(5) + 1885))^(1/4))/(843*sqrt(5) + 1885)))) + 12*(1/250)^(1
/4)*(sqrt(5)*(843*sqrt(5) + 1885))^(3/4)*sqrt(sqrt(5)*(843*sqrt(5
) - 1885))*x^3*arctan(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(843*sq
rt(5) + 1885))^(3/4)*(3*sqrt(5) - 7)/(2*sqrt(5)*sqrt(2)*sqrt(1/10
)*sqrt(sqrt(5)*(843*sqrt(5) + 1885))*x - 5*sqrt(1/10)*(1/250)^(1/
4)*(sqrt(5)*(843*sqrt(5) + 1885))^(3/4)*(3*sqrt(5) - 7) + 2*sqrt(
5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5) + 1885))*sqrt((84
3*sqrt(5)*x^2 + 1885*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5)
 + 1885))*(9*sqrt(5) + 20) + 5*(1/250)^(1/4)*(55*sqrt(5)*sqrt(2)*
x + 123*sqrt(2)*x)*(sqrt(5)*(843*sqrt(5) + 1885))^(1/4))/(843*sqr
t(5) + 1885)))) + 3*(1/250)^(1/4)*(sqrt(5)*(843*sqrt(5) + 1885))^
(3/4)*sqrt(sqrt(5)*(843*sqrt(5) - 1885))*x^3*log(843*sqrt(5)*x^2
+ 1885*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5) + 1885))*(9*s
qrt(5) + 20) + 5*(1/250)^(1/4)*(55*sqrt(5)*sqrt(2)*x + 123*sqrt(2
)*x)*(sqrt(5)*(843*sqrt(5) + 1885))^(1/4)) - 3*(1/250)^(1/4)*(sqr
t(5)*(843*sqrt(5) + 1885))^(3/4)*sqrt(sqrt(5)*(843*sqrt(5) - 1885
))*x^3*log(843*sqrt(5)*x^2 + 1885*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)
*(843*sqrt(5) + 1885))*(9*sqrt(5) + 20) - 5*(1/250)^(1/4)*(55*sqr
t(5)*sqrt(2)*x + 123*sqrt(2)*x)*(sqrt(5)*(843*sqrt(5) + 1885))^(1
/4)) + 3*(1/250)^(1/4)*sqrt(sqrt(5)*(843*sqrt(5) + 1885))*(sqrt(5
)*(843*sqrt(5) - 1885))^(3/4)*x^3*log(843*sqrt(5)*x^2 - 1885*x^2
+ 2*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(5) - 1885))*(9*sqrt(5) - 20
) + 5*(1/250)^(1/4)*(55*sqrt(5)*sqrt(2)*x - 123*sqrt(2)*x)*(sqrt(
5)*(843*sqrt(5) - 1885))^(1/4)) - 3*(1/250)^(1/4)*sqrt(sqrt(5)*(8
43*sqrt(5) + 1885))*(sqrt(5)*(843*sqrt(5) - 1885))^(3/4)*x^3*log(
843*sqrt(5)*x^2 - 1885*x^2 + 2*sqrt(1/10)*sqrt(sqrt(5)*(843*sqrt(
5) - 1885))*(9*sqrt(5) - 20) - 5*(1/250)^(1/4)*(55*sqrt(5)*sqrt(2
)*x - 123*sqrt(2)*x)*(sqrt(5)*(843*sqrt(5) - 1885))^(1/4)) + 4*sq
rt(2)*sqrt(sqrt(5)*(843*sqrt(5) + 1885))*sqrt(sqrt(5)*(843*sqrt(5
) - 1885)))/(x^3*sqrt(843*sqrt(5) + 1885)*sqrt(843*sqrt(5) - 1885
))
```

Sympy [A] time = 4.0804, size = 34, normalized size = 0.07

$$\operatorname{RootSum}\left(40960000t^8 + 5395200t^4 + 1, \left(t \mapsto t\log\left(\frac{179200t^5}{377} + \frac{23112t}{377} + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179
200*_t**5/377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^8 + 3x^4 + 1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 + 3*x^4 + 1)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 + 3*x^4 + 1)*x^4), x)
```

3.385 $\int \frac{x^m}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(
3 - Sqrt[5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(
Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rubi [A] time = 0.128404, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(
3 - Sqrt[5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(
Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rubi in Sympy [A] time = 14.9538, size = 105, normalized size = 0.9

$$-\frac{\sqrt{5}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-\frac{x^4}{-\frac{3}{2}-\frac{\sqrt{5}}{2}}\right)}{5\left(\frac{\sqrt{5}}{2}+\frac{3}{2}\right)(m+1)}+\frac{\sqrt{5}x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-\frac{x^4}{-\frac{3}{2}+\frac{\sqrt{5}}{2}}\right)}{5\left(-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(x**8-3*x**4+1),x)
```

```
[Out] -sqrt(5)*x**(m + 1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), -x**4/(-3
/2 - sqrt(5)/2))/(5*(sqrt(5)/2 + 3/2)*(m + 1)) + sqrt(5)*x**(m +
1)*hyper((1, m/4 + 1/4), (m/4 + 5/4,), -x**4/(-3/2 + sqrt(5)/2))/
(5*(-sqrt(5)/2 + 3/2)*(m + 1))
```

Mathematica [C] time = 1.0223, size = 575, normalized size = 4.91

$$x^m\left(\frac{\text{RootSum}\left[\#1^4-\#1^2-1\&,\frac{\#1^2m^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+3\#1^2m\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+2\#1^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#}{x-}\right.}{2\#1^3-\#1}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (x^m*(-RootSum[-1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 -
 m, -(#1/(x - #1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] + (RootSu
m[-1 - #1^2 + #1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (
2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x -
#1))^m + (3*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1
^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(
x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2*#1^3
) & ] - (2 + 3*m + m^2)*RootSum[-1 + #1^2 + #1^4 & , Hypergeometr
ic2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(#1 + 2*#1^3
)) & ] - RootSum[-1 + #1^2 + #1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1
 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]
*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m, 1 - m, -(#
1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m
, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m
)/(#1 + 2*#1^3) & ])/(2 + 3*m + m^2)))/(4*m)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(x^8-3*x^4+1),x)
```

```
[Out] int(x^m/(x^8-3*x^4+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - 3x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - 3*x^4 + 1),x, algorithm="maxima")

[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{x^8 - 3x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - 3*x^4 + 1),x, algorithm="fricas")

[Out] integral(x^m/(x^8 - 3*x^4 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(x**8-3*x**4+1),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{x^8 - 3x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/(x^8 - 3*x^4 + 1),x, algorithm="giac")

[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

3.386 $\int \frac{x^{11}}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7
*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi [A] time = 0.106418, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7
*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi in Sympy [A] time = 12.2276, size = 68, normalized size = 1.1

$$\frac{x^4}{4}-\frac{\sqrt{5}\left(-\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(-2x^4-\sqrt{5}+3\right)}{20}+\frac{\sqrt{5}\left(\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(-2x^4+\sqrt{5}+3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(x**8-3*x**4+1),x)
```

```
[Out] x**4/4 - sqrt(5)*(-3*sqrt(5)/2 + 7/2)*log(-2*x**4 - sqrt(5) + 3)/
20 + sqrt(5)*(3*sqrt(5)/2 + 7/2)*log(-2*x**4 + sqrt(5) + 3)/20
```

Mathematica [A] time = 0.0578139, size = 56, normalized size = 0.9

$$\frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqr
t[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.005, size = 38, normalized size = 0.6

$$\frac{x^4}{4} + \frac{3\ln\left(x^8 - 3x^4 + 1\right)}{8} - \frac{7\sqrt{5}}{20}\mathit{Artanh}\left(\frac{\left(2x^4 - 3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(x^8-3*x^4+1),x)
```

```
[Out] 1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^
(1/2))
```

Maxima [A] time = 0.82364, size = 68, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3}\right) + \frac{3}{8}\log\left(x^8 - 3x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5)
 - 3)) + 3/8*log(x^8 - 3*x^4 + 1)
```

Fricas [A] time = 0.295006, size = 96, normalized size = 1.55

$$\frac{1}{40}\sqrt{5}\left(2\sqrt{5}x^4 + 3\sqrt{5}\log\left(x^8 - 3x^4 + 1\right) + 7\log\left(-\frac{10x^4 - \sqrt{5}\left(2x^8 - 6x^4 + 7\right) - 15}{x^8 - 3x^4 + 1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*(2*sqrt(5)*x^4 + 3*sqrt(5)*log(x^8 - 3*x^4 + 1) + 7*
log(-(10*x^4 - sqrt(5)*(2*x^8 - 6*x^4 + 7) - 15)/(x^8 - 3*x^4 + 1
)))
```

Sympy [A] time = 0.348346, size = 58, normalized size = 0.94

$$\frac{x^4}{4}+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(x**8-3*x**4+1),x)
```

```
[Out] x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*s
qrt(5)/40 + 3/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

GIAC/XCAS [A] time = 0.298896, size = 72, normalized size = 1.16

$$\frac{1}{4}x^4+\frac{7}{40}\sqrt{5}\ln\left(\frac{|2x^4-\sqrt{5}-3|}{|2x^4+\sqrt{5}-3|}\right)+\frac{3}{8}\ln\left(|x^8-3x^4+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*ln(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sq
rt(5) - 3)) + 3/8*ln(abs(x^8 - 3*x^4 + 1))
```

3.387 $\int \frac{x^9}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^
2])/2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^
2])/2
```

Rubi [A] time = 0.245514, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^
2])/2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^
2])/2
```

Rubi in Sympy [A] time = 21.8075, size = 102, normalized size = 1.13

$$\frac{x^2}{2}-\frac{\sqrt{2}\left(-\frac{7\sqrt{5}}{10}+\frac{3}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{2}\left(\frac{3}{2}+\frac{7\sqrt{5}}{10}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**8-3*x**4+1),x)
```

```
[Out] x**2/2 - sqrt(2)*(-7*sqrt(5)/10 + 3/2)*atanh(sqrt(2)*x**2/sqrt(-s
qrt(5) + 3))/(2*sqrt(-sqrt(5) + 3)) - sqrt(2)*(3/2 + 7*sqrt(5)/10
)*atanh(sqrt(2)*x**2/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5) + 3))
```

Mathematica [A] time = 0.0883431, size = 103, normalized size = 1.14

$$\frac{1}{20}\left(10x^2+\left(2\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)\right.$$
$$\left.+\left(5-2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)-\left(5+2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqr
t[5])*Log[1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5]
 + 2*x^2] - (5 + 2*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A] time = 0.008, size = 67, normalized size = 0.7

$$\frac{x^2}{2}+\frac{\ln\left(x^4-x^2-1\right)}{4}-\frac{\sqrt{5}}{5}Artanh\left(\frac{\left(2\,x^2-1\right)\sqrt{5}}{5}\right)-\frac{\ln\left(x^4+x^2-1\right)}{4}-\frac{\sqrt{5}}{5}Artanh\left(\frac{\left(2\,x^2+1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^8-3*x^4+1),x)
```

```
[Out] 1/2*x^2+1/4*ln(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/
2))-1/4*ln(x^4+x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Maxima [A] time = 0.820497, size = 124, normalized size = 1.38

$$\frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{4}\,\log\left(x^4+x^2-1\right)+\frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5)
 + 1)) + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5)
- 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Fricas [A] time = 0.270642, size = 166, normalized size = 1.84

$$\frac{1}{20}\sqrt{5}\left(2\sqrt{5}x^2-\sqrt{5}\log\left(x^4+x^2-1\right)+\sqrt{5}\log\left(x^4-x^2-1\right)+2\log\left(-\frac{10x^2-\sqrt{5}\left(2x^4+2x^2+3\right)+5}{x^4+x^2-1}\right)+2\log\left(-\frac{10x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*(2*sqrt(5)*x^2 - sqrt(5)*log(x^4 + x^2 - 1) + sqrt(5
)*log(x^4 - x^2 - 1) + 2*log(-(10*x^2 - sqrt(5)*(2*x^4 + 2*x^2 +
3) + 5)/(x^4 + x^2 - 1)) + 2*log(-(10*x^2 - sqrt(5)*(2*x^4 - 2*x^
2 + 3) - 5)/(x^4 - x^2 - 1)))
```

Sympy [A] time = 1.75372, size = 170, normalized size = 1.89

$$\frac{x^2}{2}+\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-\frac{47\sqrt{5}}{20}-120\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3\right)$$
$$+\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-120\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{47\sqrt{5}}{20}\right)$$
$$+\left(-\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{47\sqrt{5}}{20}-120\left(-\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3+\frac{47}{8}\right)$$
$$+\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-120\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3+\frac{47\sqrt{5}}{20}+\frac{47}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**8-3*x**4+1),x)
```

```
[Out] x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 12
0*(-1/4 - sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 -
 120*(-1/4 + sqrt(5)/10)**3 + 47*sqrt(5)/20) + (-sqrt(5)/10 + 1/4
)*log(x**2 - 47*sqrt(5)/20 - 120*(-sqrt(5)/10 + 1/4)**3 + 47/8) +
 (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sqrt(5)/10 + 1/4)**3 + 47*sqr
t(5)/20 + 47/8)
```

GIAC/XCAS [A] time = 0.31495, size = 131, normalized size = 1.46

$$\frac{1}{2}x^2+\frac{1}{10}\sqrt{5}\ln\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)+\frac{1}{10}\sqrt{5}\ln\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{4}\ln\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\ln\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*ln(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(
5) + 1)) + 1/10*sqrt(5)*ln(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + s
qrt(5) - 1)) - 1/4*ln(abs(x^4 + x^2 - 1)) + 1/4*ln(abs(x^4 - x^2
- 1))
```

3.388 $\int \frac{x^7}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi [A] time = 0.07598, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi in Sympy [A] time = 8.80411, size = 60, normalized size = 1.09

$$-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(-2x^4-\sqrt{5}+3\right)}{20}+\frac{\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(-2x^4+\sqrt{5}+3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**8-3*x**4+1),x)
```

```
[Out] -sqrt(5)*(-sqrt(5)/2 + 3/2)*log(-2*x**4 - sqrt(5) + 3)/20 + sqrt(
5)*(sqrt(5)/2 + 3/2)*log(-2*x**4 + sqrt(5) + 3)/20
```

Mathematica [A] time = 0.033476, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*
Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.003, size = 33, normalized size = 0.6

$$\frac{\ln\left(x^8-3\,x^4+1\right)}{8}-\frac{3\,\sqrt{5}}{20}Artanh\left(\frac{\left(2\,x^4-3\right)\,\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^8-3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A] time = 0.820352, size = 61, normalized size = 1.11

$$\frac{3}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right)+\frac{1}{8}\,\log\left(x^8-3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1
/8*log(x^8 - 3*x^4 + 1)
```

Fricas [A] time = 0.281777, size = 84, normalized size = 1.53

$$\frac{1}{40}\,\sqrt{5}\left(\sqrt{5}\log\left(x^8-3\,x^4+1\right)+3\,\log\left(-\frac{10\,x^4-\sqrt{5}\left(2\,x^8-6\,x^4+7\right)-15}{x^8-3\,x^4+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*(sqrt(5)*log(x^8 - 3*x^4 + 1) + 3*log(-(10*x^4 - sqr
t(5)*(2*x^8 - 6*x^4 + 7) - 15)/(x^8 - 3*x^4 + 1)))
```

Sympy [A] time = 0.31384, size = 53, normalized size = 0.96

$$\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**8-3*x**4+1),x)
```

```
[Out] (1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/40
 + 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

GIAC/XCAS [A] time = 0.291244, size = 65, normalized size = 1.18

$$\frac{3}{40}\sqrt{5}\ln\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{8}\ln\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*ln(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)
) + 1/8*ln(abs(x^8 - 3*x^4 + 1))
```

3.389 $\int \frac{x^5}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

```
[Out] -(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 +
(Sqrt[(3 - Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.147157, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 +
(Sqrt[(3 - Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi in Sympy [A] time = 15.4152, size = 99, normalized size = 1.22

$$-\frac{\sqrt{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**8-3*x**4+1),x)
```

```
[Out] -sqrt(2)*(-3*sqrt(5)/10 + 1/2)*atanh(sqrt(2)*x**2/sqrt(-sqrt(5) +
 3))/(2*sqrt(-sqrt(5) + 3)) - sqrt(2)*(1/2 + 3*sqrt(5)/10)*atanh(
sqrt(2)*x**2/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5) + 3))
```

Mathematica [A] time = 0.0504037, size = 91, normalized size = 1.12

$$\frac{1}{40}\left(\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}-1\right)-\left(5+\sqrt{5}\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5
 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/40
```

Maple [A] time = 0.005, size = 62, normalized size = 0.8

$$\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\sqrt{5}}{20}Artanh\left(\frac{\left(2\,x^2-1\right)\sqrt{5}}{5}\right)-\frac{\ln\left(x^4+x^2-1\right)}{8}-\frac{\sqrt{5}}{20}Artanh\left(\frac{\left(2\,x^2+1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^8-3*x^4+1),x)
```

```
[Out] 1/8*ln(x^4-x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8
*ln(x^4+x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Maxima [A] time = 0.826103, size = 117, normalized size = 1.44

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1
/40*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/
8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Fricas [A] time = 0.31288, size = 155, normalized size = 1.91

$$-\frac{1}{40}\sqrt{5}\left(\sqrt{5}\log\left(x^4+x^2-1\right)-\sqrt{5}\log\left(x^4-x^2-1\right)-\log\left(-\frac{10\,x^2-\sqrt{5}\left(2\,x^4+2\,x^2+3\right)+5}{x^4+x^2-1}\right)-\log\left(-\frac{10\,x^2-\sqrt{5}\left(2\,x^4-\right.}{x^4-x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*(sqrt(5)*log(x^4 + x^2 - 1) - sqrt(5)*log(x^4 - x^2
 - 1) - log(-(10*x^2 - sqrt(5)*(2*x^4 + 2*x^2 + 3) + 5)/(x^4 + x^
2 - 1)) - log(-(10*x^2 - sqrt(5)*(2*x^4 - 2*x^2 + 3) - 5)/(x^4 -
x^2 - 1)))
```

Sympy [A] time = 1.73989, size = 165, normalized size = 2.04

$$\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right)+\left(-\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{3\sqrt{5}}{10}-640\left(-\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{3}{2}\right)+\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-640\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{3\sqrt{5}}{10}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**8-3*x**4+1),x)
```

```
[Out] (-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - s
qrt(5)/40)**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 +
 sqrt(5)/40)**3 + 3*sqrt(5)/10) + (-sqrt(5)/40 + 1/8)*log(x**2 -
3*sqrt(5)/10 - 640*(-sqrt(5)/40 + 1/8)**3 + 3/2) + (sqrt(5)/40 +
1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)**3 + 3*sqrt(5)/10 + 3/2)
```

GIAC/XCAS [A] time = 0.294872, size = 124, normalized size = 1.53

$$\frac{1}{40}\sqrt{5}\ln\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\ln\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\ln\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\ln\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(5)*ln(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) +
 1/40*sqrt(5)*ln(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1
)) - 1/8*ln(abs(x^4 + x^2 - 1)) + 1/8*ln(abs(x^4 - x^2 - 1))
```

3.390 $\int \frac{x^3}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rubi [A] time = 0.0561634, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 - 3*x^4 + x^8),x]

[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 5.98414, size = 24, normalized size = 1.04

$$-\frac{\sqrt{5}\,\text{atanh}\left(\sqrt{5}\left(\frac{2x^4}{5}-\frac{3}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**8-3*x**4+1),x)

[Out] -sqrt(5)*atanh(sqrt(5)*(2*x**4/5 - 3/5))/10
```

Mathematica [A] time = 0.0180893, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}+3\right)-\log\left(2x^4+\sqrt{5}-3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5]
)
```

Maple [A] time = 0.002, size = 19, normalized size = 0.8

$$-\frac{\sqrt{5}}{10}Artanh\left(\frac{\left(2x^{4}-3\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^8-3*x^4+1),x)
```

```
[Out] -1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A] time = 0.834201, size = 42, normalized size = 1.83

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^{4}-\sqrt{5}-3}{2x^{4}+\sqrt{5}-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3))
```

Fricas [A] time = 0.275691, size = 59, normalized size = 2.57

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{10x^{4}-\sqrt{5}\left(2x^{8}-6x^{4}+7\right)-15}{x^{8}-3x^{4}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log(-(10*x^4 - sqrt(5)*(2*x^8 - 6*x^4 + 7) - 15)/(x^
8 - 3*x^4 + 1))
```

Sympy [A] time = 0.26458, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 +
 sqrt(5)/2)/20
```

GIAC/XCAS [A] time = 0.291253, size = 45, normalized size = 1.96

$$\frac{1}{20}\,\sqrt{5}\ln\left(\frac{\left|2\,x^4-\sqrt{5}-3\right|}{\left|2\,x^4+\sqrt{5}-3\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*ln(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)
)
```

3.391 $\int \frac{x}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (S
qrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.0846217, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (S
qrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi in Sympy [A] time = 8.71116, size = 73, normalized size = 0.97

$$\frac{\sqrt{10}\,\text{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{10\sqrt{-\sqrt{5}+3}}-\frac{\sqrt{10}\,\text{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{10\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(10)*atanh(sqrt(2)*x**2/sqrt(-sqrt(5) + 3))/(10*sqrt(-sqrt(5)
 + 3)) - sqrt(10)*atanh(sqrt(2)*x**2/sqrt(sqrt(5) + 3))/(10*sqrt(
sqrt(5) + 3))
```

Mathematica [A] time = 0.0536746, size = 91, normalized size = 1.21

$$\frac{1}{40}\left(-\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)-\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5+\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+\left(\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] +
(-5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/40
```

Maple [A] time = 0.005, size = 62, normalized size = 0.8

$$\frac{\ln\left(x^4-x^2-1\right)}{8}+\frac{\sqrt{5}}{20}Artanh\left(\frac{\left(2\,x^2-1\right)\sqrt{5}}{5}\right)-\frac{\ln\left(x^4+x^2-1\right)}{8}+\frac{\sqrt{5}}{20}Artanh\left(\frac{\left(2\,x^2+1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^8-3*x^4+1),x)
```

```
[Out] 1/8*ln(x^4-x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8
*ln(x^4+x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Maxima [A] time = 0.822112, size = 117, normalized size = 1.56

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) -
1/40*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1
/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Fricas [A] time = 0.292682, size = 150, normalized size = 2.

$$-\frac{1}{40}\sqrt{5}\left(\sqrt{5}\log\left(x^4+x^2-1\right)-\sqrt{5}\log\left(x^4-x^2-1\right)-\log\left(\frac{10\,x^2+\sqrt{5}\left(2\,x^4+2\,x^2+3\right)+5}{x^4+x^2-1}\right)-\log\left(\frac{10\,x^2+\sqrt{5}\left(2\,x^4-2\,x\right.}{x^4-x^2-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*(sqrt(5)*log(x^4 + x^2 - 1) - sqrt(5)*log(x^4 - x^2
 - 1) - log((10*x^2 + sqrt(5)*(2*x^4 + 2*x^2 + 3) + 5)/(x^4 + x^2
 - 1)) - log((10*x^2 + sqrt(5)*(2*x^4 - 2*x^2 + 3) - 5)/(x^4 - x^
2 - 1)))
```

Sympy [A] time = 1.90747, size = 165, normalized size = 2.2

$$\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right)+\left(-\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}+960\left(-\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7\sqrt{5}}{10}+960\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{7}{2}\right)+\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2+960\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/4
0 + 1/8)**3) + (-sqrt(5)/40 + 1/8)*log(x**2 - 7/2 + 960*(-sqrt(5)
/40 + 1/8)**3 + 7*sqrt(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*
sqrt(5)/10 + 960*(-1/8 + sqrt(5)/40)**3 + 7/2) + (-1/8 - sqrt(5)/
40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)**3 + 7*sqrt(5)/10 + 7/2)
```

GIAC/XCAS [A] time = 0.293638, size = 124, normalized size = 1.65

$$-\frac{1}{40}\sqrt{5}\ln\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\ln\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\ln\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\ln\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/40*sqrt(5)*ln(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
- 1/40*sqrt(5)*ln(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) -
1)) - 1/8*ln(abs(x^4 + x^2 - 1)) + 1/8*ln(abs(x^4 - x^2 - 1))
```

3.392 $\int \frac{1}{x(1-3x^4+x^8)} dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*
Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi [A] time = 0.0699153, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*
Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi in Sympy [A] time = 11.6048, size = 66, normalized size = 1.16

$$\frac{\log\left(x^4\right)}{4}-\frac{\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(-2x^4-\sqrt{5}+3\right)}{20}+\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\log\left(-2x^4+\sqrt{5}+3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**8-3*x**4+1),x)
```

```
[Out] log(x**4)/4 - sqrt(5)*(sqrt(5)/2 + 3/2)*log(-2*x**4 - sqrt(5) + 3
)/20 + sqrt(5)*(-sqrt(5)/2 + 3/2)*log(-2*x**4 + sqrt(5) + 3)/20
```

Mathematica [A] time = 0.047609, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 -
3*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.015, size = 64, normalized size = 1.1

$$\ln(x) - \frac{\ln(x^4 + x^2 - 1)}{8} + \frac{3\sqrt{5}}{20} \mathit{Artanh}\left(\frac{(2x^2 + 1)\sqrt{5}}{5}\right) - \frac{\ln(x^4 - x^2 - 1)}{8} - \frac{3\sqrt{5}}{20} \mathit{Artanh}\left(\frac{(2x^2 - 1)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^8-3*x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^4+x^2-1)+3/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2
))-1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Maxima [A] time = 0.820638, size = 69, normalized size = 1.21

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3}\right) - \frac{1}{8}\log\left(x^8 - 3x^4 + 1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1
/8*log(x^8 - 3*x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.274614, size = 93, normalized size = 1.63

$$-\frac{1}{40}\sqrt{5}\left(\sqrt{5}\log\left(x^8 - 3x^4 + 1\right) - 8\sqrt{5}\log(x) - 3\log\left(-\frac{10x^4 - \sqrt{5}(2x^8 - 6x^4 + 7) - 15}{x^8 - 3x^4 + 1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*(sqrt(5)*log(x^8 - 3*x^4 + 1) - 8*sqrt(5)*log(x) -
3*log(-(10*x^4 - sqrt(5)*(2*x^8 - 6*x^4 + 7) - 15)/(x^8 - 3*x^4 +
 1)))
```

Sympy [A] time = 0.369575, size = 58, normalized size = 1.02

$$\log(x) + \left(-\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(-\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**8-3*x**4+1),x)
```

```
[Out] log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*
sqrt(5)/40 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

GIAC/XCAS [A] time = 0.292529, size = 73, normalized size = 1.28

$$\frac{3}{40}\sqrt{5}\ln\left(\frac{\left|2x^4 - \sqrt{5} - 3\right|}{\left|2x^4 + \sqrt{5} - 3\right|}\right) + \frac{1}{4}\ln\left(x^4\right) - \frac{1}{8}\ln\left(\left|x^8 - 3x^4 + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*ln(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)
) + 1/4*ln(x^4) - 1/8*ln(abs(x^8 - 3*x^4 + 1))
```

3.393 $\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/(2*x^2) - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5]
)]*x^2])/2 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x
^2])/(4*Sqrt[10])
```

Rubi [A] time = 0.156714, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5]
)]*x^2])/2 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x
^2])/(4*Sqrt[10])
```

Rubi in Sympy [A] time = 20.0375, size = 104, normalized size = 1.17

$$\frac{\sqrt{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{2\sqrt{-\sqrt{5}+3}}+\frac{\sqrt{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{2\sqrt{\sqrt{5}+3}}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(2)*(1/2 + 3*sqrt(5)/10)*atanh(sqrt(2)*x**2/sqrt(-sqrt(5) + 3
))/(2*sqrt(-sqrt(5) + 3)) + sqrt(2)*(-3*sqrt(5)/10 + 1/2)*atanh(s
```

```
qrt(2)*x**2/sqrt(sqrt(5) + 3))/(2*sqrt(sqrt(5) + 3)) - 1/(2*x**2)
```

Mathematica [A] time = 0.105769, size = 103, normalized size = 1.16

$$\frac{1}{20}\left(-\frac{10}{x^2}-\left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5-2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5+2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+\left(2\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqr
t[5])*Log[1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5]
 + 2*x^2] + (-5 + 2*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A] time = 0.013, size = 67, normalized size = 0.8

$$-\frac{1}{2\,x^2}-\frac{\ln\left(x^4+x^2-1\right)}{4}+\frac{\sqrt{5}}{5}Artanh\left(\frac{\left(2\,x^2+1\right)\sqrt{5}}{5}\right)+\frac{\ln\left(x^4-x^2-1\right)}{4}+\frac{\sqrt{5}}{5}Artanh\left(\frac{\left(2\,x^2-1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/x^2-1/4*ln(x^4+x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1
/2))+1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Maxima [A] time = 0.825652, size = 124, normalized size = 1.39

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{2\,x^2}-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) -
1/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1
/2/x^2 - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Fricas [A] time = 0.311054, size = 177, normalized size = 1.99

$$-\frac{\sqrt{5}\left(\sqrt{5}x^2\log\left(x^4+x^2-1\right)-\sqrt{5}x^2\log\left(x^4-x^2-1\right)-2\,x^2\log\left(\frac{10\,x^2+\sqrt{5}\left(2\,x^4+2\,x^2+3\right)+5}{x^4+x^2-1}\right)-2\,x^2\log\left(\frac{10\,x^2+\sqrt{5}\left(2\,x^4-2\,x^2+3\right)-5}{x^4-x^2-1}\right)\right.}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(5)*(sqrt(5)*x^2*log(x^4 + x^2 - 1) - sqrt(5)*x^2*log(x
^4 - x^2 - 1) - 2*x^2*log((10*x^2 + sqrt(5)*(2*x^4 + 2*x^2 + 3) +
 5)/(x^4 + x^2 - 1)) - 2*x^2*log((10*x^2 + sqrt(5)*(2*x^4 - 2*x^2
 + 3) - 5)/(x^4 - x^2 - 1)) + 2*sqrt(5))/x^2
```

Sympy [A] time = 1.85077, size = 172, normalized size = 1.93

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right)$$
$$+\left(-\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}+280\left(-\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3+\frac{123\sqrt{5}}{20}\right)$$
$$+\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123\sqrt{5}}{20}+280\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{123}{8}\right)$$
$$+\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2+280\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}+\frac{123}{8}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(
5)/10 + 1/4)**3) + (-sqrt(5)/10 + 1/4)*log(x**2 - 123/8 + 280*(-s
qrt(5)/10 + 1/4)**3 + 123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x
**2 - 123*sqrt(5)/20 + 280*(-1/4 + sqrt(5)/10)**3 + 123/8) + (-1/
4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 - sqrt(5)/10)**3 + 123*sqrt(
5)/20 + 123/8) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.296997, size = 131, normalized size = 1.47

$$-\frac{1}{10}\sqrt{5}\ln\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\ln\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{2x^2}-\frac{1}{4}\ln\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\ln\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^3),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*ln(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
- 1/10*sqrt(5)*ln(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) -
1)) - 1/2/x^2 - 1/4*ln(abs(x^4 + x^2 - 1)) + 1/4*ln(abs(x^4 - x^2
 - 1))
```

3.394 $\int \frac{1}{x^5(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

```
[Out] -1/(4*x^4) + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4
])/40 - ((15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi [A] time = 0.143601, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4
])/40 - ((15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rubi in Sympy [A] time = 16.7677, size = 78, normalized size = 1.18

$$\frac{3\log\left(x^4\right)}{4}-\frac{\sqrt{5}\left(\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(-2x^4-\sqrt{5}+3\right)}{20}+\frac{\sqrt{5}\left(-\frac{3\sqrt{5}}{2}+\frac{7}{2}\right)\log\left(-2x^4+\sqrt{5}+3\right)}{20}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(x**8-3*x**4+1),x)
```

```
[Out] 3*log(x**4)/4 - sqrt(5)*(3*sqrt(5)/2 + 7/2)*log(-2*x**4 - sqrt(5)
 + 3)/20 + sqrt(5)*(-3*sqrt(5)/2 + 7/2)*log(-2*x**4 + sqrt(5) + 3
)/20 - 1/(4*x**4)
```

Mathematica [A] time = 0.062669, size = 61, normalized size = 0.92

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}+3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4
] - (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] time = 0.016, size = 71, normalized size = 1.1

$$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln(x^4+x^2-1)}{8}+\frac{7\sqrt{5}}{20}\mathit{Artanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)-\frac{3\ln(x^4-x^2-1)}{8}-\frac{7\sqrt{5}}{20}\mathit{Artanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(x^8-3*x^4+1),x)
```

```
[Out] -1/4/x^4+3*ln(x)-3/8*ln(x^4+x^2-1)+7/20*5^(1/2)*arctanh(1/5*(2*x^
2+1)*5^(1/2))-3/8*ln(x^4-x^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1
)*5^(1/2))
```

Maxima [A] time = 0.829188, size = 76, normalized size = 1.15

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{4x^4}-\frac{3}{8}\log\left(x^8-3x^4+1\right)+\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^5),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1
/4/x^4 - 3/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

Fricas [A] time = 0.305835, size = 117, normalized size = 1.77

$$-\frac{\sqrt{5}\left(3\sqrt{5}x^4\log\left(x^8-3x^4+1\right)-24\sqrt{5}x^4\log(x)-7x^4\log\left(-\frac{10x^4-\sqrt{5}(2x^8-6x^4+7)-15}{x^8-3x^4+1}\right)+2\sqrt{5}\right)}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^5),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*(3*sqrt(5)*x^4*log(x^8 - 3*x^4 + 1) - 24*sqrt(5)*x^
4*log(x) - 7*x^4*log(-(10*x^4 - sqrt(5)*(2*x^8 - 6*x^4 + 7) - 15)
/(x^8 - 3*x^4 + 1)) + 2*sqrt(5))/x^4
```

Sympy [A] time = 0.532568, size = 66, normalized size = 1.

$$3\log(x) + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right) - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(x**8-3*x**4+1),x)
```

```
[Out] 3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-
7*sqrt(5)/40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

GIAC/XCAS [A] time = 0.303294, size = 89, normalized size = 1.35

$$\frac{7}{40}\sqrt{5}\ln\left(\frac{\left|2x^4 - \sqrt{5} - 3\right|}{\left|2x^4 + \sqrt{5} - 3\right|}\right) - \frac{3x^4 + 1}{4x^4} + \frac{3}{4}\ln\left(x^4\right) - \frac{3}{8}\ln\left(\left|x^8 - 3x^4 + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^5),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*ln(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)
) - 1/4*(3*x^4 + 1)/x^4 + 3/4*ln(x^4) - 3/8*ln(abs(x^8 - 3*x^4 +
1))
```

3.395 $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/(6*x^6) - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqr
t[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh
[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi [A] time = 0.261288, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqr
t[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh
[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rubi in Sympy [A] time = 26.7714, size = 110, normalized size = 1.13

$$\frac{\sqrt{2}\left(\frac{9}{2}+\frac{21\sqrt{5}}{10}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{-\sqrt{5}+3}}\right)}{6\sqrt{-\sqrt{5}+3}}+\frac{\sqrt{2}\left(-\frac{21\sqrt{5}}{10}+\frac{9}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}x^2}{\sqrt{\sqrt{5}+3}}\right)}{6\sqrt{\sqrt{5}+3}}-\frac{3}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(2)*(9/2 + 21*sqrt(5)/10)*atanh(sqrt(2)*x**2/sqrt(-sqrt(5) +
3))/(6*sqrt(-sqrt(5) + 3)) + sqrt(2)*(-21*sqrt(5)/10 + 9/2)*atanh
(sqrt(2)*x**2/sqrt(sqrt(5) + 3))/(6*sqrt(sqrt(5) + 3)) - 3/(2*x**
2) - 1/(6*x**6)
```

Mathematica [A] time = 0.121355, size = 111, normalized size = 1.14

$$\frac{1}{120}\left(-\frac{20}{x^6}-\frac{180}{x^2}-3\left(25+11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)+3\left(25-11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)\right.$$
$$\left.+3\left(25+11\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+3\left(11\sqrt{5}-25\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2
] + 3*(25 - 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqr
t[5])*Log[-1 + Sqrt[5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sq
rt[5] + 2*x^2])/120
```

Maple [A] time = 0.016, size = 72, normalized size = 0.7

$$-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{5\ln\left(x^4+x^2-1\right)}{8}+\frac{11\sqrt{5}}{20}Artanh\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)$$
$$+\frac{5\ln\left(x^4-x^2-1\right)}{8}+\frac{11\sqrt{5}}{20}Artanh\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(x^8-3*x^4+1),x)
```

```
[Out] -1/6/x^6-3/2/x^2-5/8*ln(x^4+x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x
^2+1)*5^(1/2))+5/8*ln(x^4-x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2
-1)*5^(1/2))
```

Maxima [A] time = 0.821242, size = 134, normalized size = 1.38

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)$$
$$-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^7),x, algorithm="maxima")
```

```
[Out] -11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) -
 11/40*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) -
 1/6*(9*x^4 + 1)/x^6 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2
 - 1)
```

Fricas [A] time = 0.274751, size = 188, normalized size = 1.94

$$-\frac{\sqrt{5}\left(15\sqrt{5}x^6\log\left(x^4+x^2-1\right)-15\sqrt{5}x^6\log\left(x^4-x^2-1\right)-33x^6\log\left(\frac{10x^2+\sqrt{5}\left(2x^4+2x^2+3\right)+5}{x^4+x^2-1}\right)-33x^6\log\left(\frac{10x^2+\sqrt{5}\left(2x^4-2x\right.}{x^4-x^2-1}\right.\right.}{120x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^7),x, algorithm="fricas")
```

```
[Out] -1/120*sqrt(5)*(15*sqrt(5)*x^6*log(x^4 + x^2 - 1) - 15*sqrt(5)*x^
6*log(x^4 - x^2 - 1) - 33*x^6*log((10*x^2 + sqrt(5)*(2*x^4 + 2*x^
2 + 3) + 5)/(x^4 + x^2 - 1)) - 33*x^6*log((10*x^2 + sqrt(5)*(2*x^
4 - 2*x^2 + 3) - 5)/(x^4 - x^2 - 1)) + 4*sqrt(5)*(9*x^4 + 1))/x^6
```

Sympy [A] time = 2.02835, size = 197, normalized size = 2.03

$$\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right)$$
$$+\left(-\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(-\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}+\frac{2207\sqrt{5}}{50}\right)$$
$$+\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207\sqrt{5}}{50}+\frac{1152\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207}{22}\right)$$
$$+\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2+\frac{1152\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}+\frac{2207}{22}\right)-\frac{9x^4+1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(x**8-3*x**4+1),x)
```

```
[Out] (11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152
*(11*sqrt(5)/40 + 5/8)**3/11) + (-11*sqrt(5)/40 + 5/8)*log(x**2 -
 2207/22 + 1152*(-11*sqrt(5)/40 + 5/8)**3/11 + 2207*sqrt(5)/50) +
 (-5/8 + 11*sqrt(5)/40)*log(x**2 - 2207*sqrt(5)/50 + 1152*(-5/8 +
 11*sqrt(5)/40)**3/11 + 2207/22) + (-5/8 - 11*sqrt(5)/40)*log(x**
2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50 + 2207/22
) - (9*x**4 + 1)/(6*x**6)
```

GIAC/XCAS [A] time = 0.302619, size = 140, normalized size = 1.44

$$-\frac{11}{40}\sqrt{5}\ln\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\ln\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\ln\left(\left|x^4+x^2-1\right|\right)+\frac{5}{8}\ln\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^7),x, algorithm="giac")
```

```
[Out] -11/40*sqrt(5)*ln(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
 - 11/40*sqrt(5)*ln(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5)
- 1)) - 1/6*(9*x^4 + 1)/x^6 - 5/8*ln(abs(x^4 + x^2 - 1)) + 5/8*ln
(abs(x^4 - x^2 - 1))
```

3.396 $\int \frac{x^8}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=170

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5]
)/2)^(1/4)*x])/(4*Sqrt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan
h[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^
(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
```

Rubi [A] time = 0.325785, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + (((123 - 55*Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt
[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*Arc
Tanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((123 - 55*Sqrt[5
])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 26.3513, size = 238, normalized size = 1.4

$$x - \frac{\sqrt[4]{2}\sqrt{-2\sqrt{5}+6}\left(-\frac{7\sqrt{5}}{10}+\frac{3}{2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{\sqrt[4]{2}\left(\frac{3}{2}+\frac{7\sqrt{5}}{10}\right)\sqrt{2\sqrt{5}+6}\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\left(\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{\sqrt[4]{2}\sqrt{-2\sqrt{5}+6}\left(-\frac{7\sqrt{5}}{10}+\frac{3}{2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{\sqrt[4]{2}\left(\frac{3}{2}+\frac{7\sqrt{5}}{10}\right)\sqrt{2\sqrt{5}+6}\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\left(\sqrt{5}+3\right)^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**8-3*x**4+1),x)
```

```
[Out] x - 2**(1/4)*sqrt(-2*sqrt(5) + 6)*(-7*sqrt(5)/10 + 3/2)*atan(2**(
1/4)*x/(-sqrt(5) + 3)**(1/4))/(2*(-sqrt(5) + 3)**(5/4)) - 2**(1/4
)*(3/2 + 7*sqrt(5)/10)*sqrt(2*sqrt(5) + 6)*atan(2**(1/4)*x/(sqrt(
5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(5/4)) - 2**(1/4)*sqrt(-2*sqrt(
5) + 6)*(-7*sqrt(5)/10 + 3/2)*atanh(2**(1/4)*x/(-sqrt(5) + 3)**(1
/4))/(2*(-sqrt(5) + 3)**(5/4)) - 2**(1/4)*(3/2 + 7*sqrt(5)/10)*sq
rt(2*sqrt(5) + 6)*atanh(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(2*(sqrt
(5) + 3)**(5/4))
```

Mathematica [A] time = 0.514682, size = 160, normalized size = 0.94

$$x + \frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} - \frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\left(\sqrt{5}-2\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} - \frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1
 + Sqrt[5])] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sq
rt[10*(1 + Sqrt[5])] + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[
```

```
5])]*x])/Sqrt[10*(-1 + Sqrt[5])] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/
(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])]
```

Maple [A] time = 0.082, size = 205, normalized size = 1.2

$$x - \frac{2\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right) - \frac{1}{\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right) - \frac{2\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\mathit{Artanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right) + \frac{1}{\sqrt{-2+2\sqrt{5}}}\mathit{Artanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right) - \frac{2\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right) + \frac{1}{\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right) - \frac{2\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}\mathit{Artanh}\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right) - \frac{1}{\sqrt{2\sqrt{5}+2}}\mathit{Artanh}\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^8-3*x^4+1),x)
```

```
[Out] x-2/5*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))
-1/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-2/5*5^(1/2
)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*
5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-2/5*5^(1/2)/(-2+
2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2)
)^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-2/5*5^(1/2)/(2*5^(1/2)+2
)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))-1/(2*5^(1/2)+2)^(1/2)*ar
ctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x + \frac{1}{2}\int \frac{2x^2+1}{x^4-x^2-1}\,dx - \frac{1}{2}\int \frac{2x^2-1}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate
((2*x^2 - 1)/(x^4 + x^2 - 1), x)
```

Fricas [A] time = 0.314602, size = 383, normalized size = 2.25

$$\frac{1}{40}\sqrt{10}\left(4\sqrt{10}x+4\sqrt{5\sqrt{5}+11}\arctan\left(\frac{\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{5\sqrt{5}+11}}{2\left(\sqrt{10}\sqrt{2}x+\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\right)}\right)-4\sqrt{5\sqrt{5}-11}\arctan\left(\frac{\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{}}{2\left(\sqrt{10}\sqrt{2}x+\sqrt{10}\sqrt{2}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(10)*(4*sqrt(10)*x + 4*sqrt(5*sqrt(5) + 11)*arctan(1/2*(
3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(5*sqrt(5) + 11)/(sqrt(10)*sqr
t(2)*x + sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1))) - 4*sqrt(5*sqrt(5)
- 11)*arctan(1/2*(3*sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(5*sqrt(5) -
 11)/(sqrt(10)*sqrt(2)*x + sqrt(10)*sqrt(2*x^2 + sqrt(5) - 1))) +
 sqrt(5*sqrt(5) - 11)*log(2*sqrt(10)*x + sqrt(5*sqrt(5) - 11)*(3*
sqrt(5) + 5)) - sqrt(5*sqrt(5) - 11)*log(2*sqrt(10)*x - sqrt(5*sq
rt(5) - 11)*(3*sqrt(5) + 5)) - sqrt(5*sqrt(5) + 11)*log(2*sqrt(10
)*x + sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5)) + sqrt(5*sqrt(5) + 11
)*log(2*sqrt(10)*x - sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5)))
```

Sympy [A] time = 3.24894, size = 58, normalized size = 0.34

$$x+\operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(-\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right)$$
$$+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\left(-\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**8-3*x**4+1),x)
```

```
[Out] x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*
_t**5/11 + 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1
, Lambda(_t, _t*log(-15360*_t**5/11 + 1288*_t/55 + x)))
```

GIAC/XCAS [A] time = 0.343088, size = 200, normalized size = 1.18

$$-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)$$
$$-\frac{1}{40}\sqrt{50\sqrt{5}+110}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}+110}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)$$
$$+\frac{1}{40}\sqrt{50\sqrt{5}-110}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{50\sqrt{5}-110}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) +
1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1
/40*sqrt(50*sqrt(5) + 110)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) +
 1/40*sqrt(50*sqrt(5) + 110)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2)))
 + 1/40*sqrt(50*sqrt(5) - 110)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2)
)) - 1/40*sqrt(50*sqrt(5) - 110)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/
2))) + x
```

3.397 $\int \frac{x^6}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=167

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{144-64\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4
)*Sqrt[5]) - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(
1/4)*x])/(4*Sqrt[5]) - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5]) + ((144 - 64*Sqrt[5])^(1/4)*Ar
cTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
```

Rubi [A] time = 0.224657, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{9-4\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{9-4\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4
)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4
)*x])/(2*Sqrt[5]) - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5])
)^(1/4)*x])/(2*2^(3/4)*Sqrt[5]) + ((9 - 4*Sqrt[5])^(1/4)*ArcTanh[
((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 23.368, size = 189, normalized size = 1.13

$$\frac{\sqrt[4]{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt[4]{-\sqrt{5}+3}}+\frac{\sqrt[4]{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}}-\frac{\sqrt[4]{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt[4]{-\sqrt{5}+3}}-\frac{\sqrt[4]{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**8-3*x**4+1),x)
```

```
[Out] 2**(1/4)*(-3*sqrt(5)/10 + 1/2)*atan(2**(1/4)*x/(-sqrt(5) + 3)**(1
/4))/(2*(-sqrt(5) + 3)**(1/4)) + 2**(1/4)*(1/2 + 3*sqrt(5)/10)*at
an(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)) - 2*
*(1/4)*(-3*sqrt(5)/10 + 1/2)*atanh(2**(1/4)*x/(-sqrt(5) + 3)**(1/
4))/(2*(-sqrt(5) + 3)**(1/4)) - 2**(1/4)*(1/2 + 3*sqrt(5)/10)*ata
nh(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4))
```

Mathematica [A] time = 0.25764, size = 160, normalized size = 0.96

$$\frac{\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}}+\frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}-\frac{\left(\sqrt{5}-3\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}}-\frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] (((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt
[5]] + ((3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + S
qrt[5]] - ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt
[-1 + Sqrt[5]] - ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])
/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [A] time = 0.036, size = 206, normalized size = 1.2

$$\frac{3\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{1}{2\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$
$$+\frac{3\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{1}{2\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$
$$-\frac{3\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$
$$-\frac{3\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{2\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^8-3*x^4+1),x)
```

```
[Out] 3/10*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+
1/2/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+3/10*5^(1
/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-1/2/(-
2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-3/10*5^(1/2)
/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/2/(-2+2*
5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-3/10*5^(1/2)/(2*5
^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))-1/2/(2*5^(1/2)+2
)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

Fricas [A] time = 0.303137, size = 425, normalized size = 2.54

$$\frac{1}{5}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\arctan\left(\frac{\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\left(\sqrt{5}+1\right)}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2-1)+5\right)}+\sqrt{5}x\right)}\right)-\frac{1}{5}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\arctan\left(\frac{\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\left(\sqrt{5}-1\right)}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2+1)+5\right)}+\sqrt{5}x\right)}\right)+\frac{1}{20}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\log\left(\sqrt{5}x+\frac{1}{2}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\left(\sqrt{5}+1\right)\right)-\frac{1}{20}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\log\left(\sqrt{5}x-\frac{1}{2}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\left(\sqrt{5}+1\right)\right)-\frac{1}{20}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\log\left(\sqrt{5}x+\frac{1}{2}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\left(\sqrt{5}-1\right)\right)+\frac{1}{20}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\log\left(\sqrt{5}x-\frac{1}{2}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(-sqrt(5)*(2*sqrt(5) - 5))*arctan(1/2*sqrt(-sqrt(5)*(2*sq
rt(5) - 5))*(sqrt(5) + 1)/(sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sqrt(
5)*(2*x^2 - 1) + 5)) + sqrt(5)*x)) - 1/5*sqrt(sqrt(5)*(2*sqrt(5)
+ 5))*arctan(1/2*sqrt(sqrt(5)*(2*sqrt(5) + 5))*(sqrt(5) - 1)/(sqr
t(5)*sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 + 1) + 5)) + sqrt(5)
*x)) + 1/20*sqrt(-sqrt(5)*(2*sqrt(5) - 5))*log(sqrt(5)*x + 1/2*sq
rt(-sqrt(5)*(2*sqrt(5) - 5))*(sqrt(5) + 1)) - 1/20*sqrt(-sqrt(5)*
(2*sqrt(5) - 5))*log(sqrt(5)*x - 1/2*sqrt(-sqrt(5)*(2*sqrt(5) - 5
))*(sqrt(5) + 1)) - 1/20*sqrt(sqrt(5)*(2*sqrt(5) + 5))*log(sqrt(5
)*x + 1/2*sqrt(sqrt(5)*(2*sqrt(5) + 5))*(sqrt(5) - 1)) + 1/20*sqr
t(sqrt(5)*(2*sqrt(5) + 5))*log(sqrt(5)*x - 1/2*sqrt(sqrt(5)*(2*sq
rt(5) + 5))*(sqrt(5) - 1))
```

Sympy [A] time = 3.20836, size = 53, normalized size = 0.32

$$\operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t
**7 + 4920*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lam
bda(_t, _t*log(-1792000*_t**7 + 4920*_t**3 + x)))
```

GIAC/XCAS [A] time = 0.350257, size = 198, normalized size = 1.19

$$\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) -\frac{1}{20}\sqrt{5\sqrt{5}+10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}+10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) +\frac{1}{20}\sqrt{5\sqrt{5}-10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5\sqrt{5}-10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/1
0*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*s
qrt(5*sqrt(5) + 10)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*s
qrt(5*sqrt(5) + 10)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*s
qrt(5*sqrt(5) - 10)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*s
qrt(5*sqrt(5) - 10)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

3.398 $\int \frac{x^4}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[Out] -(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*S
qrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)
*x])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3
 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi [A] time = 0.230677, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*S
qrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)
*x])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3
 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 20.263, size = 238, normalized size = 1.38

$$-\frac{\sqrt[4]{2}\sqrt{-2\sqrt{5}+6}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{\sqrt[4]{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\sqrt{2\sqrt{5}+6}\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\left(\sqrt{5}+3\right)^{\frac{5}{4}}}$$
$$-\frac{\sqrt[4]{2}\sqrt{-2\sqrt{5}+6}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{\sqrt[4]{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\sqrt{2\sqrt{5}+6}\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\left(\sqrt{5}+3\right)^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**8-3*x**4+1),x)
```

```
[Out] -2**(1/4)*sqrt(-2*sqrt(5) + 6)*(-3*sqrt(5)/10 + 1/2)*atan(2**(1/4
)*x/(-sqrt(5) + 3)**(1/4))/(2*(-sqrt(5) + 3)**(5/4)) - 2**(1/4)*(
1/2 + 3*sqrt(5)/10)*sqrt(2*sqrt(5) + 6)*atan(2**(1/4)*x/(sqrt(5)
+ 3)**(1/4))/(2*(sqrt(5) + 3)**(5/4)) - 2**(1/4)*sqrt(-2*sqrt(5)
+ 6)*(-3*sqrt(5)/10 + 1/2)*atanh(2**(1/4)*x/(-sqrt(5) + 3)**(1/4)
)/(2*(-sqrt(5) + 3)**(5/4)) - 2**(1/4)*(1/2 + 3*sqrt(5)/10)*sqrt(
2*sqrt(5) + 6)*atanh(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(2*(sqrt(5)
 + 3)**(5/4))
```

Mathematica [A] time = 0.361066, size = 132, normalized size = 0.76

$$\frac{\sqrt{\sqrt{5}-1}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{1+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\sqrt{5}-1}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{1+\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + S
qrt[5]]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcT
anh[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/
(1 + Sqrt[5])]*x])/(2*Sqrt[10])
```

Maple [A] time = 0.046, size = 206, normalized size = 1.2

$$-\frac{1}{2\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$
$$+\frac{1}{2\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$
$$+\frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$
$$-\frac{1}{2\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-1/10*5^(
1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+1/2/(-2+
2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-1/10*5^(1/2)/(
-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/2/(-2+2*5
^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-1/10*5^(1/2)/(-2+2
*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-1/2/(2*5^(1/2)+2
)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))-1/10*5^(1/2)/(2*5^(1/2)+
2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

Fricas [A] time = 0.287486, size = 374, normalized size = 2.16

$$\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\arctan\left(\frac{\sqrt{5}\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}}{5\left(\sqrt{2}x+\sqrt{2x^2+\sqrt{5}+1}\right)}\right) - \frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\arctan\left(\frac{\sqrt{5}\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}}{5\left(\sqrt{2}x+\sqrt{2x^2+\sqrt{5}-1}\right)}\right) - \frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\log\left(\frac{1}{5}\sqrt{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}+x\right) + \frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\log\left(-\frac{1}{5}\sqrt{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}+x\right) + \frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\log\left(\frac{1}{5}\sqrt{5}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}+x\right) - \frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\log\left(-\frac{1}{5}\sqrt{5}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))*arctan(1/5*sqrt(5)*sqrt
(2)*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))/(sqrt(2)*x + sqrt(2*x^2
 + sqrt(5) + 1))) - 1/5*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*ar
ctan(1/5*sqrt(5)*sqrt(2)*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))/(
sqrt(2)*x + sqrt(2*x^2 + sqrt(5) - 1))) - 1/20*sqrt(1/2)*sqrt(sqr
t(5)*(sqrt(5) + 5))*log(1/5*sqrt(5)*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(
5) + 5)) + x) + 1/20*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))*log(-1
/5*sqrt(5)*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5)) + x) + 1/20*sqrt
(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*log(1/5*sqrt(5)*sqrt(1/2)*sqrt
(-sqrt(5)*(sqrt(5) - 5)) + x) - 1/20*sqrt(1/2)*sqrt(-sqrt(5)*(sqr
t(5) - 5))*log(-1/5*sqrt(5)*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5)
) + x)
```

Sympy [A] time = 3.1531, size = 49, normalized size = 0.28

$$\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5
 + 12*_t + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _
t*log(-51200*_t**5 + 12*_t + x)))
```

GIAC/XCAS [A] time = 0.350041, size = 198, normalized size = 1.14

$$-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}+10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1
/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/4
0*sqrt(10*sqrt(5) + 10)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/
40*sqrt(10*sqrt(5) + 10)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1
/40*sqrt(10*sqrt(5) - 10)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))) -
1/40*sqrt(10*sqrt(5) - 10)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

3.399 $\int \frac{x^2}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=145

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\tan^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2}x\right)-\frac{1}{20}\sqrt{10+10\sqrt{5}}\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right)-\frac{1}{20}\sqrt{10\sqrt{5}-10}\tanh^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2}x\right)+\frac{1}{20}\sqrt{10+10\sqrt{5}}\tanh^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right)$$

```
[Out] (Sqrt[-10 + 10*Sqrt[5]]*ArcTan[(Sqrt[-2 + 2*Sqrt[5]]*x)/2])/20 -
(Sqrt[10 + 10*Sqrt[5]]*ArcTan[(Sqrt[2 + 2*Sqrt[5]]*x)/2])/20 - (S
qrt[-10 + 10*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sqrt[5]]*x)/2])/20 + (
Sqrt[10 + 10*Sqrt[5]]*ArcTanh[(Sqrt[2 + 2*Sqrt[5]]*x)/2])/20
```

Rubi [A] time = 0.141482, antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}+\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^
(1/4)) - (((3 + Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*
x])/(2*Sqrt[5]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqr
t[5]*(3 + Sqrt[5])^(1/4)) + (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[((3
+ Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 23.1541, size = 162, normalized size = 1.12

$$-\frac{\sqrt[4]{2}\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\sqrt[4]{-\sqrt{5}+3}}+\frac{\sqrt[4]{2}\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\sqrt[4]{\sqrt{5}+3}}+\frac{\sqrt[4]{2}\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\sqrt[4]{-\sqrt{5}+3}}-\frac{\sqrt[4]{2}\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\sqrt[4]{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**8-3*x**4+1),x)
```

```
[Out] -2**(1/4)*sqrt(5)*atan(2**(1/4)*x/(-sqrt(5) + 3)**(1/4))/(10*(-sq
rt(5) + 3)**(1/4)) + 2**(1/4)*sqrt(5)*atan(2**(1/4)*x/(sqrt(5) +
3)**(1/4))/(10*(sqrt(5) + 3)**(1/4)) + 2**(1/4)*sqrt(5)*atanh(2**
(1/4)*x/(-sqrt(5) + 3)**(1/4))/(10*(-sqrt(5) + 3)**(1/4)) - 2**(1
/4)*sqrt(5)*atanh(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(10*(sqrt(5) +
 3)**(1/4))
```

Mathematica [A] time = 0.0693973, size = 131, normalized size = 0.9

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + Arc
Tan[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqr
t[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(
1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])]
```

Maple [A] time = 0.037, size = 110, normalized size = 0.8

$$\frac{\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$
$$-\frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^8-3*x^4+1),x)
```

```
[Out] 1/5*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+1
/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
-1/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2)
)-1/5*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^8 - 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

Fricas [A] time = 0.297645, size = 441, normalized size = 3.04

$$\begin{aligned}
&-\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\left(\sqrt{5}+1\right)}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2\,x^2+1)+5\right)}+\sqrt{5}x\right)}\right)\\
&+\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\left(\sqrt{5}-1\right)}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2\,x^2-1)+5\right)}+\sqrt{5}x\right)}\right)\\
&-\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\log\left(\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\left(\sqrt{5}+1\right)+\sqrt{5}x\right)\\
&+\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\log\left(-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\left(\sqrt{5}+1\right)+\sqrt{5}x\right)\\
&+\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\log\left(\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\left(\sqrt{5}-1\right)+\sqrt{5}x\right)\\
&-\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\log\left(-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\left(\sqrt{5}-1\right)+\sqrt{5}x\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*arctan(1/2*sqrt(1/2)*
sqrt(-sqrt(5)*(sqrt(5) - 5))*(sqrt(5) + 1)/(sqrt(5)*sqrt(1/10)*sq
rt(sqrt(5)*(sqrt(5)*(2*x^2 + 1) + 5)) + sqrt(5)*x)) + 1/5*sqrt(1/
2)*sqrt(sqrt(5)*(sqrt(5) + 5))*arctan(1/2*sqrt(1/2)*sqrt(sqrt(5)*
(sqrt(5) + 5))*(sqrt(5) - 1)/(sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sq
rt(5)*(2*x^2 - 1) + 5)) + sqrt(5)*x)) - 1/20*sqrt(1/2)*sqrt(-sqrt
(5)*(sqrt(5) - 5))*log(1/2*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))
*(sqrt(5) + 1) + sqrt(5)*x) + 1/20*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(
5) - 5))*log(-1/2*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*(sqrt(5)
 + 1) + sqrt(5)*x) + 1/20*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))*l
og(1/2*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))*(sqrt(5) - 1) + sqrt
(5)*x) - 1/20*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))*log(-1/2*sqrt
(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))*(sqrt(5) - 1) + sqrt(5)*x)
```

Sympy [A] time = 3.14754, size = 53, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4 - 80t^2 - 1, \left(t \mapsto t\log\left(6144000t^7 - 2240t^3 + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 80t^2 - 1, \left(t \mapsto t\log\left(6144000t^7 - 2240t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**
7 - 2240*_t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda
(_t, _t*log(6144000*_t**7 - 2240*_t**3 + x)))
```

GIAC/XCAS [A] time = 0.348456, size = 198, normalized size = 1.37

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/
20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40
*sqrt(10*sqrt(5) - 10)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/4
0*sqrt(10*sqrt(5) - 10)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/
40*sqrt(10*sqrt(5) + 10)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1
/40*sqrt(10*sqrt(5) + 10)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

3.400 $\int \frac{1}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=169

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}$$

```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5]
)^(3/4))) + ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x
])/(2*2^(3/4)*Sqrt[5]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1
/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[(
(3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.137618, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 3*x^4 + x^8)^(-1),x]
```


```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5]
)^(3/4))) + ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x
])/(2*2^(3/4)*Sqrt[5]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1
/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[(
(3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
```

Rubi in Sympy [A] time = 15.4095, size = 209, normalized size = 1.24

$$\frac{\sqrt[4]{2}\sqrt{5}\sqrt{-2\sqrt{5}+6}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{\sqrt[4]{2}\sqrt{5}\sqrt{2\sqrt{5}+6}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\left(\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{\sqrt[4]{2}\sqrt{5}\sqrt{-2\sqrt{5}+6}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{\sqrt[4]{2}\sqrt{5}\sqrt{2\sqrt{5}+6}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\left(\sqrt{5}+3\right)^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**8-3*x**4+1),x)
```

```
[Out] 2**(1/4)*sqrt(5)*sqrt(-2*sqrt(5) + 6)*atan(2**(1/4)*x/(-sqrt(5) +
 3)**(1/4))/(10*(-sqrt(5) + 3)**(5/4)) - 2**(1/4)*sqrt(5)*sqrt(2*
sqrt(5) + 6)*atan(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(10*(sqrt(5) +
 3)**(5/4)) + 2**(1/4)*sqrt(5)*sqrt(-2*sqrt(5) + 6)*atanh(2**(1/4
)*x/(-sqrt(5) + 3)**(1/4))/(10*(-sqrt(5) + 3)**(5/4)) - 2**(1/4)*
sqrt(5)*sqrt(2*sqrt(5) + 6)*atanh(2**(1/4)*x/(sqrt(5) + 3)**(1/4)
)/(10*(sqrt(5) + 3)**(5/4))
```

Mathematica [A] time = 0.27938, size = 160, normalized size = 0.95

$$\frac{\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}}-\frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}+\frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}}-\frac{\left(\sqrt{5}-1\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] (((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[
5]] - ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + S
qrt[5]] + ((1 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[
-1 + Sqrt[5]] - ((-1 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])
/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [A] time = 0.035, size = 206, normalized size = 1.2

$$\frac{\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{2\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{1}{2\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{2\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^8-3*x^4+1),x)
```

```
[Out] 1/10*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-
1/2/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+1/2/(-2+2
*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/10*5^(1/2)/(-
2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/2/(-2+2*5^
(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/10*5^(1/2)/(-2+2*
5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/10*5^(1/2)/(2*5
^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))-1/2/(2*5^(1/2)+2
)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8 - 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - 3*x^4 + 1), x)
```

Fricas [A] time = 0.334191, size = 401, normalized size = 2.37

$$\frac{1}{5}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\arctan\left(\frac{\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\left(3\sqrt{5}+5\right)}{10\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2+1)+5\right)}+x\right)}\right)-\frac{1}{5}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\arctan\left(\frac{\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\left(3\sqrt{5}-5\right)}{10\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2-1)+5\right)}+x\right)}\right)-\frac{1}{20}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\log\left(\frac{1}{10}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\left(3\sqrt{5}+5\right)+x\right)+\frac{1}{20}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\log\left(-\frac{1}{10}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\left(3\sqrt{5}+5\right)+x\right)+\frac{1}{20}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\log\left(\frac{1}{10}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\left(3\sqrt{5}-5\right)+x\right)-\frac{1}{20}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\log\left(-\frac{1}{10}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\left(3\sqrt{5}-5\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(-sqrt(5)*(2*sqrt(5) - 5))*arctan(1/10*sqrt(-sqrt(5)*(2*s
qrt(5) - 5))*(3*sqrt(5) + 5)/(sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2
*x^2 + 1) + 5)) + x)) - 1/5*sqrt(sqrt(5)*(2*sqrt(5) + 5))*arctan(
1/10*sqrt(sqrt(5)*(2*sqrt(5) + 5))*(3*sqrt(5) - 5)/(sqrt(1/10)*sq
rt(sqrt(5)*(sqrt(5)*(2*x^2 - 1) + 5)) + x)) - 1/20*sqrt(-sqrt(5)*
(2*sqrt(5) - 5))*log(1/10*sqrt(-sqrt(5)*(2*sqrt(5) - 5))*(3*sqrt(
5) + 5) + x) + 1/20*sqrt(-sqrt(5)*(2*sqrt(5) - 5))*log(-1/10*sqrt
(-sqrt(5)*(2*sqrt(5) - 5))*(3*sqrt(5) + 5) + x) + 1/20*sqrt(sqrt(
5)*(2*sqrt(5) + 5))*log(1/10*sqrt(sqrt(5)*(2*sqrt(5) + 5))*(3*sqr
t(5) - 5) + x) - 1/20*sqrt(sqrt(5)*(2*sqrt(5) + 5))*log(-1/10*sqr
t(sqrt(5)*(2*sqrt(5) + 5))*(3*sqrt(5) - 5) + x)
```

Sympy [A] time = 3.2198, size = 53, normalized size = 0.31

$$\operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5
- 47*_t/2 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t,
 _t*log(9600*_t**5 - 47*_t/2 + x)))
```

GIAC/XCAS [A] time = 0.337428, size = 198, normalized size = 1.17

$$-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)$$
$$-\frac{1}{20}\sqrt{5\sqrt{5}-10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}-10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)$$
$$+\frac{1}{20}\sqrt{5\sqrt{5}+10}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5\sqrt{5}+10}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/
10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*
sqrt(5*sqrt(5) - 10)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*
sqrt(5*sqrt(5) - 10)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*
sqrt(5*sqrt(5) + 10)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*
sqrt(5*sqrt(5) + 10)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

3.401 $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}$$

```
[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/
4)*x])/(4*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2
)^(1/4)*x])/(4*2^(1/4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcT
anh[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4
)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/4)*Sqrt[5])
```

Rubi [A] time = 0.234045, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{1}{x}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + (((123 - 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^
(1/4)*x])/(2*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5]
)/2)^(1/4)*x])/(4*2^(1/4)*Sqrt[5]) - (((123 - 55*Sqrt[5])/2)^(1/4
)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5]
)^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/4)*Sqrt[5])
```

Rubi in Sympy [A] time = 31.0543, size = 192, normalized size = 1.12

$$-\frac{\sqrt[4]{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt[4]{-\sqrt{5}+3}}-\frac{\sqrt[4]{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}}+\frac{\sqrt[4]{2}\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt[4]{-\sqrt{5}+3}}+\frac{\sqrt[4]{2}\left(-\frac{3\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**8-3*x**4+1),x)
```

```
[Out] -2**(1/4)*(1/2 + 3*sqrt(5)/10)*atan(2**(1/4)*x/(-sqrt(5) + 3)**(1
/4))/(2*(-sqrt(5) + 3)**(1/4)) - 2**(1/4)*(-3*sqrt(5)/10 + 1/2)*a
tan(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)) + 2
**(1/4)*(1/2 + 3*sqrt(5)/10)*atanh(2**(1/4)*x/(-sqrt(5) + 3)**(1/
4))/(2*(-sqrt(5) + 3)**(1/4)) + 2**(1/4)*(-3*sqrt(5)/10 + 1/2)*at
anh(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)) - 1
/x
```

Mathematica [A] time = 0.589947, size = 174, normalized size = 1.01

$$-\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(\sqrt{5}-3\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqr
t[10*(-1 + Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5
])]*x])/(2*Sqrt[10*(1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[
2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5
```

```
])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Maple [A] time = 0.037, size = 211, normalized size = 1.2

$$-x^{-1}+\frac{3\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{2\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{3\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{2\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{3\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{1}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{3\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{1}{2\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^8-3*x^4+1),x)
```

```
[Out] -1/x+3/10*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1
/2))-1/2/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+3/10
*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1
/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-3/10*5^
(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-1/2/(
-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-3/10*5^(1/2)
/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))+1/2/(2*5^(1
/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\frac{1}{2}\int\frac{x^2+2}{x^4+x^2-1}\,dx-\frac{1}{2}\int\frac{x^2-2}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrat
e((x^2 - 2)/(x^4 - x^2 - 1), x)
```

Fricas [A] time = 0.316729, size = 386, normalized size = 2.24

$$-\frac{\sqrt{10}\left(4\,x\sqrt{5\,\sqrt{5}+11}\arctan\left(\frac{\left(2\,\sqrt{5}\sqrt{2}-5\,\sqrt{2}\right)\sqrt{5\,\sqrt{5}+11}}{\sqrt{10}\sqrt{2}x+\sqrt{10}\sqrt{2\,x^2+\sqrt{5}-1}}\right)+4\,x\sqrt{5\,\sqrt{5}-11}\arctan\left(\frac{\left(2\,\sqrt{5}\sqrt{2}+5\,\sqrt{2}\right)\sqrt{5\,\sqrt{5}-11}}{\sqrt{10}\sqrt{2}x+\sqrt{10}\sqrt{2\,x^2+\sqrt{5}+1}}\right)+x\sqrt{5\,\sqrt{5}-11}\log\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(10)*(4*x*sqrt(5*sqrt(5) + 11)*arctan((2*sqrt(5)*sqrt(2
) - 5*sqrt(2))*sqrt(5*sqrt(5) + 11)/(sqrt(10)*sqrt(2)*x + sqrt(10
)*sqrt(2*x^2 + sqrt(5) - 1))) + 4*x*sqrt(5*sqrt(5) - 11)*arctan((
2*sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(5*sqrt(5) - 11)/(sqrt(10)*sqr
t(2)*x + sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1))) + x*sqrt(5*sqrt(5)
- 11)*log(sqrt(10)*x + sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5)) - x*
sqrt(5*sqrt(5) - 11)*log(sqrt(10)*x - sqrt(5*sqrt(5) - 11)*(2*sqr
t(5) + 5)) + x*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*x + sqrt(5*sqrt(
5) + 11)*(2*sqrt(5) - 5)) - x*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*x
 - sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5)) + 4*sqrt(10))/x
```

Sympy [A] time = 3.39281, size = 63, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t
**7/11 - 369792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2
- 1, Lambda(_t, _t*log(19251200*_t**7/11 - 369792*_t**3/11 + x)))
 - 1/x
```

GIAC/XCAS [A] time = 0.34726, size = 205, normalized size = 1.19

$$\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}-110}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}-110}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}+110}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{50\sqrt{5}+110}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^2),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1
/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/
40*sqrt(50*sqrt(5) - 110)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) +
1/40*sqrt(50*sqrt(5) - 110)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2)))
+ 1/40*sqrt(50*sqrt(5) + 110)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))
) - 1/40*sqrt(50*sqrt(5) + 110)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2
))) - 1/x
```

3.402 $\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=182

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/(3*x^3) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5
]))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqr
t[5])/2)^(1/4)*x])/(4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)
^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sq
rt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(3/4)*Sqrt[
5])
```

Rubi [A] time = 0.297275, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```


```
[Out] -1/(3*x^3) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5
]))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqr
t[5])/2)^(1/4)*x])/(4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)
^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sq
rt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(3/4)*Sqrt[
5])
```

Rubi in Sympy [A] time = 26.5243, size = 243, normalized size = 1.34

$$\frac{\sqrt[4]{2}\left(\frac{3}{2}+\frac{9\sqrt{5}}{10}\right)\sqrt{-2\sqrt{5}+6}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{6\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{\sqrt[4]{2}\left(-\frac{9\sqrt{5}}{10}+\frac{3}{2}\right)\sqrt{2\sqrt{5}+6}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{6\left(\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{\sqrt[4]{2}\left(\frac{3}{2}+\frac{9\sqrt{5}}{10}\right)\sqrt{-2\sqrt{5}+6}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{6\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{\sqrt[4]{2}\left(-\frac{9\sqrt{5}}{10}+\frac{3}{2}\right)\sqrt{2\sqrt{5}+6}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{6\left(\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**8-3*x**4+1),x)
```

```
[Out] 2**(1/4)*(3/2 + 9*sqrt(5)/10)*sqrt(-2*sqrt(5) + 6)*atan(2**(1/4)*
x/(-sqrt(5) + 3)**(1/4))/(6*(-sqrt(5) + 3)**(5/4)) + 2**(1/4)*(-9
*sqrt(5)/10 + 3/2)*sqrt(2*sqrt(5) + 6)*atan(2**(1/4)*x/(sqrt(5) +
 3)**(1/4))/(6*(sqrt(5) + 3)**(5/4)) + 2**(1/4)*(3/2 + 9*sqrt(5)/
10)*sqrt(-2*sqrt(5) + 6)*atanh(2**(1/4)*x/(-sqrt(5) + 3)**(1/4))/
(6*(-sqrt(5) + 3)**(5/4)) + 2**(1/4)*(-9*sqrt(5)/10 + 3/2)*sqrt(2
*sqrt(5) + 6)*atanh(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(6*(sqrt(5)
+ 3)**(5/4)) - 1/(3*x**3)
```

Mathematica [A] time = 0.483378, size = 166, normalized size = 0.91

$$-\frac{1}{3x^3}+\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqr
t[10*(-1 + Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5]
)]*x])/Sqrt[10*(1 + Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1
```

```
 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan
h[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])]
```

Maple [A] time = 0.041, size = 209, normalized size = 1.2

$$-\frac{1}{3x^{3}}+\frac{2\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{2\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{2\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{2\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^8-3*x^4+1),x)
```

```
[Out] -1/3/x^3+2/5*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)
^(1/2))-1/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+2/5
*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1
/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+2/5*5^(1/
2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*
5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))+2/5*5^(1/2)/(2*5^
(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))-1/(2*5^(1/2)+2)^(
1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3x^{3}}-\frac{1}{2}\int\frac{2x^{2}+3}{x^{4}+x^{2}-1}\,dx+\frac{1}{2}\int\frac{2x^{2}-3}{x^{4}-x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*in
tegrate((2*x^2 - 3)/(x^4 - x^2 - 1), x)
```

Fricas [A] time = 0.290151, size = 482, normalized size = 2.65

$$\frac{12\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(29\sqrt{5}-65\right)}x^3\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(29\sqrt{5}-65\right)}\left(7\sqrt{5}+15\right)}{10\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2+1)+5\right)}+x\right)}\right)-12\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(29\sqrt{5}+65\right)}x^3\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(29\sqrt{5}+65\right)}\left(7\right.}{10\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2-1\right.}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] 1/60*(12*sqrt(1/2)*sqrt(-sqrt(5)*(29*sqrt(5) - 65))*x^3*arctan(1/
10*sqrt(1/2)*sqrt(-sqrt(5)*(29*sqrt(5) - 65))*(7*sqrt(5) + 15)/(s
qrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 + 1) + 5)) + x)) - 12*sqrt
(1/2)*sqrt(sqrt(5)*(29*sqrt(5) + 65))*x^3*arctan(1/10*sqrt(1/2)*s
qrt(sqrt(5)*(29*sqrt(5) + 65))*(7*sqrt(5) - 15)/(sqrt(1/10)*sqrt(
sqrt(5)*(sqrt(5)*(2*x^2 - 1) + 5)) + x)) - 3*sqrt(1/2)*sqrt(-sqrt
(5)*(29*sqrt(5) - 65))*x^3*log(1/10*sqrt(1/2)*sqrt(-sqrt(5)*(29*s
qrt(5) - 65))*(7*sqrt(5) + 15) + x) + 3*sqrt(1/2)*sqrt(-sqrt(5)*(
29*sqrt(5) - 65))*x^3*log(-1/10*sqrt(1/2)*sqrt(-sqrt(5)*(29*sqrt(
5) - 65))*(7*sqrt(5) + 15) + x) + 3*sqrt(1/2)*sqrt(sqrt(5)*(29*sq
rt(5) + 65))*x^3*log(1/10*sqrt(1/2)*sqrt(sqrt(5)*(29*sqrt(5) + 65
))*(7*sqrt(5) - 15) + x) - 3*sqrt(1/2)*sqrt(sqrt(5)*(29*sqrt(5) +
 65))*x^3*log(-1/10*sqrt(1/2)*sqrt(sqrt(5)*(29*sqrt(5) + 65))*(7*
sqrt(5) - 15) + x) - 20)/x^3
```

Sympy [A] time = 3.44402, size = 63, normalized size = 0.35

$$\operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t*
*5/377 - 23112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 -
1, Lambda(_t, _t*log(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(
3*x**3)
```

GIAC/XCAS [A] time = 0.344057, size = 205, normalized size = 1.13

$$-\frac{1}{20}\sqrt{130\sqrt{5}-290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{130\sqrt{5}+290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{130\sqrt{5}-290}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{130\sqrt{5}-290}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{130\sqrt{5}+290}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{130\sqrt{5}+290}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^4),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) +
 1/20*sqrt(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) -
 1/40*sqrt(130*sqrt(5) - 290)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))
) + 1/40*sqrt(130*sqrt(5) - 290)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/
2))) + 1/40*sqrt(130*sqrt(5) + 290)*ln(abs(x + sqrt(1/2*sqrt(5) -
 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*ln(abs(x - sqrt(1/2*sqrt(5
) - 1/2))) - 1/3/x^3
```

3.403 $\int \frac{1}{x^6(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=173

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{2889+1292\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[Out] -1/(5*x^5) - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sq
rt[5]))^(1/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcT
an[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5
])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889
 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqr
t[5])
```

Rubi [A] time = 0.371805, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{2889+1292\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sq
rt[5]))^(1/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcT
an[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5
])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889
 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqr
t[5])
```

Rubi in Sympy [A] time = 37.4362, size = 199, normalized size = 1.15

$$-\frac{\sqrt[4]{2}\left(\frac{15}{2}+\frac{7\sqrt{5}}{2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\sqrt[4]{-\sqrt{5}+3}}-\frac{\sqrt[4]{2}\left(-\frac{7\sqrt{5}}{2}+\frac{15}{2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\sqrt[4]{\sqrt{5}+3}}+\frac{\sqrt[4]{2}\left(\frac{15}{2}+\frac{7\sqrt{5}}{2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{10\sqrt[4]{-\sqrt{5}+3}}+\frac{\sqrt[4]{2}\left(-\frac{7\sqrt{5}}{2}+\frac{15}{2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{10\sqrt[4]{\sqrt{5}+3}}-\frac{3}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**8-3*x**4+1),x)
```

```
[Out] -2**(1/4)*(15/2 + 7*sqrt(5)/2)*atan(2**(1/4)*x/(-sqrt(5) + 3)**(1
/4))/(10*(-sqrt(5) + 3)**(1/4)) - 2**(1/4)*(-7*sqrt(5)/2 + 15/2)*
atan(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(10*(sqrt(5) + 3)**(1/4)) +
 2**(1/4)*(15/2 + 7*sqrt(5)/2)*atanh(2**(1/4)*x/(-sqrt(5) + 3)**(
1/4))/(10*(-sqrt(5) + 3)**(1/4)) + 2**(1/4)*(-7*sqrt(5)/2 + 15/2)
*atanh(2**(1/4)*x/(sqrt(5) + 3)**(1/4))/(10*(sqrt(5) + 3)**(1/4))
 - 3/x - 1/(5*x**5)
```

Mathematica [A] time = 0.518811, size = 189, normalized size = 1.09

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[
2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5
])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])])
```

```
 - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*
(1 + Sqrt[5])])
```

Maple [A] time = 0.051, size = 216, normalized size = 1.3

$$-\frac{1}{5x^{5}}-3x^{-1}+\frac{7\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{3}{2\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$
$$+\frac{7\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{3}{2\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$
$$-\frac{7\sqrt{5}}{10\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{3}{2\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$
$$-\frac{7\sqrt{5}}{10\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{3}{2\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^8-3*x^4+1),x)
```

```
[Out] -1/5/x^5-3/x+7/10*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/
2)+2)^(1/2))-3/2/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/
2))+7/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^
(1/2))+3/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
-7/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2
))-3/2/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-7/10
*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))+3/2
/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{15x^{4}+1}{5x^{5}}-\frac{1}{2}\int\frac{3x^{2}+5}{x^{4}+x^{2}-1}\,dx-\frac{1}{2}\int\frac{3x^{2}-5}{x^{4}-x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^6),x, algorithm="maxima")
```

```
[Out] -1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1)
, x) - 1/2*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

Fricas [A] time = 0.297299, size = 478, normalized size = 2.76

$$-\frac{4\sqrt{-\sqrt{5}\left(38\sqrt{5}-85\right)}x^5\arctan\left(\frac{\sqrt{-\sqrt{5}\left(38\sqrt{5}-85\right)}\left(5\sqrt{5}+11\right)}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2+1)+5\right)}+\sqrt{5}x\right)}\right)-4\sqrt{\sqrt{5}\left(38\sqrt{5}+85\right)}x^5\arctan\left(\frac{\sqrt{\sqrt{5}\left(38\sqrt{5}+85\right)}\left(5\sqrt{5}-1\right.}{2\left(\sqrt{5}\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2-1)+5\right.}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^6),x, algorithm="fricas")
```

```
[Out] -1/20*(4*sqrt(-sqrt(5)*(38*sqrt(5) - 85))*x^5*arctan(1/2*sqrt(-sq
rt(5)*(38*sqrt(5) - 85))*(5*sqrt(5) + 11)/(sqrt(5)*sqrt(1/10)*sqr
t(sqrt(5)*(sqrt(5)*(2*x^2 + 1) + 5)) + sqrt(5)*x)) - 4*sqrt(sqrt(
5)*(38*sqrt(5) + 85))*x^5*arctan(1/2*sqrt(sqrt(5)*(38*sqrt(5) + 8
5))*(5*sqrt(5) - 11)/(sqrt(5)*sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2
*x^2 - 1) + 5)) + sqrt(5)*x)) + sqrt(-sqrt(5)*(38*sqrt(5) - 85))*
x^5*log(sqrt(5)*x + 1/2*sqrt(-sqrt(5)*(38*sqrt(5) - 85))*(5*sqrt(
5) + 11)) - sqrt(-sqrt(5)*(38*sqrt(5) - 85))*x^5*log(sqrt(5)*x -
1/2*sqrt(-sqrt(5)*(38*sqrt(5) - 85))*(5*sqrt(5) + 11)) - sqrt(sqr
t(5)*(38*sqrt(5) + 85))*x^5*log(sqrt(5)*x + 1/2*sqrt(sqrt(5)*(38*
sqrt(5) + 85))*(5*sqrt(5) - 11)) + sqrt(sqrt(5)*(38*sqrt(5) + 85)
)*x^5*log(sqrt(5)*x - 1/2*sqrt(sqrt(5)*(38*sqrt(5) + 85))*(5*sqrt
(5) - 11)) + 60*x^4 + 4)/x^5
```

Sympy [A] time = 3.48181, size = 71, normalized size = 0.41

$$\operatorname{RootSum}\left(6400t^4-6080t^2-1,\left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+6080t^2-1,\left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right)-\frac{15x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*
_t**7/323 - 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 608
0*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323 - 194833880*_t
**3/323 + x))) - (15*x**4 + 1)/(5*x**5)
```

GIAC/XCAS [A] time = 0.344612, size = 215, normalized size = 1.24

$$\frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{85\sqrt{5}-190}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{85\sqrt{5}-190}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{85\sqrt{5}+190}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{85\sqrt{5}+190}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{15x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^6),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1
/10*sqrt(85*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/
20*sqrt(85*sqrt(5) - 190)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) +
1/20*sqrt(85*sqrt(5) - 190)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2)))
+ 1/20*sqrt(85*sqrt(5) + 190)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))
) - 1/20*sqrt(85*sqrt(5) + 190)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2
))) - 1/5*(15*x^4 + 1)/x^5
```

3.404 $\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=189

$$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[Out] -1/(7*x^7) - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(
2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5]
)/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39
603 - 17711*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])
/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTanh[((3 + S
qrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi [A] time = 0.403161, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(
2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5]
)/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39
603 - 17711*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])
/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTanh[((3 + S
qrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 31.4174, size = 248, normalized size = 1.31

$$\frac{\sqrt[4]{2}\left(\frac{63}{2}+\frac{147\sqrt{5}}{10}\right)\sqrt{-2\sqrt{5}+6}\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{42\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{\sqrt[4]{2}\left(-\frac{147\sqrt{5}}{10}+\frac{63}{2}\right)\sqrt{2\sqrt{5}+6}\operatorname{atan}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{42\left(\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{\sqrt[4]{2}\left(\frac{63}{2}+\frac{147\sqrt{5}}{10}\right)\sqrt{-2\sqrt{5}+6}\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-\sqrt{5}+3}}\right)}{42\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}+\frac{\sqrt[4]{2}\left(-\frac{147\sqrt{5}}{10}+\frac{63}{2}\right)\sqrt{2\sqrt{5}+6}\operatorname{atanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{\sqrt{5}+3}}\right)}{42\left(\sqrt{5}+3\right)^{\frac{5}{4}}}-\frac{1}{x^{3}}-\frac{1}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**8/(x**8-3*x**4+1),x)
```

```
[Out] 2**(1/4)*(63/2 + 147*sqrt(5)/10)*sqrt(-2*sqrt(5) + 6)*atan(2**(1/
4)*x/(-sqrt(5) + 3)**(1/4))/(42*(-sqrt(5) + 3)**(5/4)) + 2**(1/4)
*(-147*sqrt(5)/10 + 63/2)*sqrt(2*sqrt(5) + 6)*atan(2**(1/4)*x/(sq
rt(5) + 3)**(1/4))/(42*(sqrt(5) + 3)**(5/4)) + 2**(1/4)*(63/2 + 1
47*sqrt(5)/10)*sqrt(-2*sqrt(5) + 6)*atanh(2**(1/4)*x/(-sqrt(5) +
3)**(1/4))/(42*(-sqrt(5) + 3)**(5/4)) + 2**(1/4)*(-147*sqrt(5)/10
 + 63/2)*sqrt(2*sqrt(5) + 6)*atanh(2**(1/4)*x/(sqrt(5) + 3)**(1/4
))/(42*(sqrt(5) + 3)**(5/4)) - 1/x**3 - 1/(7*x**7)
```

Mathematica [A] time = 0.528083, size = 189, normalized size = 1.

$$-\frac{1}{7x^7}-\frac{1}{x^3}+\frac{\left(11+5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(11-5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-11-5\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(5\sqrt{5}-11\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[
5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[S
qrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*S
qrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[
5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*S
qrt[10*(1 + Sqrt[5])])
```

Maple [A] time = 0.057, size = 216, normalized size = 1.1

$$-\frac{1}{7\,x^7}-x^{-3}+\frac{11\,\sqrt{5}}{10\,\sqrt{2\,\sqrt{5}+2}}\arctan\left(2\,\frac{x}{\sqrt{2\,\sqrt{5}+2}}\right)-\frac{5}{2\,\sqrt{2\,\sqrt{5}+2}}\arctan\left(2\,\frac{x}{\sqrt{2\,\sqrt{5}+2}}\right)+\frac{11\,\sqrt{5}}{10\,\sqrt{-2+2\,\sqrt{5}}}\mathit{Artanh}\left(2\,\frac{x}{\sqrt{-2+2\,\sqrt{5}}}\right)+\frac{5}{2\,\sqrt{-2+2\,\sqrt{5}}}\mathit{Artanh}\left(2\,\frac{x}{\sqrt{-2+2\,\sqrt{5}}}\right)+\frac{11\,\sqrt{5}}{10\,\sqrt{-2+2\,\sqrt{5}}}\arctan\left(2\,\frac{x}{\sqrt{-2+2\,\sqrt{5}}}\right)+\frac{5}{2\,\sqrt{-2+2\,\sqrt{5}}}\arctan\left(2\,\frac{x}{\sqrt{-2+2\,\sqrt{5}}}\right)+\frac{11\,\sqrt{5}}{10\,\sqrt{2\,\sqrt{5}+2}}\mathit{Artanh}\left(2\,\frac{x}{\sqrt{2\,\sqrt{5}+2}}\right)-\frac{5}{2\,\sqrt{2\,\sqrt{5}+2}}\mathit{Artanh}\left(2\,\frac{x}{\sqrt{2\,\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^8/(x^8-3*x^4+1),x)
```

```
[Out] -1/7/x^7-1/x^3+11/10*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^
(1/2)+2)^(1/2))-5/2/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^
(1/2))+11/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/
2))^(1/2))+5/2/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1
/2))+11/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))
^(1/2))+5/2/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))
```

```
+11/10*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2
))-5/2/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7\,x^4+1}{7\,x^7}-\frac{1}{2}\int\frac{5\,x^2+8}{x^4+x^2-1}\,dx+\frac{1}{2}\int\frac{5\,x^2-8}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^8),x, algorithm="maxima")
```

```
[Out] -1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1),
 x) + 1/2*integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

Fricas [A] time = 0.28179, size = 489, normalized size = 2.59

$$28\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(199\sqrt{5}-445\right)}x^7\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(199\sqrt{5}-445\right)}\left(9\sqrt{5}+20\right)}{5\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2\,x^2+1)+5\right)}+x\right)}\right)-28\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(199\sqrt{5}+445\right)}x^7\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(199\sqrt{5}\cdots\right.}}{5\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}\cdots\right.}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^8),x, algorithm="fricas")
```

```
[Out] 1/140*(28*sqrt(1/2)*sqrt(-sqrt(5)*(199*sqrt(5) - 445))*x^7*arctan
(1/5*sqrt(1/2)*sqrt(-sqrt(5)*(199*sqrt(5) - 445))*(9*sqrt(5) + 20
)/(sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 + 1) + 5)) + x)) - 28*
sqrt(1/2)*sqrt(sqrt(5)*(199*sqrt(5) + 445))*x^7*arctan(1/5*sqrt(1
/2)*sqrt(sqrt(5)*(199*sqrt(5) + 445))*(9*sqrt(5) - 20)/(sqrt(1/10
)*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 - 1) + 5)) + x)) - 7*sqrt(1/2)*sqr
t(-sqrt(5)*(199*sqrt(5) - 445))*x^7*log(1/5*sqrt(1/2)*sqrt(-sqrt(
5)*(199*sqrt(5) - 445))*(9*sqrt(5) + 20) + x) + 7*sqrt(1/2)*sqrt(
-sqrt(5)*(199*sqrt(5) - 445))*x^7*log(-1/5*sqrt(1/2)*sqrt(-sqrt(5
)*(199*sqrt(5) - 445))*(9*sqrt(5) + 20) + x) + 7*sqrt(1/2)*sqrt(s
qrt(5)*(199*sqrt(5) + 445))*x^7*log(1/5*sqrt(1/2)*sqrt(sqrt(5)*(1
99*sqrt(5) + 445))*(9*sqrt(5) - 20) + x) - 7*sqrt(1/2)*sqrt(sqrt(
5)*(199*sqrt(5) + 445))*x^7*log(-1/5*sqrt(1/2)*sqrt(sqrt(5)*(199*
sqrt(5) + 445))*(9*sqrt(5) - 20) + x) - 140*x^4 - 20)/x^7
```

Sympy [A] time = 3.56258, size = 68, normalized size = 0.36

$$\operatorname{RootSum}\left(6400t^4-15920t^2-1,\left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+15920t^2-1,\left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right)-\frac{7x^4+1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**8/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t
**5/17711 - 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*
_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711 - 2842588*_t/1771
1 + x))) - (7*x**4 + 1)/(7*x**7)
```

GIAC/XCAS [A] time = 0.353901, size = 215, normalized size = 1.14

$$-\frac{1}{20}\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{890\sqrt{5}+1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{890\sqrt{5}-1990}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{890\sqrt{5}-1990}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{890\sqrt{5}+1990}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{890\sqrt{5}+1990}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{7\,x^4+1}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^8 - 3*x^4 + 1)*x^8),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2))
+ 1/20*sqrt(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2))
 - 1/40*sqrt(890*sqrt(5) - 1990)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/
2))) + 1/40*sqrt(890*sqrt(5) - 1990)*ln(abs(x - sqrt(1/2*sqrt(5)
+ 1/2))) + 1/40*sqrt(890*sqrt(5) + 1990)*ln(abs(x + sqrt(1/2*sqrt
(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 1990)*ln(abs(x - sqrt(1/2*
sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

3.405 $\int \frac{x^3}{2+3x^4+x^8} \, dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rubi [A] time = 0.0294496, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(2 + 3*x^4 + x^8),x]

[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rubi in Sympy [A] time = 5.65434, size = 15, normalized size = 0.71

$$\frac{\log\left(x^4+1\right)}{4}-\frac{\log\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**8+3*x**4+2),x)

[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4
```

Mathematica [A] time = 0.00633758, size = 21, normalized size = 1.

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Maple [A] time = 0.009, size = 18, normalized size = 0.9

$$\frac{\ln\left(x^4+1\right)}{4}-\frac{\ln\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+2),x)

[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Maxima [A] time = 0.746434, size = 23, normalized size = 1.1

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8 + 3*x^4 + 2),x, algorithm="maxima")

[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Fricas [A] time = 0.254498, size = 23, normalized size = 1.1

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8 + 3*x^4 + 2),x, algorithm="fricas")

[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] time = 0.255413, size = 15, normalized size = 0.71

$$\frac{\log\left(x^4+1\right)}{4}-\frac{\log\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**8+3*x**4+2),x)

[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4
```

GIAC/XCAS [A] time = 0.281482, size = 23, normalized size = 1.1

$$-\frac{1}{4}\ln\left(x^4+2\right)+\frac{1}{4}\ln\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^8 + 3*x^4 + 2),x, algorithm="giac")

[Out] -1/4*ln(x^4 + 2) + 1/4*ln(x^4 + 1)
```

3.406 $\int \frac{x^{11}}{2+3x^4+x^8}\,dx$

Optimal. Leaf size=26

$$\frac{x^4}{4}+\frac{1}{4}\log\left(x^4+1\right)-\log\left(x^4+2\right)$$

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rubi [A] time = 0.0448069, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^4}{4}+\frac{1}{4}\log\left(x^4+1\right)-\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(2 + 3*x^4 + x^8),x]

[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rubi in Sympy [A] time = 11.1046, size = 19, normalized size = 0.73

$$\frac{x^4}{4}+\frac{\log\left(x^4+1\right)}{4}-\log\left(x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(x**8+3*x**4+2),x)

[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

Mathematica [A] time = 0.00697979, size = 26, normalized size = 1.

$$\frac{x^4}{4}+\frac{1}{4}\log\left(x^4+1\right)-\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(2 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Maple [A] time = 0.008, size = 23, normalized size = 0.9

$$\frac{x^4}{4} + \frac{\ln(x^4+1)}{4} - \ln(x^4+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(x^8+3*x^4+2),x)
```

```
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Maxima [A] time = 0.744052, size = 30, normalized size = 1.15

$$\frac{1}{4}\,x^4 - \log(x^4+2) + \frac{1}{4}\,\log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + 3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Fricas [A] time = 0.251848, size = 30, normalized size = 1.15

$$\frac{1}{4}\,x^4 - \log(x^4+2) + \frac{1}{4}\,\log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + 3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] time = 0.298764, size = 19, normalized size = 0.73

$$\frac{x^4}{4} + \frac{\log(x^4+1)}{4} - \log(x^4+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(x**8+3*x**4+2),x)
```

```
[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

GIAC/XCAS [A] time = 0.281172, size = 30, normalized size = 1.15

$$\frac{1}{4}x^4 - \ln\left(x^4+2\right) + \frac{1}{4}\ln\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(x^8 + 3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - ln(x^4 + 2) + 1/4*ln(x^4 + 1)
```

3.407 $\int \frac{x^9}{2+x^5+x^{10}} \, dx$

Optimal. Leaf size=37

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10
```

Rubi [A] time = 0.0685276, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(2 + x^5 + x^10),x]

[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10
```

Rubi in Sympy [A] time = 8.26654, size = 34, normalized size = 0.92

$$\frac{\log\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\sqrt{7}\left(\frac{2x^5}{7}+\frac{1}{7}\right)\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(x**10+x**5+2),x)

[Out] log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(sqrt(7)*(2*x**5/7 + 1/7))
/35
```

Mathematica [A] time = 0.0187539, size = 37, normalized size = 1.

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(2 + x^5 + x^10),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10
```

Maple [A] time = 0.004, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^{10}+x^{5}+2\right)}{10}-\frac{\sqrt{7}}{35}\arctan\left(\frac{\left(2x^{5}+1\right)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(x^10+x^5+2),x)
```

```
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 0.821823, size = 41, normalized size = 1.11

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^{5}+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^{5}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^10 + x^5 + 2),x, algorithm="maxima")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x
^5 + 2)
```

Fricas [A] time = 0.298392, size = 46, normalized size = 1.24

$$\frac{1}{70}\sqrt{7}\left(\sqrt{7}\log\left(x^{10}+x^{5}+2\right)-2\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^{5}+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^10 + x^5 + 2),x, algorithm="fricas")
```

```
[Out] 1/70*sqrt(7)*(sqrt(7)*log(x^10 + x^5 + 2) - 2*arctan(1/7*sqrt(7)*
(2*x^5 + 1)))
```

Sympy [A] time = 0.351268, size = 37, normalized size = 1.

$$\frac{\log\left(x^{10}+x^{5}+2\right)}{10}-\frac{\sqrt{7}\,\operatorname{atan}\left(\frac{2\sqrt{7}x^{5}}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(x**10+x**5+2),x)
```

```
[Out] log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7
)/7)/35
```

GIAC/XCAS [A] time = 0.271432, size = 41, normalized size = 1.11

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2\,x^{5}+1\right)\right)+\frac{1}{10}\ln\left(x^{10}+x^{5}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(x^10 + x^5 + 2),x, algorithm="giac")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*ln(x^10 + x^
5 + 2)
```

3.408 $\int \frac{x^4}{2+x^5+x^{10}}\,dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rubi [A] time = 0.0448043, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(2 + x^5 + x^10),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rubi in Sympy [A] time = 5.20041, size = 24, normalized size = 1.04

$$\frac{2\sqrt{7}\,\text{atan}\left(\sqrt{7}\left(\frac{2x^5}{7}+\frac{1}{7}\right)\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**10+x**5+2),x)
```

```
[Out] 2*sqrt(7)*atan(sqrt(7)*(2*x**5/7 + 1/7))/35
```

Mathematica [A] time = 0.00923215, size = 23, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(2 + x^5 + x^10),x]

[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Maple [A] time = 0.003, size = 19, normalized size = 0.8

$$\frac{2\sqrt{7}}{35}\arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^10+x^5+2),x)

[Out] 2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 0.820614, size = 24, normalized size = 1.04

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^10 + x^5 + 2),x, algorithm="maxima")

[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Fricas [A] time = 0.260471, size = 24, normalized size = 1.04

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^10 + x^5 + 2),x, algorithm="fricas")

[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Sympy [A] time = 0.323585, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**10+x**5+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

GIAC/XCAS [A] time = 0.295419, size = 24, normalized size = 1.04

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^10 + x^5 + 2),x, algorithm="giac")
```

```
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

3.409 $\int \frac{1}{x(1+x^5+x^{10})}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 +
 x^10]/10
```

Rubi [A] time = 0.0719504, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 +
 x^10]/10
```

Rubi in Sympy [A] time = 12.4147, size = 41, normalized size = 1.05

$$\frac{\log\left(x^5\right)}{5}-\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^5}{3}+\frac{1}{3}\right)\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**10+x**5+1),x)
```

```
[Out] log(x**5)/5 - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(sqrt(3)*(2*
x**5/3 + 1/3))/15
```

Mathematica [C] time = 0.0572546, size = 197, normalized size = 5.05

$$-\frac{1}{5}\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)+3\#1^4\log(x-\#1)-\#1^3\log(x-\#1)+2\#1^2\log(x-\#1)-\#1\log(\right.$$
$$\frac{}{8\#1^7-7\#1^6+5\#1^4-4\#1^3+3\#1^2-1}$$
$$-\frac{1}{10}\log\left(x^2+x+1\right)+\log(x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{5\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]
/10 - RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Lo
g[x - #1]*#1) + 2*Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x -
 #1]*#1^4 - Log[x - #1]*#1^5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]
*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ]/5
```

Maple [B] time = 0.07, size = 66, normalized size = 1.7

$$-\frac{\ln\left(x^2+x+1\right)}{10}+\ln(x)-\frac{\sqrt{3}}{15}\arctan\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)-\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^10+x^5+1),x)
```

```
[Out] -1/10*ln(x^2+x+1)+ln(x)-1/15*3^(1/2)*arctan(2/3*3^(1/2)*x^5+1/3*3
^(1/2))-1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)
```

Maxima [A] time = 0.819528, size = 49, normalized size = 1.26

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^10 + x^5 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x
^5 + 1) + 1/5*log(x^5)
```

Fricas [A] time = 0.265033, size = 55, normalized size = 1.41

$$-\frac{1}{30}\sqrt{3}\left(\sqrt{3}\log\left(x^{10}+x^{5}+1\right)-10\sqrt{3}\log\left(x\right)+2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{5}+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^10 + x^5 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(3)*(sqrt(3)*log(x^10 + x^5 + 1) - 10*sqrt(3)*log(x) +
2*arctan(1/3*sqrt(3)*(2*x^5 + 1)))
```

Sympy [A] time = 0.406134, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^{5}+1\right)}{10}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^{5}}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**10+x**5+1),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3
 + sqrt(3)/3)/15
```

GIAC/XCAS [A] time = 0.294943, size = 45, normalized size = 1.15

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{5}+1\right)\right)-\frac{1}{10}\ln\left(x^{10}+x^{5}+1\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^10 + x^5 + 1)*x),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*ln(x^10 + x^
5 + 1) + ln(abs(x))
```

3.410 $\int \frac{1}{x^6(1+x^5+x^{10})} dx$

Optimal. Leaf size=48

$$-\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log(x)$$

```
[Out] -1/(5*x^5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + L
og[1 + x^5 + x^10]/10
```

Rubi [A] time = 0.103481, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] -1/(5*x^5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + L
og[1 + x^5 + x^10]/10
```

Rubi in Sympy [A] time = 15.3372, size = 48, normalized size = 1.

$$-\frac{\log\left(x^5\right)}{5}+\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^5}{3}+\frac{1}{3}\right)\right)}{15}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(x**10+x**5+1),x)
```

```
[Out] -log(x**5)/5 + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(sqrt(3)*(2
*x**5/3 + 1/3))/15 - 1/(5*x**5)
```

Mathematica [C] time = 0.0663843, size = 208, normalized size = 4.33

$$\frac{1}{30}\left(6\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-4\#1^6\log(x-\#1)+\#1^5\log(x-\#1)+2\#1^4\log(x-\#1)-3\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+\#1\log(\right.\right.}{8\#1^7-7\#1^6+5\#1^4-4\#1^3+3\#1^2-1}$$

$$-\frac{6}{x^5}+3\log\left(x^2+x+1\right)-30\log(x)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\Bigg)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] (-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log
[1 + x + x^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1
^8 & , (-Log[x - #1] + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[
x - #1]*#1^3 + 2*Log[x - #1]*#1^4 + Log[x - #1]*#1^5 - 4*Log[x -
#1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 - 7
*#1^6 + 8*#1^7) & ])/30
```

Maple [A] time = 0.034, size = 73, normalized size = 1.5

$$\frac{\ln\left(x^2+x+1\right)}{10}-\frac{1}{5\,x^5}-\ln(x)-\frac{\sqrt{3}}{15}\arctan\left(\frac{2\,\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)+\frac{\ln\left(4\,x^8-4\,x^7+4\,x^5-4\,x^4+4\,x^3-4\,x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(x^10+x^5+1),x)
```

```
[Out] 1/10*ln(x^2+x+1)-1/5/x^5-ln(x)-1/15*3^(1/2)*arctan(2/3*3^(1/2)*x^
5+1/3*3^(1/2))+1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)
```

Maxima [A] time = 0.826326, size = 55, normalized size = 1.15

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^5+1\right)\right)-\frac{1}{5\,x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^10 + x^5 + 1)*x^6),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*lo
g(x^10 + x^5 + 1) - 1/5*log(x^5)
```

Fricas [A] time = 0.267185, size = 78, normalized size = 1.62

$$\frac{\sqrt{3}\left(\sqrt{3}x^5 \log\left(x^{10}+x^5+1\right) - 10\sqrt{3}x^5 \log\left(x\right) - 2\,x^5 \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^5+1\right)\right) - 2\sqrt{3}\right)}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^10 + x^5 + 1)*x^6),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(3)*(sqrt(3)*x^5*log(x^10 + x^5 + 1) - 10*sqrt(3)*x^5*lo
g(x) - 2*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 2*sqrt(3))/x^5
```

Sympy [A] time = 0.609619, size = 48, normalized size = 1.

$$-\log\left(x\right) + \frac{\log\left(x^{10}+x^5+1\right)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(x**10+x**5+1),x)
```

```
[Out] -log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/
3 + sqrt(3)/3)/15 - 1/(5*x**5)
```

GIAC/XCAS [A] time = 0.2824, size = 61, normalized size = 1.27

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^5+1\right)\right) + \frac{x^5-1}{5\,x^5} + \frac{1}{10}\ln\left(x^{10}+x^5+1\right) - \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^10 + x^5 + 1)*x^6),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5
 + 1/10*ln(x^10 + x^5 + 1) - ln(abs(x))
```

3.411 $\int \frac{1}{x+x^6+x^{11}}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 +
 x^10]/10
```

Rubi [A] time = 0.066934, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^6 + x^11)^(-1),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 +
 x^10]/10
```

Rubi in Sympy [A] time = 13.4016, size = 41, normalized size = 1.05

$$\frac{\log\left(x^5\right)}{5}-\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2x^5}{3}+\frac{1}{3}\right)\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**11+x**6+x),x)
```

```
[Out] log(x**5)/5 - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(sqrt(3)*(2*
x**5/3 + 1/3))/15
```

Mathematica [C] time = 0.0274597, size = 197, normalized size = 5.05

$$-\frac{1}{5}\text{RootSum}\Big[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)+3\#1^4\log(x-\#1)-\#1^3\log(x-\#1)+2\#1^2\log(x-\#1)-\#1\log(}{8\#1^7-7\#1^6+5\#1^4-4\#1^3+3\#1^2-1}$$

$$-\frac{1}{10}\log\left(x^2+x+1\right)+\log(x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{5\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^6 + x^11)^(-1),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]
/10 - RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Lo
g[x - #1]*#1) + 2*Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x -
 #1]*#1^4 - Log[x - #1]*#1^5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]
*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ]/5
```

Maple [B] time = 0.017, size = 66, normalized size = 1.7

$$-\frac{\ln\left(x^2+x+1\right)}{10}+\ln(x)-\frac{\sqrt{3}}{15}\arctan\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)-\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^11+x^6+x),x)
```

```
[Out] -1/10*ln(x^2+x+1)+ln(x)-1/15*3^(1/2)*arctan(2/3*3^(1/2)*x^5+1/3*3
^(1/2))-1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{5}\int\frac{4x^7-3x^6-x^5+3x^4-x^3+2x^2-x}{x^8-x^7+x^5-x^4+x^3-x+1}\,dx-\frac{1}{10}\log\left(x^2+x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^11 + x^6 + x),x, algorithm="maxima")
```

```
[Out] 1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7
 - 3*x^6 - x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4
+ x^3 - x + 1), x) - 1/10*log(x^2 + x + 1) + log(x)
```

Fricas [A] time = 0.255054, size = 55, normalized size = 1.41

$$-\frac{1}{30}\sqrt{3}\left(\sqrt{3}\log\left(x^{10}+x^{5}+1\right)-10\sqrt{3}\log\left(x\right)+2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{5}+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^11 + x^6 + x),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(3)*(sqrt(3)*log(x^10 + x^5 + 1) - 10*sqrt(3)*log(x) +
2*arctan(1/3*sqrt(3)*(2*x^5 + 1)))
```

Sympy [A] time = 0.427512, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^{5}+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^{5}}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**11+x**6+x),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3
 + sqrt(3)/3)/15
```

GIAC/XCAS [A] time = 0.266234, size = 45, normalized size = 1.15

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{5}+1\right)\right)-\frac{1}{10}\ln\left(x^{10}+x^{5}+1\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^11 + x^6 + x),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*ln(x^10 + x^
5 + 1) + ln(abs(x))
```

3.412 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=147

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}-\frac{bx(b^2-2ac)}{c^4}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}$$

```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/
(3*c^2) + x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3
*a*b^2*c + a^2*c^2)*Log[a + b*x + c*x^2])/(2*c^5)
```

Rubi [A] time = 0.276181, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}-\frac{bx(b^2-2ac)}{c^4}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/
(3*c^2) + x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3
*a*b^2*c + a^2*c^2)*Log[a + b*x + c*x^2])/(2*c^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^3}{3c^2}+\frac{b(5a^2c^2-5ab^2c+b^4)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^5\sqrt{-4ac+b^2}}+\frac{x^4}{4c}+\frac{(-ac+b^2)\int x\,dx}{c^3}-\frac{(-2ac+b^2)\int b\,dx}{c^4}+\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**3/(3*c**2) + b*(5*a**2*c**2 - 5*a*b**2*c + b**4)*atanh((b +
 2*c*x)/sqrt(-4*a*c + b**2))/(c**5*sqrt(-4*a*c + b**2)) + x**4/(4
*c) + (-a*c + b**2)*Integral(x, x)/c**3 - (-2*a*c + b**2)*Integra
l(b, x)/c**4 + (a**2*c**2 - 3*a*b**2*c + b**4)*log(a + b*x + c*x*
*2)/(2*c**5)
```

Mathematica [A] time = 0.215489, size = 140, normalized size = 0.95

$$\frac{6\left(a^2c^2-3ab^2c+b^4\right)\log(a+x(b+cx))-\frac{12b\left(5a^2c^2-5ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx\left(-4bc\left(cx^2-6a\right)+3c^2x\left(cx^2-2a\right)-12b^3}{12c^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a
+ c*x^2)) - (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x
)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c +
a^2*c^2)*Log[a + x*(b + c*x)])/(12*c^5)
```

Maple [A] time = 0.009, size = 236, normalized size = 1.6

$$\frac{x^4}{4c}-\frac{bx^3}{3c^2}-\frac{ax^2}{2c^2}+\frac{x^2b^2}{2c^3}+2\frac{abx}{c^3}-\frac{b^3x}{c^4}+\frac{\ln\left(cx^2+bx+a\right)a^2}{2c^3}-\frac{3\ln\left(cx^2+bx+a\right)ab^2}{2c^4}+\frac{\ln\left(cx^2+bx+a\right)b^4}{2c^5}-5\frac{a^2b}{c^3\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)+5\frac{ab^3}{c^4\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)-\frac{b^5}{c^5}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c+a/x^2+b/x),x)
```

```
[Out] 1/4*x^4/c-1/3*b*x^3/c^2-1/2/c^2*x^2*a+1/2/c^3*x^2*b^2+2/c^3*a*b*x
-1/c^4*b^3*x+1/2/c^3*ln(c*x^2+b*x+a)*a^2-3/2/c^4*ln(c*x^2+b*x+a)*
a*b^2+1/2/c^5*ln(c*x^2+b*x+a)*b^4-5/c^3/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b+5/c^4/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/c^5/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c + b/x + a/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262378, size = 1, normalized size = 0.01

$$\left[\frac{6\left(b^5-5\,ab^3c+5\,a^2bc^2\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\left(3\,c^4x^4-4\,bc^3x^3+6\left(b^2c^2-ac^3\right)x^2-\right.}{12\,\sqrt{b^2-4\,ac}c^5}\right.$$

$$-\frac{12\left(b^5-5\,ab^3c+5\,a^2bc^2\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-\left(3\,c^4x^4-4\,bc^3x^3+6\left(b^2c^2-ac^3\right)x^2-12\left(b^3c-2\,abc^2\right)x+6\left(b^4\right.\right.}{12\,\sqrt{-b^2+4\,ac}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c + b/x + a/x^2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*log((b^3 - 4*a*b*c + 2*(
b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2
 - 4*a*c))/(c*x^2 + b*x + a)) + (3*c^4*x^4 - 4*b*c^3*x^3 + 6*(b^2
*c^2 - a*c^3)*x^2 - 12*(b^3*c - 2*a*b*c^2)*x + 6*(b^4 - 3*a*b^2*c
 + a^2*c^2)*log(c*x^2 + b*x + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 -
4*a*c)*c^5), -1/12*(12*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (3*c^4*x^4 - 4*b*c^
3*x^3 + 6*(b^2*c^2 - a*c^3)*x^2 - 12*(b^3*c - 2*a*b*c^2)*x + 6*(b
^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a))*sqrt(-b^2 + 4*a*c
))/(sqrt(-b^2 + 4*a*c)*c^5)]
```

Sympy [A] time = 3.93449, size = 600, normalized size = 4.08

$$-\frac{bx^3}{3c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2 - 5ab^2c + b^4\right)}{2c^5\left(4ac - b^2\right)} + \frac{a^2c^2 - 3ab^2c + b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2 - 4a^2b^2c + ab^4 - 4ac^5\left(-\frac{b\sqrt{-4ac+b^2}(5a^2c^2-5ab^2c+b^4)}{2c^5(4ac-b^2)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right) + b^2c^4\left(-\frac{b\sqrt{-4ac+b^2}}{2c}\cdots\right.}{5a^2bc^2 - 5ab^3c + b^5}\right.$$

$$+\left(\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2 - 5ab^2c + b^4\right)}{2c^5\left(4ac - b^2\right)} + \frac{a^2c^2 - 3ab^2c + b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2 - 4a^2b^2c + ab^4 - 4ac^5\left(\frac{b\sqrt{-4ac+b^2}(5a^2c^2-5ab^2c+b^4)}{2c^5(4ac-b^2)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right) + b^2c^4\left(\frac{b\sqrt{-4ac+b^2}(5a\cdots}{2c^5(4a\cdots}\right.}{5a^2bc^2 - 5ab^3c + b^5}\right.$$

$$+\frac{x^4}{4c} - \frac{x^2\left(ac - b^2\right)}{2c^3} + \frac{x\left(2abc - b^3\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**3/(3*c**2) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**
2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b
**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*
a*c**5*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/
(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5
)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c
+ b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)
/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c
 + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)
) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**
2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(b*sqrt(-4*a*c + b**2)*(5*a
**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**
2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(b*sqrt(-4*a*c + b**
2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a
**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**
3*c + b**5)) + x**4/(4*c) - x**2*(a*c - b**2)/(2*c**3) + x*(2*a*b
*c - b**3)/c**4
```

GIAC/XCAS [A] time = 0.278178, size = 196, normalized size = 1.33

$$\frac{3\,c^3x^4 - 4\,bc^2x^3 + 6\,b^2cx^2 - 6\,ac^2x^2 - 12\,b^3x + 24\,abcx}{12\,c^4} + \frac{\left(b^4 - 3\,ab^2c + a^2c^2\right)\ln\left(cx^2 + bx + a\right)}{2\,c^5} - \frac{\left(b^5 - 5\,ab^3c + 5\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c + b/x + a/x^2),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^
3*x + 24*a*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*ln(c*x^2
+ b*x + a)/c^5 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x +
b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

3.413 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

Optimal. Leaf size=118

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b
^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sq
rt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.220944, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b
^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sq
rt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\int x\,dx}{c^2}-\frac{b\left(-2ac+b^2\right)\log\left(a+bx+cx^2\right)}{2c^4}+\left(-ac+b^2\right)\int\frac{1}{c^3}\,dx+\frac{x^3}{3c}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^4\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c+a/x**2+b/x),x)
```

```
[Out] -b*Integral(x, x)/c**2 - b*(-2*a*c + b**2)*log(a + b*x + c*x**2)/
(2*c**4) + (-a*c + b**2)*Integral(c**(-3), x) + x**3/(3*c) - (2*a
```

```
**2*c**2 - 4*a*b**2*c + b**4)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**
2))/(c**4*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.143475, size = 112, normalized size = 0.95

$$\frac{\frac{6(2a^2c^2-4ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-3\left(b^3-2abc\right)\log(a+x(b+cx))+cx\left(-6ac+6b^2-3bcx+2c^2x^2\right)}{6c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] - 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

Maple [A] time = 0.005, size = 190, normalized size = 1.6

$$\frac{x^3}{3\,c}-\frac{bx^2}{2\,c^2}-\frac{ax}{c^2}+\frac{b^2x}{c^3}+\frac{\ln\left(cx^2+bx+a\right)ab}{c^3}-\frac{\ln\left(cx^2+bx+a\right)b^3}{2\,c^4}+2\,\frac{a^2}{c^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-4\,\frac{ab^2}{c^3\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^4}{c^4}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c+a/x^2+b/x),x)
```

```
[Out] 1/3*x^3/c-1/2*b*x^2/c^2-1/c^2*x*a+1/c^3*x*b^2+1/c^3*ln(c*x^2+b*x+
a)*a*b-1/2/c^4*ln(c*x^2+b*x+a)*b^3+2/c^2/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2-4/c^3/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2+1/c^4/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c + b/x + a/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280483, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)\log\left(-\frac{b^{3}-4\,abc+2\,\left(b^{2}c-4\,ac^{2}\right)x-\left(2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{2}+bx+a}\right)+\left(2\,c^{3}x^{3}-3\,bc^{2}x^{2}+6\,\left(b^{2}c-ac^{2}\right)x-3\right.}{6\,\sqrt{b^{2}-4\,ac}c^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c + b/x + a/x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*log(-(b^3 - 4*a*b*c + 2*(b^
2*c - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 -
 4*a*c))/(c*x^2 + b*x + a)) + (2*c^3*x^3 - 3*b*c^2*x^2 + 6*(b^2*c
 - a*c^2)*x - 3*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a))*sqrt(b^2 -
4*a*c))/(sqrt(b^2 - 4*a*c)*c^4), 1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*
c^2)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (2*c
^3*x^3 - 3*b*c^2*x^2 + 6*(b^2*c - a*c^2)*x - 3*(b^3 - 2*a*b*c)*lo
g(c*x^2 + b*x + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)]
```

Sympy [A] time = 3.48875, size = 496, normalized size = 4.2

$$-\frac{bx^2}{2c^2} + \left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\,(2a^2c^2-4ab^2c+b^4)}{2c^4(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}(2a^2c^2-4ab^2c+b^4)}{2c^4(4ac-b^2)}\right) - b^2c^3\left(\frac{b(2ac-b^2)}{2c^4}\right.}{2a^2c^2-4ab^2c+b^4}\right.$$

$$+\left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\,(2a^2c^2-4ab^2c+b^4)}{2c^4(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}(2a^2c^2-4ab^2c+b^4)}{2c^4(4ac-b^2)}\right) - b^2c^3\left(\frac{b(2ac-b^2)}{2c^4}\right.}{2a^2c^2-4ab^2c+b^4}\right.$$

$$+\frac{x^3}{3c} - \frac{x(ac-b^2)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**2/(2*c**2) + (b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**
2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log
(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4)
- sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(
4*a*c - b**2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*
a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b*
*2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + (b*(2*a*c - b**2)/(2*
c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*
c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b
*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a
*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c -
b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c +
b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)
) + x**3/(3*c) - x*(a*c - b**2)/c**3
```

GIAC/XCAS [A] time = 0.296501, size = 153, normalized size = 1.3

$$\frac{2\,c^2x^3 - 3\,bcx^2 + 6\,b^2x - 6\,acx}{6\,c^3} - \frac{(b^3 - 2\,abc)\ln\left(cx^2 + bx + a\right)}{2\,c^4} + \frac{(b^4 - 4\,ab^2c + 2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c + b/x + a/x^2),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 -
2*a*b*c)*ln(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*
arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.414 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.170739, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b(-3ac+b^2)\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^3\sqrt{-4ac+b^2}}+\frac{\int x\,dx}{c}-\frac{\int b\,dx}{c^2}+\frac{(-ac+b^2)\log(a+bx+cx^2)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c+a/x**2+b/x),x)
```

```
[Out] b*(-3*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**3*sq
rt(-4*a*c + b**2)) + Integral(x, x)/c - Integral(b, x)/c**2 + (-a
*c + b**2)*log(a + b*x + c*x**2)/(2*c**3)
```

Mathematica [A] time = 0.184402, size = 84, normalized size = 0.94

$$\frac{(b^2-ac)\log(a+x(b+cx))-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx(cx-2b)}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x
)])/(2*c^3)
```

Maple [A] time = 0.004, size = 132, normalized size = 1.5

$$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{\ln(cx^2+bx+a)\,a}{2c^2}+\frac{\ln(cx^2+bx+a)\,b^2}{2c^3}+3\frac{ab}{c^2\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)-\frac{b^3}{c^3}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c+a/x^2+b/x),x)
```

```
[Out] 1/2*x^2/c-b*x/c^2-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+
a)*b^2+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*a*b-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c + b/x + a/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259509, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^3-3\,abc\right)\log\left(-\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x-\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\left(c^2x^2-2\,bcx+\left(b^2-ac\right)\log\left(cx^2+bx+a\right)\right)\sqrt{b^2-4\,ac}}{2\,\sqrt{b^2-4\,ac}c^3}, -\frac{2\,\left(b^3-3\,abc\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-\left(c^2x^2-2\,bcx+\left(b^2-ac\right)\log\left(cx^2+bx+a\right)\right)\sqrt{-b^2+4\,ac}}{2\,\sqrt{-b^2+4\,ac}c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c + b/x + a/x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*
x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2
 + b*x + a)) - (c^2*x^2 - 2*b*c*x + (b^2 - a*c)*log(c*x^2 + b*x +
 a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^3), -1/2*(2*(b^3 - 3
*a*b*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (
c^2*x^2 - 2*b*c*x + (b^2 - a*c)*log(c*x^2 + b*x + a))*sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)]
```

Sympy [A] time = 2.88398, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right)+\left(\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right)+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c+a/x**2+b/x),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
 - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4
*a*c**3*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b
**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2
)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
)/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c*
*3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a
*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4
*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c
 + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2
*c**3)))/(3*a*b*c - b**3)) + x**2/(2*c)
```

GIAC/XCAS [A] time = 0.273518, size = 116, normalized size = 1.3

$$\frac{cx^2 - 2\,bx}{2\,c^2} + \frac{\left(b^2 - ac\right)\ln\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c + b/x + a/x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*ln(c*x^2 + b*x + a)/c^3
 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-
b^2 + 4*a*c)*c^3)
```

3.415 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}} dx$

Optimal. Leaf size=70

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}+\frac{x}{c}$$

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.105374, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi in Sympy [A] time = 21.1567, size = 65, normalized size = 0.93

$$-\frac{b\log(a+bx+cx^2)}{2c^2}+\frac{x}{c}-\frac{(-2ac+b^2)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x),x)
```

```
[Out] -b*log(a + b*x + c*x**2)/(2*c**2) + x/c - (-2*a*c + b**2)*atanh((
b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.103111, size = 73, normalized size = 1.04

$$\frac{(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2
*Sqrt[-b^2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] time = 0.004, size = 101, normalized size = 1.4

$$\frac{x}{c}-\frac{b\ln\left(cx^2+bx+a\right)}{2\,c^2}-2\,\frac{a}{c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^2}{c^2}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x),x)
```

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x + a/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280536, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\sqrt{b^2-4\,ac}\left(2\,cx-b\log\left(cx^2+bx+a\right)\right)}{2\,\sqrt{b^2-4\,ac}c^2}, \frac{2\,\left(b^2-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x + a/x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x +
 (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 +
b*x + a)) - sqrt(b^2 - 4*a*c)*(2*c*x - b*log(c*x^2 + b*x + a)))/(
sqrt(b^2 - 4*a*c)*c^2), 1/2*(2*(b^2 - 2*a*c)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(2*c*x - b
*log(c*x^2 + b*x + a)))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 2.37063, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c
- b**2)))*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2
) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2))))/
```

```
(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2))) +
b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*
(4*a*c - b**2))))/(2*a*c - b**2)) + x/c
```

GIAC/XCAS [A] time = 0.290691, size = 90, normalized size = 1.29

$$\frac{x}{c} - \frac{b\ln\left(cx^2 + bx + a\right)}{2\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x + a/x^2),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*ln(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x
 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.416 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])
+ Log[a + b*x + c*x^2]/(2*c)
```

Rubi [A] time = 0.0778973, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])
+ Log[a + b*x + c*x^2]/(2*c)
```

Rubi in Sympy [A] time = 17.7413, size = 49, normalized size = 0.88

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c\sqrt{-4ac+b^2}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)/x,x)
```

```
[Out] b*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c*sqrt(-4*a*c + b**2))
+ log(a + b*x + c*x**2)/(2*c)
```

Mathematica [A] time = 0.0553859, size = 57, normalized size = 1.02

$$\frac{\log(a+x(b+cx))-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 + Log[a + x*(b + c*x)])/(2*c)
```

Maple [A] time = 0.003, size = 56, normalized size = 1.

$$\frac{\ln\left(cx^2+bx+a\right)}{2c}-\frac{b}{c}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)/x,x)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268462, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x+(2\,c^2x^2+2\,bcx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\sqrt{b^2-4\,ac}\log\left(cx^2+bx+a\right)}{2\,\sqrt{b^2-4\,ac}c},\; -\frac{2\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\log\left(cx^2+bx+a\right)}{2\,\sqrt{-b^2+4\,ac}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 +
 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + s
qrt(b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(sqrt(b^2 - 4*a*c)*c), -1/
2*(2*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - sq
rt(-b^2 + 4*a*c)*log(c*x^2 + b*x + a))/(sqrt(-b^2 + 4*a*c)*c)]
```

Sympy [A] time = 0.909219, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)/x,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (
-4*a*c*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) +
2*a + b**2*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)
))/b) + (b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*lo
g(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*
c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/
(2*c)))/b)
```

GIAC/XCAS [A] time = 0.267124, size = 74, normalized size = 1.32

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}+\frac{\ln\left(cx^2+bx+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
+ 1/2*ln(c*x^2 + b*x + a)/c
```

3.417 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0642011, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)*x^2),x]

[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 11.2003, size = 32, normalized size = 0.89

$$\frac{2\,\mathrm{atanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)/x**2,x)

[Out] 2*atanh((2*a/x + b)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
```

Mathematica [A] time = 0.0108532, size = 38, normalized size = 1.06

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)*x^2),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.001, size = 35, normalized size = 1.

$$2\,\frac{1}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)/x^2,x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269253, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x-(2\,c^2x^2+2\,bcx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}},\ \frac{2\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a))/sqrt(b^2
- 4*a*c), 2*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 0.54815, size = 124, normalized size = 3.44

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)/x**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2))
+ b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**
2))*log(x + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c
- b**2)) + b)/(2*c))
```

GIAC/XCAS [A] time = 0.286537, size = 46, normalized size = 1.28

$$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.418 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c])
+ Log[x]/a - Log[a + b*x + c*x^2]/(2*a)
```

Rubi [A] time = 0.106115, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c])
+ Log[x]/a - Log[a + b*x + c*x^2]/(2*a)
```

Rubi in Sympy [A] time = 24.8998, size = 54, normalized size = 0.87

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a\sqrt{-4ac+b^2}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)/x**3,x)
```

```
[Out] b*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a*sqrt(-4*a*c + b**2))
+ log(x)/a - log(a + b*x + c*x**2)/(2*a)
```

Mathematica [A] time = 0.112243, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log(a+x(b+cx))-2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] -((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*Log[x] + Log[a + x*(b + c*x)])/(2*a)
```

Maple [A] time = 0.007, size = 62, normalized size = 1.

$$\frac{\ln(x)}{a}-\frac{\ln\left(cx^2+bx+a\right)}{2\,a}-\frac{b}{a}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)/x^3,x)
```

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282766, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x+(2\,c^2x^2+2\,bcx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\sqrt{b^2-4\,ac}\left(\log\left(cx^2+bx+a\right)-2\,\log\left(x\right)\right)}{2\,\sqrt{b^2-4\,ac}a},\right.$$
$$\left.-\frac{2\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(\log\left(cx^2+bx+a\right)-2\,\log\left(x\right)\right)}{2\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 +
 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) - s
qrt(b^2 - 4*a*c)*(log(c*x^2 + b*x + a) - 2*log(x)))/(sqrt(b^2 - 4
*a*c)*a), -1/2*(2*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 -
 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*x^2 + b*x + a) - 2*log(x)))/
(sqrt(-b^2 + 4*a*c)*a)]
```

Sympy [A] time = 6.62289, size = 564, normalized size = 9.1

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(-\frac{b\sqrt{-4ac+}}{2a(4ac-}\right.}{9abc^2-2b^3c}\right.$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right.}{9abc^2-2b^3c}\right.$$
$$+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)/x**3,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (
24*a**4*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*
```

```
a))**2 - 14*a**3*b**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**
2)) - 1/(2*a))**2 - 12*a**3*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*
a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-b*sqrt(-4*a*c + b**2)/(2*
a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-b*sqrt(-4*a*c +
 b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2
*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*
c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*
sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*
c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a
**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**
2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1
/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

GIAC/XCAS [A] time = 0.305839, size = 84, normalized size = 1.35

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a}-\frac{\ln\left(cx^2+bx+a\right)}{2\,a}+\frac{\ln\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^3),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
- 1/2*ln(c*x^2 + b*x + a)/a + ln(abs(x))/a
```

3.419 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*
x^2])/(2*a^2)
```

Rubi [A] time = 0.221876, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*
x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 40.3917, size = 75, normalized size = 0.93

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{(-2ac+b^2)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)/x**4,x)
```

```
[Out] -1/(a*x) - b*log(x)/a**2 + b*log(a + b*x + c*x**2)/(2*a**2) - (-2
*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**2*sqrt(-4
*a*c + b**2))
```

Mathematica [A] time = 0.141913, size = 77, normalized size = 0.95

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log(a+x(b+cx))-\frac{2a}{x}-2b\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*
a^2)
```

Maple [A] time = 0.01, size = 112, normalized size = 1.4

$$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(cx^2+bx+a)}{2a^2}-2\frac{c}{a\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)+\frac{b^2}{a^2}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)/x^4,x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-2/a/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277599, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)x\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\left(bx\log\left(cx^2+bx+a\right)-2\,bx\log\left(x\right)-2\,a\right)\sqrt{b^2-}}{2\,\sqrt{b^2-4\,ac}a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*x*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x
 + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2
+ b*x + a)) - (b*x*log(c*x^2 + b*x + a) - 2*b*x*log(x) - 2*a)*sqr
t(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x), 1/2*(2*(b^2 - 2*a*c)*x
*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b*x*log
(c*x^2 + b*x + a) - 2*b*x*log(x) - 2*a)*sqrt(-b^2 + 4*a*c))/(sqrt
(-b^2 + 4*a*c)*a^2*x)]
```

Sympy [A] time = 12.3633, size = 862, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)/x**4,x)
```

```
[Out] (b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c -
 b**2)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**
2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b
/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b
**2)))**2 - 4*a**5*c**3*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**5*(b/(2*a**2) - sqrt
(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 3*a*
*4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*
a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25
*a**2*b**3*c**2 - 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b*
*2*c**3 - 12*a*b**4*c**2 + 2*b**6*c)) + (b/(2*a**2) + sqrt(-4*a*c
 + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*log(x + (-28*a**
6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2
*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(
b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c -
b**2))) - 2*a**4*b**5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c -
```

```
b**2)/(2*a**2*(4*a*c - b**2)))**2 - 3*a**4*b**2*c**2*(b/(2*a**2)
+ sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) + a
**3*b**4*c*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a*
*2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14*a*b*
*5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2
+ 2*b**6*c)) - 1/(a*x) - b*log(x)/a**2
```

GIAC/XCAS [A] time = 0.315963, size = 107, normalized size = 1.32

$$\frac{b\ln\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\ln\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^4),x, algorithm="giac")
```

```
[Out] 1/2*b*ln(c*x^2 + b*x + a)/a^2 - b*ln(abs(x))/a^2 + (b^2 - 2*a*c)*
arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) -
 1/(a*x)
```

3.420 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

Optimal. Leaf size=104

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])
/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rubi [A] time = 0.325958, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])
/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 49.4208, size = 97, normalized size = 0.93

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(-3ac+b^2)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^3\sqrt{-4ac+b^2}}+\frac{(-ac+b^2)\log(x)}{a^3}-\frac{(-ac+b^2)\log(a+bx+cx^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)/x**5,x)
```

```
[Out] -1/(2*a*x**2) + b/(a**2*x) + b*(-3*a*c + b**2)*atanh((b + 2*c*x)/
sqrt(-4*a*c + b**2))/(a**3*sqrt(-4*a*c + b**2)) + (-a*c + b**2)*l
og(x)/a**3 - (-a*c + b**2)*log(a + b*x + c*x**2)/(2*a**3)
```

Mathematica [A] time = 0.251331, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2}+2\log(x)\left(b^2-ac\right)+\left(ac-b^2\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (
-b^2 + a*c)*Log[a + x*(b + c*x)])/(2*a^3)
```

Maple [A] time = 0.012, size = 150, normalized size = 1.4

$$-\frac{1}{2\,ax^2}-\frac{\ln(x)\,c}{a^2}+\frac{\ln(x)\,b^2}{a^3}+\frac{b}{a^2x}+\frac{c\ln\left(cx^2+bx+a\right)}{2\,a^2}-\frac{\ln\left(cx^2+bx+a\right)\,b^2}{2\,a^3}+3\,\frac{bc}{a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-\frac{b^3}{a^3}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)/x^5,x)
```

```
[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+b/a^2/x+1/2/a^2*c*ln(c*x
^2+b*x+a)-1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.308412, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^{3}-3\,abc\right)x^{2}\log\left(-\frac{b^{3}-4\,abc+2\,\left(b^{2}c-4\,ac^{2}\right)x-\left(2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{2}+bx+a}\right)+\left(\left(b^{2}-ac\right)x^{2}\log\left(cx^{2}+bx+a\right)-2\,\left(b^{2}-ac\right)\right.}{2\,\sqrt{b^{2}-4\,ac}a^{3}x^{2}}\right.$$

$$-\frac{2\,\left(b^{3}-3\,abc\right)x^{2}\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)+\left(\left(b^{2}-ac\right)x^{2}\log\left(cx^{2}+bx+a\right)-2\,\left(b^{2}-ac\right)x^{2}\log\left(x\right)-2\,abx+a^{2}\right)\sqrt{-b^{2}}}{2\,\sqrt{-b^{2}+4\,ac}a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*x^2*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c
*x^2 + b*x + a)) + ((b^2 - a*c)*x^2*log(c*x^2 + b*x + a) - 2*(b^2
 - a*c)*x^2*log(x) - 2*a*b*x + a^2)*sqrt(b^2 - 4*a*c))/(sqrt(b^2
- 4*a*c)*a^3*x^2), -1/2*(2*(b^3 - 3*a*b*c)*x^2*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2 - a*c)*x^2*log(c*x^2
+ b*x + a) - 2*(b^2 - a*c)*x^2*log(x) - 2*a*b*x + a^2)*sqrt(-b^2
+ 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3*x^2)]
```

Sympy [A] time = 17.8016, size = 1525, normalized size = 14.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)/x**5,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) +
(a*c - b**2)/(2*a**3))*log(x + (24*a**9*c**3*(-b*sqrt(-4*a*c + b*
*2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3
))**2 - 42*a**8*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 17*a**7*b**
4*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)
) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*c**4*(-b*sqrt(-4*a*c + b*
*2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3
)) - 2*a**6*b**6*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(
4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 - 15*a**6*b**2*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a
*c - b**2)/(2*a**3)) + 7*a**5*b**4*c**2*(-b*sqrt(-4*a*c + b**2)*(
```

```
3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) -
12*a**5*c**5 - a**4*b**6*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)
/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 63*a**4*b**2*
c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2 - 20*a*b**8*c + 2*b
**10)/(27*a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 1
8*a*b**7*c**2 + 2*b**9*c)) + (b*sqrt(-4*a*c + b**2)*(3*a*c - b**2
)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))*log(x + (24*a*
*9*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b*
*2)) + (a*c - b**2)/(2*a**3))**2 - 42*a**8*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/
(2*a**3))**2 + 17*a**7*b**4*c*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**
2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*
c**4*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)
) + (a*c - b**2)/(2*a**3)) - 2*a**6*b**6*(b*sqrt(-4*a*c + b**2)*(
3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2
 - 15*a**6*b**2*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**
3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 7*a**5*b**4*c**2*(b*
sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c
 - b**2)/(2*a**3)) - 12*a**5*c**5 - a**4*b**6*c*(b*sqrt(-4*a*c +
b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a*
*3)) + 63*a**4*b**2*c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2
 - 20*a*b**8*c + 2*b**10)/(27*a**4*b*c**5 - 63*a**3*b**3*c**4 + 5
4*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b**9*c)) + (-a + 2*b*x)/(2*
a**2*x**2) - (a*c - b**2)*log(x + (-12*a**5*c**5 + 63*a**4*b**2*c
**4 - 12*a**4*c**4*(a*c - b**2) - 103*a**3*b**4*c**3 + 15*a**3*b*
*2*c**3*(a*c - b**2) + 24*a**3*c**3*(a*c - b**2)**2 + 70*a**2*b**
6*c**2 - 7*a**2*b**4*c**2*(a*c - b**2) - 42*a**2*b**2*c**2*(a*c -
 b**2)**2 - 20*a*b**8*c + a*b**6*c*(a*c - b**2) + 17*a*b**4*c*(a*
c - b**2)**2 + 2*b**10 - 2*b**6*(a*c - b**2)**2)/(27*a**4*b*c**5
- 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b**9
*c))/a**3
```

GIAC/XCAS [A] time = 0.294643, size = 142, normalized size = 1.37

$$-\frac{\left(b^2-ac\right)\ln\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\left(b^2-ac\right)\ln\left(|x|\right)}{a^3}-\frac{\left(b^3-3\,abc\right)\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^5),x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*ln(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*ln(abs(x))
/a^3 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

3.421 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

Optimal. Leaf size=137

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*
b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*
c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rubi [A] time = 0.397995, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*
b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*
c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rubi in Sympy [A] time = 62.5124, size = 129, normalized size = 0.94

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{-ac+b^2}{a^3x}-\frac{b\left(-2ac+b^2\right)\log(x)}{a^4}+\frac{b\left(-2ac+b^2\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^4\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)/x**6,x)
```

```
[Out] -1/(3*a*x**3) + b/(2*a**2*x**2) - (-a*c + b**2)/(a**3*x) - b*(-2*
a*c + b**2)*log(x)/a**4 + b*(-2*a*c + b**2)*log(a + b*x + c*x**2)
/(2*a**4) - (2*a**2*c**2 - 4*a*b**2*c + b**4)*atanh((b + 2*c*x)/s
qrt(-4*a*c + b**2))/(a**4*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.17301, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3}+\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3a^2b}{x^2}-6\log(x)\left(b^3-2abc\right)+3\left(b^3-2abc\right)\log(a+x(b+cx))+\frac{6a\left(ac-b^2\right)}{x}}{6a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 -
4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sq
rt[-b^2 + 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*L
og[a + x*(b + c*x)])/(6*a^4)
```

Maple [A] time = 0.013, size = 214, normalized size = 1.6

$$-\frac{1}{3\,ax^3}+\frac{c}{a^2x}-\frac{b^2}{a^3x}+2\,\frac{b\ln(x)\,c}{a^3}-\frac{b^3\ln(x)}{a^4}+\frac{b}{2\,a^2x^2}-\frac{c\ln\left(cx^2+bx+a\right)b}{a^3}+\frac{\ln\left(cx^2+bx+a\right)b^3}{2\,a^4}+2\,\frac{c^2}{a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-4\,\frac{b^2c}{a^3\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^4}{a^4}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)/x^6,x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/
2*b/a^2/x^2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3
+2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-
4/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c
+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.331939, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)x^{3}\log\left(-\frac{b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)x-\left(2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{2}+bx+a}\right)+\left(3\left(b^{3}-2\,abc\right)x^{3}\log\left(cx^{2}+bx+a\right)\right.}{6\sqrt{b^{2}-4\,ac}a^{4}x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^6),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*log(-(b^3 - 4*a*b*c + 2
*(b^2*c - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b
^2 - 4*a*c))/(c*x^2 + b*x + a)) + (3*(b^3 - 2*a*b*c)*x^3*log(c*x^
2 + b*x + a) - 6*(b^3 - 2*a*b*c)*x^3*log(x) + 3*a^2*b*x - 2*a^3 -
 6*(a*b^2 - a^2*c)*x^2)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^4
*x^3), 1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (3*(b^3 - 2*a*b*c)*x^3*log
(c*x^2 + b*x + a) - 6*(b^3 - 2*a*b*c)*x^3*log(x) + 3*a^2*b*x - 2*
a^3 - 6*(a*b^2 - a^2*c)*x^2)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*a^4*x^3)]
```

Sympy [A] time = 28.619, size = 2105, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)/x**6,x)
```

```
[Out] (-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 -
4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*log(x + (-52*a**11*b*
```

c**3*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c*
*2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 57*a**10*b*
*3*c**2*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2
*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 - 19*a**9*
b**5*c*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*
c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 4*a**9*c*
5(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2
 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 2*a**8*b**7*(-b*
(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*
b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 23*a**8*b**2*c**4*(-
b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 26*a**7*b**4*c**3*(-b
*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a
*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 9*a**6*b**6*c**2*(-b*(
2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b
2*c + b4)/(2*a**4*(4*a*c - b**2))) - 8*a**6*b*c**6 - a**5*b**
8*c*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**
2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 166*a**5*b**3*c
5 - 361*a4*b**5*c**4 + 312*a**3*b**7*c**3 - 130*a**2*b**9*c**
2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c**7 + 60*a**5*b**2*c**6 - 20
7*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a
*b**10*c**2 - 2*b**12*c)) + (-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4
*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b
2)))*log(x + (-52*a11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*
c - b**2)))**2 + 57*a**10*b**3*c**2*(-b*(2*a*c - b**2)/(2*a**4) +
 sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4
*a*c - b**2)))**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) +
sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*
a*c - b**2)))**2 + 4*a**9*c**5*(-b*(2*a*c - b**2)/(2*a**4) + sqrt
(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c
- b**2))) + 2*a**8*b**7*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c
 + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)
))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a
*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**
2))) - 26*a**7*b**4*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*
c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2
))) + 9*a**6*b**6*c**2*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c
+ b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))
) - 8*a**6*b*c**6 - a**5*b**8*c*(-b*(2*a*c - b**2)/(2*a**4) + sqr
t(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c
 - b**2))) + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 + 312*a**3*b
7*c3 - 130*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c
7 + 60*a5*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4
 - 108*a**2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c)) + (-2*a**2
+ 3*a*b*x + x**2*(6*a*c - 6*b**2))/(6*a**3*x**3) + b*(2*a*c - b**
2)*log(x + (-8*a**6*b*c**6 + 166*a**5*b**3*c**5 + 4*a**5*b*c**5*(
2*a*c - b**2) - 361*a**4*b**5*c**4 + 23*a**4*b**3*c**4*(2*a*c - b
2) + 312*a3*b**7*c**3 - 26*a**3*b**5*c**3*(2*a*c - b**2) - 52
*a**3*b**3*c**3*(2*a*c - b**2)**2 - 130*a**2*b**9*c**2 + 9*a**2*b
7*c2*(2*a*c - b**2) + 57*a**2*b**5*c**2*(2*a*c - b**2)**2 + 2
6*a*b**11*c - a*b**9*c*(2*a*c - b**2) - 19*a*b**7*c*(2*a*c - b**2
)**2 - 2*b**13 + 2*b**9*(2*a*c - b**2)**2)/(2*a**6*c**7 + 60*a**5
*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b

```
**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c))/a**4
```

GIAC/XCAS [A] time = 0.269459, size = 184, normalized size = 1.34

$$\frac{(b^3 - 2\,abc)\ln(cx^2 + bx + a)}{2\,a^4} - \frac{(b^3 - 2\,abc)\ln(|x|)}{a^4} + \frac{(b^4 - 4\,ab^2c + 2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}a^4} + \frac{3\,a^2bx - 2\,a^3 - 6\,(ab^2 - a^2c)\,x^2}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)*x^6),x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*ln(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*ln(
abs(x))/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a
^3 - 6*(a*b^2 - a^2*c)*x^2)/(a^4*x^3)
```

3.422 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} dx$

Optimal. Leaf size=196

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4} - \frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*
x^2)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*
a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^
2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b
^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^
4)
```

Rubi [A] time = 0.439333, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4} - \frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*
x^2)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*
a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^
2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b
^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^
4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^3}{c\,(-4ac+b^2)}+\frac{b\,(30a^2c^2-20ab^2c+3b^4)\,\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^4\,(-4ac+b^2)^{\frac{3}{2}}}+\frac{x^4\,(2a+bx)}{(-4ac+b^2)\,(a+bx+cx^2)}+\frac{(-8ac+3b^2)\int x\,dx}{c^2\,(-4ac+b^2)}-\frac{(-11ac+3b^2)\int b\,dx}{c^3\,(-4ac+b^2)}+\frac{(-2ac+3b^2)\log\left(a+bx+cx^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c+a/x**2+b/x)**2,x)
```

```
[Out] -b*x**3/(c*(-4*a*c + b**2)) + b*(30*a**2*c**2 - 20*a*b**2*c + 3*b
**4)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**4*(-4*a*c + b**2)
**(3/2)) + x**4*(2*a + b*x)/((-4*a*c + b**2)*(a + b*x + c*x**2))
+ (-8*a*c + 3*b**2)*Integral(x, x)/(c**2*(-4*a*c + b**2)) - (-11*
a*c + 3*b**2)*Integral(b, x)/(c**3*(-4*a*c + b**2)) + (-2*a*c + 3
*b**2)*log(a + b*x + c*x**2)/(2*c**4)
```

Mathematica [A] time = 0.349643, size = 163, normalized size = 0.83

$$\frac{\frac{2b(30a^2c^2-20ab^2c+3b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\frac{2(2a^3c^2+a^2bc(5cx-4b)+ab^3(b-5cx)+b^5x)}{(b^2-4ac)(a+x(b+cx))}+\left(3b^2-2ac\right)\log(a+x(b+cx))-4bcx+c^2x^2}{2c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] (-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) +
 a^2*b*c*(-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*
b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
+ 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (3*b^2 - 2*a*c)*Log[a + x*(b +
c*x)])/(2*c^4)
```

Maple [B] time = 0.019, size = 662, normalized size = 3.4

$$\frac{x^2}{2c^2}-2\frac{bx}{c^3}-5\frac{bxa^2}{c^2(cx^2+bx+a)(4ac-b^2)}+5\frac{b^3xa}{c^3(cx^2+bx+a)(4ac-b^2)}-\frac{b^5x}{c^4(cx^2+bx+a)(4ac-b^2)}-2\frac{a^3}{c^2(cx^2+bx+a)(4ac-b^2)}+4\frac{a^2b^2}{c^3(cx^2+bx+a)(4ac-b^2)}-\frac{ab^4}{c^4(cx^2+bx+a)(4ac-b^2)}-4\frac{\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)a^2}{(4ac-b^2)c^2}+7\frac{\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)ab^2}{c^3(4ac-b^2)}-\frac{3\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)b^4}{2c^4(4ac-b^2)}+30\frac{a^2b}{c^2\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)-20\frac{ab^3}{c^3\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)+3\frac{b^5}{c^4\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c+a/x^2+b/x)^2,x)
```

```
[Out] 1/2*x^2/c^2-2/c^3*x*b-5/c^2/(c*x^2+b*x+a)*b/(4*a*c-b^2)*x*a^2+5/c
^3/(c*x^2+b*x+a)*b^3/(4*a*c-b^2)*x*a-1/c^4/(c*x^2+b*x+a)*b^5/(4*a
*c-b^2)*x-2/c^2/(c*x^2+b*x+a)*a^3/(4*a*c-b^2)+4/c^3/(c*x^2+b*x+a)
*a^2/(4*a*c-b^2)*b^2-1/c^4/(c*x^2+b*x+a)*a/(4*a*c-b^2)*b^4-4/c^2/
(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^2+b*x+a))*a^2+7/c^3/(4*a*c-b^2)*l
n((4*a*c-b^2)*(c*x^2+b*x+a))*a*b^2-3/2/c^4/(4*a*c-b^2)*ln((4*a*c-
b^2)*(c*x^2+b*x+a))*b^4+30/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^
4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a^2*b-20/c^3/(64*a^3*c^3
-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(
4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a
*b^3+3/c^4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arcta
n((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12
*a*b^4*c-b^6)^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c + b/x + a/x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270392, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c + b/x + a/x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b
^3*c^2 + 30*a^2*b*c^3)*x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2
)*x)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) - (2*
a*b^4 - 8*a^2*b^2*c + 4*a^3*c^2 + (b^2*c^3 - 4*a*c^4)*x^4 - 3*(b^
3*c^2 - 4*a*b*c^3)*x^3 - (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*x^2
 + 2*(b^5 - 7*a*b^3*c + 13*a^2*b*c^2)*x + (3*a*b^4 - 14*a^2*b^2*c
 + 8*a^3*c^2 + (3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^2 + (3*b^5
- 14*a*b^3*c + 8*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))*sqrt(b^2 - 4
*a*c))/((a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c
^4 - 4*a*b*c^5)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(3*a*b^5 - 20*a^2*
b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^
2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*a*b^4 - 8*a^2*b^2*c + 4*a^
3*c^2 + (b^2*c^3 - 4*a*c^4)*x^4 - 3*(b^3*c^2 - 4*a*b*c^3)*x^3 - (
4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*x^2 + 2*(b^5 - 7*a*b^3*c + 13
*a^2*b*c^2)*x + (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2 + (3*b^4*c -
14*a*b^2*c^2 + 8*a^2*c^3)*x^2 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2
)*x)*log(c*x^2 + b*x + a))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^4 - 4*a^
2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)*sqrt(-
b^2 + 4*a*c))]
```

Sympy [A] time = 7.77928, size = 1012, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c+a/x**2+b/x)**2,x)
```

```
[Out] -2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b
**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**
2 - 17*a**2*b**2*c + 16*a**2*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(3
0*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a*
```

```
*2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))
+ 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*
c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*
c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3
*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6)) - (2*a*c - 3*b**2)/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*
c + 3*b**5)) + (b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b
**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**
2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30
*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) +
 3*a*b**4 - 8*a*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c*
*2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c*
*3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6)) - (2*a*c - 3*b**2)/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c +
 3*b**5)) - (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 + x*(5*a**2*b*c
**2 - 5*a*b**3*c + b**5))/(4*a**2*c**5 - a*b**2*c**4 + x**2*(4*a*
c**6 - b**2*c**5) + x*(4*a*b*c**5 - b**3*c**4)) + x**2/(2*c**2)
```

GIAC/XCAS [A] time = 0.291716, size = 254, normalized size = 1.3

$$-\frac{\left(3\,b^{5}-20\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{(b^{2}c^{4}-4\,ac^{5})\sqrt{-b^{2}+4\,ac}}+\frac{\left(3\,b^{2}-2\,ac\right)\ln\left(cx^{2}+bx+a\right)}{2\,c^{4}}+\frac{c^{2}x^{2}-4\,bcx}{2\,c^{4}}+\frac{ab^{4}-4\,a^{2}b^{2}c+2\,a^{3}c^{2}+\left(b^{5}-5\,ab^{3}c+5\,a^{2}bc^{2}\right)x}{(cx^{2}+bx+a)(b^{2}-4\,ac)c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c + b/x + a/x^2)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2
- 2*a*c)*ln(c*x^2 + b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 +
(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^4)
```

3.423 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c
)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^
4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3
```

Rubi [A] time = 0.307181, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c
)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^
4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2b\int x\,dx}{c\left(-4ac+b^2\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}+\frac{x^3\left(2a+bx\right)}{\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}+\frac{2x\left(-3ac+b^2\right)}{c^2\left(-4ac+b^2\right)}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^3\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2,x)
```

```
[Out] -2*b*Integral(x, x)/(c*(-4*a*c + b**2)) - b*log(a + b*x + c*x**2)
/c**3 + x**3*(2*a + b*x)/((-4*a*c + b**2)*(a + b*x + c*x**2)) + 2
*x*(-3*a*c + b**2)/(c**2*(-4*a*c + b**2)) - 2*(6*a**2*c**2 - 6*a*
b**2*c + b**4)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**3*(-4*a
*c + b**2)**(3/2))
```

Mathematica [A] time = 0.299002, size = 132, normalized size = 0.88

$$\frac{-\frac{2(6a^2c^2-6ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{(b^2-4ac)(a+x(b+cx))}-b\log(a+x(b+cx))+cx}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2
 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*A
rcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*L
og[a + x*(b + c*x)])/c^3
```

Maple [B] time = 0.012, size = 569, normalized size = 3.8

$$\frac{x}{c^2}+2\frac{a^2x}{c(cx^2+bx+a)(4ac-b^2)}-4\frac{axb^2}{c^2(cx^2+bx+a)(4ac-b^2)}+\frac{xb^4}{c^3(cx^2+bx+a)(4ac-b^2)}-3\frac{a^2b}{c^2(cx^2+bx+a)(4ac-b^2)}+\frac{ab^3}{c^3(cx^2+bx+a)(4ac-b^2)}-4\frac{\ln\left((4ac-b^2)(cx^2+bx+a)\right)ab}{(4ac-b^2)c^2}+\frac{\ln\left((4ac-b^2)(cx^2+bx+a)\right)b^3}{c^3(4ac-b^2)}-12\frac{a^2}{c\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2(4ac-b^2)cx+(4ac-b^2)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)+12\frac{ab^2}{c^2\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2(4ac-b^2)cx+(4ac-b^2)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)-2\frac{b^4}{c^3\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2(4ac-b^2)cx+(4ac-b^2)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2,x)
```

```
[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*
a*c-b^2)*x*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x
^2+b*x+a)*a^2*b/(4*a*c-b^2)+1/c^3/(c*x^2+b*x+a)*a*b^3/(4*a*c-b^2)
-4/c^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^2+b*x+a))*a*b+1/c^3/(4*a*c
-b^2)*ln((4*a*c-b^2)*(c*x^2+b*x+a))*b^3-12/c/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2
)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a^2+12/c^2
/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*
c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)^(1/2))*a*b^2-2/c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + b/x + a/x^2)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271959, size = 1, normalized size = 0.01

$$\left[-\frac{\left(ab^4 - 6\,a^2b^2c + 6\,a^3c^2 + \left(b^4c - 6\,ab^2c^2 + 6\,a^2c^3\right)x^2 + \left(b^5 - 6\,ab^3c + 6\,a^2bc^2\right)x\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b\right.}{cx^2+bx+a}\right.}{(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + b/x + a/x^2)^(-2),x, algorithm="fricas")
```

```
[Out] [-((a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^
2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x)*log((b^3 - 4*a*b*
c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*s
qrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (a*b^3 - 3*a^2*b*c - (b^2*
c^2 - 4*a*c^3)*x^3 - (b^3*c - 4*a*b*c^2)*x^2 + (b^4 - 5*a*b^2*c +
 6*a^2*c^2)*x + (a*b^3 - 4*a^2*b*c + (b^3*c - 4*a*b*c^2)*x^2 + (b
^4 - 4*a*b^2*c)*x)*log(c*x^2 + b*x + a))*sqrt(b^2 - 4*a*c))/((a*b
^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c
```

```
^4)*x)*sqrt(b^2 - 4*a*c)), (2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 +
(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*
b*c^2)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 (a*b^3 - 3*a^2*b*c - (b^2*c^2 - 4*a*c^3)*x^3 - (b^3*c - 4*a*b*c^
2)*x^2 + (b^4 - 5*a*b^2*c + 6*a^2*c^2)*x + (a*b^3 - 4*a^2*b*c + (
b^3*c - 4*a*b*c^2)*x^2 + (b^4 - 4*a*b^2*c)*x)*log(c*x^2 + b*x + a
))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c
^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 6.1305, size = 842, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b
**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c
- b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a*b**
2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**
2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)
) + (-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c
 + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*(-b/c**3 + sqrt(-(4*
a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6))) - b**4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b
**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4*a*b**2*c + b**
4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4) + x*
(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

GIAC/XCAS [A] time = 0.261878, size = 217, normalized size = 1.45

$$\frac{2\left(b^{4}-6\,ab^{2}c+6\,a^{2}c^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{(b^{2}c^{3}-4\,ac^{4})\sqrt{-b^{2}+4\,ac}}+\frac{x}{c^{2}}-\frac{b\ln\left(cx^{2}+bx+a\right)}{c^{3}}-\frac{\frac{\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)x}{c}+\frac{ab^{3}-3\,a^{2}bc}{c}}{(cx^{2}+bx+a)(b^{2}-4\,ac)c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + b/x + a/x^2)^(-2),x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*ln(c*x
^2 + b*x + a)/c^3 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 -
 3*a^2*b*c)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.424 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a
+ b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2
)
```

Rubi [A] time = 0.214693, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a
+ b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2
)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^2\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x^2\left(2a+bx\right)}{\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}-\frac{\int b\,dx}{c\left(-4ac+b^2\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2/x,x)
```

```
[Out] b*(-6*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**2*(-
4*a*c + b**2)**(3/2)) + x**2*(2*a + b*x)/((-4*a*c + b**2)*(a + b*
```

```
x + c*x**2)) - Integral(b, x)/(c*(-4*a*c + b**2)) + log(a + b*x +
 c*x**2)/(2*c**2)
```

Mathematica [A] time = 0.225686, size = 109, normalized size = 0.96

$$\frac{\frac{2(-2a^2c+ab(b-3cx)+b^3x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [B] time = 0.017, size = 330, normalized size = 2.9

$$\frac{1}{cx^2+bx+a}\left(\frac{b\left(3\,ac-b^2\right)x}{\left(4\,ac-b^2\right)c^2}+\frac{a\left(2\,ac-b^2\right)}{\left(4\,ac-b^2\right)c^2}\right)+\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)}{2\,c^2}$$
$$-6\,\frac{ab}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$+\frac{b^3}{c}\arctan\left(\left(2\,c^2\left(4\,ac-b^2\right)x+c\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2/x,x)
```

```
[Out] (b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(
c*x^2+b*x+a)+1/2/c^2*ln(c*(4*a*c-b^2)*(c*x^2+b*x+a))-6/(64*a^3*c^
5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c
-b^2)*x+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-
b^6*c^2)^(1/2))*a*b+1/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6
*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2)*x+c*(4*a*c-b^2)*b)/(64*a^3*
c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2))*b^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261208, size = 1, normalized size = 0.01

$$\left[\frac{\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right) + \left(2\,ab^2 - 4\,a^2\right.}{2\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^2 + (b^3c^2}\right.$$

$$-\frac{2\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) - \left(2\,ab^2 - 4\,a^2c + 2\left(b^3 - 3\,abc\right)x + \left(ab^2\right.\right.}{2\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^2 + (b^3c^2 - 4\,abc^3)x)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b
^2*c)*x)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) +
(2*a*b^2 - 4*a^2*c + 2*(b^3 - 3*a*b*c)*x + (a*b^2 - 4*a^2*c + (b^
2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(c*x^2 + b*x + a))*sqr
t(b^2 - 4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2
 + (b^3*c^2 - 4*a*b*c^3)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(a*b^3 -
6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*a*b^2 - 4*a^
2*c + 2*(b^3 - 3*a*b*c)*x + (a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*
x^2 + (b^3 - 4*a*b*c)*x)*log(c*x^2 + b*x + a))*sqrt(-b^2 + 4*a*c)
)/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 -
4*a*b*c^3)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 4.53166, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-(4ac-b^2)^3}\,(6ac-b^2)}{2c^2\,(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2}{6abc-b^3}\right)$$
$$+\left(\frac{b\sqrt{-(4ac-b^2)^3}\,(6ac-b^2)}{2c^2\,(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b}{6abc-b^3}\right)$$
$$+\frac{2a^2c-ab^2+x\,(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2\,(4ac^4-b^2c^3)+x\,(4abc^3-b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x +
 (-16*a**2*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c*
*2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(
2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*
(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*a*c -
 b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*s
qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16
*a**2*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2
)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c -
a*b**2 + x*(3*a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4
*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

GIAC/XCAS [A] time = 0.269174, size = 169, normalized size = 1.48

$$-\frac{\left(b^{3}-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{(b^{2}c^{2}-4\,ac^{3})\sqrt{-b^{2}+4\,ac}}+\frac{\ln\left(cx^{2}+bx+a\right)}{2\,c^{2}}+\frac{ab^{2}-2\,a^{2}c+\left(b^{3}-3\,abc\right)x}{(cx^{2}+bx+a)(b^{2}-4\,ac)c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x),x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2
 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/2*ln(c*x^2 + b*x + a)/c^2 + (
a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*
a*c)*c^2)
```

3.425 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx$

Optimal. Leaf size=71

$$\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b
 + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0925557, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b
 + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 11.7961, size = 60, normalized size = 0.85

$$-\frac{4a\,\mathrm{atanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{\frac{2a}{x}+b}{(-4ac+b^2)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

```
[Out] -4*a*atanh((2*a/x + b)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/2
) + (2*a/x + b)/((-4*a*c + b**2)*(a/x**2 + b/x + c))
```

Mathematica [A] time = 0.150376, size = 81, normalized size = 1.14

$$\frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)\left(a+x(b+cx)\right)}+\frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4
*a*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.008, size = 97, normalized size = 1.4

$$\frac{1}{cx^2+bx+a}\left(-\frac{\left(2\,ac-b^2\right)x}{\left(4\,ac-b^2\right)c}+\frac{ab}{\left(4\,ac-b^2\right)c}\right)+4\,\frac{a}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2/x^2,x)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*
a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274394, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(ac^{2}x^{2}+abcx+a^{2}c\right)\log\left(-\frac{b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)x-\left(2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{2}+bx+a}\right)+\left(ab+\left(b^{2}-2\,ac\right)x\right)\sqrt{b^{2}-4\,ac}}{(ab^{2}c-4\,a^{2}c^{2}+(b^{2}c^{2}-4\,ac^{3})x^{2}+(b^{3}c-4\,abc^{2})x)\sqrt{b^{2}-4\,ac}},\right.$$
$$\left.-\frac{4\left(ac^{2}x^{2}+abcx+a^{2}c\right)\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)+\left(ab+\left(b^{2}-2\,ac\right)x\right)\sqrt{-b^{2}+4\,ac}}{(ab^{2}c-4\,a^{2}c^{2}+(b^{2}c^{2}-4\,ac^{3})x^{2}+(b^{3}c-4\,abc^{2})x)\sqrt{-b^{2}+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-(2*(a*c^2*x^2 + a*b*c*x + a^2*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c
 - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*
a*c))/(c*x^2 + b*x + a)) + (a*b + (b^2 - 2*a*c)*x)*sqrt(b^2 - 4*a
*c))/((a*b^2*c - 4*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4
*a*b*c^2)*x)*sqrt(b^2 - 4*a*c)), -(4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (a*b +
(b^2 - 2*a*c)*x)*sqrt(-b^2 + 4*a*c))/((a*b^2*c - 4*a^2*c^2 + (b^2
*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 2.83179, size = 280, normalized size = 3.94

$$-2a\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\log\left(x+\frac{-32a^{3}c^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+16a^{2}b^{2}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-2ab^{4}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+2ab}{4ac}\right)$$
$$+2a\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\log\left(x+\frac{32a^{3}c^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-16a^{2}b^{2}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+2ab^{4}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+2ab}{4ac}\right)$$
$$-\frac{-ab+x\left(2ac-b^{2}\right)}{4a^{2}c^{2}-ab^{2}c+x^{2}\left(4ac^{3}-b^{2}c^{2}\right)+x\left(4abc^{2}-b^{3}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4
*a*c - b**2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*
a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1
/(4*a*c - b**2)**3)*log(x + (32*a**3*c**2*sqrt(-1/(4*a*c - b**2)*
*3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b**4*sqrt(-
```

```
1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) - (-a*b + x*(2*a*c - b**2)
)/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*
b*c**2 - b**3*c))
```

GIAC/XCAS [A] time = 0.283716, size = 119, normalized size = 1.68

$$-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^2),x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2
 + b*x + a))
```

3.426 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0764737, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 13.7373, size = 60, normalized size = 0.91

$$-\frac{2b\,\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{2a+bx}{(-4ac+b^2)\,(a+bx+cx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

```
[Out] -2*b*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/2
) + (2*a + b*x)/((-4*a*c + b**2)*(a + b*x + c*x**2))
```

Mathematica [A] time = 0.104742, size = 69, normalized size = 1.05

$$\frac{2a+bx}{(b^2-4ac)(a+x(b+cx))}-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b +
2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.004, size = 70, normalized size = 1.1

$$\frac{-bx-2\,a}{(4\,ac-b^2)(cx^2+bx+a)}-2\,\frac{b}{(4\,ac-b^2)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2/x^3,x)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265909, size = 1, normalized size = 0.02

$$\left[-\frac{(bcx^2+b^2x+ab)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x+(2\,c^2x^2+2\,bcx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\sqrt{b^2-4\,ac}(bx+2\,a)}{(ab^2-4\,a^2c+(b^2c-4\,ac^2)x^2+(b^3-4\,abc)x)\sqrt{b^2-4\,ac}},\ \frac{2\,(bcx^2+b^2x+ab)\ \mathrm{ar}}{(ab^2-4\,a^2c+(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-((b*c*x^2 + b^2*x + a*b)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^
2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*
x^2 + b*x + a)) - sqrt(b^2 - 4*a*c)*(b*x + 2*a))/((a*b^2 - 4*a^2*
c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(b^2 - 4*a*c))
, (2*(b*c*x^2 + b^2*x + a*b)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x +
b)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(b*x + 2*a))/((a*b^2 - 4*a
^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(-b^2 + 4*a
*c))]
```

Sympy [A] time = 2.65729, size = 252, normalized size = 3.82

$$b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)-\frac{2a+bx}{4a^2c-ab^2+x^2\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*
a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sq
rt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b*
*2)**3)*log(x + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*
b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**
3) + b**2)/(2*b*c)) - (2*a + b*x)/(4*a**2*c - a*b**2 + x**2*(4*a*
c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

GIAC/XCAS [A] time = 0.271158, size = 103, normalized size = 1.56

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^3),x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b
^2 + 4*a*c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.427 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.068717, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 11.3361, size = 60, normalized size = 0.91

$$\frac{4c\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{b+2cx}{(-4ac+b^2)(a+bx+cx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

```
[Out] 4*c*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/2)
 - (b + 2*c*x)/((-4*a*c + b**2)*(a + b*x + c*x**2))
```

Mathematica [A] time = 0.122845, size = 70, normalized size = 1.06

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-
b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A] time = 0.003, size = 68, normalized size = 1.

$$\frac{2\,cx+b}{(4\,ac-b^2)\,(cx^2+bx+a)}+4\,\frac{c}{(4\,ac-b^2)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2/x^4,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277398, size = 1, normalized size = 0.02

$$\left[-\frac{2\left(c^{2}x^{2}+bcx+ac\right)\log\left(-\frac{b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)x-\left(2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{2}+bx+a}\right)+\sqrt{b^{2}-4\,ac}(2\,cx+b)}{(ab^{2}-4\,a^{2}c+(b^{2}c-4\,ac^{2})x^{2}+(b^{3}-4\,abc)x)\sqrt{b^{2}-4\,ac}},\right.$$
$$\left.-\frac{4\left(c^{2}x^{2}+bcx+ac\right)\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)+\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{(ab^{2}-4\,a^{2}c+(b^{2}c-4\,ac^{2})x^{2}+(b^{3}-4\,abc)x)\sqrt{-b^{2}+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^4),x, algorithm="fricas")
```

```
[Out] [-(2*(c^2*x^2 + b*c*x + a*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a
*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/
(c*x^2 + b*x + a)) + sqrt(b^2 - 4*a*c)*(2*c*x + b))/((a*b^2 - 4*a
^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(b^2 - 4*a*
c)), -(4*(c^2*x^2 + b*c*x + a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(2*c*x + b))/((a*b^2 -
 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(-b^2 +
 4*a*c))]
```

Sympy [A] time = 2.71092, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\log\left(x+\frac{-32a^{2}c^{3}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+16ab^{2}c^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-2b^{4}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+2bc}{4c^{2}}\right)$$
$$+2c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\log\left(x+\frac{32a^{2}c^{3}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-16ab^{2}c^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+2b^{4}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+2bc}{4c^{2}}\right)$$
$$+\frac{b+2cx}{4a^{2}c-ab^{2}+x^{2}\left(4ac^{2}-b^{2}c\right)+x\left(4abc-b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4
*a*c - b**2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*
b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-
1/(4*a*c - b**2)**3)*log(x + (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)
**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + 2*b**4*c*sqrt(
```

```
-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*c*x)/(4*a**2*c
- a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

GIAC/XCAS [A] time = 0.277385, size = 103, normalized size = 1.56

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^4),x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.428 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b
^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4
*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rubi [A] time = 0.309417, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b
^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4
*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rubi in Sympy [A] time = 45.7865, size = 102, normalized size = 0.94

$$\frac{-2ac+b^2+bcx}{a\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}+\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^2\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*(-4*a*c + b**2)*(a + b*x + c*x**2)) +
b*(-6*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**2*(-
4*a*c + b**2)**(3/2)) + log(x)/a**2 - log(a + b*x + c*x**2)/(2*a*
```

```
*2)
```

Mathematica [A] time = 0.31496, size = 107, normalized size = 0.99

$$\frac{\frac{2a(-2ac+b^2+bcx)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}-\log(a+x(b+cx))+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) +
(2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(3/2) + 2*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [B] time = 0.016, size = 389, normalized size = 3.6

$$\frac{\ln(x)}{a^2}-\frac{bcx}{a(cx^2+bx+a)(4ac-b^2)}+2\frac{c}{(4ac-b^2)(cx^2+bx+a)}-\frac{b^2}{a(cx^2+bx+a)(4ac-b^2)}$$
$$-2\frac{c\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)}{a(4ac-b^2)}+\frac{\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)b^2}{2(4ac-b^2)a^2}$$
$$-6\frac{bc}{a\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$
$$+\frac{b^3}{a^2}\arctan\left((2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2/x^5,x)
```

```
[Out] ln(x)/a^2-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*
a*c-b^2)*c-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln
((4*a*c-b^2)*(c*x^2+b*x+a))+1/2/a^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c
*x^2+b*x+a))*b^2-6/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(
1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c+1/a^2/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)
/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.34169, size = 1, normalized size = 0.01

$$\left[\frac{\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right) + \left(2\,abcx + 2\,a\right.}{2\left(a^3b^2 - 4\,a^4c + \left(a\right.\right.}\right.$$

$$-\frac{2\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) - \left(2\,abcx + 2\,ab^2 - 4\,a^2c - \left(ab^2 - 4\,a^2c\right.\right.}{2\left(a^3b^2 - 4\,a^4c + \left(a^2b^2c - 4\,a^3c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^5),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b
^2*c)*x)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) +
(2*a*b*c*x + 2*a*b^2 - 4*a^2*c - (a*b^2 - 4*a^2*c + (b^2*c - 4*a*
c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(c*x^2 + b*x + a) + 2*(a*b^2 - 4
*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(x))*sqrt(
b^2 - 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 +
 (a^2*b^3 - 4*a^3*b*c)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(a*b^3 - 6*
a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*arctan(-
sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*a*b*c*x + 2*a*
b^2 - 4*a^2*c - (a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 -
 4*a*b*c)*x)*log(c*x^2 + b*x + a) + 2*(a*b^2 - 4*a^2*c + (b^2*c -
 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(x))*sqrt(-b^2 + 4*a*c))/((
a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^
3*b*c)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 25.5113, size = 2236, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*log(x +
 (1536*a**9*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/
(2*a**2))**2 - 2112*a**8*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(
6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) - 1/(2*a**2))**2 + 1136*a**7*b**4*c**3*(-b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a**7*c**5*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a
**6*b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*
a**2))**2 + 624*a**6*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*
c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(-b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 768*a**5*c**
5 - 2*a**4*b**10*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/
(2*a**2))**2 + 23*a**4*b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*
a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6)) - 1/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(-b
*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 -
48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 952*a**3
*b**4*c**3 + 277*a**2*b**6*c**2 - 38*a*b**8*c + 2*b**10)/(864*a**
4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**2*b**5*c**3 - 36*a*b**7*c*
*2 + 2*b**9*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/
(2*a**2))*log(x + (1536*a**9*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*
a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6)) - 1/(2*a**2))**2 - 2112*a**8*b**2*c**4*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 1136*a**7*b**4*c*
*3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 -
768*a**7*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a
**2)) - 300*a**6*b**6*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b
**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b*
*6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(b*sqrt(-(4*a*c - b**2
)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 184*a**5*b**4*c*
*3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 768
```

```
*a**5*c**5 - 2*a**4*b**10*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b*
*2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6)) - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - 1/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**
8*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 95
2*a**3*b**4*c**3 + 277*a**2*b**6*c**2 - 38*a*b**8*c + 2*b**10)/(8
64*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**2*b**5*c**3 - 36*a*b
**7*c**2 + 2*b**9*c)) - (-2*a*c + b**2 + b*c*x)/(4*a**3*c - a**2*
b**2 + x**2*(4*a**2*c**2 - a*b**2*c) + x*(4*a**2*b*c - a*b**3)) +
 log(x)/a**2
```

GIAC/XCAS [A] time = 0.275373, size = 170, normalized size = 1.57

$$-\frac{\left(b^{3}-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{2}b^{2}-4\,a^{3}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{\ln\left(cx^{2}+bx+a\right)}{2\,a^{2}}+\frac{\ln\left(|x|\right)}{a^{2}}+\frac{abcx+ab^{2}-2\,a^{2}c}{\left(cx^{2}+bx+a\right)\left(b^{2}-4\,ac\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^5),x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2
 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/2*ln(c*x^2 + b*x + a)/a^2 + l
n(abs(x))/a^2 + (a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b
^2 - 4*a*c)*a^2)
```

3.429 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} \, dx$

Optimal. Leaf size=148

$$\frac{b \log \left(a+b x+c x^2\right)}{a^3}-\frac{2 b \log (x)}{a^3}-\frac{2 \left(b^2-3 a c\right)}{a^2 x \left(b^2-4 a c\right)} -\frac{2 \left(6 a^2 c^2-6 a b^2 c+b^4\right) \tanh ^{-1}\left(\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right)}{a^3 \left(b^2-4 a c\right)^{3/2}}+\frac{-2 a c+b^2+b c x}{a x \left(b^2-4 a c\right) \left(a+b x+c x^2\right)}$$

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/
(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c
)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3
```

Rubi [A] time = 0.400381, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b \log \left(a+b x+c x^2\right)}{a^3}-\frac{2 b \log (x)}{a^3}-\frac{2 \left(b^2-3 a c\right)}{a^2 x \left(b^2-4 a c\right)} -\frac{2 \left(6 a^2 c^2-6 a b^2 c+b^4\right) \tanh ^{-1}\left(\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right)}{a^3 \left(b^2-4 a c\right)^{3/2}}+\frac{-2 a c+b^2+b c x}{a x \left(b^2-4 a c\right) \left(a+b x+c x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/
(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c
)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3
```

Rubi in Sympy [A] time = 74.9223, size = 143, normalized size = 0.97

$$\frac{-2 a c+b^2+b c x}{a x \left(-4 a c+b^2\right) \left(a+b x+c x^2\right)}-\frac{2 \left(-3 a c+b^2\right)}{a^2 x \left(-4 a c+b^2\right)}-\frac{2 b \log (x)}{a^3} +\frac{b \log \left(a+b x+c x^2\right)}{a^3}-\frac{2 \left(6 a^2 c^2-6 a b^2 c+b^4\right) \operatorname{atanh}\left(\frac{b+2 c x}{\sqrt{-4 a c+b^2}}\right)}{a^3 \left(-4 a c+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*x*(-4*a*c + b**2)*(a + b*x + c*x**2))
- 2*(-3*a*c + b**2)/(a**2*x*(-4*a*c + b**2)) - 2*b*log(x)/a**3 +
b*log(a + b*x + c*x**2)/a**3 - 2*(6*a**2*c**2 - 6*a*b**2*c + b**4
)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**3*(-4*a*c + b**2)**(
3/2))
```

Mathematica [A] time = 0.462612, size = 131, normalized size = 0.89

$$-\frac{\frac{2(6a^2c^2-6ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\frac{a(-3abc-2ac^2x+b^3+b^2cx)}{(b^2-4ac)(a+x(b+cx))}-b\log(a+x(b+cx))+\frac{a}{x}+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b
+ 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] -
 b*Log[a + x*(b + c*x)])/a^3)
```

Maple [B] time = 0.021, size = 545, normalized size = 3.7

$$-\frac{1}{a^2x}-2\frac{b\ln(x)}{a^3}-2\frac{c^2x}{a(cx^2+bx+a)(4ac-b^2)}+\frac{cxb^2}{a^2(cx^2+bx+a)(4ac-b^2)}$$
$$-3\frac{bc}{a(cx^2+bx+a)(4ac-b^2)}+\frac{b^3}{a^2(cx^2+bx+a)(4ac-b^2)}$$
$$+4\frac{c\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)b}{(4ac-b^2)a^2}-\frac{\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)b^3}{a^3(4ac-b^2)}$$
$$-12\frac{c^2}{a\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$
$$+12\frac{b^2c}{a^2\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$
$$-2\frac{b^4}{a^3\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2/x^6,x)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x+1/a^2/
(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^2-3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)
*c+1/a^2/(c*x^2+b*x+a)*b^3/(4*a*c-b^2)+4/a^2/(4*a*c-b^2)*c*ln((4*
a*c-b^2)*(c*x^2+b*x+a))*b-1/a^3/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^2
+b*x+a))*b^3-12/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2
)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)^(1/2))*c^2+12/a^2/(64*a^3*c^3-48*a^2*b^2*c^2
+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(
64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^2*c-2/a^3/(64*
a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2
)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(
1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.421351, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^6),x, algorithm="fricas")
```

```
[Out] [-(((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*
a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*log((b^3 -
4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (a^2*b^2 - 4*a^3*c +
 2*(a*b^2*c - 3*a^2*c^2)*x^2 + (2*a*b^3 - 7*a^2*b*c)*x - ((b^3*c
- 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)
*log(c*x^2 + b*x + a) + 2*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b
^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*log(x))*sqrt(b^2 - 4*a*c))/(((
a^3*b^2*c - 4*a^4*c^2)*x^3 + (a^3*b^3 - 4*a^4*b*c)*x^2 + (a^4*b^2
 - 4*a^5*c)*x)*sqrt(b^2 - 4*a*c)), (2*((b^4*c - 6*a*b^2*c^2 + 6*a
^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^
2*b^2*c + 6*a^3*c^2)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) - (a^2*b^2 - 4*a^3*c + 2*(a*b^2*c - 3*a^2*c^2)*x^2 +
 (2*a*b^3 - 7*a^2*b*c)*x - ((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*
```

```
b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*log(c*x^2 + b*x + a) + 2*((b^
3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c
)*x)*log(x))*sqrt(-b^2 + 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^3 +
(a^3*b^3 - 4*a^4*b*c)*x^2 + (a^4*b^2 - 4*a^5*c)*x)*sqrt(-b^2 + 4*
a*c))]
```

Sympy [A] time = 38.6586, size = 2672, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

```
[Out] (b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b*
*4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
))*log(x + (-1728*a**11*b*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 2256*a**10*b**3*c**4*(b/a*
*3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(
a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2
 - 1172*a**9*b**5*c**3*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6)))**2 - 288*a**9*c**6*(b/a**3 - sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 303*a**8*b**7*c
**2*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c
+ b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)))**2 - 432*a**8*b**2*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 - sqrt
(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*
a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 558*a*
*7*b**4*c**4*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*
a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) + 2*a**6*b**11*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3
 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a*
*3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 57
6*a**6*b*c**6 + 34*a**5*b**8*c**2*(b/a**3 - sqrt(-(4*a*c - b**2)*
*3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a**5*b**3*c**5 - 2*a*
*4*b**10*c*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*
b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*
a**2*b**9*c**2 + 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a
**5*b**2*c**6 - 5292*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*
a**2*b**8*c**3 + 144*a*b**10*c**2 - 8*b**12*c)) + (b/a**3 + sqrt(
```

```
-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-17
28*a**11*b*c**5*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)))**2 + 2256*a**10*b**3*c**4*(b/a**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 1172*a**9*b*
*5*c**3*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**
2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)))**2 - 288*a**9*c**6*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*
(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6))) + 303*a**8*b**7*c**2*(b/a**3 + s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 432
*a**8*b**2*c**5*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 + sqrt(-(4*a*c - b**2
)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b
/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
 + 2*a**6*b**11*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 + sqrt(-(4*a*c
 - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 +
 34*a**5*b**8*c**2*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6))) + 6048*a**5*b**3*c**5 - 2*a**4*b**10*c*(b/a
**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/
(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) -
 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*c**2
+ 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 -
 5292*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3
+ 144*a*b**10*c**2 - 8*b**12*c)) - (4*a**2*c - a*b**2 + x**2*(6*a
*c**2 - 2*b**2*c) + x*(7*a*b*c - 2*b**3))/(x**3*(4*a**3*c**2 - a*
*2*b**2*c) + x**2*(4*a**3*b*c - a**2*b**3) + x*(4*a**4*c - a**3*b
**2)) - 2*b*log(x)/a**3
```

GIAC/XCAS [A] time = 0.286524, size = 231, normalized size = 1.56

$$\frac{2\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2 - 4\,a^4c)\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^2cx^2 - 6\,ac^2x^2 + 2\,b^3x - 7\,abcx + ab^2 - 4\,a^2c}{(a^2b^2 - 4\,a^3c)(cx^3 + bx^2 + ax)} + \frac{b\ln\left(cx^2 + bx + a\right)}{a^3} - \frac{2\,b\ln\left(|x|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^6),x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6
*a*c^2*x^2 + 2*b^3*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4
*a^3*c)*(c*x^3 + b*x^2 + a*x)) + b*ln(c*x^2 + b*x + a)/a^3 - 2*b*
ln(abs(x))/a^3
```

3.430 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$

Optimal. Leaf size=202

$$-\frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{\log(x)\left(3b^2-2ac\right)}{a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}+\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))
/(a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x
^2*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + (
(3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^
2])/(2*a^4)
```

Rubi [A] time = 0.507093, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{\log(x)\left(3b^2-2ac\right)}{a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}+\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))
/(a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x
^2*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + (
(3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^
2])/(2*a^4)
```

Rubi in Sympy [A] time = 85.6393, size = 192, normalized size = 0.95

$$\frac{-2ac+b^2+bcx}{ax^2\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}-\frac{-8ac+3b^2}{2a^2x^2\left(-4ac+b^2\right)}+\frac{b\left(-11ac+3b^2\right)}{a^3x\left(-4ac+b^2\right)}+\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^4\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\left(-2ac+3b^2\right)\log\left(x\right)}{a^4}-\frac{\left(-2ac+3b^2\right)\log\left(a+bx+cx^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*x**2*(-4*a*c + b**2)*(a + b*x + c*x**2
)) - (-8*a*c + 3*b**2)/(2*a**2*x**2*(-4*a*c + b**2)) + b*(-11*a*c
 + 3*b**2)/(a**3*x*(-4*a*c + b**2)) + b*(30*a**2*c**2 - 20*a*b**2
*c + 3*b**4)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**4*(-4*a*c
 + b**2)**(3/2)) + (-2*a*c + 3*b**2)*log(x)/a**4 - (-2*a*c + 3*b*
*2)*log(a + b*x + c*x**2)/(2*a**4)
```

Mathematica [A] time = 0.772291, size = 175, normalized size = 0.87

$$\frac{\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{2a\left(2a^2c^2-4ab^2c-3abc^2x+b^4+b^3cx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)}-\frac{a^2}{x^2}+2\log(x)\left(3b^2-2ac\right)+\left(2ac-3b^2\right)\log(a+x(b+c$$

$$2a^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3
*c*x - 3*a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*
b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/(-b^2 + 4*a*c)^(3/2) + 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 +
 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [B] time = 0.023, size = 646, normalized size = 3.2

$$
\begin{aligned}
&-\frac{1}{2\,a^2x^2}-2\,\frac{\ln(x)\,c}{a^3}+3\,\frac{b^2\ln(x)}{a^4}+2\,\frac{b}{a^3x}+3\,\frac{c^2bx}{a^2\,(cx^2+bx+a)\,(4\,ac-b^2)}\\
&-\frac{b^3cx}{a^3\,(cx^2+bx+a)\,(4\,ac-b^2)}-2\,\frac{c^2}{a\,(cx^2+bx+a)\,(4\,ac-b^2)}+4\,\frac{b^2c}{a^2\,(cx^2+bx+a)\,(4\,ac-b^2)}\\
&-\frac{b^4}{a^3\,(cx^2+bx+a)\,(4\,ac-b^2)}+4\,\frac{c^2\ln\left((4\,ac-b^2)\,(cx^2+bx+a)\right)}{(4\,ac-b^2)\,a^2}\\
&-7\,\frac{c\ln\left((4\,ac-b^2)\,(cx^2+bx+a)\right)b^2}{a^3\,(4\,ac-b^2)}+\frac{3\,\ln\left((4\,ac-b^2)\,(cx^2+bx+a)\right)b^4}{2\,a^4\,(4\,ac-b^2)}\\
&+30\,\frac{c^2b}{a^2\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,(4\,ac-b^2)\,cx+(4\,ac-b^2)\,b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\\
&-20\,\frac{b^3c}{a^3\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,(4\,ac-b^2)\,cx+(4\,ac-b^2)\,b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\\
&+3\,\frac{b^5}{a^4\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,(4\,ac-b^2)\,cx+(4\,ac-b^2)\,b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^2/x^7,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*ln(x)*c+3/a^4*b^2*ln(x)+2/a^3*b/x+3/a^2/(c*x^2
+b*x+a)*b*c^2/(4*a*c-b^2)*x-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)
*x-2/a/(c*x^2+b*x+a)/(4*a*c-b^2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b
^2)*b^2*c-1/a^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4+4/a^2/(4*a*c-b^2)*c
^2*ln((4*a*c-b^2)*(c*x^2+b*x+a))-7/a^3/(4*a*c-b^2)*c*ln((4*a*c-b^
2)*(c*x^2+b*x+a))*b^2+3/2/a^4/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^2+b
*x+a))*b^4+30/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2
)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)^(1/2))*b*c^2-20/a^3/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)
/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3*c+3/a^4/(6
4*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b
^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.521776, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^7),x, algorithm="fricas")
```

```
[Out] [-1/2*(((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20
*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3
*b*c^2)*x^2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a
)) + (a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a
*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x
+ ((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^4 + (3*b^5 - 14*a*b^3*c
 + 8*a^2*b*c^2)*x^3 + (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*x^2)*l
og(c*x^2 + b*x + a) - 2*((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^4
 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^3 + (3*a*b^4 - 14*a^2*b^2
*c + 8*a^3*c^2)*x^2)*log(x))*sqrt(b^2 - 4*a*c))/(((a^4*b^2*c - 4*
a^5*c^2)*x^4 + (a^4*b^3 - 4*a^5*b*c)*x^3 + (a^5*b^2 - 4*a^6*c)*x^
2)*sqrt(b^2 - 4*a*c)), -1/2*(2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*
b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5
 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*arctan(-sqrt(-b^2 + 4*a*c)*(
2*c*x + b)/(b^2 - 4*a*c)) + (a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 1
1*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(
a^2*b^3 - 4*a^3*b*c)*x + ((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^
4 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^3 + (3*a*b^4 - 14*a^2*b^
2*c + 8*a^3*c^2)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^4*c - 14*a*b
^2*c^2 + 8*a^2*c^3)*x^4 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^3
+ (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*x^2)*log(x))*sqrt(-b^2 + 4
*a*c))/(((a^4*b^2*c - 4*a^5*c^2)*x^4 + (a^4*b^3 - 4*a^5*b*c)*x^3
+ (a^5*b^2 - 4*a^6*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 59.1678, size = 4083, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4
)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
```

```
) + (2*a*c - 3*b**2)/(2*a**4))*log(x + (3072*a**14*c**6*(-b*sqrt(
-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*
c - 3*b**2)/(2*a**4))**2 - 9408*a**13*b**2*c**5*(-b*sqrt(-(4*a*c
- b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**
3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b*
*2)/(2*a**4))**2 + 9040*a**12*b**4*c**4*(-b*sqrt(-(4*a*c - b**2)*
*3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a
**4))**2 - 4116*a**11*b**6*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(30*
a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2
 + 3072*a**11*c**7*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 2
0*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 987*a**10*b**
8*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 7536*a**10*b**2*c**6*(
-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)
/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 + (2*a*c - 3*b**2)/(2*a**4)) - 121*a**9*b**10*c*(-b*sqrt(-(4*a*c
 - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b
**2)/(2*a**4))**2 + 8152*a**9*b**4*c**5*(-b*sqrt(-(4*a*c - b**2)*
*3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a
**4)) + 6*a**8*b**12*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 -
 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4343*a**
8*b**6*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**
2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 6144*a**8*c**8 + 119
8*a**7*b**8*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*
a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 50208*a**7*b**2
*c**7 - 165*a**6*b**10*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2
*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 1377
92*a**6*b**4*c**6 + 9*a**5*b**12*c*(-b*sqrt(-(4*a*c - b**2)**3)*(
30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))
 + 176474*a**5*b**6*c**5 - 119275*a**4*b**8*c**4 + 45448*a**3*b**
10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**14*c - 54*b**16)/(1728
0*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 - 88
605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**3
+ 1080*a*b**13*c**2 - 54*b**15*c)) + (b*sqrt(-(4*a*c - b**2)**3)*
(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)
)*log(x + (3072*a**14*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 940
8*a**13*b**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*
a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 9040*a**12*b
```

```
**4*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c
+ 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4116*a**11*b**6*c**3*
(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)
/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 + (2*a*c - 3*b**2)/(2*a**4))**2 + 3072*a**11*c**7*(b*sqrt(-(4*a*
c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*
b**2)/(2*a**4)) + 987*a**10*b**8*c**2*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4
))**2 - 7536*a**10*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2
*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 121*
a**9*b**10*c*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**
2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 8152*a**9*b**4*c*
*5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6)) + (2*a*c - 3*b**2)/(2*a**4)) + 6*a**8*b**12*(b*sqrt(-(4*a*c -
 b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3
*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**
2)/(2*a**4))**2 - 4343*a**8*b**6*c**4*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4
)) - 6144*a**8*c**8 + 1198*a**7*b**8*c**3*(b*sqrt(-(4*a*c - b**2)
**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*
a**4)) + 50208*a**7*b**2*c**7 - 165*a**6*b**10*c**2*(b*sqrt(-(4*a
*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*
a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3
*b**2)/(2*a**4)) - 137792*a**6*b**4*c**6 + 9*a**5*b**12*c*(b*sqrt
(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**
4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a
*c - 3*b**2)/(2*a**4)) + 176474*a**5*b**6*c**5 - 119275*a**4*b**8
*c**4 + 45448*a**3*b**10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**
14*c - 54*b**16)/(17280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 1124
28*a**5*b**5*c**6 - 88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 -
 8820*a**2*b**11*c**3 + 1080*a*b**13*c**2 - 54*b**15*c)) + (-4*a*
*3*c + a**2*b**2 + x**3*(22*a*b*c**2 - 6*b**3*c) + x**2*(-8*a**2*
c**2 + 25*a*b**2*c - 6*b**4) + x*(12*a**2*b*c - 3*a*b**3))/(x**4*
(8*a**4*c**2 - 2*a**3*b**2*c) + x**3*(8*a**4*b*c - 2*a**3*b**3) +
 x**2*(8*a**5*c - 2*a**4*b**2)) - (2*a*c - 3*b**2)*log(x + (-6144
*a**8*c**8 + 50208*a**7*b**2*c**7 - 3072*a**7*c**7*(2*a*c - 3*b**
2) - 137792*a**6*b**4*c**6 + 7536*a**6*b**2*c**6*(2*a*c - 3*b**2)
 + 3072*a**6*c**6*(2*a*c - 3*b**2)**2 + 176474*a**5*b**6*c**5 - 8
152*a**5*b**4*c**5*(2*a*c - 3*b**2) - 9408*a**5*b**2*c**5*(2*a*c
- 3*b**2)**2 - 119275*a**4*b**8*c**4 + 4343*a**4*b**6*c**4*(2*a*c
 - 3*b**2) + 9040*a**4*b**4*c**4*(2*a*c - 3*b**2)**2 + 45448*a**3
*b**10*c**3 - 1198*a**3*b**8*c**3*(2*a*c - 3*b**2) - 4116*a**3*b*
*6*c**3*(2*a*c - 3*b**2)**2 - 9846*a**2*b**12*c**2 + 165*a**2*b**
10*c**2*(2*a*c - 3*b**2) + 987*a**2*b**8*c**2*(2*a*c - 3*b**2)**2
 + 1134*a*b**14*c - 9*a*b**12*c*(2*a*c - 3*b**2) - 121*a*b**10*c*
(2*a*c - 3*b**2)**2 - 54*b**16 + 6*b**12*(2*a*c - 3*b**2)**2)/(17
```

```
280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 -
88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**
3 + 1080*a*b**13*c**2 - 54*b**15*c))/a**4
```

GIAC/XCAS [A] time = 0.276051, size = 309, normalized size = 1.53

$$-\frac{\left(3\,b^{5}-20\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{2}-4\,a^{5}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{\left(3\,b^{2}-2\,ac\right)\ln\left(cx^{2}+bx+a\right)}{2\,a^{4}}+\frac{\left(3\,b^{2}-2\,ac\right)\ln\left(|x|\right)}{a^{4}}-\frac{a^{3}b^{2}-4\,a^{4}c-2\left(3\,ab^{3}c-11\,a^{2}bc^{2}\right)x^{3}-\left(6\,ab^{4}-25\,a^{2}b^{2}c+8\,a^{3}c^{2}\right)x^{2}-3\left(a^{2}b^{3}-4\,a^{3}bc\right)x}{2\left(cx^{2}+bx+a\right)\left(b^{2}-4\,ac\right)a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^2*x^7),x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2
- 2*a*c)*ln(c*x^2 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*ln(abs(x))/a^4
 - 1/2*(a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6
*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*
x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

3.431 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3} dx$

Optimal. Leaf size=238

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(bx\left(b^2-7ac\right)+a\left(b^2-10ac\right)\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3b\log\left(a+bx+cx^2\right)}{2c^4}$$

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b
*(b^2 - 6*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(
2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b
*(b^2 - 7*a*c)*x))/(c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (3*(b^
6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[a + b*x
 + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.637574, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(bx\left(b^2-7ac\right)+a\left(b^2-10ac\right)\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3b\log\left(a+bx+cx^2\right)}{2c^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + a/x^2 + b/x)^(-3),x]
```


```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b
*(b^2 - 6*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(
2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b
*(b^2 - 7*a*c)*x))/(c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (3*(b^
6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[a + b*x
 + c*x^2])/(2*c^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{30a^2x}{c\left(-4ac+b^2\right)^2}-\frac{21ab^2x}{c^2\left(-4ac+b^2\right)^2}+\frac{3b^4x}{c^3\left(-4ac+b^2\right)^2}-\frac{3b\left(-6ac+b^2\right)\int x\,dx}{c^2\left(-4ac+b^2\right)^2}-\frac{3b\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x^5\left(2a+bx\right)}{2\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(2a\left(-10ac+b^2\right)+2bx\left(-7ac+b^2\right)\right)}{2c\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}-\frac{6\left(-10a^3c^3+15a^2b^2c^2-5ab^4c+\frac{b^6}{2}\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^4\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3,x)
```

```
[Out] 30*a**2*x/(c*(-4*a*c + b**2)**2) - 21*a*b**2*x/(c**2*(-4*a*c + b*
*2)**2) + 3*b**4*x/(c**3*(-4*a*c + b**2)**2) - 3*b*(-6*a*c + b**2
)*Integral(x, x)/(c**2*(-4*a*c + b**2)**2) - 3*b*log(a + b*x + c*
x**2)/(2*c**4) + x**5*(2*a + b*x)/(2*(-4*a*c + b**2)*(a + b*x + c
*x**2)**2) + x**3*(2*a*(-10*a*c + b**2) + 2*b*x*(-7*a*c + b**2))/
(2*c*(-4*a*c + b**2)**2*(a + b*x + c*x**2)) - 6*(-10*a**3*c**3 +
15*a**2*b**2*c**2 - 5*a*b**4*c + b**6/2)*atanh((b + 2*c*x)/sqrt(-
4*a*c + b**2))/(c**4*(-4*a*c + b**2)**(5/2))
```

Mathematica [A] time = 0.649345, size = 260, normalized size = 1.09

$$\frac{\frac{a^3c^2(2cx-5b)+a^2b^2c(5b-9cx)-ab^4(b-6cx)+b^6(-x)}{(b^2-4ac)(a+x(b+cx))^2}+\frac{6c\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+\frac{-78a^3bc^3+36a^3c^4x+61a^2b^3c^2-102a^2b^2c^3x-}{(b^2-4ac)^2(a+x(b+}}{2c^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*
b^6*c*x + 48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^
2 - 4*a*c)^2*(a + x*(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*
c*x) - a*b^4*(b - 6*c*x) + a^3*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*
c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

Maple [B] time = 0.027, size = 1524, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3,x)
```

```
[Out] x/c^3+18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^3-51/c/
(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^2*b^2+24/c^2/(c*
x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b^4-3/c^3/(c*x^2+b*
x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^6-21/c/(c*x^2+b*x+a)^2*b/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3-41/2/c^2/(c*x^2+b*x+a)^2*b^3/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2+17/c^3/(c*x^2+b*x+a)^2*b^5/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^2*a-5/2/c^4/(c*x^2+b*x+a)^2*b^7/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^2+14/c/(c*x^2+b*x+a)^2*a^4/(16*a^2*c^2-8*a*b
^2*c+b^4)*x-71/c^2/(c*x^2+b*x+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)
*x*b^2+38/c^3/(c*x^2+b*x+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^
4-5/c^4/(c*x^2+b*x+a)^2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^6-29/c^2
/(c*x^2+b*x+a)^2*b*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)+18/c^3/(c*x^2+b
*x+a)^2*b^3*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)-5/2/c^4/(c*x^2+b*x+a)^
2*b^5*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)-24/c^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^2+b*x+a))*a^2*b+12/c^3/(
16*a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^2+b*
x+a))*a*b^3-3/2/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a
*b^2*c+b^4)*(c*x^2+b*x+a))*b^5-60/c/(1024*a^5*c^5-1280*a^4*b^2*c^
4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((
2*c*x*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1
024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a
*b^8*c-b^10)^(1/2))*a^3+90/c^2/(1024*a^5*c^5-1280*a^4*b^2*c^4+640
*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x
*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a
^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*
c-b^10)^(1/2))*b^2*a^2-30/c^3/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*
a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x*
(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^
5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c
-b^10)^(1/2))*a*b^4+3/c^4/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*
b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x*(16*
a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^
5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^1
0)^(1/2))*b^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + b/x + a/x^2)^(-3),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26988, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + b/x + a/x^2)^(-3),x, algorithm="fricas")

[Out] [-1/2*(3*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 +
(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b
^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 -
 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 +
2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*log((
b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (5*a^2*b^5 - 3
6*a^3*b^3*c + 58*a^4*b*c^2 - 2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*x^5 - 4*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^4 + 2*(2*b^6*
c - 18*a*b^4*c^2 + 51*a^2*b^2*c^3 - 50*a^3*c^4)*x^3 + (5*b^7 - 38
*a*b^5*c + 73*a^2*b^3*c^2 - 22*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 39*a
^2*b^4*c + 79*a^3*b^2*c^2 - 30*a^4*c^3)*x + 3*(a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^4 +
 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x^3 + (b^7 - 6*a*b^5*c
+ 32*a^3*b*c^3)*x^2 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x)
*log(c*x^2 + b*x + a))*sqrt(b^2 - 4*a*c))/((a^2*b^4*c^4 - 8*a^3*b
^2*c^5 + 16*a^4*c^6 + (b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*x^4 +
2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*x^3 + (b^6*c^4 - 6*a*b^4
*c^5 + 32*a^3*c^7)*x^2 + 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*
c^6)*x)*sqrt(b^2 - 4*a*c)), 1/2*(6*(a^2*b^6 - 10*a^3*b^4*c + 30*a
^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^
4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 -
20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^
2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^
2 - 20*a^4*b*c^3)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - (5*a^2*b^5 - 36*a^3*b^3*c + 58*a^4*b*c^2 - 2*(b^4*c^3
 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^5 - 4*(b^5*c^2 - 8*a*b^3*c^3 + 16*
a^2*b*c^4)*x^4 + 2*(2*b^6*c - 18*a*b^4*c^2 + 51*a^2*b^2*c^3 - 50*
a^3*c^4)*x^3 + (5*b^7 - 38*a*b^5*c + 73*a^2*b^3*c^2 - 22*a^3*b*c^
3)*x^2 + 2*(5*a*b^6 - 39*a^2*b^4*c + 79*a^3*b^2*c^2 - 30*a^4*c^3)
*x + 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + (b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4)*x^4 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^
3)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(a*b^6 - 8*a^2*
b^4*c + 16*a^3*b^2*c^2)*x)*log(c*x^2 + b*x + a))*sqrt(-b^2 + 4*a*
c))/((a^2*b^4*c^4 - 8*a^3*b^2*c^5 + 16*a^4*c^6 + (b^4*c^6 - 8*a*b
^2*c^7 + 16*a^2*c^8)*x^4 + 2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^
7)*x^3 + (b^6*c^4 - 6*a*b^4*c^5 + 32*a^3*c^7)*x^2 + 2*(a*b^5*c^4
```

```
- 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 16.2446, size = 1714, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3,x)
```

```
[Out] (-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a*
*2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280
*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(
2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2
*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b
**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) -
3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10
*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))
 - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b*
*2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(
2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3
 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**
2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*
b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
 - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c - 3*
b**6)) + (-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**
3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**
2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**
6*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*
a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*
a*b**8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2
*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*
c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b*
*2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
 b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) + 3*sqrt(-(4
*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c
- b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*
b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c*
*3*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30
*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20
*a*b**8*c - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b*
*4*c - 3*b**6)) + (-58*a**4*b*c**2 + 36*a**3*b**3*c - 5*a**2*b**5
```

```
 + x**3*(36*a**3*c**4 - 102*a**2*b**2*c**3 + 48*a*b**4*c**2 - 6*b
**6*c) + x**2*(-42*a**3*b*c**3 - 41*a**2*b**3*c**2 + 34*a*b**5*c
- 5*b**7) + x*(28*a**4*c**3 - 142*a**3*b**2*c**2 + 76*a**2*b**4*c
 - 10*a*b**6))/(32*a**4*c**6 - 16*a**3*b**2*c**5 + 2*a**2*b**4*c*
*4 + x**4*(32*a**2*c**8 - 16*a*b**2*c**7 + 2*b**4*c**6) + x**3*(6
4*a**2*b*c**7 - 32*a*b**3*c**6 + 4*b**5*c**5) + x**2*(64*a**3*c**
7 - 12*a*b**4*c**5 + 2*b**6*c**4) + x*(64*a**3*b*c**6 - 32*a**2*b
**3*c**5 + 4*a*b**5*c**4)) + x/c**3
```

GIAC/XCAS [A] time = 0.268229, size = 381, normalized size = 1.6

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}c^{4}-8\,ab^{2}c^{5}+16\,a^{2}c^{6}\right)\sqrt{-b^{2}+4\,ac}}+\frac{x}{c^{3}}-\frac{3\,b\ln\left(cx^{2}+bx+a\right)}{2\,c^{4}}-\frac{5\,a^{2}b^{5}-36\,a^{3}b^{3}c+58\,a^{4}bc^{2}+6\left(b^{6}c-8\,ab^{4}c^{2}+17\,a^{2}b^{2}c^{3}-6\,a^{3}c^{4}\right)x^{3}+\left(5\,b^{7}-34\,ab^{5}c+41\,a^{2}b^{3}c^{2}+42\,a^{3}bc^{3}\right)x^{2}+2\left(5\right.}{2\left(cx^{2}+bx+a\right)^{2}\left(b^{2}-4\,ac\right)^{2}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + b/x + a/x^2)^(-3),x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x
+ b)/sqrt(-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sq
rt(-b^2 + 4*a*c)) + x/c^3 - 3/2*b*ln(c*x^2 + b*x + a)/c^4 - 1/2*(
5*a^2*b^5 - 36*a^3*b^3*c + 58*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2
+ 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 + (5*b^7 - 34*a*b^5*c + 41*a^2*
b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 38*a^2*b^4*c + 71*a^3*
b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*c^4
)
```

3.432 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$

Optimal. Leaf size=190

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{\log\left(a+bx+cx^2\right)}{2c^3}$$

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/
(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) +
b*(b^2 - 10*a*c)*x))/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b
*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[a + b*x + c*x^2]/(2*c^3)
```

Rubi [A] time = 0.575202, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{\log\left(a+bx+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/
(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) +
b*(b^2 - 10*a*c)*x))/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b
*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[a + b*x + c*x^2]/(2*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^3\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{x^4\left(2a+bx\right)}{2\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}+\frac{x^2\left(a\left(-16ac+b^2\right)+bx\left(-10ac+b^2\right)\right)}{2c\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}-\frac{\left(-7ac+b^2\right)\int b\,dx}{c^2\left(-4ac+b^2\right)^2}+\frac{\log\left(a+bx+cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x,x)
```

```
[Out] b*(30*a**2*c**2 - 10*a*b**2*c + b**4)*atanh((b + 2*c*x)/sqrt(-4*a
*c + b**2))/(c**3*(-4*a*c + b**2)**(5/2)) + x**4*(2*a + b*x)/(2*(
-4*a*c + b**2)*(a + b*x + c*x**2)**2) + x**2*(a*(-16*a*c + b**2)
+ b*x*(-10*a*c + b**2))/(2*c*(-4*a*c + b**2)**2*(a + b*x + c*x**2
)) - (-7*a*c + b**2)*Integral(b, x)/(c**2*(-4*a*c + b**2)**2) + l
og(a + b*x + c*x**2)/(2*c**3)
```

Mathematica [A] time = 0.564855, size = 221, normalized size = 1.16

$$\frac{-\frac{2bc(30a^2c^2-10ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+\frac{2a^3c^2+a^2bc(5cx-4b)+ab^3(b-5cx)+b^5x}{(b^2-4ac)(a+x(b+cx))^2}+\frac{32a^3c^3-39a^2b^2c^2+50a^2bc^3x+11ab^4c-30ab^3c^2x-b^6+4b^5cx}{(b^2-4ac)^2(a+x(b+cx))}+c}{2c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 3
0*a*b^3*c^2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x)
)) + (2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c
*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) - (2*b*c*(b^4 - 10*a*b^2
*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 +
4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [B] time = 0.027, size = 806, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x,x)
```

```
[Out] (1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^3+1/2*(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2
*c^2-8*a*b^2*c+b^4)/c^3*x+3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(
16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/2/c^3*ln(c^2*(16*a^2
*c^2-8*a*b^2*c+b^4)*(c*x^2+b*x+a))-30/(1024*a^5*c^9-1280*a^4*b^2*
c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2)*
```

```
arctan((2*(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*x+c^2*(16*a^2*c^2-8*a*b^
2*c+b^4)*b)/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^
2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2))*a^2*b*c+10/(1024*a^5*c^9-
1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^1
0*c^4)^(1/2)*arctan((2*(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*x+c^2*(16*a
^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b
^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2))*a*b^3-1/(102
4*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b
^8*c^5-b^10*c^4)^(1/2)*arctan((2*(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*x
+c^2*(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^9-1280*a^4*b^2*c^8
+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2))*b^
5/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274273, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^
3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*
c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2
+ 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*log((b^3 - 4*a*b*c
 + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sq
rt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (3*a^2*b^4 - 21*a^3*b^2*c +
 24*a^4*c^2 + 2*(2*b^5*c - 15*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*
b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*a^3*c^3)*x^2 + 2*(3*a*b^5
- 22*a^2*b^3*c + 31*a^3*b*c^2)*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^
4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2
 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a)
)*sqrt(b^2 - 4*a*c))/((a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 +
 (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*
```

```
c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^
2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x)*sqrt(b^2 - 4*
a*c)), -1/2*(2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2
- 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30
*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*
c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (3*a^2*b^4 - 21*a^
3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c - 15*a*b^3*c^2 + 25*a^2*b*c^3)*
x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*a^3*c^3)*x^2 + 2*
(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x + (a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3
*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 +
 b*x + a))*sqrt(-b^2 + 4*a*c))/((a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16
*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4
- 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a
^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x)*sqr
t(-b^2 + 4*a*c))]
```

Sympy [A] time = 10.6519, size = 1510, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/
(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(
x + (-64*a**3*c**5*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 1
0*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
+ 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
 + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*sqrt(-(4*a*
c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a
**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b*
*6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 1
2*a*b**4*c**3*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b
**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640
*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/
(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a
**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8
*c - b**10)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)
) + (b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**
4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*
c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*l
og(x + (-64*a**3*c**5*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**
```

```
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(b*sqrt(-(4*a
*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*
a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b
**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c -
12*a*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b
**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640
*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/
(2*c**3)) + a*b**4 + b**6*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a*
*2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4
*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*
c - b**10)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5))
 + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a**2*b**4 + x**3*(50*a**2*b
*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**2*(32*a**3*c**3 + 11*a**2
*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x*(62*a**3*b*c**2 - 44*a**2*
b**3*c + 6*a*b**5))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b*
*4*c**3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x*
*3*(64*a**2*b*c**6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**
3*c**6 - 12*a*b**4*c**4 + 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a
**2*b**3*c**4 + 4*a*b**5*c**3))
```

GIAC/XCAS [A] time = 0.287871, size = 331, normalized size = 1.74

$$-\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)\sqrt{-b^2+4\,ac}}+\frac{\ln\left(cx^2+bx+a\right)}{2\,c^3}+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\left(2\,b^5c-15\,ab^3c^2+25\,a^2bc^3\right)x^3+\left(3\,b^6-19\,ab^4c+11\,a^2b^2c^2+32\,a^3c^3\right)x^2+2\left(3\,ab^5-22\,a\right.}{2\left(cx^2+bx+a\right)^2\left(b^2-4\,ac\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x),x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)
) + 1/2*ln(c*x^2 + b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c +
 24*a^4*c^2 + 2*(2*b^5*c - 15*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*
b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*a^3*c^3)*x^2 + 2*(3*a*b^5
- 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a
*c)^2*c^3)
```

3.433 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx$

Optimal. Leaf size=111

$$\frac{12a^2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3a\left(\frac{2a}{x}+b\right)}{(b^2-4ac)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (
2*a)/x))/((b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b
 + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.139202, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{12a^2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3a\left(\frac{2a}{x}+b\right)}{(b^2-4ac)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (
2*a)/x))/((b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b
 + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 17.3953, size = 95, normalized size = 0.86

$$\frac{12a^2\operatorname{atanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}-\frac{3a\left(\frac{2a}{x}+b\right)}{(-4ac+b^2)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2(-4ac+b^2)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

```
[Out] 12*a**2*atanh((2*a/x + b)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(
5/2) - 3*a*(2*a/x + b)/((-4*a*c + b**2)**2*(a/x**2 + b/x + c)) +
```

```
(2*a/x + b)/(2*(-4*a*c + b**2)*(a/x**2 + b/x + c)**2)
```

Mathematica [A] time = 0.27454, size = 174, normalized size = 1.57

$$\frac{1}{2}\left(\frac{24a^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+\frac{a^2c(2cx-3b)+ab^2(b-4cx)+b^4x}{c^3(4ac-b^2)(a+x(b+cx))^2}+\frac{22a^2bc^2-20a^2c^3x-8ab^3c+16ab^2c^2x+b^5-2b^4cx}{c^3(b^2-4ac)^2(a+x(b+cx))}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 2
0*a^2*c^3*x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a
*b^2*(b - 4*c*x) + a^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a +
 x*(b + c*x))^2) + (24*a^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]
)/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.015, size = 260, normalized size = 2.3

$$\frac{1}{(cx^2+bx+a)^2}\left(-\frac{(10\,a^2c^2-8\,ab^2c+b^4)\,x^3}{(16\,a^2c^2-8\,ab^2c+b^4)\,c}+\frac{b\,(2\,a^2c^2+8\,ab^2c-b^4)\,x^2}{2\,c^2\,(16\,a^2c^2-8\,ab^2c+b^4)}-\frac{a\,(6\,a^2c^2-10\,ab^2c+b^4)\,x}{c^2\,(16\,a^2c^2-8\,ab^2c+b^4)}+\frac{a^2b\,(10\,ac}{2\,c^2\,(16\,a^2c^2-8}\right.$$
$$+12\,\frac{a^2}{(16\,a^2c^2-8\,ab^2c+b^4)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x^2,x)
```

```
[Out] (-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1
/2*b*(2*a^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2
-a*(6*a^2*c^2-10*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/
2*a^2*b*(10*a*c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a
)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269721, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2
 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c
- 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a
*c))/(c*x^2 + b*x + a)) - (a^2*b^3 - 10*a^3*b*c + 2*(b^4*c - 8*a*
b^2*c^2 + 10*a^2*c^3)*x^3 + (b^5 - 8*a*b^3*c - 2*a^2*b*c^2)*x^2 +
 2*(a*b^4 - 10*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c))/((a^2
*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + (b^4*c^4 - 8*a*b^2*c^5 +
16*a^2*c^6)*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^3 +
(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^2 + 2*(a*b^5*c^2 - 8*a^2*b
^3*c^3 + 16*a^3*b*c^4)*x)*sqrt(b^2 - 4*a*c)), 1/2*(24*(a^2*c^4*x^
4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 + (a^2*b^2*c^2 + 2*
a^3*c^3)*x^2)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) - (a^2*b^3 - 10*a^3*b*c + 2*(b^4*c - 8*a*b^2*c^2 + 10*a^2*c^3)
*x^3 + (b^5 - 8*a*b^3*c - 2*a^2*b*c^2)*x^2 + 2*(a*b^4 - 10*a^2*b^
2*c + 6*a^3*c^2)*x)*sqrt(-b^2 + 4*a*c))/((a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + 2*
(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^3 + (b^6*c^2 - 6*a*b^4*c
^3 + 32*a^3*c^5)*x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^
4)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 6.66454, size = 547, normalized size = 4.93

$$-6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+6}{12a^2c}\right)$$
$$+6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-6a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2}{12a^2c}\right)$$
$$-\frac{-10a^3bc+a^2b^3+x^3\left(20a^2c^3-16ab^2c^2+2b^4c\right)+x^2\left(-2a^2bc^2-8ab^3c+b^5\right)+x\left(12a^3c^2-20a\right.}{32a^4c^4-16a^3b^2c^3+2a^2b^4c^2+x^4\left(32a^2c^6-16ab^2c^5+2b^4c^4\right)+x^3\left(64a^2bc^5-32ab^3c^4+4b^5c^3\right)+x^2\left(64a^3c^5-12ab^4c^3+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

```
[Out] -6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-
1/(4*a*c - b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)*
*5) - 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqr
t(-1/(4*a*c - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1
/(4*a*c - b**2)**5)*log(x + (384*a**5*c**3*sqrt(-1/(4*a*c - b**2)
**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) + 72*a**3*b*
*4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/(4*a*c - b*
*2)**5) + 6*a**2*b)/(12*a**2*c)) - (-10*a**3*b*c + a**2*b**3 + x*
*3*(20*a**2*c**3 - 16*a*b**2*c**2 + 2*b**4*c) + x**2*(-2*a**2*b*c
**2 - 8*a*b**3*c + b**5) + x*(12*a**3*c**2 - 20*a**2*b**2*c + 2*a
*b**4))/(32*a**4*c**4 - 16*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x*
*4*(32*a**2*c**6 - 16*a*b**2*c**5 + 2*b**4*c**4) + x**3*(64*a**2*
b*c**5 - 32*a*b**3*c**4 + 4*b**5*c**3) + x**2*(64*a**3*c**5 - 12*
a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c**4 - 32*a**2*b**3*c**
3 + 4*a*b**5*c**2))
```

GIAC/XCAS [A] time = 0.295476, size = 273, normalized size = 2.46

$$\frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2b^2cx+12\,a^3c^2x+a^2b^3-10\,a^3bc}{2\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^2),x, algorithm="giac")
```

```
[Out] 12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^
```

```
2*x^3 + 20*a^2*c^3*x^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^
2 + 2*a*b^4*x - 20*a^2*b^2*c*x + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*
b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^2 + b*x + a)^2)
```

3.434 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} dx$

Optimal. Leaf size=107

$$\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(x^3*(b + 2*c*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x
*(2*a + b*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.113438, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -(x^3*(b + 2*c*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x
*(2*a + b*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 24.14, size = 102, normalized size = 0.95

$$\frac{6ab\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{3bx\left(2a+bx\right)}{2\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}-\frac{x^3\left(b+2cx\right)}{2\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

```
[Out] 6*a*b*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(5/
2) + 3*b*x*(2*a + b*x)/(2*(-4*a*c + b**2)**2*(a + b*x + c*x**2))
- x**3*(b + 2*c*x)/(2*(-4*a*c + b**2)*(a + b*x + c*x**2)**2)
```

Mathematica [A] time = 0.348262, size = 126, normalized size = 1.18

$$-\frac{8a^3c+a^2\left(b^2+10bcx+16c^2x^2\right)+abx\left(2b^2+bcx+6c^2x^2\right)+b^4x^2}{2c\left(b^2-4ac\right)^2(a+x(b+cx))^2}-\frac{6ab\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^
2 + 10*b*c*x + 16*c^2*x^2))/(2*c*(b^2 - 4*a*c)^2*(a + x*(b + c*x)
)^2) - (6*a*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(5/2)
```

Maple [B] time = 0.013, size = 223, normalized size = 2.1

$$\frac{1}{(cx^2+bx+a)^2}\left(-3\,\frac{abcx^3}{16\,a^2c^2-8\,ab^2c+b^4}-\frac{\left(16\,a^2c^2+ab^2c+b^4\right)x^2}{2\,\left(16\,a^2c^2-8\,ab^2c+b^4\right)c}-\frac{\left(5\,ac+b^2\right)abx}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)c}-\frac{a^2\left(8\,ac+b^2\right)}{2\,\left(16\,a^2c^2-8\,ab^2c+b\right.}\right.$$
$$-6\,\frac{ab}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x^3,x)
```

```
[Out] (-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+
b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c
^2-8*a*b^2*c+b^4)*x-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b
^4))/(c*x^2+b*x+a)^2-6*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272292, size = 1, normalized size = 0.01

$$\left[\frac{6\left(abc^3x^4+2\,ab^2c^2x^3+2\,a^2b^2cx+a^3bc+\left(ab^3c+2\,a^2bc^2\right)x^2\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\left(\cdots\right.}{2\left(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3+(b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^4+2\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)x^3+(b^6c-6\,a\cdots\right.}\right.$$

$$-\frac{12\left(abc^3x^4+2\,ab^2c^2x^3+2\,a^2b^2cx+a^3bc+\left(ab^3c+2\,a^2bc^2\right)x^2\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+\left(6\,abc^2x^3+a^2b^2+8\,a^3c+\left(\cdots\right.\right.}{2\left(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3+(b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^4+2\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)x^3+(b^6c-6\,ab^4c^2+32\,a^3c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*(6*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c
+ (a*b^3*c + 2*a^2*b*c^2)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*
a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))
/(c*x^2 + b*x + a)) - (6*a*b*c^2*x^3 + a^2*b^2 + 8*a^3*c + (b^4 +
 a*b^2*c + 16*a^2*c^2)*x^2 + 2*(a*b^3 + 5*a^2*b*c)*x)*sqrt(b^2 -
4*a*c))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*
c^4)*x^3 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c -
8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x)*sqrt(b^2 - 4*a*c)), -1/2*(12*(a*
b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c
+ 2*a^2*b*c^2)*x^2)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 -
 4*a*c)) + (6*a*b*c^2*x^3 + a^2*b^2 + 8*a^3*c + (b^4 + a*b^2*c +
16*a^2*c^2)*x^2 + 2*(a*b^3 + 5*a^2*b*c)*x)*sqrt(-b^2 + 4*a*c))/((
a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 +
 (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*
c^2 + 16*a^3*b*c^3)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.71233, size = 510, normalized size = 4.77

$$3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a}{6abc}\right.$$

$$-3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a}{6abc}\right.$$

$$-\frac{8a^3c+a^2b^2+6abc^2x^3+x^2\left(16a^2c^2+ab^2c+b^4\right)+x\left(10a^2bc+2ab^3\right)}{32a^4c^3-16a^3b^2c^2+2a^2b^4c+x^4\left(32a^2c^5-16ab^2c^4+2b^4c^3\right)+x^3\left(64a^2bc^4-32ab^3c^3+4b^5c^2\right)+x^2\left(64a^3c^4-12ab^4c^2+2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-
1/(4*a*c - b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)*
*5) - 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-
1/(4*a*c - b**2)**5) + 3*a*b**2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*
c - b**2)**5)*log(x + (192*a**4*b*c**3*sqrt(-1/(4*a*c - b**2)**5)
 - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) + 36*a**2*b**5*c
*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5)
 + 3*a*b**2)/(6*a*b*c)) - (8*a**3*c + a**2*b**2 + 6*a*b*c**2*x**3
 + x**2*(16*a**2*c**2 + a*b**2*c + b**4) + x*(10*a**2*b*c + 2*a*b
**3))/(32*a**4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(3
2*a**2*c**5 - 16*a*b**2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**
4 - 32*a*b**3*c**3 + 4*b**5*c**2) + x**2*(64*a**3*c**4 - 12*a*b**
4*c**2 + 2*b**6*c) + x*(64*a**3*b*c**3 - 32*a**2*b**3*c**2 + 4*a*
b**5*c))
```

GIAC/XCAS [A] time = 0.264445, size = 220, normalized size = 2.06

$$-\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^3),x, algorithm="giac")
```

```
[Out] -6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 +
```

```
 a*b^2*c*x^2 + 16*a^2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^
2 + 8*a^3*c)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a
)^2)
```

3.435 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx$

Optimal. Leaf size=115

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b +
(b^2 + 2*a*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 +
2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2
)
```

Rubi [A] time = 0.147836, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b +
(b^2 + 2*a*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 +
2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2
)
```

Rubi in Sympy [A] time = 29.0257, size = 110, normalized size = 0.96

$$\frac{x\left(2a+bx\right)}{2\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}+\frac{6ab+x\left(4ac+2b^2\right)}{2\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

```
[Out] x*(2*a + b*x)/(2*(-4*a*c + b**2)*(a + b*x + c*x**2)**2) + (6*a*b
+ x*(4*a*c + 2*b**2))/(2*(-4*a*c + b**2)**2*(a + b*x + c*x**2)) -
```

```
 2*(2*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c
+ b**2)**(5/2)
```

Mathematica [A] time = 0.259498, size = 131, normalized size = 1.14

$$\frac{1}{2}\left(\frac{(2ac+b^2)(b+2cx)}{c(b^2-4ac)^2(a+x(b+cx))}+\frac{a(b-2cx)+b^2x}{c(4ac-b^2)(a+x(b+cx))^2}+\frac{4(2ac+b^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] ((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) +
 ((b^2 + 2*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))
) + (4*(b^2 + 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^
2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.013, size = 262, normalized size = 2.3

$$\frac{1}{(cx^2+bx+a)^2}\left(\frac{c(2ac+b^2)x^3}{16a^2c^2-8ab^2c+b^4}+\frac{3b(2ac+b^2)x^2}{32a^2c^2-16ab^2c+2b^4}-\frac{a(2ac-5b^2)x}{16a^2c^2-8ab^2c+b^4}+3\frac{a^2b}{16a^2c^2-8ab^2c+b^4}\right)$$
$$+4\frac{ac}{(16a^2c^2-8ab^2c+b^4)\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$
$$+2\frac{b^2}{(16a^2c^2-8ab^2c+b^4)\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x^4,x)
```

```
[Out] (c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2
*c+b^4)*x+3*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+4/(
16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*a*c+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273332, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*(2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c +
 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 +
2*a^2*b*c)*x)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x +
a)) + (2*(b^2*c + 2*a*c^2)*x^3 + 6*a^2*b + 3*(b^3 + 2*a*b*c)*x^2
+ 2*(5*a*b^2 - 2*a^2*c)*x)*sqrt(b^2 - 4*a*c))/((a^2*b^4 - 8*a^3*b
^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt(b^2
 - 4*a*c)), 1/2*(4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c +
 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 +
2*(a*b^3 + 2*a^2*b*c)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) + (2*(b^2*c + 2*a*c^2)*x^3 + 6*a^2*b + 3*(b^3 + 2*a
*b*c)*x^2 + 2*(5*a*b^2 - 2*a^2*c)*x)*sqrt(-b^2 + 4*a*c))/((a^2*b^
4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*
b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*
x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.51506, size = 570, normalized size = 4.96

$$-\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)+2abc+b}{4ac^2+2b^2c}\right.$$

$$+\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)\log\left(x+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(2ac+b^2)+2abc-b^6}{4ac^2+2b^2c}\right.$$

$$+\frac{6a^2b+x^3(4ac^2+2b^2c)+x^2(6abc+3b^3)+x(-4a^2c+10ab^2)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3(64a^2bc^3-32ab^3c^2+4b^5c)+x^2(64a^3c^3-12ab^4c+2b^6)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3
*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sq
rt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*
a*c - b**2)**5)*(2*a*c + b**2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c -
b**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c**2 + 2*b**2*c)) + sqrt(-1
/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (64*a**3*c**3*sqrt(-1/
(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*sqrt(-1/(4*
a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**
2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)
*(2*a*c + b**2) + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3
*(4*a*c**2 + 2*b**2*c) + x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c +
 10*a*b**2))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*
(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*b*c
**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4
*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

GIAC/XCAS [A] time = 0.274233, size = 208, normalized size = 1.81

$$\frac{2\,(b^2+2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}+\frac{2\,b^2cx^3+4\,ac^2x^3+3\,b^3x^2+6\,abcx^2+10\,ab^2x-4\,a^2cx+6\,a^2b}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^4),x, algorithm="giac")
```

```
[Out] 2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*
a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*
a*c^2*x^3 + 3*b^3*x^2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*
a^2*b)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.436 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} \, dx$

Optimal. Leaf size=103

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c
*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0985487, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c
*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 18.1801, size = 97, normalized size = 0.94

$$\frac{6bc\,\text{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{5}{2}}}-\frac{3b\,(b+2cx)}{2\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}+\frac{2a+bx}{2\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

```
[Out] 6*b*c*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(5/
2) - 3*b*(b + 2*c*x)/(2*(-4*a*c + b**2)**2*(a + b*x + c*x**2)) +
(2*a + b*x)/(2*(-4*a*c + b**2)*(a + b*x + c*x**2)**2)
```

Mathematica [A] time = 0.169536, size = 102, normalized size = 0.99

$$\frac{\frac{(b^2-4ac)(2a+bx)}{(a+x(b+cx))^2}-\frac{12bc\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3b(b+2cx)}{a+x(b+cx)}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*
x))/(a + x*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.005, size = 130, normalized size = 1.3

$$\frac{-bx-2a}{(8ac-2b^2)(cx^2+bx+a)^2}-3\frac{bcx}{(4ac-b^2)^2(cx^2+bx+a)}-\frac{3b^2}{2(4ac-b^2)^2(cx^2+bx+a)}-6\frac{bc}{(4ac-b^2)^{5/2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x^5,x)
```

```
[Out] 1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3*b/(4*a*c-b^2)^2/(c*x
^2+b*x+a)*c*x-3/2*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)-6*b/(4*a*c-b^2)
^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26734, size = 1, normalized size = 0.01

$$\left[\frac{6\left(bc^3x^4 + 2\,b^2c^2x^3 + 2\,ab^2cx + a^2bc + \left(b^3c + 2\,abc^2\right)x^2\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right) - \left(6\,bc^2x \cdots\right.}{2\left(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^4 + 2\,(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^3 + (b^6 - 6\,ab^4c + 32\,a^3c^3) \cdots\right.}\right.$$

$$-\frac{12\left(bc^3x^4 + 2\,b^2c^2x^3 + 2\,ab^2cx + a^2bc + \left(b^3c + 2\,abc^2\right)x^2\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) + \left(6\,bc^2x^3 + 9\,b^2cx^2 + ab^2 + 8\,a \cdots\right.}{2\left(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^4 + 2\,(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^3 + (b^6 - 6\,ab^4c + 32\,a^3c^3)x^2 + 2\,(a \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^5),x, algorithm="fricas")
```

```
[Out] [1/2*(6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3
*c + 2*a*b*c^2)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x +
 (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 +
b*x + a)) - (6*b*c^2*x^3 + 9*b^2*c*x^2 + a*b^2 + 8*a^2*c + 2*(b^3
 + 5*a*b*c)*x)*sqrt(b^2 - 4*a*c))/((a^2*b^4 - 8*a^3*b^2*c + 16*a^
4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2
 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt(b^2 - 4*a*c)),
-1/2*(12*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^
3*c + 2*a*b*c^2)*x^2)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
 - 4*a*c)) + (6*b*c^2*x^3 + 9*b^2*c*x^2 + a*b^2 + 8*a^2*c + 2*(b^
3 + 5*a*b*c)*x)*sqrt(-b^2 + 4*a*c))/((a^2*b^4 - 8*a^3*b^2*c + 16*
a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x
^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt(-b^2 + 4*a*c)
)]
```

Sympy [A] time = 5.11295, size = 479, normalized size = 4.65

$$3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x + \frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}} + 144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}} - 36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}} + 3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}} + 3b \cdots}{6bc^2}\right.$$

$$-3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x + \frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}} - 144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}} + 36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}} - 3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}} + 3b \cdots}{6bc^2}\right.$$

$$-\frac{8a^2c + ab^2 + 9b^2cx^2 + 6bc^2x^3 + x\left(10abc + 2b^3\right)}{32a^4c^2 - 16a^3b^2c + 2a^2b^4 + x^4\left(32a^2c^4 - 16ab^2c^3 + 2b^4c^2\right) + x^3\left(64a^2bc^3 - 32ab^3c^2 + 4b^5c\right) + x^2\left(64a^3c^3 - 12ab^4c + 2b^6\right) + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-
1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)*
*5) - 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-
1/(4*a*c - b**2)**5) + 3*b**2*c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a
*c - b**2)**5)*log(x + (192*a**3*b*c**4*sqrt(-1/(4*a*c - b**2)**5
) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**
2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5
) + 3*b**2*c)/(6*b*c**2)) - (8*a**2*c + a*b**2 + 9*b**2*c*x**2 +
6*b*c**2*x**3 + x*(10*a*b*c + 2*b**3))/(32*a**4*c**2 - 16*a**3*b*
*2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4
*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2
*(64*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a
**2*b**3*c + 4*a*b**5))
```

GIAC/XCAS [A] time = 0.283169, size = 182, normalized size = 1.77

$$-\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^5),x, algorithm="giac")
```

```
[Out] -6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2
 + 2*b^3*x + 10*a*b*c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16
*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.437 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx$

Optimal. Leaf size=101

$$-\frac{12c^2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3c(b+2cx)}{(b^2-4ac)^2\,(a+bx+cx^2)}-\frac{b+2cx}{2\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*
c*x))/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0912883, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{12c^2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3c(b+2cx)}{(b^2-4ac)^2\,(a+bx+cx^2)}-\frac{b+2cx}{2\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*
c*x))/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 16.6955, size = 95, normalized size = 0.94

$$-\frac{12c^2 \operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}+\frac{3c\,(b+2cx)}{(-4ac+b^2)^2\,(a+bx+cx^2)}-\frac{b+2cx}{2\,(-4ac+b^2)\,(a+bx+cx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

```
[Out] -12*c**2*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**
(5/2) + 3*c*(b + 2*c*x)/((-4*a*c + b**2)**2*(a + b*x + c*x**2)) -
 (b + 2*c*x)/(2*(-4*a*c + b**2)*(a + b*x + c*x**2)**2)
```

Mathematica [A] time = 0.159161, size = 97, normalized size = 0.96

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{(b+2cx)\left(-2c\left(5a+3cx^2\right)+b^2-6bcx\right)}{(a+x(b+cx))^2}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b
+ c*x))^2) + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.005, size = 129, normalized size = 1.3

$$\frac{2\,cx+b}{\left(8\,ac-2\,b^2\right)\left(cx^2+bx+a\right)^2}+6\,\frac{c^2x}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)}+3\,\frac{bc}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)}+12\,\frac{c^2}{\left(4\,ac-b^2\right)^{5/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x^6,x)
```

```
[Out] 1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+6*c^2/(4*a*c-b^2)^2/(c*
x^2+b*x+a)*x+3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b+12*c^2/(4*a*c-b^2)
^(5/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262293, size = 1, normalized size = 0.01

$$\left[\frac{12\left(c^4x^4+2\,bc^3x^3+2\,abc^2x+a^2c^2+\left(b^2c^2+2\,ac^3\right)x^2\right)\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x-\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\left(12\,c^3x\right.}{2\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^3+\left(b^6-6\,ab^4c+32\,a^3c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^6),x, algorithm="fricas")
```

```
[Out] [1/2*(12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^
2 + 2*a*c^3)*x^2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (
2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*
x + a)) + (12*c^3*x^3 + 18*b*c^2*x^2 - b^3 + 10*a*b*c + 4*(b^2*c
+ 5*a*c^2)*x)*sqrt(b^2 - 4*a*c))/((a^2*b^4 - 8*a^3*b^2*c + 16*a^4
*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt(b^2 - 4*a*c)), 1
/2*(24*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2
+ 2*a*c^3)*x^2)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) + (12*c^3*x^3 + 18*b*c^2*x^2 - b^3 + 10*a*b*c + 4*(b^2*c + 5
*a*c^2)*x)*sqrt(-b^2 + 4*a*c))/((a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c
^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 +
2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.19414, size = 474, normalized size = 4.69

$$-6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+6b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+6b}{12c^3}\right.$$

$$+6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+6bc}{12c^3}\right.$$

$$+\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x\left(20ac^2+4b^2c\right)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\left(32a^2c^4-16ab^2c^3+2b^4c^2\right)+x^3\left(64a^2bc^3-32ab^3c^2+4b^5c\right)+x^2\left(64a^3c^3-12ab^4c+2b^6\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

```
[Out] -6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-
1/(4*a*c - b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)*
```

```
*5) - 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqr
t(-1/(4*a*c - b**2)**5) + 6*b*c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(
4*a*c - b**2)**5)*log(x + (384*a**3*c**5*sqrt(-1/(4*a*c - b**2)**
5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 72*a*b**4*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4*a*c - b**2
)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2
+ 12*c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**
3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*
b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) +
x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 -
32*a**2*b**3*c + 4*a*b**5))
```

GIAC/XCAS [A] time = 0.313823, size = 184, normalized size = 1.82

$$\frac{12\,c^{2}\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{12\,c^{3}x^{3}+18\,bc^{2}x^{2}+4\,b^{2}cx+20\,ac^{2}x-b^{3}+10\,abc}{2\,\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\left(cx^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^6),x, algorithm="giac")
```

```
[Out] 12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2
 + 4*b^2*c*x + 20*a*c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 1
6*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.438 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$

Optimal. Leaf size=185

$$-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (
2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(
b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2
*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^
(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^2]/(2*a^3)
```

Rubi [A] time = 0.482344, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (
2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(
b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2
*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^
(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^2]/(2*a^3)
```

Rubi in Sympy [A] time = 82.0019, size = 180, normalized size = 0.97

$$\frac{-2ac+b^2+bcx}{2a\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}+\frac{16a^2c^2-15ab^2c+2b^4+2bcx\left(-7ac+b^2\right)}{2a^2\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^3\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{\log(x)}{a^3}-\frac{\log\left(a+bx+cx^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x**7,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(2*a*(-4*a*c + b**2)*(a + b*x + c*x**2)**
2) + (16*a**2*c**2 - 15*a*b**2*c + 2*b**4 + 2*b*c*x*(-7*a*c + b**
2))/(2*a**2*(-4*a*c + b**2)**2*(a + b*x + c*x**2)) + b*(30*a**2*c
**2 - 10*a*b**2*c + b**4)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/
(a**3*(-4*a*c + b**2)**(5/2)) + log(x)/a**3 - log(a + b*x + c*x**
2)/(2*a**3)
```

Mathematica [A] time = 0.594925, size = 178, normalized size = 0.96

$$\frac{\frac{a^2\left(-2ac+b^2+bcx\right)}{(b^2-4ac)(a+x(b+cx))^2}-\frac{2b\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+\frac{a\left(16a^2c^2-15ab^2c-14abc^2x+2b^4+2b^3cx\right)}{(b^2-4ac)^2(a+x(b+cx))}-\log(a+x(b+cx))+2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2)
+ (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x)
)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c +
30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(5/2) + 2*Log[x] - Log[a + x*(b + c*x)])/(2*a^3)
```

Maple [B] time = 0.024, size = 1159, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x^7,x)
```

```
[Out] ln(x)/a^3-7/a/(c*x^2+b*x+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
3+1/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+8/
(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-29/2/a/(c*x^2+
b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2+2/a^2/(c*x^2+b*x+
a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4-1/(c*x^2+b*x+a)^2*b/(16
*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-6/a/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2
-8*a*b^2*c+b^4)*x*c+1/a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2-8*a*b^2
*c+b^4)*x+12*a/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2-21/
```

```
2/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+3/2/a/(c*x^2+b
*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4-8/a/(16*a^2*c^2-8*a*b^2*c+
b^4)*c^2*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^2+b*x+a))+4/a^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*c*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^2+b*x+
a))*b^2-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2
*c+b^4)*(c*x^2+b*x+a))*b^4-30/a/(1024*a^5*c^5-1280*a^4*b^2*c^4+64
0*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*
x*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*
a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8
*c-b^10)^(1/2))*b*c^2+10/a^2/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a
^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x*(
16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5
*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-
b^10)^(1/2))*b^3*c-1/a^3/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b
^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x*(16*a
^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5
-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10
)^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.644771, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^7),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^
3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*
c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2
+ 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*log((b^3 - 4*a*b*c
 + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sq
rt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (3*a^2*b^4 - 21*a^3*b^2*c +
 24*a^4*c^2 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a
^2*b^2*c^2 + 16*a^3*c^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2
```

```
)*x - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*(a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^
3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))*sqrt(b
^2 - 4*a*c))/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^2
- 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^4 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2
+ 16*a^5*b*c^3)*x^3 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^2 +
2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x)*sqrt(b^2 - 4*a*c)), -
1/2*(2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b
^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2
*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2
 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (3*a^2*b^4 - 21*a^3*b^2*c
+ 24*a^4*c^2 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*
a^2*b^2*c^2 + 16*a^3*c^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^
2)*x - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3
 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c +
16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c
^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))*sqrt(
-b^2 + 4*a*c))/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^4 + 2*(a^3*b^5*c - 8*a^4*b^3*c^
2 + 16*a^5*b*c^3)*x^3 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^2
+ 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x)*sqrt(-b^2 + 4*a*c))
]
```

Sympy [A] time = 66.5154, size = 4862, normalized size = 26.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x**7,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/
(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))*log(
x + (98304*a**14*c**8*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2
- 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c*
*4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**1
0)) - 1/(2*a**3))**2 - 211968*a**13*b**2*c**7*(-b*sqrt(-(4*a*c -
b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*
c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c
**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 + 196352*a**12*b**4*
```

c**6*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b
4)/(2*a3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**
4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))
2 - 102528*a11*b**6*c**5*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**
2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*
b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
 - b**10)) - 1/(2*a**3))**2 - 49152*a**11*c**8*(-b*sqrt(-(4*a*c -
 b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5
*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*
c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 33120*a**10*b**8*c**
4*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4
)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c
3 - 160*a2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2
 + 68544*a**10*b**2*c**7*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c*
*2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2
*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b
10)) - 1/(2*a3)) - 6796*a**9*b**10*c**3*(-b*sqrt(-(4*a*c - b*
*2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**
2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 - 41296*a**9*b**4*c**6
*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)
/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c*
*3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 8
67*a**8*b**12*c**2*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 1
0*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
+ 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
 - 1/(2*a**3))**2 + 14036*a**8*b**6*c**5*(-b*sqrt(-(4*a*c - b**2)
5)*(30*a2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5
- 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 49152*a**8*c**8 - 63*a**7*
b**14*c*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c
+ b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*
b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**
3))**2 - 2935*a**7*b**8*c**4*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**
2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*
b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
 - b**10)) - 1/(2*a**3)) + 143424*a**7*b**2*c**7 + 2*a**6*b**16*(
-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(
2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3
 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 +
382*a**6*b**10*c**3*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)
) - 1/(2*a**3)) - 155056*a**6*b**4*c**6 - 29*a**5*b**12*c**2*(-b*
sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a
3*(1024*a5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 88492*
a**5*b**6*c**5 + a**4*b**14*c*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a*
*2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4
*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*
c - b**10)) - 1/(2*a**3)) - 30185*a**4*b**8*c**4 + 6414*a**3*b**1
0*c**3 - 838*a**2*b**12*c**2 + 62*a*b**14*c - 2*b**16)/(69120*a**
7*b*c**8 - 102690*a**6*b**3*c**7 + 67554*a**5*b**5*c**6 - 25155*a
4*b7*c**5 + 5690*a**3*b**9*c**4 - 780*a**2*b**11*c**3 + 60*a*

```
b**13*c**2 - 2*b**15*c)) + (b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c
**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
b**10)) - 1/(2*a**3))*log(x + (98304*a**14*c**8*(b*sqrt(-(4*a*c -
 b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5
*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*
c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 - 211968*a**13*b**2
*c**7*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b
**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**
4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))
**2 + 196352*a**12*b**4*c**6*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2
*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b
**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10)) - 1/(2*a**3))**2 - 102528*a**11*b**6*c**5*(b*sqrt(-(4*a
*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*
a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b
**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 - 49152*a**11*c
**8*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**
4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*
c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) +
 33120*a**10*b**8*c**4*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2
- 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c*
*4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**1
0)) - 1/(2*a**3))**2 + 68544*a**10*b**2*c**7*(b*sqrt(-(4*a*c - b*
*2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**
2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 6796*a**9*b**10*c**3*(b
*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*
a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3))**2 - 41
296*a**9*b**4*c**6*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10
*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
- 1/(2*a**3)) + 867*a**8*b**12*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(
30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280
*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) - 1/(2*a**3))**2 + 14036*a**8*b**6*c**5*(b*sqrt(
-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(
1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a
**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)) - 49152*a**8*
c**8 - 63*a**7*b**14*c*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2
- 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c*
*4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**1
0)) - 1/(2*a**3))**2 - 2935*a**7*b**8*c**4*(b*sqrt(-(4*a*c - b**2
)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5
 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
+ 20*a*b**8*c - b**10)) - 1/(2*a**3)) + 143424*a**7*b**2*c**7 + 2
*a**6*b**16*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2
*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a*
*3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*
a**3))**2 + 382*a**6*b**10*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(30*a
**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8
*c - b**10)) - 1/(2*a**3)) - 155056*a**6*b**4*c**6 - 29*a**5*b**1
```

```
2*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c +
b**4)/(2*a**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b*
*4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 1/(2*a**3)
) + 88492*a**5*b**6*c**5 + a**4*b**14*c*(b*sqrt(-(4*a*c - b**2)**
5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*a**3*(1024*a**5*c**5 -
1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10)) - 1/(2*a**3)) - 30185*a**4*b**8*c**4 + 6414*
a**3*b**10*c**3 - 838*a**2*b**12*c**2 + 62*a*b**14*c - 2*b**16)/(
69120*a**7*b*c**8 - 102690*a**6*b**3*c**7 + 67554*a**5*b**5*c**6
- 25155*a**4*b**7*c**5 + 5690*a**3*b**9*c**4 - 780*a**2*b**11*c**
3 + 60*a*b**13*c**2 - 2*b**15*c)) - (-24*a**3*c**2 + 21*a**2*b**2
*c - 3*a*b**4 + x**3*(14*a*b*c**3 - 2*b**3*c**2) + x**2*(-16*a**2
*c**3 + 29*a*b**2*c**2 - 4*b**4*c) + x*(2*a**2*b*c**2 + 12*a*b**3
*c - 2*b**5))/(32*a**6*c**2 - 16*a**5*b**2*c + 2*a**4*b**4 + x**4
*(32*a**4*c**4 - 16*a**3*b**2*c**3 + 2*a**2*b**4*c**2) + x**3*(64
*a**4*b*c**3 - 32*a**3*b**3*c**2 + 4*a**2*b**5*c) + x**2*(64*a**5
*c**3 - 12*a**3*b**4*c + 2*a**2*b**6) + x*(64*a**5*b*c**2 - 32*a*
*4*b**3*c + 4*a**3*b**5)) + log(x)/a**3
```

GIAC/XCAS [A] time = 0.305493, size = 323, normalized size = 1.75

$$-\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)\sqrt{-b^2+4\,ac}}-\frac{\ln\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\ln\left(|x|\right)}{a^3}+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\left(ab^3c^2-7\,a^2bc^3\right)x^3+\left(4\,ab^4c-29\,a^2b^2c^2+16\,a^3c^3\right)x^2+2\left(ab^5-6\,a^2b^3c-a^3bc^2\right)x}{2\left(cx^2+bx+a\right)^2\left(b^2-4\,ac\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^7),x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)
) - 1/2*ln(c*x^2 + b*x + a)/a^3 + ln(abs(x))/a^3 + 1/2*(3*a^2*b^4
 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 +
(4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*x^2 + 2*(a*b^5 - 6*a^2*
b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^3)
```

3.439 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$

Optimal. Leaf size=239

$$\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3x\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 -
 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^
4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2
- 4*a*c)^2*x*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b
^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4
*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Log[a + b*x + c*x
^2])/(2*a^4)
```

Rubi [A] time = 0.633861, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3x\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 -
 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^
4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2
- 4*a*c)^2*x*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b
^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4
*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Log[a + b*x + c*x
^2])/(2*a^4)
```

Rubi in Sympy [A] time = 141.342, size = 238, normalized size = 1.

$$\frac{-2ac+b^2+bcx}{2ax\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}+\frac{20a^2c^2-20ab^2c+3b^4+3bcx\left(-6ac+b^2\right)}{2a^2x\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(-5ac+b^2\right)\left(-2ac+b^2\right)}{a^3x\left(-4ac+b^2\right)^2}-\frac{3b\log\left(x\right)}{a^4}+\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{6\left(-10a^3c^3+15a^2b^2c^2-5ab^4c+\frac{b^6}{2}\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^4\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(2*a*x*(-4*a*c + b**2)*(a + b*x + c*x**2)
**2) + (20*a**2*c**2 - 20*a*b**2*c + 3*b**4 + 3*b*c*x*(-6*a*c + b
**2))/(2*a**2*x*(-4*a*c + b**2)**2*(a + b*x + c*x**2)) - 3*(-5*a*
c + b**2)*(-2*a*c + b**2)/(a**3*x*(-4*a*c + b**2)**2) - 3*b*log(x
)/a**4 + 3*b*log(a + b*x + c*x**2)/(2*a**4) - 6*(-10*a**3*c**3 +
15*a**2*b**2*c**2 - 5*a*b**4*c + b**6/2)*atanh((b + 2*c*x)/sqrt(-
4*a*c + b**2))/(a**4*(-4*a*c + b**2)**(5/2))
```

Mathematica [A] time = 0.7415, size = 221, normalized size = 0.92

$$\frac{\frac{a^2\left(-3abc-2ac^2x+b^3+b^2cx\right)}{\left(4ac-b^2\right)\left(a+x(b+cx)\right)^2}-\frac{a\left(46a^2bc^2+28a^2c^3x-29ab^3c-26ab^2c^2x+4b^5+4b^4cx\right)}{\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)}+\frac{6\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+3b\log(a}{2a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] ((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 +
4*a*c)*(a + x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c
^2 + 4*b^4*c*x - 26*a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2
*(a + x*(b + c*x))) + (6*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*
a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [B] time = 0.03, size = 1438, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^2+b/x)^3/x^8,x)
```

```
[Out] -1/a^3/x-3*b*ln(x)/a^4-14/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^3+13/a^2/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3*b^2-2/a^3/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^3*b^4-37/a/(c*x^2+b*x+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+
55/2/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-4
/a^3/(c*x^2+b*x+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-18/(c*x
^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3-7/a/(c*x^2+b*x+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+12/a^2/(c*x^2+b*x+a)^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-2/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x*b^6-29/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^
4)*c^2+18/a/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c-5/2/
a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)+24/a^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*c^2*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^2+b*x+
a))*b-12/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln((16*a^2*c^2-8*a*b^2*
c+b^4)*(c*x^2+b*x+a))*b^3+3/2/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln((
16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^2+b*x+a))*b^5-60/a/(1024*a^5*c^5-1
280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^
(1/2)*arctan((2*c*x*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^
2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^
2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*c^3+90/a^2/(1024*a^5*c^5-1280*a
^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)
*arctan((2*c*x*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b
^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6
*c^2+20*a*b^8*c-b^10)^(1/2))*b^2*c^2-30/a^3/(1024*a^5*c^5-1280*a^
4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*
arctan((2*c*x*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^
4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*
c^2+20*a*b^8*c-b^10)^(1/2))*b^4*c+3/a^4/(1024*a^5*c^5-1280*a^4*b^
2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arct
an((2*c*x*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b
)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+
20*a*b^8*c-b^10)^(1/2))*b^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.859926, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^8),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*
x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^
4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c
^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3
)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x)
*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (2*a^3*b
^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 1
0*a^3*c^4)*x^4 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^
3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 +
 (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2)*x - 3*((b^5*c^2 - 8*a
*b^3*c^3 + 16*a^2*b*c^4)*x^5 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^
2*c^3)*x^4 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^3 + 2*(a*b^6 - 8*
a^2*b^4*c + 16*a^3*b^2*c^2)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4
*b*c^2)*x)*log(c*x^2 + b*x + a) + 6*((b^5*c^2 - 8*a*b^3*c^3 + 16*
a^2*b*c^4)*x^5 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x^4 + (
b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16
*a^3*b^2*c^2)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)*log
(x))*sqrt(b^2 - 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c
^4)*x^5 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x^4 + (a^4
*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^3 + 2*(a^5*b^5 - 8*a^6*b^3*c +
 16*a^7*b*c^2)*x^2 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x)*sqrt
(b^2 - 4*a*c)), 1/2*(6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20
*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*
c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2
- 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 -
20*a^5*c^3)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*
c)) - (2*a^3*b^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a
^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46
*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*
a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2)*x - 3*(
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^5 + 2*(b^6*c - 8*a*b^4*c
^2 + 16*a^2*b^2*c^3)*x^4 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^3 +
 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^2 + (a^2*b^5 - 8*a^3*
b^3*c + 16*a^4*b*c^2)*x)*log(c*x^2 + b*x + a) + 6*((b^5*c^2 - 8*a
*b^3*c^3 + 16*a^2*b*c^4)*x^5 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^
2*c^3)*x^4 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^3 + 2*(a*b^6 - 8*
a^2*b^4*c + 16*a^3*b^2*c^2)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4
*b*c^2)*x)*log(x))*sqrt(-b^2 + 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2
*c^3 + 16*a^6*c^4)*x^5 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*
c^3)*x^4 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^3 + 2*(a^5*b^5
- 8*a^6*b^3*c + 16*a^7*b*c^2)*x^2 + (a^6*b^4 - 8*a^7*b^2*c + 16*a
^8*c^2)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 153.597, size = 5722, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

```
[Out] (3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**
2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*
a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)))*log(x + (-108544*a**16*b*c**8*(3*b/(2*a**4) - 3*
sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a
*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 6
40*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**
2 + 224768*a**15*b**3*c**7*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)
**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a
**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 202752*a**14*b**
5*c**6*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 -
 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5
- 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
 20*a*b**8*c - b**10)))**2 + 104128*a**13*b**7*c**5*(3*b/(2*a**4)
 - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 +
 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)))**2 - 19200*a**13*c**9*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)*
*5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a*
*4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 1
60*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 33320*a**12*b**9*c**
4*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a
**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 128
0*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10)))**2 - 44736*a**12*b**2*c**8*(3*b/(2*a**4) - 3*s
qrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*
b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) +
6806*a**11*b**11*c**3*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*
(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(
1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a
**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 101232*a**11*b**4*c**
7*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a
**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 128
0*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10))) - 867*a**10*b**13*c**2*(3*b/(2*a**4) - 3*sqrt(
-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4
*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a*
*3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 7
7268*a**10*b**6*c**6*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(
20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1
```

```
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a*
*2*b**6*c**2 + 20*a*b**8*c - b**10))) + 63*a**9*b**15*c*(3*b/(2*a
**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2
*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b
**10)))**2 + 31368*a**9*b**8*c**5*(3*b/(2*a**4) - 3*sqrt(-(4*a*c
- b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**
6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*
c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 57600*a**9*b
*c**9 - 2*a**8*b**17*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(
20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a*
*2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 7545*a**8*b**10*c**4*(
3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2
*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a
**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + 842688*a**8*b**3*c**8 + 1086*a**7*b**12*c**3*(3
*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*
b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a*
*4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**
8*c - b**10))) - 1719216*a**7*b**5*c**7 - 87*a**6*b**14*c**2*(3*b
/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b*
*2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4
*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*
c - b**10))) + 1592964*a**6*b**7*c**6 + 3*a**5*b**16*c*(3*b/(2*a*
*4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**
2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10))) - 843048*a**5*b**9*c**5 + 277245*a**4*b**11*c**4 - 57996*a
**3*b**13*c**3 + 7542*a**2*b**15*c**2 - 558*a*b**17*c + 18*b**19)
/(18000*a**9*c**10 + 333720*a**8*b**2*c**9 - 991980*a**7*b**4*c**
8 + 1099710*a**6*b**6*c**7 - 651186*a**5*b**8*c**6 + 231795*a**4*
b**10*c**5 - 51480*a**3*b**12*c**4 + 7020*a**2*b**14*c**3 - 540*a
*b**16*c**2 + 18*b**18*c)) + (3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2
*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3
- 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-108544*a*
*16*b*c**8*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c*
*3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)))**2 + 224768*a**15*b**3*c**7*(3*b/(2*a
**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2
*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b
**10)))**2 - 202752*a**14*b**5*c**6*(3*b/(2*a**4) + 3*sqrt(-(4*a*
c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b
**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**
4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 104128*
a**13*b**7*c**5*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a*
*3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a
**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b*
*6*c**2 + 20*a*b**8*c - b**10)))**2 - 19200*a**13*c**9*(3*b/(2*a*
*4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**
2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*
```

```
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10))) - 33320*a**12*b**9*c**4*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b
**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/
(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 44736*a**12*
b**2*c**8*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**
3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**
2 + 20*a*b**8*c - b**10))) + 6806*a**11*b**11*c**3*(3*b/(2*a**4)
 + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 +
10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)
))**2 + 101232*a**11*b**4*c**7*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b
**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/
(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 867*a**10*b**13
*c**2*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 -
30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 -
 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
20*a*b**8*c - b**10)))**2 - 77268*a**10*b**6*c**6*(3*b/(2*a**4) +
 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
+ 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
) + 63*a**9*b**15*c*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(2
0*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**
2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 31368*a**9*b**8*c**5*(3
*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*
b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a*
*4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**
8*c - b**10))) - 57600*a**9*b*c**9 - 2*a**8*b**17*(3*b/(2*a**4) +
 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
+ 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
)**2 - 7545*a**8*b**10*c**4*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2
)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*
a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 842688*a**8*b**3*c
**8 + 1086*a**7*b**12*c**3*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)
**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a
**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 1719216*a**7*b**5*c
**7 - 87*a**6*b**14*c**2*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**
5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**
4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 16
0*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 1592964*a**6*b**7*c**
6 + 3*a**5*b**16*c*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2
*b**6*c**2 + 20*a*b**8*c - b**10))) - 843048*a**5*b**9*c**5 + 277
245*a**4*b**11*c**4 - 57996*a**3*b**13*c**3 + 7542*a**2*b**15*c**
2 - 558*a*b**17*c + 18*b**19)/(18000*a**9*c**10 + 333720*a**8*b**
2*c**9 - 991980*a**7*b**4*c**8 + 1099710*a**6*b**6*c**7 - 651186*
a**5*b**8*c**6 + 231795*a**4*b**10*c**5 - 51480*a**3*b**12*c**4 +
```

```
 7020*a**2*b**14*c**3 - 540*a*b**16*c**2 + 18*b**18*c)) - (32*a**
4*c**2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(60*a**2*c**4 - 42*a
*b**2*c**3 + 6*b**4*c**2) + x**3*(138*a**2*b*c**3 - 87*a*b**3*c**
2 + 12*b**5*c) + x**2*(100*a**3*c**3 + 14*a**2*b**2*c**2 - 36*a*b
**4*c + 6*b**6) + x*(122*a**3*b*c**2 - 68*a**2*b**3*c + 9*a*b**5)
)/(x**5*(32*a**5*c**4 - 16*a**4*b**2*c**3 + 2*a**3*b**4*c**2) + x
**4*(64*a**5*b*c**3 - 32*a**4*b**3*c**2 + 4*a**3*b**5*c) + x**3*(
64*a**6*c**3 - 12*a**4*b**4*c + 2*a**3*b**6) + x**2*(64*a**6*b*c*
*2 - 32*a**5*b**3*c + 4*a**4*b**5) + x*(32*a**7*c**2 - 16*a**6*b*
*2*c + 2*a**5*b**4)) - 3*b*log(x)/a**4
```

GIAC/XCAS [A] time = 0.298729, size = 417, normalized size = 1.74

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,b\ln\left(cx^{2}+bx+a\right)}{2\,a^{4}}-\frac{3\,b\ln\left(|x|\right)}{a^{4}}-\frac{2\,a^{3}b^{4}-16\,a^{4}b^{2}c+32\,a^{5}c^{2}+6\left(ab^{4}c^{2}-7\,a^{2}b^{2}c^{3}+10\,a^{3}c^{4}\right)x^{4}+3\left(4\,ab^{5}c-29\,a^{2}b^{3}c^{2}+46\,a^{3}bc^{3}\right)x^{3}+2\left(3\,ab^{6}-18\,a^{2}b^{4}c\right.}{2\left(cx^{2}+bx+a\right)^{2}\left(b^{2}-4\,ac\right)^{2}a^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c + b/x + a/x^2)^3*x^8),x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x
+ b)/sqrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sq
rt(-b^2 + 4*a*c)) + 3/2*b*ln(c*x^2 + b*x + a)/a^4 - 3*b*ln(abs(x)
)/a^4 - 1/2*(2*a^3*b^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2
 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3*(4*a*b^5*c - 29*a^2*b^3*c^
2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2
 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2)*x
)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

3.440 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Lo
g[1 + 5*x]/4375
```

Rubi [A] time = 0.0570015, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Lo
g[1 + 5*x]/4375
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{45}-\frac{16\log(3x+2)}{567}+\frac{\log(5x+1)}{4375}+\int\frac{139}{3375}\,dx-\frac{13\int x\,dx}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(15+2/x**2+13/x),x)
```

```
[Out] x**3/45 - 16*log(3*x + 2)/567 + log(5*x + 1)/4375 + Integral(139/
3375, x) - 13*Integral(x, x)/225
```

Mathematica [A] time = 0.00880849, size = 40, normalized size = 1.

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Lo
g[1 + 5*x]/4375
```

Maple [A] time = 0.009, size = 31, normalized size = 0.8

$$\frac{139\,x}{3375}-\frac{13\,x^{2}}{450}+\frac{x^{3}}{45}-\frac{16\,\ln\left(2+3\,x\right)}{567}+\frac{\ln\left(1+5\,x\right)}{4375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(15+2/x^2+13/x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A] time = 0.738535, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(13/x + 2/x^2 + 15),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567
*log(3*x + 2)
```

Fricas [A] time = 0.263365, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(13/x + 2/x^2 + 15),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567
*log(3*x + 2)
```

Sympy [A] time = 0.246941, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16\log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(15+2/x**2+13/x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x
 + 2/3)/567
```

GIAC/XCAS [A] time = 0.269196, size = 43, normalized size = 1.08

$$\frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\ln\left(|5\,x + 1|\right) - \frac{16}{567}\ln\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(13/x + 2/x^2 + 15),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*ln(abs(5*x + 1)) - 16
/567*ln(abs(3*x + 2))
```

3.441 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] time = 0.0475965, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{8\log(3x+2)}{189}-\frac{\log(5x+1)}{875}+\int\left(-\frac{13}{225}\right)dx+\frac{\int x\,dx}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(15+2/x**2+13/x),x)
```

```
[Out] 8*log(3*x + 2)/189 - log(5*x + 1)/875 + Integral(-13/225, x) + In
tegral(x, x)/15
```

Mathematica [A] time = 0.00715482, size = 33, normalized size = 1.

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(15 + 2/x^2 + 13/x),x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] time = 0.008, size = 26, normalized size = 0.8

$$-\frac{13\,x}{225}+\frac{x^2}{30}+\frac{8\,\ln\left(2+3\,x\right)}{189}-\frac{\ln\left(1+5\,x\right)}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(15+2/x^2+13/x),x)

[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A] time = 0.751829, size = 34, normalized size = 1.03

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(13/x + 2/x^2 + 15),x, algorithm="maxima")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A] time = 0.264427, size = 34, normalized size = 1.03

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(13/x + 2/x^2 + 15),x, algorithm="fricas")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] time = 0.25695, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x + \frac{1}{5}\right)}{875} + \frac{8\log\left(x + \frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(15+2/x**2+13/x),x)
```

```
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

GIAC/XCAS [A] time = 0.288082, size = 36, normalized size = 1.09

$$\frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\ln\left(|5\,x + 1|\right) + \frac{8}{189}\ln\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(13/x + 2/x^2 + 15),x, algorithm="giac")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*ln(abs(5*x + 1)) + 8/189*ln(abs(3*x +
 2))
```

3.442 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] time = 0.0353568, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi in Sympy [A] time = 7.45568, size = 20, normalized size = 0.77

$$\frac{x}{15}-\frac{4\log(3x+2)}{63}+\frac{\log(5x+1)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(15+2/x**2+13/x),x)
```

```
[Out] x/15 - 4*log(3*x + 2)/63 + log(5*x + 1)/175
```

Mathematica [A] time = 0.00649054, size = 26, normalized size = 1.

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(15 + 2/x^2 + 13/x)^(-1),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] time = 0.008, size = 21, normalized size = 0.8

$$\frac{x}{15}-\frac{4\ln\left(2+3x\right)}{63}+\frac{\ln\left(1+5x\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(15+2/x^2+13/x),x)
```

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A] time = 0.743061, size = 27, normalized size = 1.04

$$\frac{1}{15}x+\frac{1}{175}\log\left(5x+1\right)-\frac{4}{63}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(13/x + 2/x^2 + 15),x, algorithm="maxima")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A] time = 0.25933, size = 27, normalized size = 1.04

$$\frac{1}{15}x+\frac{1}{175}\log\left(5x+1\right)-\frac{4}{63}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(13/x + 2/x^2 + 15),x, algorithm="fricas")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] time = 0.240016, size = 20, normalized size = 0.77

$$\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15+2/x**2+13/x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

GIAC/XCAS [A] time = 0.259606, size = 30, normalized size = 1.15

$$\frac{1}{15}\,x+\frac{1}{175}\,\ln\left(|5\,x+1|\right)-\frac{4}{63}\,\ln\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(13/x + 2/x^2 + 15),x, algorithm="giac")
```

```
[Out] 1/15*x + 1/175*ln(abs(5*x + 1)) - 4/63*ln(abs(3*x + 2))
```

3.443 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] time = 0.030687, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((15 + 2/x^2 + 13/x)*x),x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi in Sympy [A] time = 9.10233, size = 17, normalized size = 0.81

$$\frac{2\log{(3x+2)}}{21}-\frac{\log{(5x+1)}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(15+2/x**2+13/x)/x,x)

[Out] 2*log(3*x + 2)/21 - log(5*x + 1)/35
```

Mathematica [A] time = 0.00506181, size = 21, normalized size = 1.

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((15 + 2/x^2 + 13/x)*x),x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] time = 0.008, size = 18, normalized size = 0.9

$$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(15+2/x^2+13/x)/x,x)

[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A] time = 0.743437, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(13/x + 2/x^2 + 15)),x, algorithm="maxima")

[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A] time = 0.286427, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(13/x + 2/x^2 + 15)),x, algorithm="fricas")

[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] time = 0.217867, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15+2/x**2+13/x)/x,x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

GIAC/XCAS [A] time = 0.297453, size = 26, normalized size = 1.24

$$-\frac{1}{35}\ln\left(|5x+1|\right)+\frac{2}{21}\ln\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(13/x + 2/x^2 + 15)),x, algorithm="giac")
```

```
[Out] -1/35*ln(abs(5*x + 1)) + 2/21*ln(abs(3*x + 2))
```

3.444 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

```
[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rubi [A] time = 0.0302982, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((15 + 2/x^2 + 13/x)*x^2),x]

[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rubi in Sympy [A] time = 6.82851, size = 15, normalized size = 0.65

$$-\frac{\log\left(3+\frac{2}{x}\right)}{7}+\frac{\log\left(5+\frac{1}{x}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(15+2/x**2+13/x)/x**2,x)

[Out] -log(3 + 2/x)/7 + log(5 + 1/x)/7
```

Mathematica [A] time = 0.00448552, size = 21, normalized size = 0.91

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]

[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Maple [A] time = 0.007, size = 18, normalized size = 0.8

$$\frac{\ln(1+5x)}{7} - \frac{\ln(2+3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(15+2/x^2+13/x)/x^2,x)

[Out] 1/7*ln(1+5*x)-1/7*ln(2+3*x)
```

Maxima [A] time = 0.750124, size = 23, normalized size = 1.

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(13/x + 2/x^2 + 15)),x, algorithm="maxima")

[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A] time = 0.246921, size = 23, normalized size = 1.

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(13/x + 2/x^2 + 15)),x, algorithm="fricas")

[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] time = 0.216806, size = 15, normalized size = 0.65

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15+2/x**2+13/x)/x**2,x)

[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

GIAC/XCAS [A] time = 0.301019, size = 26, normalized size = 1.13

$$\frac{1}{7}\ln\left(|5x+1|\right)-\frac{1}{7}\ln\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(13/x + 2/x^2 + 15)),x, algorithm="giac")

[Out] 1/7*ln(abs(5*x + 1)) - 1/7*ln(abs(3*x + 2))
```

3.445 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3} \, dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0437314, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((15 + 2/x^2 + 13/x)*x^3),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi in Sympy [A] time = 12.1255, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2}+\frac{3\log(3x+2)}{14}-\frac{5\log(5x+1)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(15+2/x**2+13/x)/x**3,x)

[Out] log(x)/2 + 3*log(3*x + 2)/14 - 5*log(5*x + 1)/7
```

Mathematica [A] time = 0.00693499, size = 27, normalized size = 1.

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] time = 0.009, size = 22, normalized size = 0.8

$$\frac{\ln(x)}{2} + \frac{3\ln(2+3x)}{14} - \frac{5\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(15+2/x^2+13/x)/x^3,x)

[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A] time = 0.744873, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1) + \frac{3}{14}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3*(13/x + 2/x^2 + 15)),x, algorithm="maxima")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A] time = 0.254148, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1) + \frac{3}{14}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3*(13/x + 2/x^2 + 15)),x, algorithm="fricas")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.313965, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2} - \frac{5\log\left(x+\frac{1}{5}\right)}{7} + \frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15+2/x**2+13/x)/x**3,x)

[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

GIAC/XCAS [A] time = 0.265431, size = 32, normalized size = 1.19

$$-\frac{5}{7}\ln(|5x+1|) + \frac{3}{14}\ln(|3x+2|) + \frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3*(13/x + 2/x^2 + 15)),x, algorithm="giac")

[Out] -5/7*ln(abs(5*x + 1)) + 3/14*ln(abs(3*x + 2)) + 1/2*ln(abs(x))
```

3.446 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x]
)/7
```

Rubi [A] time = 0.0751486, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x]
)/7
```

Rubi in Sympy [A] time = 17.3745, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4}-\frac{9\log(3x+2)}{28}+\frac{25\log(5x+1)}{7}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(15+2/x**2+13/x)/x**4,x)
```

```
[Out] -13*log(x)/4 - 9*log(3*x + 2)/28 + 25*log(5*x + 1)/7 - 1/(2*x)
```

Mathematica [A] time = 0.0064387, size = 34, normalized size = 1.

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x]
)/7
```

Maple [A] time = 0.012, size = 27, normalized size = 0.8

$$-\frac{1}{2\,x}-\frac{13\,\ln\left(x\right)}{4}-\frac{9\,\ln\left(2+3\,x\right)}{28}+\frac{25\,\ln\left(1+5\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(15+2/x^2+13/x)/x^4,x)
```

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A] time = 0.742661, size = 35, normalized size = 1.03

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4*(13/x + 2/x^2 + 15)),x, algorithm="maxima")
```

```
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Fricas [A] time = 0.257558, size = 41, normalized size = 1.21

$$\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4*(13/x + 2/x^2 + 15)),x, algorithm="fricas")
```

```
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] time = 0.364334, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15+2/x**2+13/x)/x**4,x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

GIAC/XCAS [A] time = 0.284736, size = 39, normalized size = 1.15

$$-\frac{1}{2\,x}+\frac{25}{7}\,\ln\left(|5\,x+1|\right)-\frac{9}{28}\,\ln\left(|3\,x+2|\right)-\frac{13}{4}\,\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4*(13/x + 2/x^2 + 15)),x, algorithm="giac")
```

```
[Out] -1/2/x + 25/7*ln(abs(5*x + 1)) - 9/28*ln(abs(3*x + 2)) - 13/4*ln(
abs(x))
```

3.447 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (
125*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0817365, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (
125*Log[1 + 5*x])/7
```

Rubi in Sympy [A] time = 17.6277, size = 37, normalized size = 0.9

$$\frac{139\log(x)}{8}+\frac{27\log(3x+2)}{56}-\frac{125\log(5x+1)}{7}+\frac{13}{4x}-\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(15+2/x**2+13/x)/x**5,x)
```

```
[Out] 139*log(x)/8 + 27*log(3*x + 2)/56 - 125*log(5*x + 1)/7 + 13/(4*x)
 - 1/(4*x**2)
```

Mathematica [A] time = 0.00765143, size = 41, normalized size = 1.

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (
125*Log[1 + 5*x])/7
```

Maple [A] time = 0.012, size = 32, normalized size = 0.8

$$-\frac{1}{4\,x^{2}}+\frac{13}{4\,x}+\frac{139\,\ln\left(x\right)}{8}+\frac{27\,\ln\left(2+3\,x\right)}{56}-\frac{125\,\ln\left(1+5\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(15+2/x^2+13/x)/x^5,x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A] time = 0.740028, size = 42, normalized size = 1.02

$$\frac{13\,x-1}{4\,x^{2}}-\frac{125}{7}\,\log\left(5\,x+1\right)+\frac{27}{56}\,\log\left(3\,x+2\right)+\frac{139}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5*(13/x + 2/x^2 + 15)),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 13
9/8*log(x)
```

Fricas [A] time = 0.253013, size = 53, normalized size = 1.29

$$-\frac{1000\,x^{2}\log\left(5\,x+1\right)-27\,x^{2}\log\left(3\,x+2\right)-973\,x^{2}\log\left(x\right)-182\,x+14}{56\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5*(13/x + 2/x^2 + 15)),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(
x) - 182*x + 14)/x^2
```

Sympy [A] time = 0.396951, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8} - \frac{125\log\left(x+\frac{1}{5}\right)}{7} + \frac{27\log\left(x+\frac{2}{3}\right)}{56} + \frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15+2/x**2+13/x)/x**5,x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x -
1)/(4*x**2)
```

GIAC/XCAS [A] time = 0.280195, size = 46, normalized size = 1.12

$$\frac{13\,x-1}{4\,x^2} - \frac{125}{7}\ln\left(|5\,x+1|\right) + \frac{27}{56}\ln\left(|3\,x+2|\right) + \frac{139}{8}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5*(13/x + 2/x^2 + 15)),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*ln(abs(5*x + 1)) + 27/56*ln(abs(3*x +
2)) + 139/8*ln(abs(x))
```

3.448 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[
2 + 3*x])/112 + (625*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0943076, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[
2 + 3*x])/112 + (625*Log[1 + 5*x])/7
```

Rubi in Sympy [A] time = 17.4766, size = 44, normalized size = 0.92

$$-\frac{1417\log(x)}{16}-\frac{81\log(3x+2)}{112}+\frac{625\log(5x+1)}{7}-\frac{139}{8x}+\frac{13}{8x^2}-\frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(15+2/x**2+13/x)/x**6,x)
```

```
[Out] -1417*log(x)/16 - 81*log(3*x + 2)/112 + 625*log(5*x + 1)/7 - 139/
(8*x) + 13/(8*x**2) - 1/(6*x**3)
```

Mathematica [A] time = 0.00803701, size = 48, normalized size = 1.

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[
2 + 3*x])/112 + (625*Log[1 + 5*x])/7
```

Maple [A] time = 0.012, size = 37, normalized size = 0.8

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(15+2/x^2+13/x)/x^6,x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln
(1+5*x)
```

Maxima [A] time = 0.750557, size = 49, normalized size = 1.02

$$-\frac{417x^2-39x+4}{24x^3}+\frac{625}{7}\log(5x+1)-\frac{81}{112}\log(3x+2)-\frac{1417}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6*(13/x + 2/x^2 + 15)),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(
3*x + 2) - 1417/16*log(x)
```

Fricas [A] time = 0.253734, size = 59, normalized size = 1.23

$$\frac{30000x^3\log(5x+1)-243x^3\log(3x+2)-29757x^3\log(x)-5838x^2+546x-56}{336x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6*(13/x + 2/x^2 + 15)),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*
log(x) - 5838*x^2 + 546*x - 56)/x^3
```

Sympy [A] time = 0.443151, size = 41, normalized size = 0.85

$$-\frac{1417\log(x)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}-\frac{417x^2-39x+4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15+2/x**2+13/x)/x**6,x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 - (417
*x**2 - 39*x + 4)/(24*x**3)
```

GIAC/XCAS [A] time = 0.281253, size = 53, normalized size = 1.1

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\ln\left(|5\,x+1|\right)-\frac{81}{112}\ln\left(|3\,x+2|\right)-\frac{1417}{16}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6*(13/x + 2/x^2 + 15)),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*ln(abs(5*x + 1)) - 81/112*
ln(abs(3*x + 2)) - 1417/16*ln(abs(x))
```

3.449 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=204

$$\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}-\frac{5\left(\frac{2c\left(12ac+b^2\right)}{x}+b\left(44ac+b^2\right)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{64c}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{24}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}$$

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x
^2 + b/x]*(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (
a + c/x^2 + b/x)^(5/2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sq
rt[a]*Sqrt[a + c/x^2 + b/x]])])/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*
c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(1
28*c^(3/2))
```

Rubi [A] time = 0.607295, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}-\frac{5\left(\frac{2c\left(12ac+b^2\right)}{x}+b\left(44ac+b^2\right)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{64c}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{24}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x
^2 + b/x]*(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (
a + c/x^2 + b/x)^(5/2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sq
rt[a]*Sqrt[a + c/x^2 + b/x]])])/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*
c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(1
28*c^(3/2))
```

Rubi in Sympy [A] time = 61.5345, size = 178, normalized size = 0.87

$$\frac{5a^{\frac{3}{2}}b\operatorname{atanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}-\frac{5\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}{24}-\frac{5\left(\frac{b(44ac+b^2)}{2}+\frac{c(12ac+b^2)}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{32c}+\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2+b/x)**(5/2),x)
```

```
[Out] 5*a**(3/2)*b*atanh((2*a + b/x)/(2*sqrt(a)*sqrt(a + b/x + c/x**2))
)/2 + x*(a + b/x + c/x**2)**(5/2) - 5*(7*b + 6*c/x)*(a + b/x + c/
x**2)**(3/2)/24 - 5*(b*(44*a*c + b**2)/2 + c*(12*a*c + b**2)/x)*s
qrt(a + b/x + c/x**2)/(32*c) + 5*(-48*a**2*c**2 - 24*a*b**2*c + b
**4)*atanh((b + 2*c/x)/(2*sqrt(c)*sqrt(a + b/x + c/x**2)))/(128*c
**(3/2))
```

Mathematica [A] time = 1.02075, size = 233, normalized size = 1.14

$$x\left(a+\frac{bx+c}{x^2}\right)^{5/2}\left(960a^{3/2}bc^{3/2}x^4\log\left(2\sqrt{a}\sqrt{x(ax+b)+c}+2ax+b\right)-15x^4\log(x)\left(-48a^2c^2-24ab^2c+b^4\right)+15x^4\left(-48a^2\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (x*(a + (c + b*x)/x^2)^(5/2)*(-2*Sqrt[c]*Sqrt[c + x*(b + a*x)]*(4
8*c^3 + 15*b^3*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2 +
278*a*b*x - 96*a^2*x^2)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4
*Log[x] + 960*a^(3/2)*b*c^(3/2)*x^4*Log[b + 2*a*x + 2*Sqrt[a]*Sqr
t[c + x*(b + a*x)]] + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*Log[
2*c + b*x + 2*Sqrt[c]*Sqrt[c + x*(b + a*x)]]))/(384*c^(3/2)*(c +
x*(b + a*x))^(5/2))
```

Maple [B] time = 0.02, size = 701, normalized size = 3.4

$$-\frac{x}{384\,c^4}\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{5}{2}}\left(720\,a^{7/2}\ln\left(\frac{2\,c+bx+2\,\sqrt{c}\sqrt{ax^2+bx+c}}{x}\right)c^{9/2}x^4+96\,\left(ax^2+bx+c\right)^{7/2}c^3a^{3/2}+6\,\left(ax^2+bx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2+b/x)^(5/2),x)
```

```
[Out] -1/384*((a*x^2+b*x+c)/x^2)^(5/2)*x*(720*a^(7/2)*ln((2*c+b*x+2*c^(
1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(9/2)*x^4+96*(a*x^2+b*x+c)^(7/2)*c
^3*a^(3/2)+6*(a*x^2+b*x+c)^(5/2)*a^(5/2)*x^5*b^3-144*(a*x^2+b*x+c
)^(5/2)*a^(7/2)*x^4*c^2+144*(a*x^2+b*x+c)^(7/2)*a^(5/2)*x^2*c^2-2
40*(a*x^2+b*x+c)^(3/2)*a^(7/2)*x^4*c^3-720*(a*x^2+b*x+c)^(1/2)*a^
(7/2)*x^4*c^4-6*(a*x^2+b*x+c)^(7/2)*a^(3/2)*x^3*b^3+6*(a*x^2+b*x+
c)^(5/2)*a^(3/2)*x^4*b^4-660*(a*x^2+b*x+c)^(1/2)*a^(5/2)*x^4*b^2*
c^3-960*a^3*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*x^4*b*c^4-4*(a*x^2+b*x+c)^(7/2)*a^(3/2)*x^2*b^2*c+10*(a*x^2+b*
x+c)^(3/2)*a^(3/2)*x^4*b^4*c-16*(a*x^2+b*x+c)^(7/2)*a^(3/2)*x*b*c
^2+30*(a*x^2+b*x+c)^(1/2)*a^(3/2)*x^4*b^4*c^2+360*a^(5/2)*ln((2*c
+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(7/2)*x^4*b^2-15*a^(3/2)
*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(5/2)*x^4*b^4-60
0*(a*x^2+b*x+c)^(1/2)*a^(7/2)*x^5*b*c^3-152*(a*x^2+b*x+c)^(5/2)*a
^(7/2)*x^5*b*c+152*(a*x^2+b*x+c)^(7/2)*a^(5/2)*x^3*b*c-148*(a*x^2
+b*x+c)^(5/2)*a^(5/2)*x^4*b^2*c-280*(a*x^2+b*x+c)^(3/2)*a^(7/2)*x
^5*b*c^2+10*(a*x^2+b*x+c)^(3/2)*a^(5/2)*x^5*b^3*c-260*(a*x^2+b*x+
c)^(3/2)*a^(5/2)*x^4*b^2*c^2+30*(a*x^2+b*x+c)^(1/2)*a^(5/2)*x^5*b
^3*c^2)/(a*x^2+b*x+c)^(5/2)/c^4/a^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.423722, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*
a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15
```

```
*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*log(-((8*b*c*x + (b^
2 + 4*a*c)*x^2 + 8*c^2)*sqrt(c) - 4*(b*c*x^2 + 2*c^2*x)*sqrt((a*x
^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*
c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x
^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), 1/768*(1920*sqrt(-a)*
a*b*c^2*x^3*arctan(1/2*(2*a*x + b)/(sqrt(-a)*x*sqrt((a*x^2 + b*x
+ c)/x^2))) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*log(
-((8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2)*sqrt(c) - 4*(b*c*x^2 + 2*
c^2*x)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4 - 1
36*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^
2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), 1/384
*(480*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c -
4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4
- 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x + 2*c)*sq
rt(-c)/(c*x*sqrt((a*x^2 + b*x + c)/x^2))) + 2*(192*a^2*c^2*x^4 -
136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c
^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), 1/38
4*(960*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x + b)/(sqrt(-a)*x*sq
rt((a*x^2 + b*x + c)/x^2))) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*
sqrt(-c)*x^3*arctan(1/2*(b*x + 2*c)*sqrt(-c)/(c*x*sqrt((a*x^2 + b
*x + c)/x^2))) + 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*
b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((
a*x^2 + b*x + c)/x^2))/(c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(5/2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.450 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=145

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^
(3/2)*x + (3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/2 - (3*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[
c]*Sqrt[a + c/x^2 + b/x])])/(8*Sqrt[c])
```

Rubi [A] time = 0.370456, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^
(3/2)*x + (3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/2 - (3*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[
c]*Sqrt[a + c/x^2 + b/x])])/(8*Sqrt[c])
```

Rubi in Sympy [A] time = 37.6517, size = 124, normalized size = 0.86

$$\frac{3\sqrt{a}b\operatorname{atanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}-\frac{3\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{4}-\frac{3\left(4ac+b^2\right)\operatorname{atanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*atanh((2*a + b/x)/(2*sqrt(a)*sqrt(a + b/x + c/x**2)))
/2 + x*(a + b/x + c/x**2)**(3/2) - 3*(3*b + 2*c/x)*sqrt(a + b/x +
 c/x**2)/4 - 3*(4*a*c + b**2)*atanh((b + 2*c/x)/(2*sqrt(c)*sqrt(a
 + b/x + c/x**2)))/(8*sqrt(c))
```

Mathematica [A] time = 0.286969, size = 236, normalized size = 1.63

$$\frac{\sqrt{a+\frac{bx+c}{x^2}}\left(3x^2\log(x)\left(4ac+b^2\right)-3b^2x^2\log\left(2\sqrt{c}\sqrt{x(ax+b)+c}+bx+2c\right)-4c^{3/2}\sqrt{x(ax+b)+c}+8a\sqrt{c}x^2\sqrt{x(ax+b)+\right.}{8\sqrt{c}x\sqrt{x(}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(-4*c^(3/2)*Sqrt[c + x*(b + a*x)] - 10*b
*Sqrt[c]*x*Sqrt[c + x*(b + a*x)] + 8*a*Sqrt[c]*x^2*Sqrt[c + x*(b
+ a*x)] + 3*(b^2 + 4*a*c)*x^2*Log[x] + 12*Sqrt[a]*b*Sqrt[c]*x^2*L
og[b + 2*a*x + 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]] - 3*b^2*x^2*Log[2
*c + b*x + 2*Sqrt[c]*Sqrt[c + x*(b + a*x)]] - 12*a*c*x^2*Log[2*c
+ b*x + 2*Sqrt[c]*Sqrt[c + x*(b + a*x)]]))/(8*Sqrt[c]*x*Sqrt[c +
x*(b + a*x)])
```

Maple [B] time = 0.011, size = 334, normalized size = 2.3

$$-\frac{x}{8c^2}\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{3}{2}}\left(12a^{5/2}\ln\left(\frac{2c+bx+2\sqrt{c}\sqrt{ax^2+bx+c}}{x}\right)c^{5/2}x^2-2a^{5/2}\left(ax^2+bx+c\right)^{3/2}x^3b-4a^{5/2}\left(ax^2+bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2+b/x)^(3/2),x)
```

```
[Out] -1/8*((a*x^2+b*x+c)/x^2)^(3/2)*x*(12*a^(5/2)*ln((2*c+b*x+2*c^(1/2
)*(a*x^2+b*x+c)^(1/2))/x)*c^(5/2)*x^2-2*a^(5/2)*(a*x^2+b*x+c)^(3/
2)*x^3*b-4*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^2*c-6*a^(5/2)*(a*x^2+b*x
+c)^(1/2)*x^3*b*c+3*a^(3/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(
1/2))/x)*c^(3/2)*x^2*b^2-12*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^2*c^2+2
*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x*b-2*a^(3/2)*(a*x^2+b*x+c)^(3/2)*x^
2*b^2+4*(a*x^2+b*x+c)^(5/2)*c*a^(3/2)-6*a^(3/2)*(a*x^2+b*x+c)^(1/
2)*x^2*b^2*c-12*a^2*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*x^2*b*c^2)/(a*x^2+b*x+c)^(3/2)/c^2/a^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.344944, size = 1, normalized size = 0.01

$$\left[\frac{12\sqrt{a}bcx\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac-4\left(2\,ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+3\left(b^2+4\,ac\right)\sqrt{c}x\log\left(-\frac{\left(8\,bcx+\left(b^2+4\,ac\right)x^2+8\,c^2\right)}{}\right.}{16\,cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c -
4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 +
 4*a*c)*sqrt(c)*x*log(-((8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2)*sqr
t(c) - 4*(b*c*x^2 + 2*c^2*x)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) +
4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x
), 1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x + b)/(sqrt(-a)*x*sqr
t((a*x^2 + b*x + c)/x^2))) + 3*(b^2 + 4*a*c)*sqrt(c)*x*log(-((8*b
```

```
*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2)*sqrt(c) - 4*(b*c*x^2 + 2*c^2*x)
*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c
^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(
-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sq
rt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/
2*(b*x + 2*c)*sqrt(-c)/(c*x*sqrt((a*x^2 + b*x + c)/x^2))) + 2*(4*
a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/
8*(12*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x + b)/(sqrt(-a)*x*sqrt((a*x
^2 + b*x + c)/x^2))) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x
 + 2*c)*sqrt(-c)/(c*x*sqrt((a*x^2 + b*x + c)/x^2))) + 2*(4*a*c*x^
2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(3/2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.451 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=105

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[
a + c/x^2 + b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(
2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])]
```

Rubi [A] time = 0.216392, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[
a + c/x^2 + b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(
2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])]
```

Rubi in Sympy [A] time = 22.4312, size = 85, normalized size = 0.81

$$-\sqrt{c}\operatorname{atanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\operatorname{atanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2+b/x)**(1/2),x)
```

```
[Out] -sqrt(c)*atanh((b + 2*c/x)/(2*sqrt(c)*sqrt(a + b/x + c/x**2))) +
x*sqrt(a + b/x + c/x**2) + b*atanh((2*a + b/x)/(2*sqrt(a)*sqrt(a
+ b/x + c/x**2)))/(2*sqrt(a))
```

Mathematica [A] time = 0.15374, size = 138, normalized size = 1.31

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\Big(b\log\Big(2\sqrt{a}\sqrt{x(ax+b)+c}+2ax+b\Big)+2\sqrt{a}\Big(\sqrt{x(ax+b)+c}-\sqrt{c}\log\Big(2\sqrt{c}\sqrt{x(ax+b)+c}+bx+2c\Big)\Big)+2\sqrt{a}}{2\sqrt{a}\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(2*Sqrt[a]*Sqrt[c]*Log[x] + b*Log[b +
2*a*x + 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]] + 2*Sqrt[a]*(Sqrt[c + x*
(b + a*x)] - Sqrt[c]*Log[2*c + b*x + 2*Sqrt[c]*Sqrt[c + x*(b + a*
x)]])))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

Maple [A] time = 0.01, size = 121, normalized size = 1.2

$$\frac{x}{2}\sqrt{\frac{ax^2+bx+c}{x^2}}\Bigg(-2\sqrt{c}\ln\Bigg(\frac{2c+bx+2\sqrt{c}\sqrt{ax^2+bx+c}}{x}\Bigg)\sqrt{a}+b\ln\Bigg(\frac{1}{2}\Big(2\sqrt{ax^2+bx+c}\sqrt{a}+2ax+b\Big)\frac{1}{\sqrt{a}}\Bigg)+2\sqrt{ax^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((2*c+b*x+2*c^(1/2)
*(a*x^2+b*x+c)^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b
*x+c)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x + c/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295776, size = 1, normalized size = 0.01

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-\left(8\,a^2x^2+8\,abx+b^2+4\,ac\right)\sqrt{a}-4\left(2\,a^2x^2+abx\right)\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+2\,a\sqrt{c}\log\left(-\frac{8\,bcx+\left(b^2+4\,ac\right)x^2}{\quad}\right.}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x + c/x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-(8*a^2*x
^2 + 8*a*b*x + b^2 + 4*a*c)*sqrt(a) - 4*(2*a^2*x^2 + a*b*x)*sqrt(
(a*x^2 + b*x + c)/x^2)) + 2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*
c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)
/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) - sqrt(-a)
*b*arctan(1/2*(2*a*x + b)*sqrt(-a)/(a*x*sqrt((a*x^2 + b*x + c)/x^
2))) + a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b
*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/4*(
4*a*x*sqrt((a*x^2 + b*x + c)/x^2) - 4*a*sqrt(-c)*arctan(1/2*(b*x
+ 2*c)/(sqrt(-c)*x*sqrt((a*x^2 + b*x + c)/x^2))) + sqrt(a)*b*log(
-(8*a^2*x^2 + 8*a*b*x + b^2 + 4*a*c)*sqrt(a) - 4*(2*a^2*x^2 + a*b
*x)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x
 + c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x + b)*sqrt(-a)/(a*x*sqrt
((a*x^2 + b*x + c)/x^2))) - 2*a*sqrt(-c)*arctan(1/2*(b*x + 2*c)/(
sqrt(-c)*x*sqrt((a*x^2 + b*x + c)/x^2))))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x + c/x^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.452 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*S
qrt[a + c/x^2 + b/x])])/(2*a^(3/2))
```

Rubi [A] time = 0.0956167, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*S
qrt[a + c/x^2 + b/x])])/(2*a^(3/2))
```

Rubi in Sympy [A] time = 11.689, size = 53, normalized size = 0.79

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\,\mathrm{atanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)**(1/2),x)
```

```
[Out] x*sqrt(a + b/x + c/x**2)/a - b*atanh((2*a + b/x)/(2*sqrt(a)*sqrt(
a + b/x + c/x**2)))/(2*a**(3/2))
```

Mathematica [A] time = 0.0970812, size = 87, normalized size = 1.3

$$\frac{2\sqrt{a}(x(ax+b)+c)-b\sqrt{x(ax+b)+c}\log\left(2\sqrt{a}\sqrt{x(ax+b)+c}+2ax+b\right)}{2a^{3/2}x\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*Log[b + 2*
a*x + 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]])/(2*a^(3/2)*x*Sqrt[a + (c
+ b*x)/x^2])
```

Maple [A] time = 0.01, size = 88, normalized size = 1.3

$$\frac{1}{2x}\sqrt{ax^2+bx+c}\left(2\sqrt{ax^2+bx+c}a^{3/2}-b\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx+c}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)a\right)\frac{1}{\sqrt{\frac{ax^2+bx+c}{x^2}}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)^(1/2),x)
```

```
[Out] 1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(
2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a)/((a*x^2+b*x+c)
/x^2)^(1/2)/x/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a + b/x + c/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276129, size = 1, normalized size = 0.01

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-\left(8\,a^2x^2+8\,abx+b^2+4\,ac\right)\sqrt{a}+4\,\left(2\,a^2x^2+abx\right)\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a^2},\ \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{-a}b\,\mathrm{arcta}}{2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a + b/x + c/x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-(8*a^2*x
^2 + 8*a*b*x + b^2 + 4*a*c)*sqrt(a) + 4*(2*a^2*x^2 + a*b*x)*sqrt(
(a*x^2 + b*x + c)/x^2)))/a^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x
^2) + sqrt(-a)*b*arctan(1/2*(2*a*x + b)*sqrt(-a)/(a*x*sqrt((a*x^2
 + b*x + c)/x^2))))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b/x + c/x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a + b/x + c/x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.453 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}+\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (
2*(b^2 - 2*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/
x]) - (3*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])]
)/(2*a^(5/2))
```

Rubi [A] time = 0.250395, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}+\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (
2*(b^2 - 2*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/
x]) - (3*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])]
)/(2*a^(5/2))
```

Rubi in Sympy [A] time = 30.2972, size = 117, normalized size = 0.88

$$-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(-4ac+b^2\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(-16ac+6b^2\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{2a^2\left(-4ac+b^2\right)}-\frac{3b\,\text{atanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

```
[Out] -2*x*(-2*a*c + b**2 + b*c/x)/(a*(-4*a*c + b**2)*sqrt(a + b/x + c/
x**2)) + x*(-16*a*c + 6*b**2)*sqrt(a + b/x + c/x**2)/(2*a**2*(-4*
a*c + b**2)) - 3*b*atanh((2*a + b/x)/(2*sqrt(a)*sqrt(a + b/x + c/
x**2)))/(2*a**(5/2))
```

Mathematica [A] time = 0.193161, size = 136, normalized size = 1.02

$$-\frac{3b\left(b^2-4ac\right)\sqrt{x(ax+b)+c}\log\left(2\sqrt{a}\sqrt{x(ax+b)+c}+2ax+b\right)+2\sqrt{a}\left(-b^2\left(ax^2+3c\right)+10abcx+4ac\left(ax^2+2c\right)-3b^3

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.313032, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^{3}c-4\,abc^{2}+\left(ab^{3}-4\,a^{2}bc\right)x^{2}+\left(b^{4}-4\,ab^{2}c\right)x\right)\sqrt{a}\log\left(-\left(8\,a^{2}x^{2}+8\,abx+b^{2}+4\,ac\right)\sqrt{a}+4\left(2\,a^{2}x^{2}+abx\right)\sqrt{\frac{ax^{2}+bx\cdot}{x^{2}}}\right.}{4\left(a^{3}b^{2}c-4\,a^{4}c^{2}+\left(a^{4}b^{2}-4\,a^{5}c\right)x^{2}+\left(a^{3}b^{3}-4\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a
*b^2*c)*x)*sqrt(a)*log(-(8*a^2*x^2 + 8*a*b*x + b^2 + 4*a*c)*sqrt(
a) + 4*(2*a^2*x^2 + a*b*x)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2
*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*x^2 + (3*a*b^2*c - 8
*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c - 4*a^4*c^2
+ (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x), 1/2*(3*(b^3
*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*s
qrt(-a)*arctan(1/2*(2*a*x + b)*sqrt(-a)/(a*x*sqrt((a*x^2 + b*x +
c)/x^2))) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*x
^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3
*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b
*c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^{2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(-3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.454 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3
*a^3*(b^2 - 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 -
 4*a*c)*(a + c/x^2 + b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^
2*c^2 + (b*c*(5*b^2 - 28*a*c))/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[
a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a +
c/x^2 + b/x])])/(2*a^(7/2))
```

Rubi [A] time = 0.473482, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3
*a^3*(b^2 - 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 -
 4*a*c)*(a + c/x^2 + b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^
2*c^2 + (b*c*(5*b^2 - 28*a*c))/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[
a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a +
c/x^2 + b/x])])/(2*a^(7/2))
```

Rubi in Sympy [A] time = 67.4174, size = 206, normalized size = 0.94

$$-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(-4ac+b^2\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}-\frac{4x\left(16a^2c^2-16ab^2c+\frac{5b^4}{2}+\frac{bc(-28ac+5b^2)}{2x}\right)}{3a^2\left(-4ac+b^2\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(128a^2c^2-100ab^2c+15b^4\right)}{3a^3\left(-4ac+b^2\right)^2}-\frac{5b\,\text{atanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)**(5/2),x)
```

```
[Out] -2*x*(-2*a*c + b**2 + b*c/x)/(3*a*(-4*a*c + b**2)*(a + b/x + c/x*
*2)**(3/2)) - 4*x*(16*a**2*c**2 - 16*a*b**2*c + 5*b**4/2 + b*c*(-
28*a*c + 5*b**2)/(2*x))/(3*a**2*(-4*a*c + b**2)**2*sqrt(a + b/x +
 c/x**2)) + x*sqrt(a + b/x + c/x**2)*(128*a**2*c**2 - 100*a*b**2*
c + 15*b**4)/(3*a**3*(-4*a*c + b**2)**2) - 5*b*atanh((2*a + b/x)/
(2*sqrt(a)*sqrt(a + b/x + c/x**2)))/(2*a**(7/2))
```

Mathematica [A] time = 0.527389, size = 235, normalized size = 1.07

$$\frac{\frac{4\left(5a^2bc^2x+2a^2c^3-5ab^3cx-4ab^2c^2+b^5x+b^4c\right)(x(ax+b)+c)}{b^2-4ac}-\frac{4\left(-80a^3bc^2x-48a^3c^3+50a^2b^3cx+56a^2b^2c^2-7ab^5x-14ab^4c+b^6\right)(x(ax+b)+c)^2}{\left(b^2-4ac\right)^2}+6a(x(ax\ldots}{6a^4x^5\left(a+\frac{bx+c}{x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3 + b^5*x - 5*a*b^3*c*x + 5*a^
2*b*c^2*x)*(c + x*(b + a*x)))/(b^2 - 4*a*c) - (4*(b^6 - 14*a*b^4*
c + 56*a^2*b^2*c^2 - 48*a^3*c^3 - 7*a*b^5*x + 50*a^2*b^3*c*x - 80
*a^3*b*c^2*x)*(c + x*(b + a*x))^2)/(b^2 - 4*a*c)^2 + 6*a*(c + x*(
b + a*x))^3 - 15*Sqrt[a]*b*(c + x*(b + a*x))^(5/2)*Log[b + 2*a*x
+ 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]])/(6*a^4*x^5*(a + (c + b*x)/x^2
)^(5/2))
```

Maple [A] time = 0.016, size = 376, normalized size = 1.7

$$-\frac{ax^2+bx+c}{6\,x^5\left(4\,ac-b^2\right)^2}\left(-96\,a^{13/2}x^4c^2+48\,a^{11/2}x^4b^2c-512\,a^{11/2}x^3bc^2-6\,a^{9/2}x^4b^4-384\,a^{11/2}x^2c^3+296\,a^{9/2}x^3b^3c-96\,a^{9/2}x\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)^(5/2),x)
```

```
[Out] -1/6*(a*x^2+b*x+c)*(-96*a^(13/2)*x^4*c^2+48*a^(11/2)*x^4*b^2*c-51
2*a^(11/2)*x^3*b*c^2-6*a^(9/2)*x^4*b^4-384*a^(11/2)*x^2*c^3+296*a
^(9/2)*x^3*b^3*c-96*a^(9/2)*x^2*b^2*c^2-40*a^(7/2)*x^3*b^5-624*a^
(9/2)*x*b*c^3+180*a^(7/2)*x^2*b^4*c-256*a^(9/2)*c^4+420*a^(7/2)*x
*b^3*c^2-30*a^(5/2)*x^2*b^6+200*a^(7/2)*b^2*c^3-60*a^(5/2)*x*b^5*
c+240*(a*x^2+b*x+c)^(3/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*a^4*b*c^2-120*(a*x^2+b*x+c)^(3/2)*ln(1/2*(2*(a*x
^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^3*c+15*(a*x^2+b*x
+c)^(3/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*a^2*b^5-30*a^(5/2)*b^4*c^2)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x
^5/(4*a*c-b^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(-5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(-5/2), x)
```

Fricas [A] time = 0.422722, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(-5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^
3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2
*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a
*b^4*c^2 + 16*a^2*b^2*c^3)*x)*sqrt(a)*log(-(8*a^2*x^2 + 8*a*b*x +
 b^2 + 4*a*c)*sqrt(a) + 4*(2*a^2*x^2 + a*b*x)*sqrt((a*x^2 + b*x +
 c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^5 + 4*(5*
a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*
b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*
```

```
b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 1
28*a^3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*
b^2*c^3 + 16*a^6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 +
 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*
b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b
*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^
5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16
*a^3*b^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6
*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x
+ b)*sqrt(-a)/(a*x*sqrt((a*x^2 + b*x + c)/x^2))) + 2*(3*(a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 6
4*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 6
4*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^
2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((a*x^2
 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^
6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c
+ 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 +
2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-5/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x + c/x^2)^(-5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.455 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

Optimal. Leaf size=73

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b
^2/x^2 + (2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rubi [A] time = 0.0864296, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b
^2/x^2 + (2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rubi in Sympy [A] time = 5.40982, size = 58, normalized size = 0.79

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}+\frac{b\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\log(x)}{a+\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

```
[Out] a*x*sqrt(a**2 + 2*a*b/x + b**2/x**2)/(a + b/x) + b*sqrt(a**2 + 2*
a*b/x + b**2/x**2)*log(x)/(a + b/x)
```

Mathematica [A] time = 0.0300928, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(ax+b)^2}{x^2}}(ax+b\log(x))}{ax+b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

Maple [A] time = 0.014, size = 40, normalized size = 0.6

$$\frac{x\left(ax+b\ln(x)\right)}{ax+b}\sqrt{\frac{a^2x^2+2\,abx+b^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x^2+2*a*b/x)^(1/2),x)
```

```
[Out] ((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))
```

Maxima [A] time = 0.75148, size = 11, normalized size = 0.15

$$ax+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a^2 + 2*a*b/x + b^2/x^2),x, algorithm="maxima")
```

```
[Out] a*x + b*log(x)
```

Fricas [A] time = 0.267432, size = 11, normalized size = 0.15

$$ax+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a^2 + 2*a*b/x + b^2/x^2),x, algorithm="fricas")
```

```
[Out] a*x + b*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + \frac{2ab}{x} + \frac{b^2}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

GIAC/XCAS [A] time = 0.293726, size = 39, normalized size = 0.53

$$ax\mathrm{sign}\left(ax^2 + bx\right) + b\ln\left(|x|\right)\mathrm{sign}\left(ax^2 + bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a^2 + 2*a*b/x + b^2/x^2),x, algorithm="giac")
```

```
[Out] a*x*sign(a*x^2 + b*x) + b*ln(abs(x))*sign(a*x^2 + b*x)
```

3.456 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.601272, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 53.1511, size = 189, normalized size = 1.06

$$\frac{x}{c}-\frac{\sqrt{2}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**4+b/x**2),x)
```

```
[Out] x/c - sqrt(2)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2
)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b + s
qrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(-2*a*c + b**2
 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4
*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*
a*c + b**2))
```

Mathematica [A] time = 0.221972, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.013, size = 343, normalized size = 1.9

$$\frac{x}{c}-\frac{b\sqrt{2}}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\sqrt{2}a\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$-\frac{\sqrt{2}b^{2}}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\frac{b\sqrt{2}}{2\,c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\sqrt{2}aArtanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$-\frac{\sqrt{2}b^{2}}{2\,c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^4+b/x^2),x)
```

```
[Out] x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c} - \frac{\int \frac{bx^2+a}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^2 + a/x^4),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [A] time = 0.273947, size = 1430, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^2 + a/x^4),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*
a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^
2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b
^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3
 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))
)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2
*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c -
(b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/
2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*
c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 5.94838, size = 129, normalized size = 0.72

$$\mathrm{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3-4ta}{a^2c-ab}\right.\right.\right.$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**4+b/x**2),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) +
_t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t,
_t*log(x + (32*_t**3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**
2 + 8*_t*a*b**2*c - 2*_t*b**4)/(a**2*c - a*b**2)))) + x/c
```

GIAC/XCAS [A] time = 0.829529, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^2 + a/x^4),x, algorithm="giac")
```

```
[Out] Done
```

3.457 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*S
qrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt
[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqr
t[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)
*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(
2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((
b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 2.59974, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*S
qrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt
[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqr
t[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)
*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(
2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((
b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**6+b/x**3),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0594439, size = 70, normalized size = 0.11

$$\frac{x}{c}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - 
#1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] time = 0.023, size = 59, normalized size = 0.1

$$\frac{x}{c}+\frac{1}{3\,c}\sum_{\_R=RootOf(\_Z^6c+\_Z^3b+a)}\frac{\left(-\_R^3b-a\right)\ln\left(x-\_R\right)}{2\,\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^6+b/x^3),x)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z
^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c}-\frac{\int\frac{bx^3+a}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^3 + a/x^6),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [A] time = 0.418434, size = 6885, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^3 + a/x^6),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^
5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^
4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))
/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan((1/2)^(1/3)*(sqrt(3)*(b^5*c^4
- 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*
c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^
2*c^2 - 8*a^3*c^3))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(1/3)/(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + 4*sqr
t(1/2)*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*sqrt((2*(a^2*b^4 - 4*a^3
*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*
b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 +
 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*
a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11))
)/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^
3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*
c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*
a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 -
 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c
^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) - (1/2)^(1/3)*(b
^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6
*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 -
 4*a*c^5))^(1/3))) - 4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (
b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
```

```
3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
- 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan((1/2)^(1/3)*(s
qrt(3)*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a
*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^6 - 8*a*b
^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3))*(-(b^3 - 2*a*b*c - (b^2*c^4 -
 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*
c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)/(4*(a*b^4 - 4*a^2*b^2*c + 2*a^
3*c^2)*x + 4*sqrt(1/2)*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*sqrt((2*
(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a
*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4
- 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c -
(b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^
5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*
a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5
)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/
(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2))
- (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + (b^
5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4
*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^
11)))/(b^2*c^4 - 4*a*c^5))^(1/3))) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b
*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2
*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4
- 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c +
34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*a*b^
5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^
9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4
- 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^
3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3
*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8
*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a
^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c
^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^
6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
- 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 -
4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c
^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c +
2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 -
```

```
 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b
^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt(
(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^
4 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 1
6*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64
*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*
x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3
)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt(
(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^
4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (
1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - (b^5*c
^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48
*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*
c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*
a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)
))/(b^2*c^4 - 4*a*c^5))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c
 - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*
b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48
*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) + 6*x)
/c
```

Sympy [A] time = 9.0671, size = 196, normalized size = 0.31

$$\operatorname{RootSum}\left(t^6\left(46656a^3c^7 - 34992a^2b^2c^6 + 8748ab^4c^5 - 729b^6c^4\right) + t^3\left(864a^3bc^3 - 864a^2b^3c^2 + 270ab^5c - 27b^7\right) + a^4, \left(t \mapsto\right.\right.$$
$$+ \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**6+b/x**3),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b*
*4*c**5 - 729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3
```

```
*c**2 + 270*a*b**5*c - 27*b**7) + a**4, Lambda(_t, _t*log(x + (12
96*_t**4*a**2*b*c**6 - 648*_t**4*a*b**3*c**5 + 81*_t**4*b**5*c**4
 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**2 - 21*_t*a*b**4*c + 3*_t
*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) + x/c
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c + \frac{b}{x^3} + \frac{a}{x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^3 + a/x^6),x, algorithm="giac")
```

```
[Out] integrate(1/(c + b/x^3 + a/x^6), x)
```

3.458 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b
 + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.36441, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b
 + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 137.529, size = 382, normalized size = 1.02

$$\frac{x}{c}-\frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+a/x**8+b/x**4),x)
```

```
[Out] x/c - 2**(3/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(2**(1
/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(5/4)*(-b
 + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) - 2**(3/4)*(-
2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(
-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(5/4)*(-b + sqrt(-4*a*c +
 b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*(-2*a*c + b**2 + b
```

```
*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c
+ b**2))**(1/4))/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*sq
rt(-4*a*c + b**2)) + 2**(3/4)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b*
*2))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))
/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2
))
```

Mathematica [C] time = 0.0617522, size = 70, normalized size = 0.19

$$\frac{x}{c}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x -
#1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] time = 0.037, size = 59, normalized size = 0.2

$$\frac{x}{c}+\frac{1}{4\,c}\sum_{\_R=RootOf\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\,\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+a/x^8+b/x^4),x)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c}-\frac{\int\frac{bx^4+a}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^4 + a/x^8),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [A] time = 0.440067, size = 5154, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^4 + a/x^8),x, algorithm="fricas")
```

```
[Out] 1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 4
8*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)))*arctan(-1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 -
 (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/((a*b^4 - 3*a^2*b^2*c + a^3*c^2
)*x + sqrt(1/2)*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt((2*(a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^
2*b^4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6
+ 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^
2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16
*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))) - 4*c*sqrt(sqrt(
1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/2*(b
^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*
c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*
c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))/((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + sqrt(1/2)*(a*b
^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt((2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^3*b
^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 6
4*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b
```

```
^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^2*b^4
 - 3*a^3*b^2*c + a^4*c^2)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5
- 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)
*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5
 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^
2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^1
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x
- 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 -
8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b
^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 1
2*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^
2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1
/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a
*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a
^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c
^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12
*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*
(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^
3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6
+ 16*a^2*c^7)))) + 4*x)/c
```

Sympy [A] time = 52.0304, size = 218, normalized size = 0.58

$$\operatorname{RootSum}\left(t^{8}\left(16777216a^{4}c^{9}-16777216a^{3}b^{2}c^{8}+6291456a^{2}b^{4}c^{7}-1048576ab^{6}c^{6}+65536b^{8}c^{5}\right)+t^{4}\left(20480a^{4}bc^{4}-30720a^{3}\right.\right.$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+a/x**8+b/x**4),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**9 - 16777216*a**3*b**2*c**8 + 629
1456*a**2*b**4*c**7 - 1048576*a*b**6*c**6 + 65536*b**8*c**5) + _t
**4*(20480*a**4*b*c**4 - 30720*a**3*b**3*c**3 + 15616*a**2*b**5*c
**2 - 3328*a*b**7*c + 256*b**9) + a**5, Lambda(_t, _t*log(x + (16
384*_t**5*a**2*b*c**7 - 8192*_t**5*a*b**3*c**6 + 1024*_t**5*b**5*
c**5 - 8*_t*a**3*c**3 + 36*_t*a**2*b**2*c**2 - 24*_t*a*b**4*c + 4
*_t*b**6)/(a**3*c**2 - 3*a**2*b**2*c + a*b**4)))) + x/c
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c+\frac{b}{x^{4}}+\frac{a}{x^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c + b/x^4 + a/x^8),x, algorithm="giac")
```

```
[Out] integrate(1/(c + b/x^4 + a/x^8), x)
```

3.459 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

Optimal. Leaf size=106

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])
/(2*Sqrt[a]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqr
t[x])/(2*Sqrt[c]*Sqrt[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rubi [A] time = 0.234997, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])
/(2*Sqrt[a]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqr
t[x])/(2*Sqrt[c]*Sqrt[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rubi in Sympy [A] time = 28.4034, size = 97, normalized size = 0.92

$$-2\sqrt{a}\,\mathrm{atanh}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\,\mathrm{atanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}+2\sqrt{a+b\sqrt{x}+cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*atanh((2*a + b*sqrt(x))/(2*sqrt(a)*sqrt(a + b*sqrt(x)
+ c*x))) + b*atanh((b + 2*c*sqrt(x))/(2*sqrt(c)*sqrt(a + b*sqrt(x
```

```
) + c*x)))/sqrt(c) + 2*sqrt(a + b*sqrt(x) + c*x)
```

Mathematica [A] time = 0.124372, size = 110, normalized size = 1.04

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\log\left(2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}+2a+b\sqrt{x}\right)+\frac{b\log\left(2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}+b+2c\sqrt{x}\right)}{\sqrt{c}}+\sqrt{a}\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] + Sqrt[a]*Log[x] - 2*Sqrt[a]*Log[2*a
+ b*Sqrt[x] + 2*Sqrt[a]*Sqrt[a + b*Sqrt[x] + c*x]] + (b*Log[b + 2
*c*Sqrt[x] + 2*Sqrt[c]*Sqrt[a + b*Sqrt[x] + c*x]])/Sqrt[c]
```

Maple [A] time = 0.034, size = 84, normalized size = 0.8

$$2\sqrt{a+cx+b\sqrt{x}}+b\ln\left(1\left(\frac{b}{2}+c\sqrt{x}\right)\frac{1}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)\frac{1}{\sqrt{c}}-2\sqrt{a}\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c*x+b*x^(1/2))^(1/2)/x,x)
```

```
[Out] 2*(a+c*x+b*x^(1/2))^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b
*x^(1/2))^(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a
+c*x+b*x^(1/2))^(1/2))/x^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx+b\sqrt{x}+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x + b*sqrt(x) + a)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x + b*sqrt(x) + a)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sqrt{x} + cx}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x + b*sqrt(x) + a)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.460 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2\,dx$

Optimal. Leaf size=40

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

```
[Out] -(b*(b + 2*c*Sqrt[x])^5)/(160*c^4) + (b + 2*c*Sqrt[x])^6/(192*c^4
)
```

Rubi [A] time = 0.0464199, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -(b*(b + 2*c*Sqrt[x])^5)/(160*c^4) + (b + 2*c*Sqrt[x])^6/(192*c^4
)
```

Rubi in Sympy [A] time = 13.3516, size = 34, normalized size = 0.85

$$-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/4/c*b**2+c*x+b*x**(1/2))**2,x)
```

```
[Out] -b*(b + 2*c*sqrt(x))**5/(160*c**4) + (b + 2*c*sqrt(x))**6/(192*c*
*4)
```

Mathematica [A] time = 0.0165751, size = 60, normalized size = 1.5

$$\frac{b^4x+\frac{16}{3}b^3cx^{3/2}+12b^2c^2x^2+\frac{64}{5}bc^3x^{5/2}+\frac{16c^4x^3}{3}}{16c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] (b^4*x + (16*b^3*c*x^(3/2))/3 + 12*b^2*c^2*x^2 + (64*b*c^3*x^(5/2
))/5 + (16*c^4*x^3)/3)/(16*c^2)
```

Maple [A] time = 0.003, size = 52, normalized size = 1.3

$$\frac{x^2b^2}{2}+\frac{b}{2c}\left(\frac{8c^2}{5}x^{\frac{5}{2}}+\frac{2b^2}{3}x^{\frac{3}{2}}\right)+\frac{1}{3c}\left(\frac{b^2}{4c}+cx\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/4*b^2/c+c*x+b*x^(1/2))^2,x)
```

```
[Out] 1/2*x^2*b^2+1/2*b/c*(8/5*x^(5/2)*c^2+2/3*x^(3/2)*b^2)+1/3*(1/4*b^
2/c+c*x)^3/c
```

Maxima [A] time = 0.747215, size = 73, normalized size = 1.82

$$\frac{1}{3}c^2x^3+\frac{4}{5}bcx^{\frac{5}{2}}+\frac{1}{2}b^2x^2+\frac{b^4x}{16c^2}+\frac{\left(3cx^2+4bx^{\frac{3}{2}}\right)b^2}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/16*(4*c*x + 4*b*sqrt(x) + b^2/c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/
12*(3*c*x^2 + 4*b*x^(3/2))*b^2/c
```

Fricas [A] time = 0.274786, size = 72, normalized size = 1.8

$$\frac{80c^4x^3+180b^2c^2x^2+15b^4x+16\left(12bc^3x^2+5b^3cx\right)\sqrt{x}}{240c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/16*(4*c*x + 4*b*sqrt(x) + b^2/c)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2
 + 5*b^3*c*x)*sqrt(x))/c^2
```

Sympy [A] time = 1.63181, size = 58, normalized size = 1.45

$$\frac{b^4x + \frac{16b^3cx^{\frac{3}{2}}}{3} + 12b^2c^2x^2 + \frac{64bc^3x^{\frac{5}{2}}}{5} + \frac{16c^4x^3}{3}}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/4/c*b**2+c*x+b*x**(1/2))**2,x)
```

```
[Out] (b**4*x + 16*b**3*c*x**(3/2)/3 + 12*b**2*c**2*x**2 + 64*b*c**3*x*
*(5/2)/5 + 16*c**4*x**3/3)/(16*c**2)
```

GIAC/XCAS [A] time = 0.295246, size = 66, normalized size = 1.65

$$\frac{80\,c^4x^3 + 192\,bc^3x^{\frac{5}{2}} + 180\,b^2c^2x^2 + 80\,b^3cx^{\frac{3}{2}} + 15\,b^4x}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/16*(4*c*x + 4*b*sqrt(x) + b^2/c)^2,x, algorithm="giac")
```

```
[Out] 1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*
c*x^(3/2) + 15*b^4*x)/c^2
```

3.461 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*
Log[a + b*Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rubi [A] time = 0.0851436, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*
Log[a + b*Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rubi in Sympy [A] time = 9.52971, size = 73, normalized size = 0.97

$$-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}+\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)
```

```
[Out] -2*a*(a + b*sqrt(x))*log(a + b*sqrt(x))/(b**2*sqrt(a**2 + 2*a*b*s
qrt(x) + b**2*x)) + 2*sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x)/b**2
```

Mathematica [A] time = 0.0383138, size = 50, normalized size = 0.67

$$\frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[
(a + b*Sqrt[x])^2])
```

Maple [A] time = 0.033, size = 50, normalized size = 0.7

$$2\,\frac{\sqrt{a^{2}+b^{2}x+2\,ab\sqrt{x}}\left(b\sqrt{x}-a\ln\left(a+b\sqrt{x}\right)\right)}{\left(a+b\sqrt{x}\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x)
```

```
[Out] 2*(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)*(b*x^(1/2)-a*ln(a+b*x^(1/2)))/(
a+b*x^(1/2))/b^2
```

Maxima [A] time = 0.761172, size = 31, normalized size = 0.41

$$-\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^{2}}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x + 2*a*b*sqrt(x) + a^2),x, algorithm="maxima")
```

```
[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x + 2*a*b*sqrt(x) + a^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x), x)
```

GIAC/XCAS [A] time = 0.29918, size = 65, normalized size = 0.87

$$-\frac{2\,a\ln\left(\left|b\sqrt{x}+a\right|\right)}{b^2\mathrm{sign}\left(bx+a\sqrt{x}\right)}+\frac{2\,\sqrt{x}}{b\mathrm{sign}\left(bx+a\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x + 2*a*b*sqrt(x) + a^2),x, algorithm="giac")
```

```
[Out] -2*a*ln(abs(b*sqrt(x) + a))/(b^2*sign(b*x + a*sqrt(x))) + 2*sqrt(
x)/(b*sign(b*x + a*sqrt(x)))
```

3.462 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}$$

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
/(8*b^3) - (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)])/(3*b^3) + (3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3)
 + b^2*x^(2/3)])/(10*b^3)
```

Rubi [A] time = 0.171075, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
/(8*b^3) - (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)])/(3*b^3) + (3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3)
 + b^2*x^(2/3)])/(10*b^3)
```

Rubi in Sympy [A] time = 15.006, size = 126, normalized size = 0.92

$$\frac{3a^2\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}{80b^3}-\frac{a\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}{15b^3}+\frac{3x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] 3*a**2*(2*a + 2*b*x**(1/3))*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3
))**(7/2)/(80*b**3) - a*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**
(9/2)/(15*b**3) + 3*x**(2/3)*(2*a + 2*b*x**(1/3))*(a**2 + 2*a*b*x
**(1/3) + b**2*x**(2/3))**(7/2)/(20*b)
```

Mathematica [A] time = 0.0630229, size = 115, normalized size = 0.84

$$\frac{x\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(120a^7+630a^6b\sqrt[3]{x}+1512a^5b^2x^{2/3}+2100a^4b^3x+1800a^3b^4x^{4/3}+945a^2b^5x^{5/3}+280ab^6x^2+36b^7x^{7/3}\right)}{120\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*x*(120*a^7 + 630*a^6*b*x^(1/3) + 1512*a^
5*b^2*x^(2/3) + 2100*a^4*b^3*x + 1800*a^3*b^4*x^(4/3) + 945*a^2*b
^5*x^(5/3) + 280*a*b^6*x^2 + 36*b^7*x^(7/3)))/(120*(a + b*x^(1/3)
))
```

Maple [A] time = 0.014, size = 109, normalized size = 0.8

$$\frac{1}{120}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(36\,b^7x^{10/3}+945\,a^2b^5x^{8/3}+1800\,a^3b^4x^{7/3}+1512\,a^5b^2x^{5/3}+630\,a^6bx^{4/3}+280\,ab^6x^3+2100\,a^4b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)
```

```
[Out] 1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*b^7*x^(10/3)+945*
a^2*b^5*x^(8/3)+1800*a^3*b^4*x^(7/3)+1512*a^5*b^2*x^(5/3)+630*a^6
*b*x^(4/3)+280*a*b^6*x^3+2100*a^4*b^3*x^2+120*a^7*x)/(a+b*x^(1/3)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274036, size = 113, normalized size = 0.82

$$\frac{7}{3}\,ab^{6}x^{3} + \frac{35}{2}\,a^{4}b^{3}x^{2} + a^{7}x + \frac{63}{40}\left(5\,a^{2}b^{5}x^{2} + 8\,a^{5}b^{2}x\right)x^{\frac{2}{3}} + \frac{3}{20}\left(2\,b^{7}x^{3} + 100\,a^{3}b^{4}x^{2} + 35\,a^{6}bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 +
 8*a^5*b^2*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^
6*b*x)*x^(1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.304417, size = 189, normalized size = 1.38

$$\frac{3}{10}\,b^{7}x^{\frac{10}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{7}{3}\,ab^{6}x^{3}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{63}{8}\,a^{2}b^{5}x^{\frac{8}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + 15\,a^{3}b^{4}x^{\frac{7}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{35}{2}\,a^{4}b^{3}x^{2}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{63}{5}\,a^{5}b^{2}x^{\frac{5}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{21}{4}\,a^{6}bx^{\frac{4}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + a^{7}x\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2),x, algorithm="giac")
```

```
[Out] 3/10*b^7*x^(10/3)*sign(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sign(b*x^(1
/3) + a) + 63/8*a^2*b^5*x^(8/3)*sign(b*x^(1/3) + a) + 15*a^3*b^4*
x^(7/3)*sign(b*x^(1/3) + a) + 35/2*a^4*b^3*x^2*sign(b*x^(1/3) + a
) + 63/5*a^5*b^2*x^(5/3)*sign(b*x^(1/3) + a) + 21/4*a^6*b*x^(4/3)
*sign(b*x^(1/3) + a) + a^7*x*sign(b*x^(1/3) + a)
```

3.463 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(
2*b^3) - (6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^
(2/3)])/(7*b^3) + (3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) +
 b^2*x^(2/3)])/(8*b^3)
```

Rubi [A] time = 0.156943, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(
2*b^3) - (6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^
(2/3)])/(7*b^3) + (3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) +
 b^2*x^(2/3)])/(8*b^3)
```

Rubi in Sympy [A] time = 14.9486, size = 126, normalized size = 0.92

$$\frac{a^2\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{16b^3}-\frac{3a\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}{28b^3}+\frac{3x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] a**2*(2*a + 2*b*x**(1/3))*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))
**(5/2)/(16*b**3) - 3*a*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**
(7/2)/(28*b**3) + 3*x**(2/3)*(2*a + 2*b*x**(1/3))*(a**2 + 2*a*b*x
**(1/3) + b**2*x**(2/3))**(5/2)/(16*b)
```

Mathematica [A] time = 0.0423923, size = 91, normalized size = 0.66

$$\frac{x\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(56a^5+210a^4b\sqrt[3]{x}+336a^3b^2x^{2/3}+280a^2b^3x+120ab^4x^{4/3}+21b^5x^{5/3}\right)}{56\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*x*(56*a^5 + 210*a^4*b*x^(1/3) + 336*a^3*
b^2*x^(2/3) + 280*a^2*b^3*x + 120*a*b^4*x^(4/3) + 21*b^5*x^(5/3))
)/(56*(a + b*x^(1/3)))
```

Maple [A] time = 0.004, size = 87, normalized size = 0.6

$$\frac{1}{56}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(21\,b^5x^{8/3}+120\,ab^4x^{7/3}+336\,a^3b^2x^{5/3}+210\,a^4bx^{4/3}+280\,a^2b^3x^2+56\,a^5x\right)\left(a+b\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x)
```

```
[Out] 1/56*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(21*b^5*x^(8/3)+120*a*
b^4*x^(7/3)+336*a^3*b^2*x^(5/3)+210*a^4*b*x^(4/3)+280*a^2*b^3*x^2
+56*a^5*x)/(a+b*x^(1/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274681, size = 82, normalized size = 0.6

$$5\,a^2b^3x^2 + a^5x + \frac{3}{8}\left(b^5x^2 + 16\,a^3b^2x\right)x^{\frac{2}{3}} + \frac{15}{28}\left(4\,ab^4x^2 + 7\,a^4bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15
/28*(4*a*b^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(5/2), x)
```

GIAC/XCAS [A] time = 0.2843, size = 138, normalized size = 1.01

$$\frac{3}{8}\,b^5x^{\frac{8}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{15}{7}\,ab^4x^{\frac{7}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + 5\,a^2b^3x^2\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + 6\,a^3b^2x^{\frac{5}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{15}{4}\,a^4bx^{\frac{4}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + a^5x\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/8*b^5*x^(8/3)*sign(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sign(b*x
^(1/3) + a) + 5*a^2*b^3*x^2*sign(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/
```

```
3)*sign(b*x^(1/3) + a) + 15/4*a^4*b*x^(4/3)*sign(b*x^(1/3) + a) +
 a^5*x*sign(b*x^(1/3) + a)
```

3.464 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^3}{4b^3}$$

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
/(4*b^3) - (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)])/(5*b^3) + ((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) +
 b^2*x^(2/3)])/(2*b^3)
```

Rubi [A] time = 0.122619, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^3}{4b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
/(4*b^3) - (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)])/(5*b^3) + ((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) +
 b^2*x^(2/3)])/(2*b^3)
```

Rubi in Sympy [A] time = 15.0208, size = 122, normalized size = 0.89

$$\frac{a^2\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{8b^3}-\frac{a\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{5b^3}+\frac{x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] a**2*(2*a + 2*b*x**(1/3))*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))
**(3/2)/(8*b**3) - a*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(5/
2)/(5*b**3) + x**(2/3)*(2*a + 2*b*x**(1/3))*(a**2 + 2*a*b*x**(1/3
) + b**2*x**(2/3))**(3/2)/(4*b)
```

Mathematica [A] time = 0.0332571, size = 65, normalized size = 0.47

$$\frac{x\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(20a^3+45a^2b\sqrt[3]{x}+36ab^2x^{2/3}+10b^3x\right)}{20\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*x*(20*a^3 + 45*a^2*b*x^(1/3) + 36*a*b^2*
x^(2/3) + 10*b^3*x))/(20*(a + b*x^(1/3)))
```

Maple [A] time = 0.004, size = 65, normalized size = 0.5

$$\frac{1}{20}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(36\,ax^{5/3}b^2+45\,x^{4/3}a^2b+10\,b^3x^2+20\,a^3x\right)\left(a+b\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x)
```

```
[Out] 1/20*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*a*x^(5/3)*b^2+45*x
^(4/3)*a^2*b+10*b^3*x^2+20*a^3*x)/(a+b*x^(1/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272695, size = 43, normalized size = 0.31

$$\frac{1}{2}\,b^3x^2 + \frac{9}{5}\,ab^2x^{\frac{5}{3}} + \frac{9}{4}\,a^2bx^{\frac{4}{3}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(3/2), x)
```

GIAC/XCAS [A] time = 0.282533, size = 86, normalized size = 0.63

$$\frac{1}{2}\,b^3x^2\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{5}\,ab^2x^{\frac{5}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{4}\,a^2bx^{\frac{4}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + a^3x\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sign(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sign(b*x^(1/3
) + a) + 9/4*a^2*b*x^(4/3)*sign(b*x^(1/3) + a) + a^3*x*sign(b*x^(
1/3) + a)
```

3.465 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (
3*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^
(1/3)))
```

Rubi [A] time = 0.0908995, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (
3*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^
(1/3)))
```

Rubi in Sympy [A] time = 9.32815, size = 70, normalized size = 0.8

$$\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{3x\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] a*x*sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))/(4*(a + b*x**(1/3
))) + 3*x*sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))/4
```

Mathematica [A] time = 0.0162209, size = 43, normalized size = 0.49

$$\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)
))
```

Maple [A] time = 0.004, size = 43, normalized size = 0.5

$$\frac{1}{4}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(3\,bx^{4/3}+4\,ax\right)\left(a+b\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x)
```

```
[Out] 1/4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(3*b*x^(4/3)+4*a*x)/(a+
b*x^(1/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276833, size = 14, normalized size = 0.16

$$\frac{3}{4}bx^{\frac{4}{3}}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2),x, algorithm="fricas")
```

```
[Out] 3/4*b*x^(4/3) + a*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

GIAC/XCAS [A] time = 0.270966, size = 35, normalized size = 0.4

$$\frac{3}{4}\,bx^{\frac{4}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right) + ax\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2),x, algorithm="giac")
```

```
[Out] 3/4*b*x^(4/3)*sign(b*x^(1/3) + a) + a*x*sign(b*x^(1/3) + a)
```

3.466 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

Optimal. Leaf size=147

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^
2*x^(2/3)]) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3)]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)
])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.15305, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^
2*x^(2/3)]) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3)]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)
])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rubi in Sympy [A] time = 16.1625, size = 131, normalized size = 0.89

$$\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}-\frac{3a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}{b^3}+\frac{3x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)}{4b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] 3*a**2*(a + b*x**(1/3))*log(a + b*x**(1/3))/(b**3*sqrt(a**2 + 2*a
*b*x**(1/3) + b**2*x**(2/3))) - 3*a*sqrt(a**2 + 2*a*b*x**(1/3) +
```

```
b**2*x**(2/3))/b**3 + 3*x**(2/3)*(2*a + 2*b*x**(1/3))/(4*b*sqrt(a
**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)))
```

Mathematica [A] time = 0.0406836, size = 65, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(2a^2\log\left(a+b\sqrt[3]{x}\right)+b\sqrt[3]{x}\left(b\sqrt[3]{x}-2a\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a +
b*x^(1/3)]))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.044, size = 103, normalized size = 0.7

$$\frac{1}{2\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(3\,b^2x^{2/3}+2\,a^2\ln\left(b^3x+a^3\right)-6\,ab\sqrt[3]{x}+4\,a^2\ln\left(a+b\sqrt[3]{x}\right)-2\,a^2\ln\left(b^2x^{2/3}-ab\sqrt[3]{x}+a^2\right)\right)\left(a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x)
```

```
[Out] 1/2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(3*b^2*x^(2/3)+2*a^2*ln
(b^3*x+a^3)-6*a*b*x^(1/3)+4*a^2*ln(a+b*x^(1/3))-2*a^2*ln(b^2*x^(2
/3)-a*b*x^(1/3)+a^2))/(a+b*x^(1/3))/b^3
```

Maxima [A] time = 0.743531, size = 62, normalized size = 0.42

$$\frac{3\,a^2b^2\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{5}{2}}}-\frac{3\,abx^{\frac{1}{3}}}{\left(b^2\right)^{\frac{3}{2}}}+\frac{3\,x^{\frac{2}{3}}}{2\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2),x, algorithm="maxima")
```

```
[Out] 3*a^2*b^2*log(x^(1/3) + a/b)/(b^2)^(5/2) - 3*a*b*x^(1/3)/(b^2)^(3
/2) + 3/2*x^(2/3)/sqrt(b^2)
```

Fricas [A] time = 0.276755, size = 45, normalized size = 0.31

$$\frac{3\left(2\,a^2\log\left(bx^{\frac{1}{3}}+a\right)+b^2x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2),x, algorithm="fricas")
```

```
[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

GIAC/XCAS [A] time = 0.283168, size = 82, normalized size = 0.56

$$\frac{3\left(bx^{\frac{2}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}}+a\right)-2\,ax^{\frac{1}{3}}\mathrm{sign}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2}+\frac{3\,a^2\ln\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sign}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^(2/3)*sign(b*x^(1/3) + a) - 2*a*x^(1/3)*sign(b*x^(1/3) +
 a))/b^2 + 3*a^2*ln(abs(b*x^(1/3) + a))/(b^3*sign(b*x^(1/3) + a))
```

3.467 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*
b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3
*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3
) + b^2*x^(2/3)])
```

Rubi [A] time = 0.15396, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*
b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3
*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3
) + b^2*x^(2/3)])
```

Rubi in Sympy [A] time = 11.9434, size = 131, normalized size = 1.01

$$-\frac{3x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)}{4b\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}-\frac{3\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] -3*x**(2/3)*(2*a + 2*b*x**(1/3))/(4*b*(a**2 + 2*a*b*x**(1/3) + b*
*2*x**(2/3))**(3/2)) - 3*x**(1/3)/(b**2*sqrt(a**2 + 2*a*b*x**(1/3
```

```
) + b**2*x**(2/3))) + 3*(a + b*x**(1/3))*log(a + b*x**(1/3))/(b**
3*sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)))
```

Mathematica [A] time = 0.0469399, size = 72, normalized size = 0.55

$$\frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)]
)/(2*b^3*(a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.012, size = 92, normalized size = 0.7

$$\frac{3}{2\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(2\,x^{2/3}\ln\left(a+b\sqrt[3]{x}\right)b^2+4\,\sqrt[3]{x}\ln\left(a+b\sqrt[3]{x}\right)ab+4\,ab\sqrt[3]{x}+2\,a^2\ln\left(a+b\sqrt[3]{x}\right)+3\,a^2\right)\left(a+b\sqrt[3]{x}\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x)
```

```
[Out] 3/2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(2*x^(2/3)*ln(a+b*x^(1/
3))*b^2+4*x^(1/3)*ln(a+b*x^(1/3))*a*b+4*a*b*x^(1/3)+2*a^2*ln(a+b*
x^(1/3))+3*a^2)/(a+b*x^(1/3))^3/b^3
```

Maxima [A] time = 0.74631, size = 88, normalized size = 0.68

$$\frac{3\,\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}}+\frac{9\,a^2b^2}{2\left(b^2\right)^{\frac{7}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{6\,abx^{\frac{1}{3}}}{\left(b^2\right)^{\frac{5}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] 3*log(x^(1/3) + a/b)/(b^2)^(3/2) + 9/2*a^2*b^2/((b^2)^(7/2)*(x^(1
/3) + a/b)^2) + 6*a*b*x^(1/3)/((b^2)^(5/2)*(x^(1/3) + a/b)^2)
```

Fricas [A] time = 0.272356, size = 93, normalized size = 0.72

$$\frac{3\left(4\,abx^{\frac{1}{3}} + 3\,a^2 + 2\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)\log\left(bx^{\frac{1}{3}} + a\right)\right)}{2\left(b^5x^{\frac{2}{3}} + 2\,ab^4x^{\frac{1}{3}} + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] 3/2*(4*a*b*x^(1/3) + 3*a^2 + 2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)*log(b*x^(1/3) + a))/(b^5*x^(2/3) + 2*a*b^4*x^(1/3) + a^2*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

GIAC/XCAS [A] time = 0.284756, size = 86, normalized size = 0.66

$$\frac{3\ln\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{b^3\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right)} + \frac{3\left(4\,ax^{\frac{1}{3}} + \frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}} + a\right)^2 b^2\mathrm{sign}\left(bx^{\frac{1}{3}} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-3/2),x, algorithm="giac")
```

```
[Out] 3*ln(abs(b*x^(1/3) + a))/(b^3*sign(b*x^(1/3) + a)) + 3/2*(4*a*x^(
1/3) + 3*a^2/b)/((b*x^(1/3) + a)^2*b^2*sign(b*x^(1/3) + a))
```

3.468 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)]) + (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3)
 + b^2*x^(2/3)]) - 3/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1
/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.157291, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)]) + (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3)
 + b^2*x^(2/3)]) - 3/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1
/3) + b^2*x^(2/3)])
```

Rubi in Sympy [A] time = 8.86532, size = 75, normalized size = 0.56

$$\frac{3x\left(2a+2b\sqrt[3]{x}\right)}{8a\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}+\frac{x}{4a^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] 3*x*(2*a + 2*b*x**(1/3))/(8*a*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2
/3))**(5/2)) + x/(4*a**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*
*(3/2))
```

Mathematica [A] time = 0.0307225, size = 58, normalized size = 0.43

$$\frac{-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3))/(4*b^3*(a + b*x^(1/3))^3*S
qrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.01, size = 54, normalized size = 0.4

$$-\frac{1}{4\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(6\,b^2x^{2/3}+4\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x)
```

```
[Out] -1/4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(6*b^2*x^(2/3)+4*a*b*x
^(1/3)+a^2)/(a+b*x^(1/3))^5/b^3
```

Maxima [A] time = 0.750146, size = 85, normalized size = 0.63

$$-\frac{3\,a^2b^2}{4\,(b^2)^{\frac{9}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}+\frac{2\,ab}{(b^2)^{\frac{7}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3}{2\,(b^2)^{\frac{5}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-5/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*b^2/((b^2)^(9/2)*(x^(1/3) + a/b)^4) + 2*a*b/((b^2)^(7/2)
*(x^(1/3) + a/b)^3) - 3/2/((b^2)^(5/2)*(x^(1/3) + a/b)^2)
```

Fricas [A] time = 0.274416, size = 90, normalized size = 0.67

$$-\frac{6\,b^2x^{\frac{2}{3}}+4\,abx^{\frac{1}{3}}+a^2}{4\left(4\,ab^6x+6\,a^2b^5x^{\frac{2}{3}}+a^4b^3+\left(b^7x+4\,a^3b^4\right)x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/(4*a*b^6*x + 6*a^2*b^5
*x^(2/3) + a^4*b^3 + (b^7*x + 4*a^3*b^4)*x^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-5/2),x, algorithm="giac")
```

```
[Out] undef
```

3.469 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -a^2/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2
/3)]) + (6*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) +
 b^2*x^(2/3)]) - 3/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1
/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.159789, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] -a^2/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2
/3)]) + (6*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) +
 b^2*x^(2/3)]) - 3/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1
/3) + b^2*x^(2/3)])
```

Rubi in Sympy [A] time = 10.9411, size = 124, normalized size = 0.91

$$-\frac{x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)}{4b\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}-\frac{\sqrt[3]{x}}{5b^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}-\frac{2a+2b\sqrt[3]{x}}{40b^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] -x**(2/3)*(2*a + 2*b*x**(1/3))/(4*b*(a**2 + 2*a*b*x**(1/3) + b**2
*x**(2/3))**(7/2)) - x**(1/3)/(5*b**2*(a**2 + 2*a*b*x**(1/3) + b*
*2*x**(2/3))**(5/2)) - (2*a + 2*b*x**(1/3))/(40*b**3*(a**2 + 2*a*
b*x**(1/3) + b**2*x**(2/3))**(5/2))
```

Mathematica [A] time = 0.0410084, size = 58, normalized size = 0.42

$$\frac{-a^2-6ab\sqrt[3]{x}-15b^2x^{2/3}}{20b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] (-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3))/(20*b^3*(a + b*x^(1/3))^5
*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.01, size = 54, normalized size = 0.4

$$-\frac{1}{20\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(15\,b^2x^{2/3}+6\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)
```

```
[Out] -1/20*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(15*b^2*x^(2/3)+6*a*b
*x^(1/3)+a^2)/(a+b*x^(1/3))^7/b^3
```

Maxima [A] time = 0.744911, size = 85, normalized size = 0.62

$$-\frac{a^2b^2}{2\,(b^2)^{\frac{11}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}+\frac{6\,ab}{5\,(b^2)^{\frac{9}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^5}-\frac{3}{4\,(b^2)^{\frac{7}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-7/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*b^2/((b^2)^(11/2)*(x^(1/3) + a/b)^6) + 6/5*a*b/((b^2)^(9
/2)*(x^(1/3) + a/b)^5) - 3/4/((b^2)^(7/2)*(x^(1/3) + a/b)^4)
```

Fricas [A] time = 0.275195, size = 123, normalized size = 0.9

$$-\frac{15\,b^2x^{\frac{2}{3}} + 6\,abx^{\frac{1}{3}} + a^2}{20\left(b^9x^2 + 20\,a^3b^6x + a^6b^3 + 3\,(2\,ab^8x + 5\,a^4b^5)x^{\frac{2}{3}} + 3\,(5\,a^2b^7x + 2\,a^5b^4)x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(15*b^2*x^(2/3) + 6*a*b*x^(1/3) + a^2)/(b^9*x^2 + 20*a^3*b^
6*x + a^6*b^3 + 3*(2*a*b^8*x + 5*a^4*b^5)*x^(2/3) + 3*(5*a^2*b^7*
x + 2*a^5*b^4)*x^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-7/2),x, algorithm="giac")
```

```
[Out] undef
```

3.470 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)]) + (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)]) - 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*
x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.158895, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3)]) + (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)]) - 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*
x^(1/3) + b^2*x^(2/3)])
```

Rubi in Sympy [A] time = 11.1277, size = 128, normalized size = 0.93

$$-\frac{3x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)}{16b\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}-\frac{3\sqrt[3]{x}}{28b^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}-\frac{2a+2b\sqrt[3]{x}}{112b^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
```

```
[Out] -3*x**(2/3)*(2*a + 2*b*x**(1/3))/(16*b*(a**2 + 2*a*b*x**(1/3) + b
**2*x**(2/3))**(9/2)) - 3*x**(1/3)/(28*b**2*(a**2 + 2*a*b*x**(1/3
) + b**2*x**(2/3))**(7/2)) - (2*a + 2*b*x**(1/3))/(112*b**3*(a**2
 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(7/2))
```

Mathematica [A] time = 0.0315545, size = 58, normalized size = 0.42

$$\frac{-a^2-8ab\sqrt[3]{x}-28b^2x^{2/3}}{56b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3))/(56*b^3*(a + b*x^(1/3))^7
*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.011, size = 54, normalized size = 0.4

$$-\frac{1}{56\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(28\,b^2x^{2/3}+8\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x)
```

```
[Out] -1/56*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(28*b^2*x^(2/3)+8*a*b
*x^(1/3)+a^2)/(a+b*x^(1/3))^9/b^3
```

Maxima [A] time = 0.7512, size = 85, normalized size = 0.62

$$-\frac{3\,a^2b^2}{8\,(b^2)^{\frac{13}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^8}+\frac{6\,ab}{7\,(b^2)^{\frac{11}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^7}-\frac{1}{2\,(b^2)^{\frac{9}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-9/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b^2/((b^2)^(13/2)*(x^(1/3) + a/b)^8) + 6/7*a*b/((b^2)^(1
1/2)*(x^(1/3) + a/b)^7) - 1/2/((b^2)^(9/2)*(x^(1/3) + a/b)^6)
```

Fricas [A] time = 0.275477, size = 151, normalized size = 1.1

$$-\frac{28\,b^2x^{\frac{2}{3}}+8\,abx^{\frac{1}{3}}+a^2}{56\left(28\,a^2b^9x^2+56\,a^5b^6x+a^8b^3+(b^{11}x^2+56\,a^3b^8x+28\,a^6b^5)x^{\frac{2}{3}}+2\,(4\,ab^{10}x^2+35\,a^4b^7x+4\,a^7b^4)x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-9/2),x, algorithm="fricas")
```

```
[Out] -1/56*(28*b^2*x^(2/3) + 8*a*b*x^(1/3) + a^2)/(28*a^2*b^9*x^2 + 56
*a^5*b^6*x + a^8*b^3 + (b^11*x^2 + 56*a^3*b^8*x + 28*a^6*b^5)*x^(
2/3) + 2*(4*a*b^10*x^2 + 35*a^4*b^7*x + 4*a^7*b^4)*x^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-9/2),x, algorithm="giac")
```

```
[Out] undef
```

3.471 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3)]) + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1
/3) + b^2*x^(2/3)]) - 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b
*x^(1/3) + b^2*x^(2/3)])
```

Rubi [A] time = 0.156081, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3)]) + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1
/3) + b^2*x^(2/3)]) - 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b
*x^(1/3) + b^2*x^(2/3)])
```

Rubi in Sympy [A] time = 11.3736, size = 126, normalized size = 0.92

$$-\frac{3x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)}{20b\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{11}{2}}}-\frac{\sqrt[3]{x}}{15b^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}-\frac{2a+2b\sqrt[3]{x}}{240b^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
```

```
[Out] -3*x**(2/3)*(2*a + 2*b*x**(1/3))/(20*b*(a**2 + 2*a*b*x**(1/3) + b
**2*x**(2/3))**(11/2)) - x**(1/3)/(15*b**2*(a**2 + 2*a*b*x**(1/3)
 + b**2*x**(2/3))**(9/2)) - (2*a + 2*b*x**(1/3))/(240*b**3*(a**2
+ 2*a*b*x**(1/3) + b**2*x**(2/3))**(9/2))
```

Mathematica [A] time = 0.0385327, size = 58, normalized size = 0.42

$$\frac{-a^2-10ab\sqrt[3]{x}-45b^2x^{2/3}}{120b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3))/(120*b^3*(a + b*x^(1/3))
^9*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] time = 0.011, size = 54, normalized size = 0.4

$$-\frac{1}{120\,b^3}\sqrt{a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(45\,b^2x^{2/3}+10\,ab\sqrt[3]{x}+a^2\right)\left(a+b\sqrt[3]{x}\right)^{-11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x)
```

```
[Out] -1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(45*b^2*x^(2/3)+10*a
*b*x^(1/3)+a^2)/(a+b*x^(1/3))^11/b^3
```

Maxima [A] time = 0.767573, size = 85, normalized size = 0.62

$$-\frac{3\,a^2b^2}{10\,(b^2)^{\frac{15}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}+\frac{2\,ab}{3\,(b^2)^{\frac{13}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{9}}-\frac{3}{8\,(b^2)^{\frac{11}{2}}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-11/2),x, algorithm="maxima")
```

```
[Out] -3/10*a^2*b^2/((b^2)^(15/2)*(x^(1/3) + a/b)^10) + 2/3*a*b/((b^2)^
(13/2)*(x^(1/3) + a/b)^9) - 3/8/((b^2)^(11/2)*(x^(1/3) + a/b)^8)
```

Fricas [A] time = 0.276703, size = 181, normalized size = 1.32

$$-\frac{45\,b^2x^{\frac{2}{3}} + 10\,abx^{\frac{1}{3}} + a^2}{120\left(10\,ab^{12}x^3 + 210\,a^4b^9x^2 + 120\,a^7b^6x + a^{10}b^3 + 9\,(5\,a^2b^{11}x^2 + 28\,a^5b^8x + 5\,a^8b^5)x^{\frac{2}{3}} + (b^{13}x^3 + 120\,a^3b^{10}x^2 + 210\,a^6b^7x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-11/2),x, algorithm="fricas")
```

```
[Out] -1/120*(45*b^2*x^(2/3) + 10*a*b*x^(1/3) + a^2)/(10*a*b^12*x^3 + 2
10*a^4*b^9*x^2 + 120*a^7*b^6*x + a^10*b^3 + 9*(5*a^2*b^11*x^2 + 2
8*a^5*b^8*x + 5*a^8*b^5)*x^(2/3) + (b^13*x^3 + 120*a^3*b^10*x^2 +
 210*a^6*b^7*x + 10*a^9*b^4)*x^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(-11/2),x, algorithm="giac")
```

```
[Out] undef
```

3.472 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx$

Optimal. Leaf size=77

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(3(m+1),-2p;3m+4;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

```
[Out] ((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F
1[3*(1 + m), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x
^(1/3))/a)^(2*p))
```

Rubi [A] time = 0.0837354, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(3(m+1),-2p;3m+4;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] ((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F
1[3*(1 + m), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x
^(1/3))/a)^(2*p))
```

Rubi in Sympy [A] time = 26.5421, size = 75, normalized size = 0.97

$$\frac{x^{-m}x^{m+1}(dx)^m\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,{}_2F_1\left(\begin{matrix}-2p,3m+3\\3m+4\end{matrix}\middle|-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)
```

```
[Out] x**(-m)*x**(m + 1)*(d*x)**m*(1 + b*x**(1/3)/a)**(-2*p)*(a**2 + 2*
a*b*x**(1/3) + b**2*x**(2/3))**p*hyper((-2*p, 3*m + 3), (3*m + 4,
), -b*x**(1/3)/a)/(m + 1)
```

Mathematica [A] time = 0.0964403, size = 68, normalized size = 0.88

$$\frac{x(dx)^m \left(\left(a+b\sqrt[3]{x}\right)^2\right)^p \left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p} {}_2F_1\left(3(m+1),-2p;3(m+1)+1;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] (((a + b*x^(1/3))^2)^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m), -2*
p, 1 + 3*(1 + m), -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)
^(2*p))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

3.473 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

Optimal. Leaf size=468

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+7)}-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+3)}+\frac{210a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+5)}-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+2)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+3)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+1)}+\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+1)}$$

```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)
/(b^9*(1 + 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(
1/3) + b^2*x^(2/3))^p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a
)^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*
a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/
(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a
)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^
9*(1 + (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b
^9*(7 + 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3
) + b^2*x^(2/3))^p)/(b^9*(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*
(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(9 + 2*p))
```

Rubi [A] time = 0.44979, antiderivative size = 468, normalized size of antiderivative = 1., number of

steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}$$
$$+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+7)}-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+3)}$$
$$+\frac{210a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+5)}$$
$$-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+2)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+3)}$$
$$-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+1)}+\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)
/(b^9*(1 + 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(
1/3) + b^2*x^(2/3))^p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a
)^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*
a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/
(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a
)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^
9*(1 + (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b
^9*(7 + 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3
) + b^2*x^(2/3))^p)/(b^9*(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*
(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(9 + 2*p))
```

Rubi in Sympy [A] time = 97.9039, size = 614, normalized size = 1.31

$$\frac{3a^8\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+1}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{10}\left(2p+1\right)} - \frac{12a^7\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+2}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{11}\left(p+1\right)} + \frac{84a^6\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+3}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{12}\left(2p+3\right)} - \frac{84a^5\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+4}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{13}\left(p+2\right)} + \frac{210a^4\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+5}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{14}\left(2p+5\right)} - \frac{84a^3\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+6}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{15}\left(p+3\right)} + \frac{84a^2\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+7}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{16}\left(2p+7\right)} - \frac{12a\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+8}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{17}\left(p+4\right)} + \frac{3\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+9}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{18}\left(2p+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

```
[Out] 3*a**8*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))**(2*p
+ 1)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**10*(2*p + 1))
 - 12*a**7*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))**(
2*p + 2)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**11*(p + 1
)) + 84*a**6*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))*
*(2*p + 3)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**12*(2*p
 + 3)) - 84*a**5*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/
3))**(2*p + 4)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**13*
(p + 2)) + 210*a**4*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**
(1/3))**(2*p + 5)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**
14*(2*p + 5)) - 84*a**3*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2
*x**(1/3))**(2*p + 6)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/
(b**15*(p + 3)) + 84*a**2*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b*
*2*x**(1/3))**(2*p + 7)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**
p/(b**16*(2*p + 7)) - 12*a*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b
**2*x**(1/3))**(2*p + 8)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*
```

```
*p/(b**17*(p + 4)) + 3*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*
x**(1/3))**(2*p + 9)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(
b**18*(2*p + 9))
```

Mathematica [A] time = 0.337015, size = 367, normalized size = 0.78

$$\frac{3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(2520a^9-5040a^8bp\sqrt[3]{x}+2520a^7b^2p(2p+1)x^{2/3}-1680a^6b^3p\left(2p^2+3p+1\right)x+420a^5b^4p\left(4p^3+12p^2+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*((a + b*x^(1/3))^2)^p*(2520*a^9 - 5040*a^8*b*p*x^(1/3) + 2520*
a^7*b^2*p*(1 + 2*p)*x^(2/3) - 1680*a^6*b^3*p*(1 + 3*p + 2*p^2)*x
+ 420*a^5*b^4*p*(3 + 11*p + 12*p^2 + 4*p^3)*x^(4/3) - 168*a^4*b^5
*p*(6 + 25*p + 35*p^2 + 20*p^3 + 4*p^4)*x^(5/3) + 28*a^3*b^6*p*(3
0 + 137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5)*x^2 - 8*a^2*b^7*p
*(90 + 441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6)*x^(7
/3) + a*b^8*p*(630 + 3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 12
88*p^5 + 224*p^6 + 16*p^7)*x^(8/3) + b^9*(2520 + 13698*p + 29531*
p^2 + 33642*p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*
p^8)*x^3))/(b^9*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(1 + 2*p)*(3 + 2*
p)*(5 + 2*p)*(7 + 2*p)*(9 + 2*p))
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

Maxima [A] time = 0.794829, size = 489, normalized size = 1.04

$$\frac{3\left(\left(16\,p^8+288\,p^7+2184\,p^6+9072\,p^5+22449\,p^4+33642\,p^3+29531\,p^2+13698\,p+2520\right)b^9x^3+\left(16\,p^8+224\,p^7+1288\,p^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*x^2,x, algorithm="maxima")
```

```
[Out] 3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^
3 + 29531*p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 128
8*p^6 + 3920*p^5 + 6769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*
x^(8/3) - 8*(8*p^7 + 84*p^6 + 350*p^5 + 735*p^4 + 812*p^3 + 441*p
^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p^6 + 60*p^5 + 170*p^4 + 225*p
^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5 + 20*p^4 + 35*p^3 +
 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 + 11*p^2 + 3
*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520*(
2*p^2 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*
x^(1/3) + a)^(2*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 12
6546*p^5 + 269325*p^4 + 361840*p^3 + 293175*p^2 + 128322*p + 2268
0)*b^9)
```

Fricas [A] time = 0.447277, size = 782, normalized size = 1.67

$$3\left(2520\,a^9 + \left(16\,b^9p^8 + 288\,b^9p^7 + 2184\,b^9p^6 + 9072\,b^9p^5 + 22449\,b^9p^4 + 33642\,b^9p^3 + 29531\,b^9p^2 + 13698\,b^9p + 2520\,b^9\right)x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*x^2,x, algorithm="fricas")
```

```
[Out] 3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9
*p^5 + 22449*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*
p + 2520*b^9)*x^3 + 28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*
b^6*p^4 + 225*a^3*b^6*p^3 + 137*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 -
 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2 + a^6*b^3*p)*x + (5040*a^7*b
^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224*a*b^8*p^7 + 1288*a*
b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8*p^3 + 3267
*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5*p
^4 + 35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) -
4*(1260*a^8*b*p + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7
*p^5 + 735*a^2*b^7*p^4 + 812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a
^2*b^7*p)*x^2 - 105*(4*a^5*b^4*p^4 + 12*a^5*b^4*p^3 + 11*a^5*b^4*
p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^9*p^6 + 12
6546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 +
 128322*b^9*p + 22680*b^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.29006, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*x^2,x, algorithm="giac")
```

```
[Out] Done
```

3.474 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

Optimal. Leaf size=315

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(p+2)}-\frac{30a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+3)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+1)}-\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+1)}$$

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p
)/(b^6*(1 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^
(1/3) + b^2*x^(2/3))^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3)
)/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (
15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(a^2 + 2*a*b*x^(
1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1/3))/
a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rubi [A] time = 0.279159, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(p+2)}-\frac{30a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+3)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+1)}-\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p
)/(b^6*(1 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^
(1/3) + b^2*x^(2/3))^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3)
```

```
)/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (
15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(a^2 + 2*a*b*x^(
1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1/3))/
a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rubi in Sympy [A] time = 65.3842, size = 410, normalized size = 1.3

$$-\frac{3a^5\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+1}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^7\left(2p+1\right)}+\frac{15a^4\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+2}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{2b^8\left(p+1\right)}-\frac{30a^3\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+3}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^9\left(2p+3\right)}+\frac{15a^2\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+4}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{10}\left(p+2\right)}-\frac{15a\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+5}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^{11}\left(2p+5\right)}+\frac{3\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+6}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{2b^{12}\left(p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
```

```
[Out] -3*a**5*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))**(2*p
 + 1)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**7*(2*p + 1))
 + 15*a**4*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))**(
2*p + 2)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(2*b**8*(p +
1)) - 30*a**3*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))
**(2*p + 3)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**9*(2*p
 + 3)) + 15*a**2*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/
3))**(2*p + 4)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**10*
(p + 2)) - 15*a*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3
))**(2*p + 5)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(b**11*(
2*p + 5)) + 3*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))
**(2*p + 6)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(2*b**12*(
p + 3))
```

Mathematica [A] time = 0.268329, size = 199, normalized size = 0.63

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(-30a^5+30a^4b(2p+1)\sqrt[3]{x}-30a^3b^2\left(2p^2+3p+1\right)x^{2/3}+10a^2b^3\left(4p^3+12p^2+11p+3\right)x-5a\right.}{2b^6(p+1)(p+2)(p+3)(2p+1)(2p+3)(}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(-30*a^5 + 30*a^4*b*(1 +
 2*p)*x^(1/3) - 30*a^3*b^2*(1 + 3*p + 2*p^2)*x^(2/3) + 10*a^2*b^3
*(3 + 11*p + 12*p^2 + 4*p^3)*x - 5*a*b^4*(6 + 25*p + 35*p^2 + 20*
p^3 + 4*p^4)*x^(4/3) + b^5*(30 + 137*p + 225*p^2 + 170*p^3 + 60*p
^4 + 8*p^5)*x^(5/3)))/(2*b^6*(1 + p)*(2 + p)*(3 + p)*(1 + 2*p)*(3
 + 2*p)*(5 + 2*p))
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

Maxima [A] time = 0.790703, size = 267, normalized size = 0.85

$$\frac{3\left(\left(8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30\right)b^6x^2+2\left(4\,p^5+20\,p^4+35\,p^3+25\,p^2+6\,p\right)ab^5x^{\frac{5}{3}}-5\left(4\,p^4+12\,p^3+11\,p^2+\right.\right.}{2\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*x,x, algorithm="maxima")
```

```
[Out] 3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 +
2*(4*p^5 + 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p
^4 + 12*p^3 + 11*p^2 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 +
 p)*a^3*b^3*x - 30*(2*p^2 + p)*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/
3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/((8*p^6 + 84*p^5 + 350*p^4 + 7
```

```
35*p^3 + 812*p^2 + 441*p + 90)*b^6)
```

Fricas [A] time = 0.363618, size = 401, normalized size = 1.27

$$-\frac{3\left(30\,a^6-\left(8\,b^6p^5+60\,b^6p^4+170\,b^6p^3+225\,b^6p^2+137\,b^6p+30\,b^6\right)x^2-20\left(2\,a^3b^3p^3+3\,a^3b^3p^2+a^3b^3p\right)x+2\left(30\,a^4b^2p\right.\right.}{2\left(8\,b^6p^6+84\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*x,x, algorithm="fricas")
```

```
[Out] -3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^
2 + 137*b^6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 +
 a^3*b^3*p)*x + 2*(30*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 +
 20*a*b^5*p^4 + 35*a*b^5*p^3 + 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/
3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4 + 12*a^2*b^4*p^3 + 11*a^2*b^4
*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^
6*p^2 + 441*b^6*p + 90*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.284555, size = 1065, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*x,x, algorithm="giac")
```

```
[Out] 3/2*(8*b^6*p^5*x^2*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) +
8*a*b^5*p^5*x^(5/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) +
```

```
 60*b^6*p^4*x^2*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) + 40*
a*b^5*p^4*x^(5/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) - 2
0*a^2*b^4*p^4*x^(4/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2))
 + 170*b^6*p^3*x^2*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) +
70*a*b^5*p^3*x^(5/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2))
- 60*a^2*b^4*p^3*x^(4/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)) + 40*a^3*b^3*p^3*x*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)
) + 225*b^6*p^2*x^2*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) +
 50*a*b^5*p^2*x^(5/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2))
 - 55*a^2*b^4*p^2*x^(4/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)) + 60*a^3*b^3*p^2*x*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)) + 137*b^6*p*x^2*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) -
60*a^4*b^2*p^2*x^(2/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)
) + 12*a*b^5*p*x^(5/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)
) - 15*a^2*b^4*p*x^(4/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)) + 20*a^3*b^3*p*x*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2))
+ 30*b^6*x^2*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) - 30*a^4
*b^2*p*x^(2/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) + 60*a
^5*b*p*x^(1/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) - 30*a
^6*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)))/(8*b^6*p^6 + 84*b
^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 441*b^6*p + 90
*b^6)
```

3.475 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

Optimal. Leaf size=142

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+1)}$$

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^
3*(1 + 2*p)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3))^p)/(b^3*(1 + p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(
1/3) + b^2*x^(2/3))^p)/(b^3*(3 + 2*p))
```

Rubi [A] time = 0.138828, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^
3*(1 + 2*p)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3))^p)/(b^3*(1 + p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(
1/3) + b^2*x^(2/3))^p)/(b^3*(3 + 2*p))
```

Rubi in Sympy [A] time = 24.417, size = 144, normalized size = 1.01

$$\frac{3a^2\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{b^3\left(2p+1\right)\left(2p+3\right)}-\frac{3a\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{p+1}}{b^3\left(p+1\right)\left(2p+3\right)}+\frac{3x^{\frac{2}{3}}\left(2a+2b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{2b\left(2p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

```
[Out] 3*a**2*(2*a + 2*b*x**(1/3))*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3
))**p/(b**3*(2*p + 1)*(2*p + 3)) - 3*a*(a**2 + 2*a*b*x**(1/3) + b
**2*x**(2/3))**(p + 1)/(b**3*(p + 1)*(2*p + 3)) + 3*x**(2/3)*(2*a
 + 2*b*x**(1/3))*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/(2*b*
(2*p + 3))
```

Mathematica [A] time = 0.0563148, size = 83, normalized size = 0.58

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(2p+1)\sqrt[3]{x}+b^2\left(2p^2+3p+1\right)x^{2/3}\right)}{b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(
1/3) + b^2*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3
+ 2*p))
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

Maxima [A] time = 0.780029, size = 104, normalized size = 0.73

$$\frac{3\left(\left(2\,p^2+3\,p+1\right)b^3x+\left(2\,p^2+p\right)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{(4\,p^3+12\,p^2+11\,p+3)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p,x, algorithm="maxima")
```

```
[Out] 3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*
p*x^(1/3) + a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p +
3)*b^3)
```

Fricas [A] time = 0.314024, size = 149, normalized size = 1.05

$$-\frac{3\left(2\,a^2bpx^{\frac{1}{3}} - a^3 - \left(2\,b^3p^2 + 3\,b^3p + b^3\right)x - \left(2\,ab^2p^2 + ab^2p\right)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{4\,b^3p^3 + 12\,b^3p^2 + 11\,b^3p + 3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p,x, algorithm="fricas")
```

```
[Out] -3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*
a*b^2*p^2 + a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)
^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277844, size = 328, normalized size = 2.31

$$\frac{3\left(2\,b^3p^2xe^{\left(p\ln\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)\right)} + 2\,ab^2p^2x^{\frac{2}{3}}e^{\left(p\ln\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)\right)} + 3\,b^3pxe^{\left(p\ln\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)\right)} + ab^2px^{\frac{2}{3}}e^{\left(p\ln\left(b^2x^{\frac{2}{3}}+2\,abx\right.\right.}\right.}{4\,b^3p^3 + 12\,b^3p^2 + 11\,b^3p + 3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p,x, algorithm="giac")
```

```
[Out] 3*(2*b^3*p^2*x*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) + 2*a*
b^2*p^2*x^(2/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) + 3*b
^3*p*x*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) + a*b^2*p*x^(2
/3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) - 2*a^2*b*p*x^(1/
3)*e^(p*ln(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) + b^3*x*e^(p*ln(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)) + a^3*e^(p*ln(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)))/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.476 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx$

Optimal. Leaf size=69

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

```
[Out] (-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hyp
ergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*
p)
```

Rubi [A] time = 0.0681823, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

```
[Out] (-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hyp
ergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*
p)
```

Rubi in Sympy [A] time = 17.8096, size = 85, normalized size = 1.23

$$-\frac{3\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+1}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,{}_2F_1\left(\begin{matrix}1,2p+1\\2p+2\end{matrix}\middle|1+\frac{b\sqrt[3]{x}}{a}\right)}{ab\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
```

```
[Out] -3*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))**(2*p + 1)
*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p*hyper((1, 2*p + 1), (
```

```
2*p + 2,), 1 + b*x**(1/3)/a)/(a*b*(2*p + 1))
```

Mathematica [A] time = 0.0376492, size = 59, normalized size = 0.86

$$\frac{3\left(\frac{a}{b\sqrt[3]{x}}+1\right)^{-2p}\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\,{}_2F_1\left(-2p,-2p;1-2p;-\frac{a}{b\sqrt[3]{x}}\right)}{2p}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

```
[Out] (3*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[-2*p, -2*p, 1 - 2*p, -
(a/(b*x^(1/3)))])/(2*p*(1 + a/(b*x^(1/3)))^(2*p))
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2} x^{\frac{2}{3}} + 2\, a b x^{\frac{1}{3}} + a^{2}\right)^{p}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^{2} x^{\frac{2}{3}} + 2\, a b x^{\frac{1}{3}} + a^{2}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

3.477 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$

Optimal. Leaf size=75

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

```
[Out] (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*
Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3
*(1 + 2*p))
```

Rubi [A] time = 0.0791315, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*
Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3
*(1 + 2*p))
```

Rubi in Sympy [A] time = 18.9054, size = 87, normalized size = 1.16

$$\frac{3b^2\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+1}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,{}_2F_1\left(\begin{matrix}4,2p+1\\2p+2\end{matrix}\middle|1+\frac{b\sqrt[3]{x}}{a}\right)}{a^4\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
```

```
[Out] 3*b**2*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3))**(2*p
+ 1)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p*hyper((4, 2*p + 1
```

```
), (2*p + 2,), 1 + b*x**(1/3)/a)/(a**4*(2*p + 1))
```

Mathematica [A] time = 0.037549, size = 66, normalized size = 0.88

$$\frac{3\left(\frac{a}{b\sqrt[3]{x}}+1\right)^{-2p}\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p {}_2F_1\left(3-2p,-2p;4-2p;-\frac{a}{b\sqrt[3]{x}}\right)}{(2p-3)x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[3 - 2*p, -2*p, 4 - 2*p
, -(a/(b*x^(1/3)))])/((-3 + 2*p)*(1 + a/(b*x^(1/3)))^(2*p)*x)
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.478 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x} \right) dx$

Optimal. Leaf size=146

$$\frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}} - \frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{ax} - \frac{b^2(1-2p)(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^3\sqrt[3]{x}}$$

```
[Out] -(((a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(a*x))
+ (b*(1 - p)*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(a^2*x^(2/3)) - (b^2*(1 - 2*p)*(1 - p)*(a + b*x^(1/3))*(a^2 +
2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(a^3*x^(1/3))
```

Rubi [C] time = 0.199595, antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 4, integrand size = 77, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)} + \frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*(a^2 +
```

```
[Out] (2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 +
(b*x^(1/3))/a])/(a^3*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2
 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometric2F1[4, 1 + 2*p, 2
*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```

Rubi in Sympy [A] time = 50.0171, size = 185, normalized size = 1.27

$$\frac{2b^2p(-2p+1)(-p+1)\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+1}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p {}_2F_1\left(\begin{matrix}1,2p+1\\2p+2\end{matrix}\middle|1+\frac{b\sqrt[3]{x}}{a}\right)}{a^4(2p+1)}+\frac{3b^2\left(ab+b^2\sqrt[3]{x}\right)^{-2p}\left(ab+b^2\sqrt[3]{x}\right)^{2p+1}\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p {}_2F_1\left(\begin{matrix}4,2p+1\\2p+2\end{matrix}\middle|1+\frac{b\sqrt[3]{x}}{a}\right)}{a^4(2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-
2*p)*(1-p)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

```
[Out] 2*b**2*p*(-2*p + 1)*(-p + 1)*(a*b + b**2*x**(1/3))**(-2*p)*(a*b +
 b**2*x**(1/3))**(2*p + 1)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)
)**p*hyper((1, 2*p + 1), (2*p + 2,), 1 + b*x**(1/3)/a)/(a**4*(2*p
 + 1)) + 3*b**2*(a*b + b**2*x**(1/3))**(-2*p)*(a*b + b**2*x**(1/3
))**(2*p + 1)*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p*hyper((4
, 2*p + 1), (2*p + 2,), 1 + b*x**(1/3)/a)/(a**4*(2*p + 1))
```

Mathematica [A] time = 0.0844742, size = 64, normalized size = 0.44

$$-\frac{\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2+ab(p-1)\sqrt[3]{x}+b^2\left(2p^2-3p+1\right)x^{2/3}\right)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*
```

```
[Out] -(((a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 + a*b*(-1 + p)*x^(1
/3) + b^2*(1 - 3*p + 2*p^2)*x^(2/3)))/(a^3*x))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p-\frac{2\,b^3(1-2\,p)(1-p)\,p}{3\,a^3x}\left(a^2+2\,ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*b*x
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p
*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

Maxima [A] time = 1.21257, size = 85, normalized size = 0.58

$$-\frac{\left(\left(2\,p^2-3\,p+1\right)b^3x+2\left(p^2-p\right)ab^2x^{\frac{2}{3}}+a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*p/(a^3
```

```
[Out] -((2*p^2 - 3*p + 1)*b^3*x + 2*(p^2 - p)*a*b^2*x^(2/3) + a^2*b*p*x
^(1/3) + a^3)*(b*x^(1/3) + a)^(2*p)/(a^3*x)
```

Fricas [A] time = 0.375884, size = 111, normalized size = 0.76

$$-\frac{\left(a^2bpx^{\frac{1}{3}}+a^3+\left(2\,b^3p^2-3\,b^3p+b^3\right)x+2\left(ab^2p^2-ab^2p\right)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*p/(a^3
```

```
[Out] -(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^
2*p^2 - a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(
a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-
2*p)*(1-p)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x} + \frac{\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*p/(a^3
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1
)*(p - 1)*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2,
x)
```

3.479 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)
/(b^4*(a + b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) +
(4*(a + b*x^(1/4))*x^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*S
qrt[x]]) - (12*a*(a + b*x^(1/4))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^
2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rubi [A] time = 0.198086, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```


```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)
/(b^4*(a + b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) +
(4*(a + b*x^(1/4))*x^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*S
qrt[x]]) - (12*a*(a + b*x^(1/4))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^
2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rubi in Sympy [A] time = 17.8797, size = 160, normalized size = 0.91

$$-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{x^{\frac{3}{4}}\left(2a+2b\sqrt[4]{x}\right)}{b\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{\frac{3}{2}}}-\frac{6\sqrt{x}}{b^2\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{12\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

```
[Out] -12*a*(a + b*x**(1/4))*log(a + b*x**(1/4))/(b**4*sqrt(a**2 + 2*a*
b*x**(1/4) + b**2*sqrt(x))) - x**(3/4)*(2*a + 2*b*x**(1/4))/(b*(a
**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(3/2)) - 6*sqrt(x)/(b**2*sq
rt(a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))) + 12*sqrt(a**2 + 2*a*b*
x**(1/4) + b**2*sqrt(x))/b**4
```

Mathematica [A] time = 0.0732028, size = 93, normalized size = 0.53

$$\frac{2\left(-5a^3-4a^2b\sqrt[4]{x}+4ab^2\sqrt{x}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)+2b^3x^{3/4}\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) -
6*a*(a + b*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*S
qrt[(a + b*x^(1/4))^2])
```

Maple [A] time = 0.021, size = 114, normalized size = 0.7

$$2\,\frac{\sqrt{a^2+2\,ab\sqrt[4]{x}+b^2\sqrt{x}}\left(2\,x^{3/4}b^3-6\,\sqrt{x}\ln\left(a+b\sqrt[4]{x}\right)ab^2+4\,\sqrt{x}ab^2-12\,\sqrt[4]{x}\ln\left(a+b\sqrt[4]{x}\right)a^2b-4\,\sqrt[4]{x}a^2b-6\,\ln\left(a+b\sqrt[4]{x}\right)\right.}{\left(a+b\sqrt[4]{x}\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x)
```

```
[Out] 2*(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)*(2*x^(3/4)*b^3-6*x^(1/2)*
ln(a+b*x^(1/4))*a*b^2+4*x^(1/2)*a*b^2-12*x^(1/4)*ln(a+b*x^(1/4))*
a^2*b-4*x^(1/4)*a^2*b-6*ln(a+b*x^(1/4))*a^3-5*a^3)/(a+b*x^(1/4))^
3/b^4
```

Maxima [A] time = 0.780037, size = 200, normalized size = 1.14

$$-\frac{12\,a\log\left(x^{\frac{1}{4}}+\frac{a}{b}\right)}{\left(b^{2}\right)^{\frac{3}{2}}b}-\frac{18\,a^{3}b}{\left(b^{2}\right)^{\frac{7}{2}}\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^{2}}+\frac{4\,\sqrt{x}}{\sqrt{b^{2}\sqrt{x}+2\,abx^{\frac{1}{4}}+a^{2}}b^{2}}-\frac{24\,a^{2}x^{\frac{1}{4}}}{\left(b^{2}\right)^{\frac{5}{2}}\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^{2}}+\frac{8\,a^{2}}{\sqrt{b^{2}\sqrt{x}+2\,abx^{\frac{1}{4}}+a^{2}}b^{4}}-\frac{4\,a^{3}}{\left(b^{2}\right)^{\frac{3}{2}}b^{3}\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] -12*a*log(x^(1/4) + a/b)/((b^2)^(3/2)*b) - 18*a^3*b/((b^2)^(7/2)*
(x^(1/4) + a/b)^2) + 4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4)
+ a^2)*b^2) - 24*a^2*x^(1/4)/((b^2)^(5/2)*(x^(1/4) + a/b)^2) + 8*
a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 4*a^3/((b^2)^
(3/2)*b^3*(x^(1/4) + a/b)^2)
```

Fricas [A] time = 0.266339, size = 123, normalized size = 0.7

$$\frac{2\left(2\,b^{3}x^{\frac{3}{4}}+4\,ab^{2}\sqrt{x}-4\,a^{2}bx^{\frac{1}{4}}-5\,a^{3}-6\left(ab^{2}\sqrt{x}+2\,a^{2}bx^{\frac{1}{4}}+a^{3}\right)\log\left(bx^{\frac{1}{4}}+a\right)\right)}{b^{6}\sqrt{x}+2\,ab^{5}x^{\frac{1}{4}}+a^{2}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] 2*(2*b^3*x^(3/4) + 4*a*b^2*sqrt(x) - 4*a^2*b*x^(1/4) - 5*a^3 - 6*
(a*b^2*sqrt(x) + 2*a^2*b*x^(1/4) + a^3)*log(b*x^(1/4) + a))/(b^6*
sqrt(x) + 2*a*b^5*x^(1/4) + a^2*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)^(-3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.480 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{6\sqrt[6]{x}\left(a+b\sqrt[6]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)
/(2*b^6*(a + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]
) - (10*a^4)/(b^6*(a + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^
2*x^(1/3)]) + (30*a^3)/(b^6*(a + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1
/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))*x^(1/6))/(b^5*Sqrt[a^2 +
 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1/6))*Log[a + b*
x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rubi [A] time = 0.294771, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{6\sqrt[6]{x}\left(a+b\sqrt[6]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```


```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)
/(2*b^6*(a + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]
) - (10*a^4)/(b^6*(a + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^
2*x^(1/3)]) + (30*a^3)/(b^6*(a + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1
/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))*x^(1/6))/(b^5*Sqrt[a^2 +
 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1/6))*Log[a + b*
x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rubi in Sympy [A] time = 27.63, size = 250, normalized size = 0.93

$$-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^{6}\sqrt{a^{2}+2ab\sqrt[6]{x}+b^{2}\sqrt[3]{x}}}-\frac{3x^{\frac{5}{6}}\left(2a+2b\sqrt[6]{x}\right)}{4b\left(a^{2}+2ab\sqrt[6]{x}+b^{2}\sqrt[3]{x}\right)^{\frac{5}{2}}}-\frac{5x^{\frac{2}{3}}}{2b^{2}\left(a^{2}+2ab\sqrt[6]{x}+b^{2}\sqrt[3]{x}\right)^{\frac{3}{2}}}-\frac{5\sqrt{x}\left(2a+2b\sqrt[6]{x}\right)}{2b^{3}\left(a^{2}+2ab\sqrt[6]{x}+b^{2}\sqrt[3]{x}\right)^{\frac{3}{2}}}-\frac{15\sqrt[3]{x}}{b^{4}\sqrt{a^{2}+2ab\sqrt[6]{x}+b^{2}\sqrt[3]{x}}}+\frac{30\sqrt{a^{2}+2ab\sqrt[6]{x}+b^{2}\sqrt[3]{x}}}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

```
[Out] -30*a*(a + b*x**(1/6))*log(a + b*x**(1/6))/(b**6*sqrt(a**2 + 2*a*
b*x**(1/6) + b**2*x**(1/3))) - 3*x**(5/6)*(2*a + 2*b*x**(1/6))/(4
*b*(a**2 + 2*a*b*x**(1/6) + b**2*x**(1/3))**(5/2)) - 5*x**(2/3)/(
2*b**2*(a**2 + 2*a*b*x**(1/6) + b**2*x**(1/3))**(3/2)) - 5*sqrt(x
)*(2*a + 2*b*x**(1/6))/(2*b**3*(a**2 + 2*a*b*x**(1/6) + b**2*x**(
1/3))**(3/2)) - 15*x**(1/3)/(b**4*sqrt(a**2 + 2*a*b*x**(1/6) + b*
*2*x**(1/3))) + 30*sqrt(a**2 + 2*a*b*x**(1/6) + b**2*x**(1/3))/b*
*6
```

Mathematica [A] time = 0.0911913, size = 121, normalized size = 0.45

$$\frac{-77a^{5}-248a^{4}b\sqrt[6]{x}-252a^{3}b^{2}\sqrt[3]{x}-48a^{2}b^{3}\sqrt{x}+48ab^{4}x^{2/3}-60a\left(a+b\sqrt[6]{x}\right)^{4}\log\left(a+b\sqrt[6]{x}\right)+12b^{5}x^{5/6}}{2b^{6}\left(a+b\sqrt[6]{x}\right)^{3}\sqrt{\left(a+b\sqrt[6]{x}\right)^{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*S
qrt[x] + 48*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))
^4*Log[a + b*x^(1/6)])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/
6))^2])
```

Maple [A] time = 0.022, size = 174, normalized size = 0.7

$$\frac{1}{2\,b^{6}}\sqrt{a^{2}+2\,ab\sqrt[6]{x}+b^{2}\sqrt[3]{x}}\left(12\,x^{5/6}b^{5}-60\,x^{2/3}\ln\left(a+b\sqrt[6]{x}\right)ab^{4}+48\,x^{2/3}ab^{4}-240\,\sqrt{x}\ln\left(a+b\sqrt[6]{x}\right)a^{2}b^{3}-48\,\sqrt{x}a^{2}b^{3}-36\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x)
```

```
[Out] 1/2*(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)*(12*x^(5/6)*b^5-60*x^(2
/3)*ln(a+b*x^(1/6))*a*b^4+48*x^(2/3)*a*b^4-240*x^(1/2)*ln(a+b*x^(
1/6))*a^2*b^3-48*x^(1/2)*a^2*b^3-360*x^(1/3)*ln(a+b*x^(1/6))*a^3*
b^2-252*x^(1/3)*a^3*b^2-240*x^(1/6)*ln(a+b*x^(1/6))*a^4*b-248*x^(
1/6)*a^4*b-60*ln(a+b*x^(1/6))*a^5-77*a^5)/(a+b*x^(1/6))^5/b^6
```

Maxima [A] time = 0.794399, size = 161, normalized size = 0.6

$$\frac{12\,b^5x^{\frac{5}{6}} + 48\,ab^4x^{\frac{2}{3}} - 48\,a^2b^3\sqrt{x} - 252\,a^3b^2x^{\frac{1}{3}} - 248\,a^4bx^{\frac{1}{6}} - 77\,a^5}{2\left(b^{10}x^{\frac{2}{3}} + 4\,ab^9\sqrt{x} + 6\,a^2b^8x^{\frac{1}{3}} + 4\,a^3b^7x^{\frac{1}{6}} + a^4b^6\right)} - \frac{30\,a\log\left(bx^{\frac{1}{6}} + a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(1/3) + 2*a*b*x^(1/6) + a^2)^(-5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252
*a^3*b^2*x^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*
a*b^9*sqrt(x) + 6*a^2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6)
- 30*a*log(b*x^(1/6) + a)/b^6
```

Fricas [A] time = 0.276934, size = 212, normalized size = 0.79

$$\frac{12\,b^5x^{\frac{5}{6}} + 48\,ab^4x^{\frac{2}{3}} - 48\,a^2b^3\sqrt{x} - 252\,a^3b^2x^{\frac{1}{3}} - 248\,a^4bx^{\frac{1}{6}} - 77\,a^5 - 60\left(ab^4x^{\frac{2}{3}} + 4\,a^2b^3\sqrt{x} + 6\,a^3b^2x^{\frac{1}{3}} + 4\,a^4bx^{\frac{1}{6}} + a^5\right)\log}{2\left(b^{10}x^{\frac{2}{3}} + 4\,ab^9\sqrt{x} + 6\,a^2b^8x^{\frac{1}{3}} + 4\,a^3b^7x^{\frac{1}{6}} + a^4b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(1/3) + 2*a*b*x^(1/6) + a^2)^(-5/2),x, algorithm="fricas")
```

```
[Out] 1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252
*a^3*b^2*x^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5 - 60*(a*b^4*x^(2/3)
 + 4*a^2*b^3*sqrt(x) + 6*a^3*b^2*x^(1/3) + 4*a^4*b*x^(1/6) + a^5)
*log(b*x^(1/6) + a))/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*a^2*b^8*
x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(1/3) + 2*a*b*x^(1/6) + a^2)^(-5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.481 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx$

Optimal. Leaf size=179

$$\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqr
t[x]) + (6*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a
+ b/Sqrt[x]) + (a^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b
/Sqrt[x]) + (6*a*b^2*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt
[x]])/(a + b/Sqrt[x])
```

Rubi [A] time = 0.200567, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqr
t[x]) + (6*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a
+ b/Sqrt[x]) + (a^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b
/Sqrt[x]) + (6*a*b^2*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt
[x]])/(a + b/Sqrt[x])
```

Rubi in Sympy [A] time = 22.7061, size = 134, normalized size = 0.75

$$-\frac{6ab^2\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\log\left(\frac{1}{\sqrt{x}}\right)}{a+\frac{b}{\sqrt{x}}}-6b^2\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}+3b\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}+x\left(a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

```
[Out] -6*a*b**2*sqrt(a**2 + 2*a*b/sqrt(x) + b**2/x)*log(1/sqrt(x))/(a +
 b/sqrt(x)) - 6*b**2*sqrt(a**2 + 2*a*b/sqrt(x) + b**2/x) + 3*b*sq
rt(x)*(a + b/sqrt(x))*sqrt(a**2 + 2*a*b/sqrt(x) + b**2/x) + x*(a*
*2 + 2*a*b/sqrt(x) + b**2/x)**(3/2)
```

Mathematica [A] time = 0.0519086, size = 66, normalized size = 0.37

$$\frac{\sqrt{\frac{\left(a\sqrt{x}+b\right)^2}{x}}\left(a^3x^{3/2}+6a^2bx+3ab^2\sqrt{x}\log(x)-2b^3\right)}{a\sqrt{x}+b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*
a*b^2*Sqrt[x]*Log[x]))/(b + a*Sqrt[x])
```

Maple [A] time = 0.036, size = 68, normalized size = 0.4

$$1\sqrt{1\left(a^2x^{\frac{3}{2}}+b^2\sqrt{x}+2\,abx\right)x^{-\frac{3}{2}}}\left(x^{\frac{3}{2}}a^3+6\,a^2bx+3\,\sqrt{x}\ln(x)\,ab^2-2\,b^3\right)\left(\sqrt{x}a+b\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x)
```

```
[Out] ((a^2*x^(3/2)+b^2*x^(1/2)+2*a*b*x)/x^(3/2))^(1/2)*(x^(3/2)*a^3+6*
a^2*b*x+3*x^(1/2)*ln(x)*a*b^2-2*b^3)/(x^(1/2)*a+b)
```

Maxima [A] time = 0.786421, size = 42, normalized size = 0.23

$$a^3x+3\,ab^2\log(x)+6\,a^2b\sqrt{x}-\frac{2\,b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/sqrt(x) + b^2/x)^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + 3*a*b^2*log(x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/sqrt(x) + b^2/x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291724, size = 108, normalized size = 0.6

$$a^3 x \operatorname{sign}\left(ax + b\sqrt{x}\right) \operatorname{sign}(x) + 3\,ab^2 \ln\left(|x|\right) \operatorname{sign}\left(ax + b\sqrt{x}\right) \operatorname{sign}(x) + 6\,a^2 b\sqrt{x} \operatorname{sign}\left(ax + b\sqrt{x}\right) \operatorname{sign}(x) - \frac{2\,b^3 \operatorname{sign}\left(ax + b\sqrt{x}\right) \operatorname{sign}(x)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/sqrt(x) + b^2/x)^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sign(a*x + b*sqrt(x))*sign(x) + 3*a*b^2*ln(abs(x))*sign(a*x
 + b*sqrt(x))*sign(x) + 6*a^2*b*sqrt(x)*sign(a*x + b*sqrt(x))*sig
n(x) - 2*b^3*sign(a*x + b*sqrt(x))*sign(x)/sqrt(x)
```

3.482 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

Optimal. Leaf size=391

$$-\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{4x^{4/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{7ab^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{105a^4b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}$$

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1
/3))*x^(4/3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)
])/((a + b/x^(1/3))*x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*
a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*x^(1/3)) + (
63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a
+ b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)
]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rubi [A] time = 0.398084, antiderivative size = 391, normalized size of antiderivative = 1., number

of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{4x^{4/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{7ab^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{105a^4b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1
/3))*x^(4/3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)
])/((a + b/x^(1/3))*x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*
a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*x^(1/3)) + (
63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a
+ b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)
]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rubi in Sympy [A] time = 45.7902, size = 313, normalized size = 0.8

$$-\frac{105a^4b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{a+\frac{b}{\sqrt[3]{x}}}-105a^3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}-\frac{35a^2b^3\left(3a+\frac{3b}{\sqrt[3]{x}}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{2}-35ab^3\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}-\frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}{4}+21b^2\sqrt[3]{x}\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}+\frac{bx^{\frac{2}{3}}\left(7a+\frac{7b}{\sqrt[3]{x}}\right)\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}{2}+x\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
```

```
[Out] -105*a**4*b**3*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))*log(x*
*(-1/3))/(a + b/x**(1/3)) - 105*a**3*b**3*sqrt(a**2 + 2*a*b/x**(1
/3) + b**2/x**(2/3)) - 35*a**2*b**3*(3*a + 3*b/x**(1/3))*sqrt(a**
2 + 2*a*b/x**(1/3) + b**2/x**(2/3))/2 - 35*a*b**3*(a**2 + 2*a*b/x
**(1/3) + b**2/x**(2/3))**(3/2) - 105*b**3*(a + b/x**(1/3))*(a**2
 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2)/4 + 21*b**2*x**(1/3)*(a
**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2) + b*x**(2/3)*(7*a +
7*b/x**(1/3))*(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2)/2 +
x*(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2)
```

Mathematica [A] time = 0.0866972, size = 125, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(4a^7x^{7/3}+42a^6bx^2+252a^5b^2x^{5/3}+140a^4b^3x^{4/3}\log(x)-420a^3b^4x-126a^2b^5x^{2/3}-28ab^6\sqrt[3]{x}-3b^7\right)}{4x\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126
*a^2*b^5*x^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b
*x^2 + 4*a^7*x^(7/3) + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(
```

```
1/3))*x)
```

Maple [A] time = 0.031, size = 115, normalized size = 0.3

$$\frac{1}{4}\left(1\left(a^{2}x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^{2}\right)x^{-\frac{2}{3}}\right)^{\frac{7}{2}}\left(42\,a^{6}bx^{3}+252\,a^{5}b^{2}x^{8/3}+140\,a^{4}b^{3}\ln(x)\,x^{7/3}+4\,a^{7}x^{10/3}-28\,ab^{6}x^{4/3}-420\,x^{2}a^{3}b^{4}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x)
```

```
[Out] 1/4*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*(42*a^6*b*x^3
+252*a^5*b^2*x^(8/3)+140*a^4*b^3*ln(x)*x^(7/3)+4*a^7*x^(10/3)-28*
a*b^6*x^(4/3)-420*x^2*a^3*b^4-126*a^2*b^5*x^(5/3)-3*b^7*x)/(b+a*x
^(1/3))^7
```

Maxima [A] time = 0.754751, size = 107, normalized size = 0.27

$$35\,a^{4}b^{3}\log(x)+\frac{4\,a^{7}x^{\frac{7}{3}}+42\,a^{6}bx^{2}+252\,a^{5}b^{2}x^{\frac{5}{3}}-420\,a^{3}b^{4}x-126\,a^{2}b^{5}x^{\frac{2}{3}}-28\,ab^{6}x^{\frac{1}{3}}-3\,b^{7}}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b
^2*x^(5/3) - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/
3) - 3*b^7)/x^(4/3)
```

Fricas [A] time = 0.273952, size = 116, normalized size = 0.3

$$\frac{420\,a^{4}b^{3}x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}\right)+42\,a^{6}bx^{2}-420\,a^{3}b^{4}x-3\,b^{7}+126\left(2\,a^{5}b^{2}x-a^{2}b^{5}\right)x^{\frac{2}{3}}+4\left(a^{7}x^{2}-7\,ab^{6}\right)x^{\frac{1}{3}}}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4*(420*a^4*b^3*x^(4/3)*log(x^(1/3)) + 42*a^6*b*x^2 - 420*a^3*b^
4*x - 3*b^7 + 126*(2*a^5*b^2*x - a^2*b^5)*x^(2/3) + 4*(a^7*x^2 -
```

```
7*a*b^6)*x^(1/3))/x^(4/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.305446, size = 234, normalized size = 0.6

$$a^7 x \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + 35\, a^4 b^3 \ln(|x|) \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + \frac{21}{2}\, a^6 b x^{\frac{2}{3}} \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + 63\, a^5 b^2 x^{\frac{1}{3}} \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) - \frac{420\, a^3 b^4 x \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + 126\, a^2 b^5 x^{\frac{2}{3}} \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + 28\, ab^6 x^{\frac{1}{3}} \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + 3\, b^7 \operatorname{sign}\left(\right.}{4\, x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(7/2),x, algorithm="giac")

[Out] a^7*x*sign(a*x + b*x^(2/3))*sign(x) + 35*a^4*b^3*ln(abs(x))*sign(
a*x + b*x^(2/3))*sign(x) + 21/2*a^6*b*x^(2/3)*sign(a*x + b*x^(2/3
))*sign(x) + 63*a^5*b^2*x^(1/3)*sign(a*x + b*x^(2/3))*sign(x) - 1
/4*(420*a^3*b^4*x*sign(a*x + b*x^(2/3))*sign(x) + 126*a^2*b^5*x^(
2/3)*sign(a*x + b*x^(2/3))*sign(x) + 28*a*b^6*x^(1/3)*sign(a*x +
b*x^(2/3))*sign(x) + 3*b^7*sign(a*x + b*x^(2/3))*sign(x))/x^(4/3)
```

3.483 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx$

Optimal. Leaf size=291

$$-\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^2b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1
/3))*x^(2/3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)])/((a + b/x^(1/3))*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3
) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) +
 (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3)
) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(
1/3)])/(a + b/x^(1/3))
```

Rubi [A] time = 0.304123, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^2b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1
/3))*x^(2/3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)])/((a + b/x^(1/3))*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3
) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^
```

```
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) +
 (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3)
) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(
1/3)])/(a + b/x^(1/3))
```

Rubi in Sympy [A] time = 32.7387, size = 228, normalized size = 0.78

$$-\frac{30a^2b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{a+\frac{b}{\sqrt[3]{x}}}-30ab^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}-15b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}+10b^2\sqrt[3]{x}\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}+\frac{5bx^{\frac{2}{3}}\left(a+\frac{b}{\sqrt[3]{x}}\right)\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}{2}+x\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] -30*a**2*b**3*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))*log(x**
(-1/3))/(a + b/x**(1/3)) - 30*a*b**3*sqrt(a**2 + 2*a*b/x**(1/3) +
 b**2/x**(2/3)) - 15*b**3*(a + b/x**(1/3))*sqrt(a**2 + 2*a*b/x**(
1/3) + b**2/x**(2/3)) + 10*b**2*x**(1/3)*(a**2 + 2*a*b/x**(1/3) +
 b**2/x**(2/3))**(3/2) + 5*b*x**(2/3)*(a + b/x**(1/3))*(a**2 + 2*
a*b/x**(1/3) + b**2/x**(2/3))**(3/2)/2 + x*(a**2 + 2*a*b/x**(1/3)
 + b**2/x**(2/3))**(5/2)
```

Mathematica [A] time = 0.0797503, size = 99, normalized size = 0.34

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^5x^{5/3}+15a^4bx^{4/3}+60a^3b^2x+20a^2b^3x^{2/3}\log(x)-30ab^4\sqrt[3]{x}-3b^5\right)}{2x\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a
^4*b*x^(4/3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqr
t[(b + a*x^(1/3))^2/x^(2/3)]*x)
```

Maple [A] time = 0.016, size = 91, normalized size = 0.3

$$\frac{x}{2}\left(1\left(a^{2}x^{\frac{2}{3}}+2ab\sqrt[3]{x}+b^{2}\right)x^{-\frac{2}{3}}\right)^{\frac{5}{2}}\left(15\,a^{4}bx^{4/3}+60\,a^{3}b^{2}x+20\,a^{2}b^{3}\ln\left(x\right)x^{2/3}+2\,a^{5}x^{5/3}-30\,ab^{4}\sqrt[3]{x}-3\,b^{5}\right)\left(b+a\sqrt[3]{x}\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(15*a^4*b*x
^(4/3)+60*a^3*b^2*x+20*a^2*b^3*ln(x)*x^(2/3)+2*a^5*x^(5/3)-30*a*b
^4*x^(1/3)-3*b^5)/(b+a*x^(1/3))^5
```

Maxima [A] time = 0.755331, size = 77, normalized size = 0.26

$$10\,a^{2}b^{3}\log\left(x\right)+\frac{2\,a^{5}x^{\frac{5}{3}}+15\,a^{4}bx^{\frac{4}{3}}+60\,a^{3}b^{2}x-30\,ab^{4}x^{\frac{1}{3}}-3\,b^{5}}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^
3*b^2*x - 30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

Fricas [A] time = 0.273202, size = 82, normalized size = 0.28

$$\frac{2\,a^{5}x^{\frac{5}{3}}+60\,a^{2}b^{3}x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)+60\,a^{3}b^{2}x-3\,b^{5}+15\left(a^{4}bx-2\,ab^{4}\right)x^{\frac{1}{3}}}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5*x^(5/3) + 60*a^2*b^3*x^(2/3)*log(x^(1/3)) + 60*a^3*b^2
*x - 3*b^5 + 15*(a^4*b*x - 2*a*b^4)*x^(1/3))/x^(2/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.298184, size = 173, normalized size = 0.59

$$a^5 x \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + 10\, a^2 b^3 \ln(|x|) \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + \frac{15}{2}\, a^4 b x^{\frac{2}{3}} \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + 30\, a^3 b^2 x^{\frac{1}{3}} \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) - \frac{3\left(10\, ab^4 x^{\frac{1}{3}} \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x) + b^5 \operatorname{sign}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sign}(x)\right)}{2\, x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sign(a*x + b*x^(2/3))*sign(x) + 10*a^2*b^3*ln(abs(x))*sign(
a*x + b*x^(2/3))*sign(x) + 15/2*a^4*b*x^(2/3)*sign(a*x + b*x^(2/3
))*sign(x) + 30*a^3*b^2*x^(1/3)*sign(a*x + b*x^(2/3))*sign(x) - 3
/2*(10*a*b^4*x^(1/3)*sign(a*x + b*x^(2/3))*sign(x) + b^5*sign(a*x
 + b*x^(2/3))*sign(x))/x^(2/3)
```

3.484 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

Optimal. Leaf size=189

$$\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a +
b/x^(1/3)) + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x
^(2/3))/(2*(a + b/x^(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*
b)/x^(1/3)]*x)/(a + b/x^(1/3)) + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) +
(2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rubi [A] time = 0.208691, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```


```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a +
b/x^(1/3)) + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x
^(2/3))/(2*(a + b/x^(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*
b)/x^(1/3)]*x)/(a + b/x^(1/3)) + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) +
(2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rubi in Sympy [A] time = 27.3739, size = 163, normalized size = 0.86

$$\frac{3ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3bx^{\frac{2}{3}}\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{2}+x\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] 3*a*b**2*x**(1/3)*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))/(a
+ b/x**(1/3)) - 3*b**3*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)
)*log(x**(-1/3))/(a + b/x**(1/3)) + 3*b*x**(2/3)*(a + b/x**(1/3))
*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))/2 + x*(a**2 + 2*a*b/
x**(1/3) + b**2/x**(2/3))**(3/2)
```

Mathematica [A] time = 0.0487907, size = 77, normalized size = 0.41

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2a^3x^{4/3}+9a^2bx+18ab^2x^{2/3}+2b^3\sqrt[3]{x}\log(x)\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(18*a*b^2*x^(2/3) + 9*a^2*b*x +
2*a^3*x^(4/3) + 2*b^3*x^(1/3)*Log[x]))/(2*(b + a*x^(1/3)))
```

Maple [A] time = 0.008, size = 69, normalized size = 0.4

$$\frac{x}{2}\left(1\left(a^2x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}\right)^{\frac{3}{2}}\left(9\,x^{2/3}a^2b+2\,b^3\ln(x)+18\,ab^2\sqrt[3]{x}+2\,a^3x\right)\left(b+a\sqrt[3]{x}\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(9*x^(2/3)*
a^2*b+2*b^3*ln(x)+18*a*b^2*x^(1/3)+2*a^3*x)/(b+a*x^(1/3))^3
```

Maxima [A] time = 0.750367, size = 41, normalized size = 0.22

$$a^3x + b^3\log(x) + \frac{9}{2}a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Fricas [A] time = 0.277231, size = 45, normalized size = 0.24

$$a^3x + 3b^3\log\left(x^{\frac{1}{3}}\right) + \frac{9}{2}a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] a^3*x + 3*b^3*log(x^(1/3)) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

GIAC/XCAS [A] time = 0.293263, size = 107, normalized size = 0.57

$$a^{3}x\operatorname{sign}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sign}\left(x\right)+b^{3}\ln\left(|x|\right)\operatorname{sign}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sign}\left(x\right)+\frac{9}{2}\,a^{2}bx^{\frac{2}{3}}\operatorname{sign}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sign}\left(x\right)+9\,ab^{2}x^{\frac{1}{3}}\operatorname{sign}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sign(a*x + b*x^(2/3))*sign(x) + b^3*ln(abs(x))*sign(a*x + b
*x^(2/3))*sign(x) + 9/2*a^2*b*x^(2/3)*sign(a*x + b*x^(2/3))*sign(
x) + 9*a*b^2*x^(1/3)*sign(a*x + b*x^(2/3))*sign(x)
```

3.485 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b
/x^(1/3))) + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a +
 b/x^(1/3))
```

Rubi [A] time = 0.127019, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b
/x^(1/3))) + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a +
 b/x^(1/3))
```

Rubi in Sympy [A] time = 9.80796, size = 70, normalized size = 0.8

$$-\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] -a*x*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))/(2*(a + b/x**(1/
3))) + 3*x*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))/2
```

Mathematica [A] time = 0.018807, size = 49, normalized size = 0.56

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2ax^{4/3}+3bx\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(3*b*x + 2*a*x^(4/3)))/(2*(b + a
*x^(1/3)))
```

Maple [A] time = 0.008, size = 50, normalized size = 0.6

$$\frac{1}{2}\sqrt{1\left(a^2x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}}\sqrt[3]{x}\left(3\,x^{2/3}b+2\,ax\right)\left(b+a\sqrt[3]{x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x^(1/3)*(3*x^
(2/3)*b+2*a*x)/(b+a*x^(1/3))
```

Maxima [A] time = 0.749743, size = 14, normalized size = 0.16

$$ax+\frac{3}{2}\,bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3)),x, algorithm="maxima")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

Fricas [A] time = 0.269271, size = 14, normalized size = 0.16

$$ax + \frac{3}{2}bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3)),x, algorithm="fricas")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

GIAC/XCAS [A] time = 0.27529, size = 46, normalized size = 0.52

$$ax\mathrm{sign}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sign}\left(x\right) + \frac{3}{2}bx^{\frac{2}{3}}\mathrm{sign}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3)),x, algorithm="giac")
```

```
[Out] a*x*sign(a*x + b*x^(2/3))*sign(x) + 3/2*b*x^(2/3)*sign(a*x + b*x^
(2/3))*sign(x)
```

3.486 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

Optimal. Leaf size=190

$$-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*
a*b)/x^(1/3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 +
b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2
 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (3*b^3*(a + b/x^(1/3))*Log[b
 + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi [A] time = 0.260605, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*
a*b)/x^(1/3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 +
b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2
 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (3*b^3*(a + b/x^(1/3))*Log[b
 + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi in Sympy [A] time = 43.3507, size = 235, normalized size = 1.24

$$\frac{x\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{2a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}-\frac{3bx^{\frac{2}{3}}\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{4a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}+\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] x*(2*a + 2*b/x**(1/3))/(2*a*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**
(2/3))) - 3*b*x**(2/3)*(2*a + 2*b/x**(1/3))/(4*a**2*sqrt(a**2 + 2
*a*b/x**(1/3) + b**2/x**(2/3))) + 3*b**3*sqrt(a**2 + 2*a*b/x**(1/
3) + b**2/x**(2/3))*log(x**(-1/3))/(a**4*(a + b/x**(1/3))) - 3*b*
*3*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))*log(a + b/x**(1/3)
)/(a**4*(a + b/x**(1/3))) + 3*b**2*x**(1/3)*sqrt(a**2 + 2*a*b/x**
(1/3) + b**2/x**(2/3))/a**4
```

Mathematica [A] time = 0.0389314, size = 86, normalized size = 0.45

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^3x-3a^2bx^{2/3}-6b^3\log\left(a\sqrt[3]{x}+b\right)+6ab^2\sqrt[3]{x}\right)}{2a^4\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] ((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6
*b^3*Log[b + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*
x^(1/3))
```

Maple [A] time = 0.009, size = 78, normalized size = 0.4

$$-\frac{1}{2\,a^4}\left(b+a\sqrt[3]{x}\right)\left(3\,x^{2/3}a^2b+6\,b^3\ln\left(b+a\sqrt[3]{x}\right)-6\,ab^2\sqrt[3]{x}-2\,a^3x\right)\frac{1}{\sqrt{1\left(a^2x^{\frac{2}{3}}+2\,ab\sqrt[3]{x}+b^2\right)x^{-\frac{2}{3}}}}\frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

```
[Out] -1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)*(b+a
*x^(1/3))*(3*x^(2/3)*a^2*b+6*b^3*ln(b+a*x^(1/3))-6*a*b^2*x^(1/3)-
2*a^3*x)/a^4
```

Maxima [A] time = 0.753073, size = 59, normalized size = 0.31

$$-\frac{3\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^4}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3)),x, algorithm="maxima")
```

```
[Out] -3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*
b^2*x^(1/3))/a^3
```

Fricas [A] time = 0.271788, size = 58, normalized size = 0.31

$$\frac{2\,a^3x-6\,b^3\log\left(ax^{\frac{1}{3}}+b\right)-3\,a^2bx^{\frac{2}{3}}+6\,ab^2x^{\frac{1}{3}}}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*x - 6*b^3*log(a*x^(1/3) + b) - 3*a^2*b*x^(2/3) + 6*a*b
^2*x^(1/3))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

GIAC/XCAS [A] time = 0.308499, size = 104, normalized size = 0.55

$$-\frac{3\,b^3\ln\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4\mathrm{sign}\left(ax+bx^{\frac{2}{3}}\right)\,\mathrm{sign}\left(x\right)}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3\mathrm{sign}\left(ax+bx^{\frac{2}{3}}\right)\,\mathrm{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3)),x, algorithm="giac")
```

```
[Out] -3*b^3*ln(abs(a*x^(1/3) + b))/(a^4*sign(a*x + b*x^(2/3))*sign(x))
 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sign(a*x +
b*x^(2/3))*sign(x))
```

3.487 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}-\frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x
^(1/3)]*(b + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a
^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a
 + b/x^(1/3))*x^(1/3))/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1
/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2*a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*Sqrt[a^2 + b^2/
x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b + a*x
^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi [A] time = 0.383812, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}-\frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x
^(1/3)]*(b + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a
^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a
```

```
 + b/x^(1/3))*x^(1/3))/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1
/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2*a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*Sqrt[a^2 + b^2/
x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b + a*x
^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi in Sympy [A] time = 53.8155, size = 314, normalized size = 1.05

$$-\frac{3x\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{4a\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}-\frac{15x}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}+\frac{5x\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}-\frac{15bx^{\frac{2}{3}}\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}+\frac{30b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{a^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{30b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] -3*x*(2*a + 2*b/x**(1/3))/(4*a*(a**2 + 2*a*b/x**(1/3) + b**2/x**(
2/3))**(3/2)) - 15*x/(2*a**2*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x*
*(2/3))) + 5*x*(2*a + 2*b/x**(1/3))/(a**3*sqrt(a**2 + 2*a*b/x**(1
/3) + b**2/x**(2/3))) - 15*b*x**(2/3)*(2*a + 2*b/x**(1/3))/(2*a**
4*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))) + 30*b**3*sqrt(a**
2 + 2*a*b/x**(1/3) + b**2/x**(2/3))*log(x**(-1/3))/(a**6*(a + b/x
**(1/3))) - 30*b**3*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))*l
og(a + b/x**(1/3))/(a**6*(a + b/x**(1/3))) + 30*b**2*x**(1/3)*sqr
t(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))/a**6
```

Mathematica [A] time = 0.0864092, size = 126, normalized size = 0.42

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^5x^{5/3}-5a^4bx^{4/3}+20a^3b^2x+63a^2b^3x^{2/3}+6ab^4\sqrt[3]{x}-60b^3\left(a\sqrt[3]{x}+b\right)^2\log\left(a\sqrt[3]{x}+b\right)-27b^5\right)}{2a^6x\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3)
+ 20*a^3*b^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*
x^(1/3))^2*Log[b + a*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3)
)^(3/2)*x)
```

Maple [A] time = 0.017, size = 141, normalized size = 0.5

$$\frac{1}{2\,xa^6}\left(2\,a^5x^{5/3}-5\,a^4bx^{4/3}-60\,x^{2/3}\ln\left(b+a\sqrt[3]{x}\right)a^2b^3+63\,x^{2/3}a^2b^3-120\,\sqrt[3]{x}\ln\left(b+a\sqrt[3]{x}\right)ab^4+6\,ab^4\sqrt[3]{x}-60\,\ln\left(b+a\sqrt[3]{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)/x*(2*a^5*x^(5
/3)-5*a^4*b*x^(4/3)-60*x^(2/3)*ln(b+a*x^(1/3))*a^2*b^3+63*x^(2/3)
*a^2*b^3-120*x^(1/3)*ln(b+a*x^(1/3))*a*b^4+6*a*b^4*x^(1/3)-60*ln(
b+a*x^(1/3))*b^5+20*a^3*b^2*x-27*b^5)*(b+a*x^(1/3))/a^6
```

Maxima [A] time = 0.752276, size = 131, normalized size = 0.44

$$\frac{2\,a^5x^{\frac{5}{3}}-5\,a^4bx^{\frac{4}{3}}+20\,a^3b^2x+63\,a^2b^3x^{\frac{2}{3}}+6\,ab^4x^{\frac{1}{3}}-27\,b^5}{2\left(a^8x^{\frac{2}{3}}+2\,a^7bx^{\frac{1}{3}}+a^6b^2\right)}-\frac{30\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(-3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*
x^(2/3) + 6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3
) + a^6*b^2) - 30*b^3*log(a*x^(1/3) + b)/a^6
```

Fricas [A] time = 0.27443, size = 154, normalized size = 0.51

$$\frac{20\,a^3b^2x-27\,b^5-60\left(a^2b^3x^{\frac{2}{3}}+2\,ab^4x^{\frac{1}{3}}+b^5\right)\log\left(ax^{\frac{1}{3}}+b\right)+\left(2\,a^5x+63\,a^2b^3\right)x^{\frac{2}{3}}-\left(5\,a^4bx-6\,ab^4\right)x^{\frac{1}{3}}}{2\left(a^8x^{\frac{2}{3}}+2\,a^7bx^{\frac{1}{3}}+a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(-3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(20*a^3*b^2*x - 27*b^5 - 60*(a^2*b^3*x^(2/3) + 2*a*b^4*x^(1/3
) + b^5)*log(a*x^(1/3) + b) + (2*a^5*x + 63*a^2*b^3)*x^(2/3) - (5
*a^4*b*x - 6*a*b^4)*x^(1/3))/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6
*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

GIAC/XCAS [A] time = 0.317177, size = 163, normalized size = 0.54

$$-\frac{30\, b^3 \ln\left(\left|a x^{\frac{1}{3}} + b\right|\right)}{a^6 \mathrm{sign}\left(a x^{\frac{2}{3}} + b x^{\frac{1}{3}}\right)} - \frac{3\left(10\, a b^4 x^{\frac{1}{3}} + 9\, b^5\right)}{2\left(a x^{\frac{1}{3}} + b\right)^2 a^6 \mathrm{sign}\left(a x^{\frac{2}{3}} + b x^{\frac{1}{3}}\right)} + \frac{2\, a^6 x - 9\, a^5 b x^{\frac{2}{3}} + 36\, a^4 b^2 x^{\frac{1}{3}}}{2\, a^9 \mathrm{sign}\left(a x^{\frac{2}{3}} + b x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(-3/2),x, algorithm="giac")
```

```
[Out] -30*b^3*ln(abs(a*x^(1/3) + b))/(a^6*sign(a*x^(2/3) + b*x^(1/3)))
- 3/2*(10*a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sign(a*x^
(2/3) + b*x^(1/3))) + 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2
*x^(1/3))/(a^9*sign(a*x^(2/3) + b*x^(1/3)))
```

3.488 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

Optimal. Leaf size=410

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}-\frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{45b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{15bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x
^(1/3)]*(b + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(
a + b/x^(1/3)))/(2*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*
(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^
2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (45*b^2*(a + b/x^
(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) -
 (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3
) + (2*a*b)/x^(1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3
)])/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi [A] time = 0.521337, antiderivative size = 410, normalized size of antiderivative = 1., number

of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}-\frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{45b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{15bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x
^(1/3)]*(b + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(
a + b/x^(1/3)))/(2*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*
(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^
2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (45*b^2*(a + b/x^
(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) -
 (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3
) + (2*a*b)/x^(1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3
)])/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rubi in Sympy [A] time = 68.3089, size = 396, normalized size = 0.97

$$-\frac{3x\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{8a\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}-\frac{7x}{4a^2\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}-\frac{21x\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{8a^3\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}-\frac{105x}{4a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}+\frac{35x\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{2a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}-\frac{105bx^{\frac{2}{3}}\left(2a+\frac{2b}{\sqrt[3]{x}}\right)}{4a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}+\frac{105b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{a^8\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{105b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{105b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] -3*x*(2*a + 2*b/x**(1/3))/(8*a*(a**2 + 2*a*b/x**(1/3) + b**2/x**(
2/3))**(5/2)) - 7*x/(4*a**2*(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3
))**(3/2)) - 21*x*(2*a + 2*b/x**(1/3))/(8*a**3*(a**2 + 2*a*b/x**(
1/3) + b**2/x**(2/3))**(3/2)) - 105*x/(4*a**4*sqrt(a**2 + 2*a*b/x
**(1/3) + b**2/x**(2/3))) + 35*x*(2*a + 2*b/x**(1/3))/(2*a**5*sqr
t(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))) - 105*b*x**(2/3)*(2*a +
 2*b/x**(1/3))/(4*a**6*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)
)) + 105*b**3*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))*log(x**
(-1/3))/(a**8*(a + b/x**(1/3))) - 105*b**3*sqrt(a**2 + 2*a*b/x**(
1/3) + b**2/x**(2/3))*log(a + b/x**(1/3))/(a**8*(a + b/x**(1/3)))
 + 105*b**2*x**(1/3)*sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))/
a**8
```

Mathematica [A] time = 0.128563, size = 152, normalized size = 0.37

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(4a^7x^{7/3}-14a^6bx^2+84a^5b^2x^{5/3}+556a^4b^3x^{4/3}+544a^3b^4x-444a^2b^5x^{2/3}-856ab^6\sqrt[3]{x}-420b^3\left(a\sqrt[3]{x}+b\right)^4\log\right.}{4a^8x^{5/3}\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2
/3) + 544*a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) -
14*a^6*b*x^2 + 4*a^7*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b +
a*x^(1/3)]))/(4*a^8*((b + a*x^(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

Maple [A] time = 0.018, size = 199, normalized size = 0.5

$$\frac{1}{4\,a^{8}}\left(4\,x^{7/3}a^{7}+84\,a^{5}b^{2}x^{5/3}-420\,x^{4/3}\ln\left(b+a\sqrt[3]{x}\right)a^{4}b^{3}+556\,x^{4/3}a^{4}b^{3}-2520\,x^{2/3}\ln\left(b+a\sqrt[3]{x}\right)a^{2}b^{5}-444\,x^{2/3}a^{2}b^{5}-16\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] 1/4/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)/x^(5/3)*(4*x^
(7/3)*a^7+84*a^5*b^2*x^(5/3)-420*x^(4/3)*ln(b+a*x^(1/3))*a^4*b^3+
556*x^(4/3)*a^4*b^3-2520*x^(2/3)*ln(b+a*x^(1/3))*a^2*b^5-444*x^(2
/3)*a^2*b^5-1680*x^(1/3)*ln(b+a*x^(1/3))*a*b^6-1680*x*ln(b+a*x^(1
/3))*a^3*b^4-14*a^6*b*x^2-856*x^(1/3)*a*b^6-420*ln(b+a*x^(1/3))*b
^7+544*x*a^3*b^4-319*b^7)*(b+a*x^(1/3))/a^8
```

Maxima [A] time = 0.750663, size = 188, normalized size = 0.46

$$\frac{4\,a^{7}x^{\frac{7}{3}}-14\,a^{6}bx^{2}+84\,a^{5}b^{2}x^{\frac{5}{3}}+556\,a^{4}b^{3}x^{\frac{4}{3}}+544\,a^{3}b^{4}x-444\,a^{2}b^{5}x^{\frac{2}{3}}-856\,ab^{6}x^{\frac{1}{3}}-319\,b^{7}}{4\left(a^{12}x^{\frac{4}{3}}+4\,a^{11}bx+6\,a^{10}b^{2}x^{\frac{2}{3}}+4\,a^{9}b^{3}x^{\frac{1}{3}}+a^{8}b^{4}\right)}-\frac{105\,b^{3}\log\left(ax^{\frac{1}{3}}+b\right)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(-5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*
b^3*x^(4/3) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(
1/3) - 319*b^7)/(a^12*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) +
 4*a^9*b^3*x^(1/3) + a^8*b^4) - 105*b^3*log(a*x^(1/3) + b)/a^8
```

Fricas [A] time = 0.275565, size = 238, normalized size = 0.58

$$-\frac{14\,a^6bx^2 - 544\,a^3b^4x + 319\,b^7 + 420\left(4\,a^3b^4x + 6\,a^2b^5x^{\frac{2}{3}} + b^7 + \left(a^4b^3x + 4\,ab^6\right)x^{\frac{1}{3}}\right)\log\left(ax^{\frac{1}{3}} + b\right) - 12\left(7\,a^5b^2x - 37\,a^2b\right.}{4\left(4\,a^{11}bx + 6\,a^{10}b^2x^{\frac{2}{3}} + a^8b^4 + (a^{12}x + 4\,a^9b^3)x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(14*a^6*b*x^2 - 544*a^3*b^4*x + 319*b^7 + 420*(4*a^3*b^4*x +
 6*a^2*b^5*x^(2/3) + b^7 + (a^4*b^3*x + 4*a*b^6)*x^(1/3))*log(a*x
^(1/3) + b) - 12*(7*a^5*b^2*x - 37*a^2*b^5)*x^(2/3) - 4*(a^7*x^2
+ 139*a^4*b^3*x - 214*a*b^6)*x^(1/3))/(4*a^11*b*x + 6*a^10*b^2*x^
(2/3) + a^8*b^4 + (a^12*x + 4*a^9*b^3)*x^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.319103, size = 190, normalized size = 0.46

$$-\frac{105\,b^3\ln\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^8\mathrm{sign}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} - \frac{420\,a^3b^4x + 1134\,a^2b^5x^{\frac{2}{3}} + 1036\,ab^6x^{\frac{1}{3}} + 319\,b^7}{4\left(ax^{\frac{1}{3}} + b\right)^4a^8\mathrm{sign}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} + \frac{2\,a^{10}x - 15\,a^9bx^{\frac{2}{3}} + 90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sign}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/3) + b^2/x^(2/3))^(-5/2),x, algorithm="giac")
```

```
[Out] -105*b^3*ln(abs(a*x^(1/3) + b))/(a^8*sign(a*x^(2/3) + b*x^(1/3)))
 - 1/4*(420*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3)
 + 319*b^7)/((a*x^(1/3) + b)^4*a^8*sign(a*x^(2/3) + b*x^(1/3))) +
 1/2*(2*a^10*x - 15*a^9*b*x^(2/3) + 90*a^8*b^2*x^(1/3))/(a^15*sig
n(a*x^(2/3) + b*x^(1/3)))
```

3.489 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx$

Optimal. Leaf size=289

$$-\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20ab^4\log\left(\sqrt[4]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4
))*x^(1/4)) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4
)]*x^(1/4))/(a + b/x^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x]
+ (2*a*b)/x^(1/4)]*Sqrt[x])/(a + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2
+ b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4))/(3*(a + b/x^(1/4))) + (
a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(a + b/x^(1/4))
+ (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^(1/4)
])/(a + b/x^(1/4))
```

Rubi [A] time = 0.302434, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20ab^4\log\left(\sqrt[4]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4
))*x^(1/4)) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4
)]*x^(1/4))/(a + b/x^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x]
+ (2*a*b)/x^(1/4)]*Sqrt[x])/(a + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2
```

```
+ b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4))/(3*(a + b/x^(1/4))) + (
a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(a + b/x^(1/4))
+ (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^(1/4)
])/(a + b/x^(1/4))
```

Rubi in Sympy [A] time = 32.5215, size = 231, normalized size = 0.8

$$-\frac{20ab^4\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\log\left(\frac{1}{\sqrt[4]{x}}\right)}{a+\frac{b}{\sqrt[4]{x}}}-20b^4\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}+10b^3\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}+\frac{10b^2\sqrt{x}\left(a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}\right)^{\frac{3}{2}}}{3}+\frac{5bx^{\frac{3}{4}}\left(a+\frac{b}{\sqrt[4]{x}}\right)\left(a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}\right)^{\frac{3}{2}}}{3}+x\left(a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
```

```
[Out] -20*a*b**4*sqrt(a**2 + 2*a*b/x**(1/4) + b**2/sqrt(x))*log(x**(-1/
4))/(a + b/x**(1/4)) - 20*b**4*sqrt(a**2 + 2*a*b/x**(1/4) + b**2/
sqrt(x)) + 10*b**3*x**(1/4)*(a + b/x**(1/4))*sqrt(a**2 + 2*a*b/x*
*(1/4) + b**2/sqrt(x)) + 10*b**2*sqrt(x)*(a**2 + 2*a*b/x**(1/4) +
 b**2/sqrt(x))**(3/2)/3 + 5*b*x**(3/4)*(a + b/x**(1/4))*(a**2 + 2
*a*b/x**(1/4) + b**2/sqrt(x))**(3/2)/3 + x*(a**2 + 2*a*b/x**(1/4)
 + b**2/sqrt(x))**(5/2)
```

Mathematica [A] time = 0.0699848, size = 98, normalized size = 0.34

$$\frac{\sqrt{\frac{\left(a\sqrt[4]{x}+b\right)^2}{\sqrt{x}}}\left(3a^5x^{5/4}+20a^4bx+60a^3b^2x^{3/4}+120a^2b^3\sqrt{x}+15ab^4\sqrt[4]{x}\log(x)-12b^5\right)}{3\left(a\sqrt[4]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] +
 60*a^3*b^2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/
4)*Log[x]))/(3*(b + a*x^(1/4)))
```

Maple [A] time = 0.044, size = 94, normalized size = 0.3

$$\frac{1}{3}\sqrt{1\left(a^{2}x^{\frac{3}{4}}+2\,ab\sqrt{x}+b^{2}\sqrt[4]{x}\right)x^{-\frac{3}{4}}}\left(20\,a^{4}bx+120\,\sqrt{x}a^{2}b^{3}+60\,x^{3/4}a^{3}b^{2}+15\,\sqrt[4]{x}\ln\left(x\right)ab^{4}+3\,x^{5/4}a^{5}-12\,b^{5}\right)\left(\sqrt[4]{x}a+b\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x)
```

```
[Out] 1/3*((a^2*x^(3/4)+2*a*b*x^(1/2)+b^2*x^(1/4))/x^(3/4))^(1/2)*(20*a
^4*b*x+120*x^(1/2)*a^2*b^3+60*x^(3/4)*a^3*b^2+15*x^(1/4)*ln(x)*a*
b^4+3*x^(5/4)*a^5-12*b^5)/(x^(1/4)*a+b)
```

Maxima [A] time = 0.755199, size = 77, normalized size = 0.27

$$5\,ab^{4}\log\left(x\right)+\frac{3\,a^{5}x^{\frac{5}{4}}+20\,a^{4}bx+60\,a^{3}b^{2}x^{\frac{3}{4}}+120\,a^{2}b^{3}\sqrt{x}-12\,b^{5}}{3\,x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/4) + b^2/sqrt(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(
3/4) + 120*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

Fricas [A] time = 0.275378, size = 82, normalized size = 0.28

$$\frac{3\,a^{5}x^{\frac{5}{4}}+60\,ab^{4}x^{\frac{1}{4}}\log\left(x^{\frac{1}{4}}\right)+20\,a^{4}bx+60\,a^{3}b^{2}x^{\frac{3}{4}}+120\,a^{2}b^{3}\sqrt{x}-12\,b^{5}}{3\,x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/4) + b^2/sqrt(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^5*x^(5/4) + 60*a*b^4*x^(1/4)*log(x^(1/4)) + 20*a^4*b*x +
 60*a^3*b^2*x^(3/4) + 120*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.307157, size = 170, normalized size = 0.59

$$a^5 x \operatorname{sign}\left(ax + bx^{\frac{3}{4}}\right)\operatorname{sign}(x) + 5\,ab^4\ln(|x|)\operatorname{sign}\left(ax + bx^{\frac{3}{4}}\right)\operatorname{sign}(x) + \frac{20}{3}\,a^4 b x^{\frac{3}{4}}\operatorname{sign}\left(ax + bx^{\frac{3}{4}}\right)\operatorname{sign}(x) + 20\,a^3 b^2\sqrt{x}\operatorname{sign}\left(ax + bx^{\frac{3}{4}}\right)\operatorname{sign}(x) + 40\,a^2 b^3 x^{\frac{1}{4}}\operatorname{sign}\left(ax + bx^{\frac{3}{4}}\right)\operatorname{sign}(x) - \frac{4\,b^5\operatorname{sign}\left(ax + bx^{\frac{3}{4}}\right)\operatorname{sign}(x)}{x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/4) + b^2/sqrt(x))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sign(a*x + b*x^(3/4))*sign(x) + 5*a*b^4*ln(abs(x))*sign(a*x
 + b*x^(3/4))*sign(x) + 20/3*a^4*b*x^(3/4)*sign(a*x + b*x^(3/4))*
sign(x) + 20*a^3*b^2*sqrt(x)*sign(a*x + b*x^(3/4))*sign(x) + 40*a
^2*b^3*x^(1/4)*sign(a*x + b*x^(3/4))*sign(x) - 4*b^5*sign(a*x + b
*x^(3/4))*sign(x)/x^(1/4)
```

3.490 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}\,dx$

Optimal. Leaf size=291

$$\frac{5b^5\log\left(\sqrt[5]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}$$

```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a +
 b/x^(1/5)) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5
)]*x^(2/5))/(a + b/x^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5)
+ (2*a*b)/x^(1/5)]*x^(3/5))/(3*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[
a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(4/5))/(4*(a + b/x^(1/5)))
 + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x)/(a + b/x^(1/
5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x^(1/5
)])/(a + b/x^(1/5))
```

Rubi [A] time = 0.289275, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{5b^5\log\left(\sqrt[5]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a +
 b/x^(1/5)) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5
)]*x^(2/5))/(a + b/x^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5)
+ (2*a*b)/x^(1/5)]*x^(3/5))/(3*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[
a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(4/5))/(4*(a + b/x^(1/5)))
```

```
 + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x)/(a + b/x^(1/
5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x^(1/5
)])/(a + b/x^(1/5))
```

Rubi in Sympy [A] time = 37.9872, size = 246, normalized size = 0.85

$$\frac{5ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}}}{a+\frac{b}{\sqrt[5]{x}}}-\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}}\log\left(\frac{1}{\sqrt[5]{x}}\right)}{a+\frac{b}{\sqrt[5]{x}}}+\frac{5b^3x^{\frac{2}{5}}\left(a+\frac{b}{\sqrt[5]{x}}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}}}{2}+\frac{5b^2x^{\frac{3}{5}}\left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}\right)^{\frac{3}{2}}}{3}+\frac{5bx^{\frac{4}{5}}\left(a+\frac{b}{\sqrt[5]{x}}\right)\left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}\right)^{\frac{3}{2}}}{4}+x\left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

```
[Out] 5*a*b**4*x**(1/5)*sqrt(a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))/(a
+ b/x**(1/5)) - 5*b**5*sqrt(a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5)
)*log(x**(-1/5))/(a + b/x**(1/5)) + 5*b**3*x**(2/5)*(a + b/x**(1/
5))*sqrt(a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))/2 + 5*b**2*x**(3/
5)*(a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(3/2)/3 + 5*b*x**(4/5
)*(a + b/x**(1/5))*(a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(3/2)
/4 + x*(a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2)
```

Mathematica [A] time = 0.074444, size = 103, normalized size = 0.35

$$\frac{\sqrt{\frac{\left(a\sqrt[5]{x}+b\right)^2}{x^{2/5}}}\left(12a^5x^{6/5}+75a^4bx+200a^3b^2x^{4/5}+300a^2b^3x^{3/5}+300ab^4x^{2/5}+12b^5\sqrt[5]{x}\log(x)\right)}{12\left(a\sqrt[5]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/5))^2/x^(2/5)]*(300*a*b^4*x^(2/5) + 300*a^2*b^3
*x^(3/5) + 200*a^3*b^2*x^(4/5) + 75*a^4*b*x + 12*a^5*x^(6/5) + 12
*b^5*x^(1/5)*Log[x]))/(12*(b + a*x^(1/5)))
```

Maple [A] time = 0.025, size = 91, normalized size = 0.3

$$\frac{x}{12}\left(1\left(a^{2}x^{\frac{2}{5}}+2\,ab\sqrt[5]{x}+b^{2}\right)x^{-\frac{2}{5}}\right)^{\frac{5}{2}}\left(75\,a^{4}bx^{4/5}+200\,a^{3}b^{2}x^{3/5}+300\,a^{2}b^{3}x^{2/5}+300\,ab^{4}\sqrt[5]{x}+12\,b^{5}\ln\left(x\right)+12\,a^{5}x\right)\left(\sqrt[5]{x}a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x)
```

```
[Out] 1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(75*a^4*b*
x^(4/5)+200*a^3*b^2*x^(3/5)+300*a^2*b^3*x^(2/5)+300*a*b^4*x^(1/5)
+12*b^5*ln(x)+12*a^5*x)/(x^(1/5)*a+b)^5
```

Maxima [A] time = 0.759689, size = 70, normalized size = 0.24

$$a^{5}x+b^{5}\log\left(x\right)+\frac{25}{4}\,a^{4}bx^{\frac{4}{5}}+\frac{50}{3}\,a^{3}b^{2}x^{\frac{3}{5}}+25\,a^{2}b^{3}x^{\frac{2}{5}}+25\,ab^{4}x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/5) + b^2/x^(2/5))^(5/2),x, algorithm="maxima")
```

```
[Out] a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) +
25*a^2*b^3*x^(2/5) + 25*a*b^4*x^(1/5)
```

Fricas [A] time = 0.275612, size = 74, normalized size = 0.25

$$a^{5}x+5\,b^{5}\log\left(x^{\frac{1}{5}}\right)+\frac{25}{4}\,a^{4}bx^{\frac{4}{5}}+\frac{50}{3}\,a^{3}b^{2}x^{\frac{3}{5}}+25\,a^{2}b^{3}x^{\frac{2}{5}}+25\,ab^{4}x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/5) + b^2/x^(2/5))^(5/2),x, algorithm="fricas")
```

```
[Out] a^5*x + 5*b^5*log(x^(1/5)) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^
(3/5) + 25*a^2*b^3*x^(2/5) + 25*a*b^4*x^(1/5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.301823, size = 169, normalized size = 0.58

$$a^5 x \operatorname{sign}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sign}(x) + b^5 \ln(|x|) \operatorname{sign}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sign}(x) + \frac{25}{4} a^4 b x^{\frac{4}{5}} \operatorname{sign}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sign}(x) + \frac{50}{3} a^3 b^2 x^{\frac{3}{5}} \operatorname{sign}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sign}(x) + 25 a^2 b^3 x^{\frac{2}{5}} \operatorname{sign}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sign}(x) + 25 a b^4 x^{\frac{1}{5}} \operatorname{sign}\left(ax + bx^{\frac{4}{5}}\right) \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/5) + b^2/x^(2/5))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sign(a*x + b*x^(4/5))*sign(x) + b^5*ln(abs(x))*sign(a*x + b
*x^(4/5))*sign(x) + 25/4*a^4*b*x^(4/5)*sign(a*x + b*x^(4/5))*sign
(x) + 50/3*a^3*b^2*x^(3/5)*sign(a*x + b*x^(4/5))*sign(x) + 25*a^2
*b^3*x^(2/5)*sign(a*x + b*x^(4/5))*sign(x) + 25*a*b^4*x^(1/5)*sig
n(a*x + b*x^(4/5))*sign(x)
```

3.491 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4
*b^5*(a + b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) +
 (20*a^3)/(3*b^5*(a + b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2
*x^(2/5)]) - (15*a^2)/(b^5*(a + b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/
5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*Log[a + b*x^(1/5)])/(b^5*
Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rubi [A] time = 0.252717, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4
*b^5*(a + b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) +
 (20*a^3)/(3*b^5*(a + b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2
*x^(2/5)]) - (15*a^2)/(b^5*(a + b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/
5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*Log[a + b*x^(1/5)])/(b^5*
Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rubi in Sympy [A] time = 20.8836, size = 218, normalized size = 0.98

$$-\frac{5x^{\frac{4}{5}}\left(2a+2b\sqrt[5]{x}\right)}{8b\left(a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}\right)^{\frac{5}{2}}}-\frac{5x^{\frac{3}{5}}}{3b^2\left(a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}\right)^{\frac{3}{2}}}-\frac{5x^{\frac{2}{5}}\left(2a+2b\sqrt[5]{x}\right)}{4b^3\left(a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}\right)^{\frac{3}{2}}}-\frac{5\sqrt[5]{x}}{b^4\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

```
[Out] -5*x**(4/5)*(2*a + 2*b*x**(1/5))/(8*b*(a**2 + 2*a*b*x**(1/5) + b*
*2*x**(2/5))**(5/2)) - 5*x**(3/5)/(3*b**2*(a**2 + 2*a*b*x**(1/5)
+ b**2*x**(2/5))**(3/2)) - 5*x**(2/5)*(2*a + 2*b*x**(1/5))/(4*b**
3*(a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(3/2)) - 5*x**(1/5)/(b
**4*sqrt(a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))) + 5*(a + b*x**(1
/5))*log(a + b*x**(1/5))/(b**5*sqrt(a**2 + 2*a*b*x**(1/5) + b**2*
x**(2/5)))
```

Mathematica [A] time = 0.0903197, size = 98, normalized size = 0.44

$$\frac{5a\left(25a^3+88a^2b\sqrt[5]{x}+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+b\sqrt[5]{x}\right)}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3
/5)) + 60*(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^
(1/5))^3*Sqrt[(a + b*x^(1/5))^2])
```

Maple [A] time = 0.018, size = 152, normalized size = 0.7

$$\frac{5}{12\,b^5}\sqrt{a^2+2\,ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}}\left(12\,x^{4/5}\ln\left(a+b\sqrt[5]{x}\right)b^4+48\,x^{3/5}\ln\left(a+b\sqrt[5]{x}\right)ab^3+48\,x^{3/5}ab^3+72\,x^{2/5}\ln\left(a+b\sqrt[5]{x}\right)a^2b^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x)
```

```
[Out] 5/12*(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)*(12*x^(4/5)*ln(a+b*x^(
1/5))*b^4+48*x^(3/5)*ln(a+b*x^(1/5))*a*b^3+48*x^(3/5)*a*b^3+72*x^
(2/5)*ln(a+b*x^(1/5))*a^2*b^2+108*x^(2/5)*a^2*b^2+48*x^(1/5)*ln(a
+b*x^(1/5))*a^3*b+88*x^(1/5)*a^3*b+12*ln(a+b*x^(1/5))*a^4+25*a^4)
/(a+b*x^(1/5))^5/b^5
```

Maxima [A] time = 0.760118, size = 134, normalized size = 0.6

$$\frac{5\left(48\,ab^{3}x^{\frac{3}{5}}+108\,a^{2}b^{2}x^{\frac{2}{5}}+88\,a^{3}bx^{\frac{1}{5}}+25\,a^{4}\right)}{12\left(b^{9}x^{\frac{4}{5}}+4\,ab^{8}x^{\frac{3}{5}}+6\,a^{2}b^{7}x^{\frac{2}{5}}+4\,a^{3}b^{6}x^{\frac{1}{5}}+a^{4}b^{5}\right)}+\frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/5) + 2*a*b*x^(1/5) + a^2)^(-5/2),x, algorithm="maxima")
```

```
[Out] 5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) +
 25*a^4)/(b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a
^3*b^6*x^(1/5) + a^4*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

Fricas [A] time = 0.27568, size = 182, normalized size = 0.82

$$\frac{5\left(48\,ab^{3}x^{\frac{3}{5}}+108\,a^{2}b^{2}x^{\frac{2}{5}}+88\,a^{3}bx^{\frac{1}{5}}+25\,a^{4}+12\left(b^{4}x^{\frac{4}{5}}+4\,ab^{3}x^{\frac{3}{5}}+6\,a^{2}b^{2}x^{\frac{2}{5}}+4\,a^{3}bx^{\frac{1}{5}}+a^{4}\right)\log\left(bx^{\frac{1}{5}}+a\right)\right)}{12\left(b^{9}x^{\frac{4}{5}}+4\,ab^{8}x^{\frac{3}{5}}+6\,a^{2}b^{7}x^{\frac{2}{5}}+4\,a^{3}b^{6}x^{\frac{1}{5}}+a^{4}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2/5) + 2*a*b*x^(1/5) + a^2)^(-5/2),x, algorithm="fricas")
```

```
[Out] 5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) +
 25*a^4 + 12*(b^4*x^(4/5) + 4*a*b^3*x^(3/5) + 6*a^2*b^2*x^(2/5) +
 4*a^3*b*x^(1/5) + a^4)*log(b*x^(1/5) + a))/(b^9*x^(4/5) + 4*a*b^
8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a^4*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[5]{x} + b^2x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

GIAC/XCAS [A] time = 0.288077, size = 113, normalized size = 0.51

$$\frac{5\ln\left(\left|bx^{\frac{1}{5}} + a\right|\right)}{b^5\mathrm{sign}\left(bx^{\frac{1}{5}} + a\right)} + \frac{5\left(48\,ab^2x^{\frac{3}{5}} + 108\,a^2bx^{\frac{2}{5}} + 88\,a^3x^{\frac{1}{5}} + \frac{25\,a^4}{b}\right)}{12\left(bx^{\frac{1}{5}} + a\right)^4 b^4\mathrm{sign}\left(bx^{\frac{1}{5}} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2/5) + 2*a*b*x^(1/5) + a^2)^(-5/2),x, algorithm="giac")
```

```
[Out] 5*ln(abs(b*x^(1/5) + a))/(b^5*sign(b*x^(1/5) + a)) + 5/12*(48*a*b
^2*x^(3/5) + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x
^(1/5) + a)^4*b^4*sign(b*x^(1/5) + a))
```

3.492 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx$

Optimal. Leaf size=391

$$-\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{42ab^6\log\left(\sqrt[6]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}$$

```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6
))*x^(1/6)) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/
6)]*x^(1/6))/(a + b/x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3
) + (2*a*b)/x^(1/6)]*x^(1/3))/(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[
a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqrt[x])/(a + b/x^(1/6)) + (
63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(2/3))/(2*
(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(
1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3)
+ (2*a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/
x^(1/3) + (2*a*b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```

Rubi [A] time = 0.391675, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{42ab^6\log\left(\sqrt[6]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6
))*x^(1/6)) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/
6)]*x^(1/6))/(a + b/x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3
) + (2*a*b)/x^(1/6)]*x^(1/3))/(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[
a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqrt[x])/(a + b/x^(1/6)) + (
63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(2/3))/(2*
(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(
1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3)
+ (2*a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/
x^(1/3) + (2*a*b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```

Rubi in Sympy [A] time = 44.4084, size = 313, normalized size = 0.8

$$- \frac{42ab^6 \sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}} \log\left(\frac{1}{\sqrt[6]{x}}\right)}{a + \frac{b}{\sqrt[6]{x}}} - 42b^6 \sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}} + 21b^5 \sqrt[6]{x} \left(a + \frac{b}{\sqrt[6]{x}}\right) \sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}} + 7b^4 \sqrt[3]{x} \left(a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}\right)^{\frac{3}{2}} + \frac{7b^3 \sqrt{x} \left(a + \frac{b}{\sqrt[6]{x}}\right) \left(a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}\right)^{\frac{3}{2}}}{2} + \frac{21b^2 x^{\frac{2}{3}} \left(a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}\right)^{\frac{5}{2}}}{10} + \frac{7bx^{\frac{5}{6}} \left(a + \frac{b}{\sqrt[6]{x}}\right) \left(a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}\right)^{\frac{5}{2}}}{5} + x \left(a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

```
[Out] -42*a*b**6*sqrt(a**2 + 2*a*b/x**(1/6) + b**2/x**(1/3))*log(x**(-1
/6))/(a + b/x**(1/6)) - 42*b**6*sqrt(a**2 + 2*a*b/x**(1/6) + b**2
/x**(1/3)) + 21*b**5*x**(1/6)*(a + b/x**(1/6))*sqrt(a**2 + 2*a*b/
x**(1/6) + b**2/x**(1/3)) + 7*b**4*x**(1/3)*(a**2 + 2*a*b/x**(1/6
) + b**2/x**(1/3))**(3/2) + 7*b**3*sqrt(x)*(a + b/x**(1/6))*(a**2
 + 2*a*b/x**(1/6) + b**2/x**(1/3))**(3/2)/2 + 21*b**2*x**(2/3)*(a
**2 + 2*a*b/x**(1/6) + b**2/x**(1/3))**(5/2)/10 + 7*b*x**(5/6)*(a
 + b/x**(1/6))*(a**2 + 2*a*b/x**(1/6) + b**2/x**(1/3))**(5/2)/5 +
 x*(a**2 + 2*a*b/x**(1/6) + b**2/x**(1/3))**(7/2)
```

Mathematica [A] time = 0.0857193, size = 124, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[6]{x}+b\right)^2}{\sqrt[3]{x}}}\left(10a^7x^{7/6}+84a^6bx+315a^5b^2x^{5/6}+700a^4b^3x^{2/3}+1050a^3b^4\sqrt{x}+1260a^2b^5\sqrt[3]{x}+70ab^6\sqrt[6]{x}\log(x)-60b^7\right)}{10\left(a\sqrt[6]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3)
+ 1050*a^3*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6
) + 84*a^6*b*x + 10*a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(
b + a*x^(1/6)))
```

Maple [A] time = 0.035, size = 116, normalized size = 0.3

$$\frac{1}{10}\sqrt{1\left(a^2\sqrt{x}+2ab\sqrt[3]{x}+b^2\sqrt[6]{x}\right)\frac{1}{\sqrt{x}}}\Big(84a^6bx+315a^5b^2x^{5/6}+1260a^2b^5\sqrt[3]{x}+700a^4b^3x^{2/3}+1050\sqrt{x}a^3b^4+70ab^6\ln(x)\sqrt[6]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

```
[Out] 1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+b^2*x^(1/6))/x^(1/2))^(1/2)*(84*
a^6*b*x+315*a^5*b^2*x^(5/6)+1260*a^2*b^5*x^(1/3)+700*a^4*b^3*x^(2
/3)+1050*x^(1/2)*a^3*b^4+70*a*b^6*ln(x)*x^(1/6)+10*a^7*x^(7/6)-60
*b^7)/(x^(1/6)*a+b)
```

Maxima [A] time = 0.757155, size = 107, normalized size = 0.27

$$7\,ab^6\log(x)+\frac{10\,a^7x^{\frac{7}{6}}+84\,a^6bx+315\,a^5b^2x^{\frac{5}{6}}+700\,a^4b^3x^{\frac{2}{3}}+1050\,a^3b^4\sqrt{x}+1260\,a^2b^5x^{\frac{1}{3}}-60\,b^7}{10\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/6) + b^2/x^(1/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*
x^(5/6) + 700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b
```

```
^5*x^(1/3) - 60*b^7)/x^(1/6)
```

Fricas [A] time = 0.274199, size = 112, normalized size = 0.29

$$\frac{10\,a^7x^{\frac{7}{6}} + 420\,ab^6x^{\frac{1}{6}}\log\left(x^{\frac{1}{6}}\right) + 84\,a^6bx + 315\,a^5b^2x^{\frac{5}{6}} + 700\,a^4b^3x^{\frac{2}{3}} + 1050\,a^3b^4\sqrt{x} + 1260\,a^2b^5x^{\frac{1}{3}} - 60\,b^7}{10\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/6) + b^2/x^(1/3))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(10*a^7*x^(7/6) + 420*a*b^6*x^(1/6)*log(x^(1/6)) + 84*a^6*b*
x + 315*a^5*b^2*x^(5/6) + 700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt
(x) + 1260*a^2*b^5*x^(1/3) - 60*b^7)/x^(1/6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.389965, size = 232, normalized size = 0.59

$$a^7x\,\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right) + 7\,ab^6\ln\left(|x|\right)\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right) + \frac{42}{5}\,a^6bx^{\frac{5}{6}}\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right) + \frac{63}{2}\,a^5b^2x^{\frac{2}{3}}\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right) + 70\,a^4b^3\sqrt{x}\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right) + 105\,a^3b^4x^{\frac{1}{3}}\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right) + 126\,a^2b^5x^{\frac{1}{6}}\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right) - \frac{6\,b^7\mathrm{sign}\left(ax + bx^{\frac{5}{6}}\right)\mathrm{sign}\left(x\right)}{x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/6) + b^2/x^(1/3))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sign(a*x + b*x^(5/6))*sign(x) + 7*a*b^6*ln(abs(x))*sign(a*x
 + b*x^(5/6))*sign(x) + 42/5*a^6*b*x^(5/6)*sign(a*x + b*x^(5/6))*
sign(x) + 63/2*a^5*b^2*x^(2/3)*sign(a*x + b*x^(5/6))*sign(x) + 70
*a^4*b^3*sqrt(x)*sign(a*x + b*x^(5/6))*sign(x) + 105*a^3*b^4*x^(1
/3)*sign(a*x + b*x^(5/6))*sign(x) + 126*a^2*b^5*x^(1/6)*sign(a*x
+ b*x^(5/6))*sign(x) - 6*b^7*sign(a*x + b*x^(5/6))*sign(x)/x^(1/6
)
```

3.493 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=46

$$\frac{b^2\log\left(b+cx^n\right)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*
n)
```

Rubi [A] time = 0.0716772, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{b^2\log\left(b+cx^n\right)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*
n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2\log\left(b+cx^n\right)}{c^3n}+\frac{\int^{x^n}x\,dx}{cn}-\frac{\int^{x^n}b\,dx}{c^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] b**2*log(b + c*x**n)/(c**3*n) + Integral(x, (x, x**n))/(c*n) - In
tegral(b, (x, x**n))/(c**2*n)
```

Mathematica [A] time = 0.0326373, size = 38, normalized size = 0.83

$$\frac{2b^2\log\left(b+cx^n\right)+cx^n\left(cx^n-2b\right)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

Maple [A] time = 0.04, size = 62, normalized size = 1.4

$$\frac{1}{e^{n\ln(x)}}\left(\frac{\left(e^{n\ln(x)}\right)^3}{2\,cn}-\frac{b\left(e^{n\ln(x)}\right)^2}{c^2n}\right)+\frac{b^2\ln\left(ce^{n\ln(x)}+b\right)}{c^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (1/2/c/n*exp(n*ln(x))^3-b/c^2/n*exp(n*ln(x))^2)/exp(n*ln(x))+b^2/
c^3/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.754667, size = 61, normalized size = 1.33

$$\frac{b^2\log\left(\frac{cx^n+b}{c}\right)}{c^3n}+\frac{cx^{2\,n}-2\,bx^n}{2\,c^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n
)
```

Fricas [A] time = 0.303133, size = 51, normalized size = 1.11

$$\frac{c^2x^{2\,n}-2\,bcx^n+2\,b^2\log\left(cx^n+b\right)}{2\,c^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.494 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=28

$$\frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}$$

```
[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)
```

Rubi [A] time = 0.0479923, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]

[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\log\left(b+cx^n\right)}{c^2n}+\frac{\int^{x^n}\frac{1}{c}\,dx}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)

[Out] -b*log(b + c*x**n)/(c**2*n) + Integral(1/c, (x, x**n))/n
```

Mathematica [A] time = 0.017544, size = 24, normalized size = 0.86

$$\frac{cx^n-b\log\left(b+cx^n\right)}{c^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n - b*Log[b + c*x^n])/(c^2*n)
```

Maple [A] time = 0.032, size = 33, normalized size = 1.2

$$\frac{e^{n\ln(x)}}{cn} - \frac{b\ln\left(ce^{n\ln(x)} + b\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c/n*exp(n*ln(x))-b/c^2/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.745154, size = 43, normalized size = 1.54

$$\frac{x^n}{cn} - \frac{b\log\left(\frac{cx^n+b}{c}\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

Fricas [A] time = 0.298037, size = 32, normalized size = 1.14

$$\frac{cx^n - b\log\left(cx^n + b\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] (c*x^n - b*log(c*x^n + b))/(c^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{cx^{2n} + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.495 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{cn}$$

```
[Out] Log[b + c*x^n]/(c*n)
```

Rubi [A] time = 0.0230653, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]

[Out] Log[b + c*x^n]/(c*n)
```

Rubi in Sympy [A] time = 4.48543, size = 10, normalized size = 0.67

$$\frac{\log(b+cx^n)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)

[Out] log(b + c*x**n)/(c*n)
```

Mathematica [A] time = 0.00442025, size = 15, normalized size = 1.

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[b + c*x^n]/(c*n)
```

Maple [A] time = 0.029, size = 18, normalized size = 1.2

$$\frac{\ln\left(c e^{n\ln(x)} + b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.747025, size = 26, normalized size = 1.73

$$\frac{\log\left(\frac{cx^n+b}{c}\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] log((c*x^n + b)/c)/(c*n)
```

Fricas [A] time = 0.286718, size = 20, normalized size = 1.33

$$\frac{\log\left(cx^n + b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] log(c*x^n + b)/(c*n)
```

Sympy [A] time = 58.4706, size = 37, normalized size = 2.47

$$\begin{cases} \frac{\log(x)}{b} & \text{for } c = 0 \wedge n = 0 \\ \frac{x^n}{bn} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ -\frac{\log(x)}{c} + \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{cn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x**n/(b*n), Eq(c, 0))
, (log(x)/(b + c), Eq(n, 0)), (-log(x)/c + log(b*x**n/c + x**(2*n
))/(c*n), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{cx^{2n} + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.496 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^n\right)}{bn}$$

```
[Out] Log[x]/b - Log[b + c*x^n]/(b*n)
```

Rubi [A] time = 0.0357283, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]

[Out] Log[x]/b - Log[b + c*x^n]/(b*n)
```

Rubi in Sympy [A] time = 7.81461, size = 19, normalized size = 0.83

$$\frac{\log\left(x^n\right)}{bn}-\frac{\log\left(b+cx^n\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)

[Out] log(x**n)/(b*n) - log(b + c*x**n)/(b*n)
```

Mathematica [A] time = 0.0144911, size = 22, normalized size = 0.96

$$\frac{n\log(x)-\log\left(b+cx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (n*Log[x] - Log[b + c*x^n])/(b*n)
```

Maple [A] time = 0.029, size = 26, normalized size = 1.1

$$\frac{\ln(x)}{b} - \frac{\ln\left(c e^{n \ln(x)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.749337, size = 36, normalized size = 1.57

$$\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] log(x)/b - log((c*x^n + b)/c)/(b*n)
```

Fricas [A] time = 0.296322, size = 30, normalized size = 1.3

$$\frac{n\log(x) - \log(cx^n + b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(c*x^n + b))/(b*n)
```

Sympy [A] time = 95.7024, size = 66, normalized size = 2.87

$$\begin{cases} \tilde{\infty} \log(x) & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ -\frac{x^{-n}}{cn} & \text{for } b = 0 \\ \frac{\frac{n^2 \log(x)}{n^2-n} - \frac{n \log(x)}{n^2-n}}{b} & \text{for } c = 0 \\ \frac{2\log(x)}{b} - \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (log(x)/(
b + c), Eq(n, 0)), (-x**(-n)/(c*n), Eq(b, 0)), ((n**2*log(x)/(n**
2 - n) - n*log(x)/(n**2 - n))/b, Eq(c, 0)), (2*log(x)/b - log(b*x
**n/c + x**(2*n))/(b*n), True))
```

GIAC/XCAS [A] time = 0.271302, size = 34, normalized size = 1.48

$$\frac{\ln(|x|)}{b} - \frac{\ln(|cx^n + b|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] ln(abs(x))/b - ln(abs(c*x^n + b))/(b*n)
```

3.497 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=57

$$-\frac{c^2\log\left(b+cx^n\right)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

```
[Out] -1/(2*b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[
b + c*x^n])/(b^3*n)
```

Rubi [A] time = 0.0799551, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{c^2\log\left(b+cx^n\right)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[
b + c*x^n])/(b^3*n)
```

Rubi in Sympy [A] time = 14.7116, size = 51, normalized size = 0.89

$$-\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log\left(x^n\right)}{b^3n}-\frac{c^2\log\left(b+cx^n\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-2*n)/(2*b*n) + c*x**(-n)/(b**2*n) + c**2*log(x**n)/(b**3*n)
 - c**2*log(b + c*x**n)/(b**3*n)
```

Mathematica [A] time = 0.0432396, size = 46, normalized size = 0.81

$$-\frac{x^{-2n}\left(2c^2x^{2n}\log\left(bx^{-n}+c\right)+b\left(b-2cx^n\right)\right)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -(b*(b - 2*c*x^n) + 2*c^2*x^(2*n)*Log[c + b/x^n])/(2*b^3*n*x^(2*n
))
```

Maple [A] time = 0.034, size = 69, normalized size = 1.2

$$\frac{1}{\left(e^{n\ln(x)}\right)^{2}}\left(\frac{c e^{n\ln(x)}}{b^{2}n}-\frac{1}{2\,bn}+\frac{c^{2}\ln\left(x\right)\left(e^{n\ln(x)}\right)^{2}}{b^{3}}\right)-\frac{c^{2}\ln\left(c e^{n\ln(x)}+b\right)}{b^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c/b^2/n*exp(n*ln(x))-1/2/b/n+c^2/b^3*ln(x)*exp(n*ln(x))^2)/exp(n
*ln(x))^2-c^2/b^3/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.74915, size = 76, normalized size = 1.33

$$\frac{c^{2}\log\left(x\right)}{b^{3}}+\frac{\left(2\,cx^{n}-b\right)x^{-2\,n}}{2\,b^{2}n}-\frac{c^{2}\log\left(\frac{cx^{n}+b}{c}\right)}{b^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log(x)/b^3 + 1/2*(2*c*x^n - b)*x^(-2*n)/(b^2*n) - c^2*log((c*
x^n + b)/c)/(b^3*n)
```

Fricas [A] time = 0.290453, size = 80, normalized size = 1.4

$$\frac{2\,c^{2}nx^{2\,n}\log\left(x\right)-2\,c^{2}x^{2\,n}\log\left(cx^{n}+b\right)+2\,bcx^{n}-b^{2}}{2\,b^{3}nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*
c*x^n - b^2)/(b^3*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-n-1}}{cx^{2n} + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.498 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=76

$$\frac{c^3\log\left(b+cx^n\right)}{b^4n}-\frac{c^3\log(x)}{b^4}-\frac{c^2x^{-n}}{b^3n}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

```
[Out] -1/(3*b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3
*Log[x])/b^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rubi [A] time = 0.09314, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{c^3\log\left(b+cx^n\right)}{b^4n}-\frac{c^3\log(x)}{b^4}-\frac{c^2x^{-n}}{b^3n}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3
*Log[x])/b^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rubi in Sympy [A] time = 17.2952, size = 66, normalized size = 0.87

$$-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log\left(x^n\right)}{b^4n}+\frac{c^3\log\left(b+cx^n\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-3*n)/(3*b*n) + c*x**(-2*n)/(2*b**2*n) - c**2*x**(-n)/(b**3*
n) - c**3*log(x**n)/(b**4*n) + c**3*log(b + c*x**n)/(b**4*n)
```

Mathematica [A] time = 0.0386904, size = 61, normalized size = 0.8

$$\frac{x^{-3n}\left(b\left(-2b^2+3bcx^n-6c^2x^{2n}\right)+6c^3x^{3n}\log\left(bx^{-n}+c\right)\right)}{6b^4n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*(-2*b^2 + 3*b*c*x^n - 6*c^2*x^(2*n)) + 6*c^3*x^(3*n)*Log[c + b
/x^n])/(6*b^4*n*x^(3*n))
```

Maple [A] time = 0.041, size = 88, normalized size = 1.2

$$\frac{1}{\left(e^{n\ln(x)}\right)^{3}}\left(-\frac{1}{3\,bn}+\frac{ce^{n\ln(x)}}{2\,b^{2}n}-\frac{c^{2}\left(e^{n\ln(x)}\right)^{2}}{b^{3}n}-\frac{c^{3}\ln(x)\left(e^{n\ln(x)}\right)^{3}}{b^{4}}\right)+\frac{c^{3}\ln\left(ce^{n\ln(x)}+b\right)}{b^{4}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (-1/3/b/n+1/2*c/b^2/n*exp(n*ln(x))-c^2/b^3/n*exp(n*ln(x))^2-c^3/b
^4*ln(x)*exp(n*ln(x))^3)/exp(n*ln(x))^3+c^3/b^4/n*ln(c*exp(n*ln(x
))+b)
```

Maxima [A] time = 0.755393, size = 93, normalized size = 1.22

$$-\frac{c^{3}\log(x)}{b^{4}}+\frac{c^{3}\log\left(\frac{cx^{n}+b}{c}\right)}{b^{4}n}-\frac{\left(6\,c^{2}x^{2\,n}-3\,bcx^{n}+2\,b^{2}\right)x^{-3\,n}}{6\,b^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] -c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(
2*n) - 3*b*c*x^n + 2*b^2)*x^(-3*n)/(b^3*n)
```

Fricas [A] time = 0.291245, size = 97, normalized size = 1.28

$$-\frac{6\,c^{3}nx^{3\,n}\log(x)-6\,c^{3}x^{3\,n}\log\left(cx^{n}+b\right)+6\,bc^{2}x^{2\,n}-3\,b^{2}cx^{n}+2\,b^{3}}{6\,b^{4}nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b
*c^2*x^(2*n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-2\,n-1}}{cx^{2\,n} + bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.499 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=93

$$-\frac{c^4\log\left(b+cx^n\right)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{c^3x^{-n}}{b^4n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

```
[Out] -1/(4*b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n))
+ c^3/(b^4*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*
n)
```

Rubi [A] time = 0.108819, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{c^4\log\left(b+cx^n\right)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{c^3x^{-n}}{b^4n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(4*b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n))
+ c^3/(b^4*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*
n)
```

Rubi in Sympy [A] time = 20.2316, size = 82, normalized size = 0.88

$$-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log\left(x^n\right)}{b^5n}-\frac{c^4\log\left(b+cx^n\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-4*n)/(4*b*n) + c*x**(-3*n)/(3*b**2*n) - c**2*x**(-2*n)/(2*b
**3*n) + c**3*x**(-n)/(b**4*n) + c**4*log(x**n)/(b**5*n) - c**4*l
og(b + c*x**n)/(b**5*n)
```

Mathematica [A] time = 0.0588833, size = 74, normalized size = 0.8

$$-\frac{x^{-4n}\left(b\left(3b^3-4b^2cx^n+6bc^2x^{2n}-12c^3x^{3n}\right)+12c^4x^{4n}\log\left(bx^{-n}+c\right)\right)}{12b^5n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -(b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)) + 12
*c^4*x^(4*n)*Log[c + b/x^n])/(12*b^5*n*x^(4*n))
```

Maple [A] time = 0.045, size = 105, normalized size = 1.1

$$\frac{1}{\left(e^{n\ln(x)}\right)^{4}}\left(\frac{c^{3}\left(e^{n\ln(x)}\right)^{3}}{b^{4}n}-\frac{1}{4\,bn}+\frac{ce^{n\ln(x)}}{3\,b^{2}n}-\frac{c^{2}\left(e^{n\ln(x)}\right)^{2}}{2\,b^{3}n}+\frac{c^{4}\ln(x)\left(e^{n\ln(x)}\right)^{4}}{b^{5}}\right)-\frac{c^{4}\ln\left(ce^{n\ln(x)}+b\right)}{b^{5}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c^3/b^4/n*exp(n*ln(x))^3-1/4/b/n+1/3*c/b^2/n*exp(n*ln(x))-1/2*c^
2/b^3/n*exp(n*ln(x))^2+c^4/b^5*ln(x)*exp(n*ln(x))^4)/exp(n*ln(x))
^4-c^4/b^5/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.749778, size = 111, normalized size = 1.19

$$\frac{c^{4}\log(x)}{b^{5}}-\frac{c^{4}\log\left(\frac{cx^{n}+b}{c}\right)}{b^{5}n}+\frac{\left(12\,c^{3}x^{3\,n}-6\,bc^{2}x^{2\,n}+4\,b^{2}cx^{n}-3\,b^{3}\right)x^{-4\,n}}{12\,b^{4}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^
(3*n) - 6*b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)*x^(-4*n)/(b^4*n)
```

Fricas [A] time = 0.291579, size = 115, normalized size = 1.24

$$\frac{12\,c^{4}nx^{4\,n}\log(x)-12\,c^{4}x^{4\,n}\log\left(cx^{n}+b\right)+12\,bc^{3}x^{3\,n}-6\,b^{2}c^{2}x^{2\,n}+4\,b^{3}cx^{n}-3\,b^{4}}{12\,b^{5}nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 1
2*b*c^3*x^(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n
*x^(4*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-3\,n-1}}{cx^{2\,n} + bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.500 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=236

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}+\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{b^{7/4}n}-\frac{4x^{-3n/4}}{3bn}$$

```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x^(n/4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x^(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[S
qrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqr
t[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*
x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```

Rubi [A] time = 0.386315, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}+\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{b^{7/4}n}-\frac{4x^{-3n/4}}{3bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x^(n/4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x^(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[S
qrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqr
t[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*
x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```

Rubi in Sympy [A] time = 64.2371, size = 206, normalized size = 0.87

$$-\frac{4x^{-\frac{3n}{4}}}{3bn}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{\frac{n}{4}}+\sqrt{b}+\sqrt{c}x^{\frac{n}{2}}\right)}{2b^{\frac{7}{4}}n}-\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{\frac{n}{4}}+\sqrt{b}+\sqrt{c}x^{\frac{n}{2}}\right)}{2b^{\frac{7}{4}}n}+\frac{\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{\frac{n}{4}}}{\sqrt[4]{b}}\right)}{b^{\frac{7}{4}}n}-\frac{\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x^{\frac{n}{4}}}{\sqrt[4]{b}}\right)}{b^{\frac{7}{4}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -4*x**(-3*n/4)/(3*b*n) + sqrt(2)*c**(3/4)*log(-sqrt(2)*b**(1/4)*c
**(1/4)*x**(n/4) + sqrt(b) + sqrt(c)*x**(n/2))/(2*b**(7/4)*n) - s
qrt(2)*c**(3/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*x**(n/4) + sqrt(b)
+ sqrt(c)*x**(n/2))/(2*b**(7/4)*n) + sqrt(2)*c**(3/4)*atan(1 - sq
rt(2)*c**(1/4)*x**(n/4)/b**(1/4))/(b**(7/4)*n) - sqrt(2)*c**(3/4)
*atan(1 + sqrt(2)*c**(1/4)*x**(n/4)/b**(1/4))/(b**(7/4)*n)
```

Mathematica [C] time = 0.0436591, size = 60, normalized size = 0.25

$$\frac{3c\text{RootSum}\left[\#1^4b+c\&,\frac{4\log\left(x^{-n/4}-\#1\right)+n\log(x)}{\#1}\&\right]-16bx^{-3n/4}}{12b^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-16*b)/x^((3*n)/4) + 3*c*RootSum[c + b*#1^4 & , (n*Log[x] + 4*L
og[x^(-n/4) - #1])/#1 & ])/(12*b^2*n)
```

Maple [C] time = 0.322, size = 54, normalized size = 0.2

$$-\frac{4}{3\,bn}\left(x^{\frac{n}{4}}\right)^{-3}+\sum_{\_R=RootOf\left(b^7n^4\_Z^4+c^3\right)}\_R\ln\left(x^{\frac{n}{4}}-\frac{b^2n\_R}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(
_Z^4*b^7*n^4+c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.306976, size = 335, normalized size = 1.42

$$\frac{12\,bnx^3x^{\frac{3}{4}\,n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\arctan\left(\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}}{cxx^{\frac{1}{4}\,n-1}+x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^3}{b^7n^4}}+c^2x^2x^{\frac{1}{2}\,n-2}}{x^2}}}\right)-3\,bnx^3x^{\frac{3}{4}\,n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}+cxx^{\frac{1}{4}\,n-1}}{x}\right)+3\,\ldots}{3\,bnx^3x^{\frac{3}{4}\,n-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] 1/3*(12*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*arctan(b^2*n
*(-c^3/(b^7*n^4))^(1/4)/(c*x*x^(1/4*n - 1) + x*sqrt((b^4*n^2*sqrt
(-c^3/(b^7*n^4)) + c^2*x^2*x^(1/2*n - 2))/x^2))) - 3*b*n*x^3*x^(3
/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7*n^4))^(1/4
) + c*x*x^(1/4*n - 1))/x) + 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^
4))^(1/4)*log(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))
/x) - 4)/(b*n*x^3*x^(3/4*n - 3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.501 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(
1/3)*x^(n/3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1
/3) + c^(1/3)*x^(n/3)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1
/3)*c^(1/3)*x^(n/3) + c^(2/3)*x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rubi [A] time = 0.245219, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(
1/3)*x^(n/3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1
/3) + c^(1/3)*x^(n/3)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1
/3)*c^(1/3)*x^(n/3) + c^(2/3)*x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rubi in Sympy [A] time = 40.6811, size = 138, normalized size = 0.86

$$-\frac{3x^{-\frac{2n}{3}}}{2bn}-\frac{c^{\frac{2}{3}}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{\frac{n}{3}}\right)}{b^{\frac{5}{3}}n}+\frac{c^{\frac{2}{3}}\log\left(b^{\frac{2}{3}}-\sqrt[3]{b}\sqrt[3]{c}x^{\frac{n}{3}}+c^{\frac{2}{3}}x^{\frac{2n}{3}}\right)}{2b^{\frac{5}{3}}n}+\frac{\sqrt{3}c^{\frac{2}{3}}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{b}}{3}-\frac{2\sqrt[3]{c}x^{\frac{n}{3}}}{3}\right)}{\sqrt[3]{b}}\right)}{b^{\frac{5}{3}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -3*x**(-2*n/3)/(2*b*n) - c**(2/3)*log(b**(1/3) + c**(1/3)*x**(n/3
))/(b**(5/3)*n) + c**(2/3)*log(b**(2/3) - b**(1/3)*c**(1/3)*x**(n
/3) + c**(2/3)*x**(2*n/3))/(2*b**(5/3)*n) + sqrt(3)*c**(2/3)*atan
(sqrt(3)*(b**(1/3)/3 - 2*c**(1/3)*x**(n/3)/3)/b**(1/3))/(b**(5/3)
*n)
```

Mathematica [C] time = 0.0436198, size = 60, normalized size = 0.38

$$\frac{2c\text{RootSum}\left[\#1^3b+c\&,\frac{3\log\left(x^{-n/3}-\#1\right)+n\log(x)}{\#1}\&\right]-9bx^{-2n/3}}{6b^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-9*b)/x^((2*n)/3) + 2*c*RootSum[c + b*#1^3 & , (n*Log[x] + 3*Lo
g[x^(-n/3) - #1])/#1 & ])/(6*b^2*n)
```

Maple [C] time = 0.091, size = 54, normalized size = 0.3

$$-\frac{3}{2\,bn}\left(x^{\frac{n}{3}}\right)^{-2}+\sum_{\_R=RootOf\left(b^5n^3\_Z^3+c^2\right)}\_R\,\ln\left(x^{\frac{n}{3}}-\frac{b^2n\_R}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-b^2*n/c*_R),_R=RootOf(
_Z^3*b^5*n^3+c^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301421, size = 293, normalized size = 1.83

$$-\frac{2\sqrt{3}x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,cxx^{\frac{1}{3}n-1}+b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\right)}{3\,b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}\right)-2\,x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)+x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log}{2\,bnx^2x^{\frac{2}{3}n-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*sqr
t(3)*(2*c*x*x^(1/3*n - 1) + b*(-c^2/b^2)^(1/3))/(b*(-c^2/b^2)^(1/
3))) - 2*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*log((c*x*x^(1/3*n - 1
) - b*(-c^2/b^2)^(1/3))/x) + x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*l
og((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*(-c^2/b^2)^(1/3)
+ b^2*(-c^2/b^2)^(2/3))/x^2) + 3)/(b*n*x^2*x^(2/3*n - 2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")

[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.502 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/
(b^(3/2)*n)
```

Rubi [A] time = 0.0684434, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/
(b^(3/2)*n)
```

Rubi in Sympy [A] time = 13.4537, size = 39, normalized size = 0.78

$$-\frac{2x^{-\frac{n}{2}}}{bn}+\frac{2\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{b}x^{-\frac{n}{2}}}{\sqrt{c}}\right)}{b^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*x**(-n/2)/(b*n) + 2*sqrt(c)*atan(sqrt(b)*x**(-n/2)/sqrt(c))/(b
**(3/2)*n)
```

Mathematica [A] time = 0.0406458, size = 50, normalized size = 1.

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/
(b^(3/2)*n)
```

Maple [A] time = 0.093, size = 79, normalized size = 1.6

$$-2\frac{1}{bnx^{n/2}}+\frac{1}{b^2n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}}-\frac{1}{c}\sqrt{-bc}\right)-\frac{1}{b^2n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}}+\frac{1}{c}\sqrt{-bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-1/c*(-b*c)^(
1/2))-1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)+1/c*(-b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298401, size = 1, normalized size = 0.02

$$\left[\frac{xx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2x^{n-2}-2\,bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right)-2}{bnxx^{\frac{1}{2}n-1}},\ \frac{2\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right)-1\right)}{bnxx^{\frac{1}{2}n-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] [(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*
n - 1)*sqrt(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*
n - 1)), 2*(x*x^(1/2*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(
1/2*n - 1))) - 1)/(b*n*x*x^(1/2*n - 1))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}n-1}}{cx^{2n} + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.503 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTa
n[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rubi [A] time = 0.089816, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTa
n[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rubi in Sympy [A] time = 17.8674, size = 56, normalized size = 0.82

$$-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{2cx^{-\frac{n}{2}}}{b^2n}-\frac{2c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{-\frac{n}{2}}}{\sqrt{c}}\right)}{b^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*x**(-3*n/2)/(3*b*n) + 2*c*x**(-n/2)/(b**2*n) - 2*c**(3/2)*atan
(sqrt(b)*x**(-n/2)/sqrt(c))/(b**(5/2)*n)
```

Mathematica [A] time = 0.0892205, size = 62, normalized size = 0.91

$$\frac{2\left(\sqrt{b}x^{-3n/2}\left(3cx^n-b\right)-3c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)\right)}{3b^{5/2}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*((Sqrt[b]*(-b + 3*c*x^n))/x^((3*n)/2) - 3*c^(3/2)*ArcTan[Sqrt[
b]/(Sqrt[c]*x^(n/2))]))/(3*b^(5/2)*n)
```

Maple [A] time = 0.122, size = 97, normalized size = 1.4

$$2\,\frac{c}{nb^2x^{n/2}}-\frac{2}{3\,bn}\left(x^{\frac{n}{2}}\right)^{-3}+\frac{c}{b^3n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}}+\frac{1}{c}\sqrt{-bc}\right)-\frac{c}{b^3n}\sqrt{-bc}\ln\left(x^{\frac{n}{2}}-\frac{1}{c}\sqrt{-bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/
n*ln(x^(1/2*n)+1/c*(-b*c)^(1/2))-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2
*n)-1/c*(-b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300984, size = 1, normalized size = 0.01

$$\left[-\frac{2\,bx^3x^{-\frac{3}{2}\,n-3}-6\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{-\frac{c}{b}}\log\left(\frac{bx^2x^{-n-2}-2\,bxx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}-c}{bx^2x^{-n-2}+c}\right)}{3\,b^2n},\ -\frac{2\left(bx^3x^{-\frac{3}{2}\,n-3}-3\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{\frac{c}{b}}\arctan\left(\frac{\sqrt{\frac{c}{b}}}{xx^{-\frac{1}{2}\,n-1}}\right)\right)}{3\,b^2n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-
c/b)*log((b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)
/(b*x^2*x^(-n - 2) + c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3
*c*x*x^(-1/2*n - 1) - 3*c*sqrt(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n
 - 1))))/(b^2*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.504 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=176

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\frac{3x^{-4n/3}}{4bn}$$

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)
*ArcTan[(c^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7
/3)*n) - (c^(4/3)*Log[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (
c^(4/3)*Log[c^(2/3) + b^(2/3)/x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(
n/3)])/(2*b^(7/3)*n)
```

Rubi [A] time = 0.271777, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\frac{3x^{-4n/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)
*ArcTan[(c^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7
/3)*n) - (c^(4/3)*Log[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (
c^(4/3)*Log[c^(2/3) + b^(2/3)/x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(
n/3)])/(2*b^(7/3)*n)
```

Rubi in Sympy [A] time = 48.8427, size = 151, normalized size = 0.86

$$-\frac{3x^{-\frac{4n}{3}}}{4bn}+\frac{3cx^{-\frac{n}{3}}}{b^2n}-\frac{c^{\frac{4}{3}}\log\left(\sqrt[3]{b}x^{-\frac{n}{3}}+\sqrt[3]{c}\right)}{b^{\frac{7}{3}}n}+\frac{c^{\frac{4}{3}}\log\left(b^{\frac{2}{3}}x^{-\frac{2n}{3}}-\sqrt[3]{b}\sqrt[3]{c}x^{-\frac{n}{3}}+c^{\frac{2}{3}}\right)}{2b^{\frac{7}{3}}n}+\frac{\sqrt{3}c^{\frac{4}{3}}\operatorname{atan}\left(\frac{\sqrt{3}\left(-\frac{2\sqrt[3]{b}x^{-\frac{n}{3}}}{3}+\frac{\sqrt[3]{c}}{3}\right)}{\sqrt[3]{c}}\right)}{b^{\frac{7}{3}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -3*x**(-4*n/3)/(4*b*n) + 3*c*x**(-n/3)/(b**2*n) - c**(4/3)*log(b*
*(1/3)*x**(-n/3) + c**(1/3))/(b**(7/3)*n) + c**(4/3)*log(b**(2/3)
*x**(-2*n/3) - b**(1/3)*c**(1/3)*x**(-n/3) + c**(2/3))/(2*b**(7/3
)*n) + sqrt(3)*c**(4/3)*atan(sqrt(3)*(-2*b**(1/3)*x**(-n/3)/3 + c
**(1/3)/3)/c**(1/3))/(b**(7/3)*n)
```

Mathematica [C] time = 0.0772193, size = 70, normalized size = 0.4

$$-\frac{4c^2\text{RootSum}\left[\#1^3b+c\&,\frac{3\log\left(x^{-n/3}-\#1\right)+n\log(x)}{\#1^2}\&\right]+9bx^{-4n/3}\left(b-4cx^n\right)}{12b^3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((9*b*(b - 4*c*x^n))/x^((4*n)/3) + 4*c^2*RootSum[c + b*#1^3 & ,
(n*Log[x] + 3*Log[x^(-n/3) - #1])/#1^2 & ])/(12*b^3*n)
```

Maple [C] time = 0.106, size = 73, normalized size = 0.4

$$3\,\frac{c}{b^2nx^{n/3}}-\frac{3}{4\,bn}\left(x^{\frac{n}{3}}\right)^{-4}+\sum_{\_R=RootOf\left(b^7n^3\_Z^3+c^4\right)}\_R\,\ln\left(x^{\frac{n}{3}}+\frac{b^5n^2\_R^2}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b
^5*n^2/c^3*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301893, size = 228, normalized size = 1.3

$$-\frac{3\,bx^4x^{-\frac{4}{3}\,n-4}-12\,cxx^{-\frac{1}{3}\,n-1}+4\,\sqrt{3}c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,xx^{-\frac{1}{3}\,n-1}+\left(-\frac{c}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{b}\right)^{\frac{1}{3}}}\right)-4\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}\,n-1}-\left(-\frac{c}{b}\right)^{\frac{1}{3}}}{x}\right)+2\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\,\mathrm{l}}{4\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) + 4*sqrt(3)*
c*(-c/b)^(1/3)*arctan(1/3*sqrt(3)*(2*x*x^(-1/3*n - 1) + (-c/b)^(1
/3))/(-c/b)^(1/3)) - 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c
/b)^(1/3))/x) + 2*c*(-c/b)^(1/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-
1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2/3))/x^2))/(b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.505 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=252

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}+\frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{b^{9/4}n}+\frac{4cx^{-n/4}}{b^2n}-\frac{4x^{-5n/4}}{5bn}$$

```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (
Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/
(b^(9/4)*n) + (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b
^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/4)*n) - (c^(5/4)*Log[Sqrt
[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt
[2]*b^(9/4)*n)
```

Rubi [A] time = 0.426692, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}+\frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{b^{9/4}n}+\frac{4cx^{-n/4}}{b^2n}-\frac{4x^{-5n/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (
Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/
(b^(9/4)*n) + (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b
^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/4)*n) - (c^(5/4)*Log[Sqrt
[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt
[2]*b^(9/4)*n)
```

Rubi in Sympy [A] time = 73.7828, size = 219, normalized size = 0.87

$$-\frac{4x^{-\frac{5n}{4}}}{5bn}+\frac{4cx^{-\frac{n}{4}}}{b^2n}+\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-\frac{n}{4}}+\sqrt{b}x^{-\frac{n}{2}}+\sqrt{c}\right)}{2b^{\frac{9}{4}}n}-\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-\frac{n}{4}}+\sqrt{b}x^{-\frac{n}{2}}+\sqrt{c}\right)}{2b^{\frac{9}{4}}n}-\frac{\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-\frac{n}{4}}}{\sqrt[4]{c}}-1\right)}{b^{\frac{9}{4}}n}-\frac{\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-\frac{n}{4}}}{\sqrt[4]{c}}+1\right)}{b^{\frac{9}{4}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -4*x**(-5*n/4)/(5*b*n) + 4*c*x**(-n/4)/(b**2*n) + sqrt(2)*c**(5/4
)*log(-sqrt(2)*b**(1/4)*c**(1/4)*x**(-n/4) + sqrt(b)*x**(-n/2) +
sqrt(c))/(2*b**(9/4)*n) - sqrt(2)*c**(5/4)*log(sqrt(2)*b**(1/4)*c
**(1/4)*x**(-n/4) + sqrt(b)*x**(-n/2) + sqrt(c))/(2*b**(9/4)*n) -
 sqrt(2)*c**(5/4)*atan(sqrt(2)*b**(1/4)*x**(-n/4)/c**(1/4) - 1)/(
b**(9/4)*n) - sqrt(2)*c**(5/4)*atan(sqrt(2)*b**(1/4)*x**(-n/4)/c*
*(1/4) + 1)/(b**(9/4)*n)
```

Mathematica [C] time = 0.0805128, size = 70, normalized size = 0.28

$$-\frac{5c^2\text{RootSum}\left[\#1^4b+c\&,\frac{4\log\left(x^{-n/4}-\#1\right)+n\log(x)}{\#1^3}\&\right]+16bx^{-5n/4}\left(b-5cx^n\right)}{20b^3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((16*b*(b - 5*c*x^n))/x^((5*n)/4) + 5*c^2*RootSum[c + b*#1^4 & ,
 (n*Log[x] + 4*Log[x^(-n/4) - #1])/#1^3 & ])/(20*b^3*n)
```

Maple [C] time = 0.118, size = 73, normalized size = 0.3

$$4\,\frac{c}{b^2nx^{n/4}}-\frac{4}{5\,bn}\left(x^{\frac{n}{4}}\right)^{-5}+\sum_{\_R=RootOf\left(b^9n^4\_Z^4+c^5\right)}\_R\,\ln\left(x^{\frac{n}{4}}+\frac{b^7n^3\_R^3}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b
^7*n^3/c^4*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.312018, size = 317, normalized size = 1.26

$$-\frac{4\,bx^5x^{-\frac{5}{4}\,n-5}-20\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\arctan\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}}{cxx^{-\frac{1}{4}\,n-1}+x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^5}{b^9n^4}}+c^2x^2x^{-\frac{1}{2}\,n-2}}{x^2}}}\right)+5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}+cxx^{-\frac{1}{4}\,n-1}}{x}\right)}{5\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")
```

```
[Out] -1/5*(4*b*x^5*x^(-5/4*n - 5) - 20*b^2*n*(-c^5/(b^9*n^4))^(1/4)*ar
ctan(b^2*n*(-c^5/(b^9*n^4))^(1/4)/(c*x*x^(-1/4*n - 1) + x*sqrt((b
^4*n^2*sqrt(-c^5/(b^9*n^4)) + c^2*x^2*x^(-1/2*n - 2))/x^2))) + 5*
b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(-c^5/(b^9*n^4))^(1/4) +
c*x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log(-(b^2
*n*(-c^5/(b^9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1
/4*n - 1))/(b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.506 $\int x^{-1-n(-1+p)} \left(bx^n + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=37

$$\frac{x^{-n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{cn(p+1)}$$

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rubi [A] time = 0.0673135, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{x^{-n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]

[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rubi in Sympy [A] time = 9.39798, size = 27, normalized size = 0.73

$$\frac{x^{-n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{cn\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)

[Out] x**(-n*(p + 1))*(b*x**n + c*x**(2*n))**(p + 1)/(c*n*(p + 1))
```

Mathematica [A] time = 0.0490163, size = 38, normalized size = 1.03

$$\frac{x^{-np}\,(b+cx^n)\,(x^n\,(b+cx^n))^p}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x^{-1-n(-1+p)} \left(bx^n + cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

Maxima [A] time = 0.81335, size = 58, normalized size = 1.57

$$\frac{(cx^n + b)e^{(-np\log(x)+p\log(cx^n+b)+p\log(x^n))}}{cn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1),x, algorithm="maxima")
```

```
[Out] (c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*
(p + 1))
```

Fricas [A] time = 0.290735, size = 80, normalized size = 2.16

$$\frac{\left(cxx^{-np+n-1}x^n + bxx^{-np+n-1}\right)\left(cx^{2n} + bx^n\right)^p}{(cnp + cn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1),x, algorithm="fricas")
```

```
[Out] (c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*
x^n)^p/((c*n*p + c*n)*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2\,n} + bx^{n}\right)^{p} x^{-n(p-1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

3.507 $\int x^{-1-n(1+2p)} \left(bx^n + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=38

$$-\frac{x^{-2n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{bn(p+1)}$$

```
[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rubi [A] time = 0.0668931, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$-\frac{x^{-2n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]

[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rubi in Sympy [A] time = 9.56211, size = 31, normalized size = 0.82

$$-\frac{x^{-2n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{bn\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)

[Out] -x**(-2*n*(p + 1))*(b*x**n + c*x**(2*n))**(p + 1)/(b*n*(p + 1))
```

Mathematica [A] time = 0.0855039, size = 43, normalized size = 1.13

$$-\frac{x^{-n(2p+1)} \left(b + cx^n\right) \left(x^n \left(b + cx^n\right)\right)^p}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p)))
)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int x^{-1-n(1+2p)} \left(bx^n + cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n\right)^p x^{-n(2p+1)-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Fricas [A] time = 0.292187, size = 80, normalized size = 2.11

$$-\frac{\left(cxx^{-2\,np-n-1}x^n + bxx^{-2\,np-n-1}\right)\left(cx^{2\,n} + bx^n\right)^p}{bnp + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1),x, algorithm="fricas")
```

```
[Out] -(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n)
 + b*x^n)^p/(b*n*p + b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2\,n} + bx^{n}\right)^{p} x^{-n(2\,p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

3.508 $\int x^{-1+2n} \left(a^2 + 2abx^n + b^2x^{2n}\right)^{5/2} \, dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^7 \sqrt{a^2+2abx^n+b^2x^{2n}}}{7n(ab^2+b^3x^n)} - \frac{a(a+bx^n)^6 \sqrt{a^2+2abx^n+b^2x^{2n}}}{6n(ab^2+b^3x^n)}$$

```
[Out] -(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*n*(a*b^
2 + b^3*x^n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)
])/(7*n*(a*b^2 + b^3*x^n))
```

Rubi [A] time = 0.114579, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\frac{(a+bx^n)^7 \sqrt{a^2+2abx^n+b^2x^{2n}}}{7n(ab^2+b^3x^n)} - \frac{a(a+bx^n)^6 \sqrt{a^2+2abx^n+b^2x^{2n}}}{6n(ab^2+b^3x^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*n*(a*b^
2 + b^3*x^n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)
])/(7*n*(a*b^2 + b^3*x^n))
```

Rubi in Sympy [A] time = 15.4336, size = 71, normalized size = 0.63

$$-\frac{a(2a+2bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{5}{2}}}{12b^2n} + \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{7}{2}}}{7b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] -a*(2*a + 2*b*x**n)*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(5/2)/(1
2*b**2*n) + (a**2 + 2*a*b*x**n + b**2*x**(2*n))**(7/2)/(7*b**2*n)
```

Mathematica [A] time = 0.0813003, size = 96, normalized size = 0.86

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\left(21a^5+70a^4bx^n+105a^3b^2x^{2n}+84a^2b^3x^{3n}+35ab^4x^{4n}+6b^5x^{5n}\right)}{42n(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2
*x^(2*n) + 84*a^2*b^3*x^(3*n) + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n))
)/(42*n*(a + b*x^n))
```

Maple [A] time = 0.102, size = 208, normalized size = 1.9

$$\frac{b^5 (x^n)^7}{(7a + 7bx^n) n}\sqrt{(a + bx^n)^2} + \frac{5ab^4 (x^n)^6}{(6a + 6bx^n) n}\sqrt{(a + bx^n)^2} + 2\frac{\sqrt{(a + bx^n)^2}a^2b^3 (x^n)^5}{(a + bx^n) n} + \frac{5a^3b^2 (x^n)^4}{(2a + 2bx^n) n}\sqrt{(a + bx^n)^2} + \frac{5a^4b (x^n)^3}{(3a + 3bx^n) n}\sqrt{(a + bx^n)^2} + \frac{a^5 (x^n)^2}{(2a + 2bx^n) n}\sqrt{(a + bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] 1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)
^(1/2)/(a+b*x^n)*a*b^4/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*
a^2*b^3/n*(x^n)^5+5/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^
n)^4+5/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^4*b/n*(x^n)^3+1/2*((a+b*
x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^2
```

Maxima [A] time = 0.771022, size = 100, normalized size = 0.89

$$\frac{6b^5x^{7n} + 35ab^4x^{6n} + 84a^2b^3x^{5n} + 105a^3b^2x^{4n} + 70a^4bx^{3n} + 21a^5x^{2n}}{42n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1),x, algorithm="maxima
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105
*a^3*b^2*x^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Fricas [A] time = 0.297136, size = 100, normalized size = 0.89

$$\frac{6b^5x^{7n} + 35ab^4x^{6n} + 84a^2b^3x^{5n} + 105a^3b^2x^{4n} + 70a^4bx^{3n} + 21a^5x^{2n}}{42n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1),x, algorithm="fricas
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105
*a^3*b^2*x^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{5}{2}} x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

3.509 $\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\,(ab^2+b^3x^n)}-\frac{a\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab^2+b^3x^n)}$$

```
[Out] -(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(4*n*(a*b^
2 + b^3*x^n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)
])/(5*n*(a*b^2 + b^3*x^n))
```

Rubi [A] time = 0.109497, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\,(ab^2+b^3x^n)}-\frac{a\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab^2+b^3x^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(4*n*(a*b^
2 + b^3*x^n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)
])/(5*n*(a*b^2 + b^3*x^n))
```

Rubi in Sympy [A] time = 15.1287, size = 71, normalized size = 0.63

$$-\frac{a\,(2a+2bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{3}{2}}}{8b^2n}+\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{5}{2}}}{5b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] -a*(2*a + 2*b*x**n)*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2)/(8
*b**2*n) + (a**2 + 2*a*b*x**n + b**2*x**(2*n))**(5/2)/(5*b**2*n)
```

Mathematica [A] time = 0.0505532, size = 70, normalized size = 0.62

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\left(10a^3+20a^2bx^n+15ab^2x^{2n}+4b^3x^{3n}\right)}{20n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^
(2*n) + 4*b^3*x^(3*n)))/(20*n*(a + b*x^n))
```

Maple [A] time = 0.038, size = 135, normalized size = 1.2

$$\frac{b^3 (x^n)^5}{(5a + 5bx^n) n}\sqrt{(a + bx^n)^2} + \frac{3ab^2 (x^n)^4}{(4a + 4bx^n) n}\sqrt{(a + bx^n)^2} + \frac{a^2 b (x^n)^3}{(a + bx^n) n}\sqrt{(a + bx^n)^2} + \frac{a^3 (x^n)^2}{(2a + 2bx^n) n}\sqrt{(a + bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/5*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/n*(x^n)^5+3/4*((a+b*x^n)^2)
^(1/2)/(a+b*x^n)*a*b^2/n*(x^n)^4+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^
2*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/n*(x^n)^2
```

Maxima [A] time = 0.765018, size = 65, normalized size = 0.58

$$\frac{4b^3 x^{5n} + 15ab^2 x^{4n} + 20a^2 b x^{3n} + 10a^3 x^{2n}}{20n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1),x, algorithm="maxima
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^
3*x^(2*n))/n
```

Fricas [A] time = 0.289621, size = 65, normalized size = 0.58

$$\frac{4b^3 x^{5n} + 15ab^2 x^{4n} + 20a^2 b x^{3n} + 10a^3 x^{2n}}{20n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1),x, algorithm="fricas
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^
3*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}} x^{2n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

3.510 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=99

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}$$

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n))
 + (b^2*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b +
b^2*x^n))
```

Rubi [A] time = 0.0809695, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n))
 + (b^2*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b +
b^2*x^n))
```

Rubi in Sympy [A] time = 9.1947, size = 80, normalized size = 0.81

$$\frac{abx^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(2ab+2b^2x^n)}+\frac{x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] a*b*x**(2*n)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(3*n*(2*a*b
+ 2*b**2*x**n)) + x**(2*n)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)
)/(3*n)
```

Mathematica [A] time = 0.0274296, size = 44, normalized size = 0.44

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\,(3a+2bx^n)}{6n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

Maple [A] time = 0.034, size = 64, normalized size = 0.7

$$\frac{b\,(x^n)^3}{(3\,a+3\,bx^n)\,n}\sqrt{(a+bx^n)^2}+\frac{a\,(x^n)^2}{(2\,a+2\,bx^n)\,n}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(
1/2)/(a+b*x^n)*a/n*(x^n)^2
```

Maxima [A] time = 0.760705, size = 30, normalized size = 0.3

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Fricas [A] time = 0.268874, size = 30, normalized size = 0.3

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b^2 x^{2\,n} + 2\,abx^n + a^2}\,x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

3.511 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \, dx$

Optimal. Leaf size=90

$$\frac{x^n (a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*
(a + b*x^n)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(
2*n)])
```

Rubi [A] time = 0.114335, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\frac{x^n (a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*
(a + b*x^n)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(
2*n)])
```

Rubi in Sympy [A] time = 16.7741, size = 73, normalized size = 0.81

$$-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}} + \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] -a*(a + b*x**n)*log(a + b*x**n)/(b**2*n*sqrt(a**2 + 2*a*b*x**n +
b**2*x**(2*n))) + sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(b**2*n
)
```

Mathematica [A] time = 0.0479725, size = 44, normalized size = 0.49

$$\frac{(a+bx^n)\,(bx^n-a\log{(a+bx^n)})}{b^2n\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((a + b*x^n)*(b*x^n - a*Log[a + b*x^n]))/(b^2*n*Sqrt[(a + b*x^n)^
2])
```

Maple [A] time = 0.048, size = 71, normalized size = 0.8

$$\frac{x^n}{(a+bx^n)\,bn}\sqrt{(a+bx^n)^2}-\frac{a}{(a+bx^n)\,b^2n}\sqrt{(a+bx^n)^2}\ln\left(x^n+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)/b/n*x^n-((a+b*x^n)^2)^(1/2)/(a+b*x^
n)*a/b^2/n*ln(x^n+a/b)
```

Maxima [A] time = 0.760084, size = 43, normalized size = 0.48

$$\frac{x^n}{bn}-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="maxima")
```

```
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

Fricas [A] time = 0.272115, size = 32, normalized size = 0.36

$$\frac{bx^n-a\log{(bx^n+a)}}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="fricas")
```

```
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2\,n-1}}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.512 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^{2n}}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0714896, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{x^{2n}}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 8.92025, size = 42, normalized size = 0.88

$$\frac{x^{2n}\,(2a+2bx^n)}{4an\,(a^2+2abx^n+b^2x^{2n})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] x**(2*n)*(2*a + 2*b*x**n)/(4*a*n*(a**2 + 2*a*b*x**n + b**2*x**(2*
n))**(3/2))
```

Mathematica [A] time = 0.0410631, size = 40, normalized size = 0.83

$$-\frac{a+2bx^n}{2b^2n\,(a+bx^n)\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]

[Out] -(a + 2*b*x^n)/(2*b^2*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.038, size = 37, normalized size = 0.8

$$-\frac{2\,bx^{n}+a}{2\,\left(a+bx^{n}\right)^{3}b^{2}n}\sqrt{\left(a+bx^{n}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)

[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3*(2*b*x^n+a)/b^2/n
```

Maxima [A] time = 0.764608, size = 55, normalized size = 1.15

$$-\frac{2\,bx^{n}+a}{2\,\left(b^{4}nx^{2\,n}+2\,ab^{3}nx^{n}+a^{2}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="maxima

[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Fricas [A] time = 0.267415, size = 55, normalized size = 1.15

$$-\frac{2\,bx^{n}+a}{2\,\left(b^{4}nx^{2\,n}+2\,ab^{3}nx^{n}+a^{2}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="fricas

[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.513 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{4b^2n(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1
/(3*b^2*n*(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.124071, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{a}{4b^2n(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1
/(3*b^2*n*(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 15.3651, size = 73, normalized size = 0.83

$$\frac{a(2a+2bx^n)}{8b^2n(a^2+2abx^n+b^2x^{2n})^{\frac{5}{2}}}-\frac{1}{3b^2n(a^2+2abx^n+b^2x^{2n})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] a*(2*a + 2*b*x**n)/(8*b**2*n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))*
*(5/2)) - 1/(3*b**2*n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2))
```

Mathematica [A] time = 0.0415744, size = 40, normalized size = 0.45

$$-\frac{a+4bx^n}{12b^2n(a+bx^n)^3\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -(a + 4*b*x^n)/(12*b^2*n*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.044, size = 37, normalized size = 0.4

$$-\frac{4\,bx^{n}+a}{12\,\left(a+bx^{n}\right)^{5}b^{2}n}\sqrt{\left(a+bx^{n}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] -1/12*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^5*(4*b*x^n+a)/b^2/n
```

Maxima [A] time = 0.758359, size = 93, normalized size = 1.06

$$-\frac{4\,bx^{n}+a}{12\,\left(b^{6}nx^{4\,n}+4\,ab^{5}nx^{3\,n}+6\,a^{2}b^{4}nx^{2\,n}+4\,a^{3}b^{3}nx^{n}+a^{4}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2),x, algorithm="maxima
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^
4*n*x^(2*n) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Fricas [A] time = 0.273235, size = 93, normalized size = 1.06

$$-\frac{4\,bx^{n}+a}{12\,\left(b^{6}nx^{4\,n}+4\,ab^{5}nx^{3\,n}+6\,a^{2}b^{4}nx^{2\,n}+4\,a^{3}b^{3}nx^{n}+a^{4}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2),x, algorithm="fricas
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^
4*n*x^(2*n) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\left(b^2 x^{2n} + 2\,abx^n + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

3.514 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{6b^2n\,(a+bx^n)^5\,\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\,(a+bx^n)^4\,\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1
/(5*b^2*n*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.122044, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{a}{6b^2n\,(a+bx^n)^5\,\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\,(a+bx^n)^4\,\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1
/(5*b^2*n*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 15.2018, size = 73, normalized size = 0.83

$$\frac{a\,(2a+2bx^n)}{12b^2n\,(a^2+2abx^n+b^2x^{2n})^{\frac{7}{2}}}-\frac{1}{5b^2n\,(a^2+2abx^n+b^2x^{2n})^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

```
[Out] a*(2*a + 2*b*x**n)/(12*b**2*n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))
**(7/2)) - 1/(5*b**2*n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(5/2)
)
```

Mathematica [A] time = 0.0492915, size = 40, normalized size = 0.45

$$-\frac{a+6bx^n}{30b^2n\,(a+bx^n)^5\,\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] -(a + 6*b*x^n)/(30*b^2*n*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.048, size = 37, normalized size = 0.4

$$-\frac{6\,bx^{n}+a}{30\,\left(a+bx^{n}\right)^{7}b^{2}n}\sqrt{\left(a+bx^{n}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x)
```

```
[Out] -1/30*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^7*(6*b*x^n+a)/b^2/n
```

Maxima [A] time = 0.767999, size = 131, normalized size = 1.49

$$-\frac{6\,bx^{n}+a}{30\,\left(b^{8}nx^{6\,n}+6\,ab^{7}nx^{5\,n}+15\,a^{2}b^{6}nx^{4\,n}+20\,a^{3}b^{5}nx^{3\,n}+15\,a^{4}b^{4}nx^{2\,n}+6\,a^{5}b^{3}nx^{n}+a^{6}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2),x, algorithm="maxima
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b
^6*n*x^(4*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^
5*b^3*n*x^n + a^6*b^2*n)
```

Fricas [A] time = 0.271406, size = 131, normalized size = 1.49

$$-\frac{6\,bx^{n}+a}{30\,\left(b^{8}nx^{6\,n}+6\,ab^{7}nx^{5\,n}+15\,a^{2}b^{6}nx^{4\,n}+20\,a^{3}b^{5}nx^{3\,n}+15\,a^{4}b^{4}nx^{2\,n}+6\,a^{5}b^{3}nx^{n}+a^{6}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2),x, algorithm="fricas
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b
^6*n*x^(4*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^
5*b^3*n*x^n + a^6*b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

3.515 $\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=108

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*
(a + b*x^n)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*
x^(2*n)])/((1 + m + n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.103808, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*
(a + b*x^n)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*
x^(2*n)])/((1 + m + n)*(a*b + b^2*x^n))
```

Rubi in Sympy [A] time = 11.4122, size = 97, normalized size = 0.9

$$\frac{2abn\,(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d\,(m+1)\left(2ab+2b^2x^n\right)(m+n+1)}+\frac{(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d\,(m+n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] 2*a*b*n*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(d
*(m + 1)*(2*a*b + 2*b**2*x**n)*(m + n + 1)) + (d*x)**(m + 1)*sqrt
(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(d*(m + n + 1))
```

Mathematica [A] time = 0.0497311, size = 55, normalized size = 0.51

$$\frac{x(dx)^m\sqrt{(a+bx^n)^2}\,(a(m+n+1)+b(m+1)x^n)}{(m+1)(m+n+1)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/(
(1 + m)*(1 + m + n)*(a + b*x^n))
```

Maple [C] time = 0.064, size = 132, normalized size = 1.2

$$\frac{x\,(mbx^n+am+an+bx^n+a)}{(a+bx^n)\,(1+m)\,(1+m+n)}\sqrt{(a+bx^n)^2}\,e^{-\frac{m\left(i\pi\,(\mathrm{csgn}(idx))^3-i\pi\,(\mathrm{csgn}(idx))^2\mathrm{csgn}(id)-i\pi\,(\mathrm{csgn}(idx))^2\mathrm{csgn}(ix)+i\pi\,\mathrm{csgn}(idx)\mathrm{csgn}(id)\mathrm{csgn}(ix)-2\ln(x)-2\ln(d)\right)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(1+m)/(
1+m+n)*exp(-1/2*m*(I*Pi*csgn(I*d*x)^3-I*Pi*csgn(I*d*x)^2*csgn(I*d
)-I*Pi*csgn(I*d*x)^2*csgn(I*x)+I*Pi*csgn(I*d*x)*csgn(I*d)*csgn(I*
x)-2*ln(x)-2*ln(d)))
```

Maxima [A] time = 0.76477, size = 63, normalized size = 0.58

$$\frac{ad^m(m+n+1)xx^m+bd^m(m+1)xe^{(m\log(x)+n\log(x))}}{m^2+m(n+2)+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] (a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)
))/(m^2 + m*(n + 2) + n + 1)
```

Fricas [A] time = 0.274498, size = 77, normalized size = 0.71

$$\frac{(bm+b)xx^n e^{(m\log(d)+m\log(x))}+(am+an+a)xe^{(m\log(d)+m\log(x))}}{m^2+(m+1)n+2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m
*log(d) + m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

GIAC/XCAS [A] time = 0.285689, size = 236, normalized size = 2.19

$$\frac{bmxe^{(m\ln(d)+m\ln(x)+n\ln(x))}\mathrm{sign}(bx^n+a)+amxe^{(m\ln(d)+m\ln(x))}\mathrm{sign}(bx^n+a)+bmxe^{(m\ln(d)+m\ln(x))}\mathrm{sign}(bx^n+a)+anxe^{(m\ln(d)}}{m^2+mn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b*m*x*e^(m*ln(d) + m*ln(x) + n*ln(x))*sign(b*x^n + a) + a*m*x*e^
(m*ln(d) + m*ln(x))*sign(b*x^n + a) + b*m*x*e^(m*ln(d) + m*ln(x))
*sign(b*x^n + a) + a*n*x*e^(m*ln(d) + m*ln(x))*sign(b*x^n + a) +
b*x*e^(m*ln(d) + m*ln(x) + n*ln(x))*sign(b*x^n + a) + a*x*e^(m*ln
(d) + m*ln(x))*sign(b*x^n + a) + b*x*e^(m*ln(d) + m*ln(x))*sign(b
*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

3.516 $\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\,(ab+b^2x^n)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\,(a+bx^n)}$$

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^
2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b +
b^2*x^n))
```

Rubi [A] time = 0.0786598, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\,(ab+b^2x^n)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^
2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b +
b^2*x^n))
```

Rubi in Sympy [A] time = 9.44331, size = 80, normalized size = 0.86

$$\frac{2abnx^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\,(6ab+6b^2x^n)}+\frac{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] 2*a*b*n*x**3*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/((n + 3)*(6*
a*b + 6*b**2*x**n)) + x**3*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)
)/(n + 3)
```

Mathematica [A] time = 0.0277947, size = 46, normalized size = 0.49

$$\frac{x^3\sqrt{(a+bx^n)^2}\,(a(n+3)+3bx^n)}{3(n+3)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b
*x^n))
```

Maple [A] time = 0.021, size = 61, normalized size = 0.7

$$\frac{ax^3}{3\,a+3\,bx^n}\sqrt{(a+bx^n)^2}+\frac{bx^3x^n}{(a+bx^n)\,(3+n)}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^3+((a+b*x^n)^2)^(1/2)/(a+b*
x^n)*b/(3+n)*x^3*x^n
```

Maxima [A] time = 0.754133, size = 34, normalized size = 0.37

$$\frac{3\,bx^3x^n+a(n+3)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

Fricas [A] time = 0.283448, size = 38, normalized size = 0.41

$$\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a + b*x**n)**2), x)
```

GIAC/XCAS [A] time = 0.279095, size = 74, normalized size = 0.8

$$\frac{anx^3\operatorname{sign}(bx^n+a)+3\,bx^3e^{(n\ln(x))}\operatorname{sign}(bx^n+a)+3\,ax^3\operatorname{sign}(bx^n+a)}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(a*n*x^3*sign(b*x^n + a) + 3*b*x^3*e^(n*ln(x))*sign(b*x^n + a
) + 3*a*x^3*sign(b*x^n + a))/(n + 3)
```

3.517 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\,(ab+b^2x^n)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\,(a+bx^n)}$$

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^
2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b +
b^2*x^n))
```

Rubi [A] time = 0.0662483, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\,(ab+b^2x^n)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^
2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b +
b^2*x^n))
```

Rubi in Sympy [A] time = 8.61075, size = 80, normalized size = 0.86

$$\frac{2abnx^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\,(4ab+4b^2x^n)}+\frac{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] 2*a*b*n*x**2*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/((n + 2)*(4*
a*b + 4*b**2*x**n)) + x**2*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)
)/(n + 2)
```

Mathematica [A] time = 0.0266408, size = 46, normalized size = 0.49

$$\frac{x^2\sqrt{(a+bx^n)^2}\,(a(n+2)+2bx^n)}{2(n+2)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b
*x^n))
```

Maple [A] time = 0.022, size = 61, normalized size = 0.7

$$\frac{ax^2}{2\,a+2\,bx^n}\sqrt{(a+bx^n)^2}+\frac{bx^2x^n}{(a+bx^n)\,(2+n)}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^2+((a+b*x^n)^2)^(1/2)/(a+b*
x^n)*b/(2+n)*x^2*x^n
```

Maxima [A] time = 0.755021, size = 34, normalized size = 0.37

$$\frac{2\,bx^2x^n+a(n+2)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

Fricas [A] time = 0.272414, size = 38, normalized size = 0.41

$$\frac{2\,bx^2x^n+(an+2\,a)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{(a + bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a + b*x**n)**2), x)
```

GIAC/XCAS [A] time = 0.278873, size = 74, normalized size = 0.8

$$\frac{anx^2\operatorname{sign}(bx^n + a) + 2\,bx^2e^{(n\ln(x))}\operatorname{sign}(bx^n + a) + 2\,ax^2\operatorname{sign}(bx^n + a)}{2\,(n + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*(a*n*x^2*sign(b*x^n + a) + 2*b*x^2*e^(n*ln(x))*sign(b*x^n + a
) + 2*a*x^2*sign(b*x^n + a))/(n + 2)
```

3.518 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=88

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\,(ab+b^2x^n)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1
 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^
n))
```

Rubi [A] time = 0.0467569, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\,(ab+b^2x^n)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1
 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^
n))
```

Rubi in Sympy [A] time = 3.28758, size = 76, normalized size = 0.86

$$\frac{2abnx\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\,(2ab+2b^2x^n)}+\frac{x\sqrt{a^2+2abx^n+b^2x^{2n}}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] 2*a*b*n*x*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/((n + 1)*(2*a*b
 + 2*b**2*x**n)) + x*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(n +
 1)
```

Mathematica [A] time = 0.0200341, size = 39, normalized size = 0.44

$$\frac{x\sqrt{(a+bx^n)^2}\,(an+a+bx^n)}{(n+1)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

Maple [A] time = 0.02, size = 56, normalized size = 0.6

$$\frac{ax}{a+bx^n}\sqrt{(a+bx^n)^2}+\frac{bxx^n}{(a+bx^n)\,(1+n)}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b
/(1+n)*x*x^n
```

Maxima [A] time = 0.749602, size = 26, normalized size = 0.3

$$\frac{a(n+1)x+bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="maxima")
```

```
[Out] (a*(n + 1)*x + b*x*x^n)/(n + 1)
```

Fricas [A] time = 0.27853, size = 27, normalized size = 0.31

$$\frac{bxx^n+(an+a)x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

GIAC/XCAS [A] time = 0.27933, size = 34, normalized size = 0.39

$$\left(ax + \frac{bx^{n+1}}{n+1}\right)\operatorname{sign}\left(bx^n + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="giac")
```

```
[Out] (a*x + b*x^(n + 1)/(n + 1))*sign(b*x^n + a)
```

3.519 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

Optimal. Leaf size=85

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n))
 + (a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rubi [A] time = 0.0740633, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n))
 + (a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rubi in Sympy [A] time = 8.7676, size = 70, normalized size = 0.82

$$\frac{2ab\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{2ab+2b^2x^n}+\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

```
[Out] 2*a*b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*log(x)/(2*a*b + 2*b
**2*x**n) + sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/n
```

Mathematica [A] time = 0.0208341, size = 37, normalized size = 0.44

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(an\log(x)+bx^n\right)}{n\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n + a*n*Log[x]))/(n*(a + b*x^n))
```

Maple [A] time = 0.026, size = 54, normalized size = 0.6

$$\frac{a\ln(x)}{a+bx^n}\sqrt{(a+bx^n)^2}+\frac{bx^n}{(a+bx^n)\,n}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*ln(x)+((a+b*x^n)^2)^(1/2)/(a+b*x^
n)*b/n*x^n
```

Maxima [A] time = 0.759137, size = 18, normalized size = 0.21

$$a\log(x)+\frac{bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x,x, algorithm="maxima")
```

```
[Out] a*log(x) + b*x^n/n
```

Fricas [A] time = 0.275207, size = 20, normalized size = 0.24

$$\frac{an\log(x)+bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x,x, algorithm="fricas")
```

```
[Out] (a*n*log(x) + b*x^n)/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a + bx^n)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)
```

3.520 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2
*x^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b +
b^2*x^n))
```

Rubi [A] time = 0.0793567, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2
*x^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b +
b^2*x^n))
```

Rubi in Sympy [A] time = 9.31757, size = 76, normalized size = 0.81

$$\frac{2abn\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(-n+1\right)\left(2ab+2b^2x^n\right)}-\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(-n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] 2*a*b*n*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(x*(-n + 1)*(2*a*
b + 2*b**2*x**n)) - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(x*(-
n + 1))
```

Mathematica [A] time = 0.0321772, size = 42, normalized size = 0.45

$$\frac{\sqrt{(a+bx^n)^2}\,(-an+a+bx^n)}{(n-1)x\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

Maple [A] time = 0.029, size = 61, normalized size = 0.7

$$-\frac{a}{(a+bx^n)\,x}\sqrt{(a+bx^n)^2}+\frac{bx^n}{(a+bx^n)\,(-1+n)\,x}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x)
```

```
[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/
(-1+n)*b/x*x^n
```

Maxima [A] time = 0.753678, size = 30, normalized size = 0.32

$$-\frac{a(n-1)-bx^n}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2,x, algorithm="maxima")
```

```
[Out] -(a*(n - 1) - b*x^n)/((n - 1)*x)
```

Fricas [A] time = 0.277106, size = 31, normalized size = 0.33

$$-\frac{an-bx^n-a}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*n - b*x^n - a)/((n - 1)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^n)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

3.521 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (b
^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b
+ b^2*x^n))
```

Rubi [A] time = 0.078336, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (b
^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b
+ b^2*x^n))
```

Rubi in Sympy [A] time = 9.37828, size = 80, normalized size = 0.83

$$\frac{2abn\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2\left(-n+2\right)\left(4ab+4b^2x^n\right)}-\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2\left(-n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] 2*a*b*n*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(x**2*(-n + 2)*(4
*a*b + 4*b**2*x**n)) - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(x
**2*(-n + 2))
```

Mathematica [A] time = 0.0348301, size = 47, normalized size = 0.49

$$\frac{\sqrt{(a+bx^n)^2}\,(2bx^n-a(n-2))}{2(n-2)x^2\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(
a + b*x^n))
```

Maple [A] time = 0.028, size = 61, normalized size = 0.6

$$-\frac{a}{(2\,a+2\,bx^n)\,x^2}\sqrt{(a+bx^n)^2}+\frac{bx^n}{(a+bx^n)\,(-2+n)\,x^2}\sqrt{(a+bx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x^2+((a+b*x^n)^2)^(1/2)/(a+b
*x^n)/(-2+n)*b/x^2*x^n
```

Maxima [A] time = 0.759294, size = 30, normalized size = 0.31

$$-\frac{a(n-2)-2\,bx^n}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

Fricas [A] time = 0.272887, size = 31, normalized size = 0.32

$$-\frac{an-2\,bx^n-2\,a}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a + bx^n)^2}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

3.522 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=238

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\,(ab+b^2x^n)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\,(ab+b^2x^n)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\,(ab+b^2x^n)}+\frac{a^3(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\,(a+bx^n)}$$

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m
)*(a + b*x^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^
n + b^2*x^(2*n)])/((1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 +
 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 2*n)
*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n
 + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.227979, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\,(ab+b^2x^n)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\,(ab+b^2x^n)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\,(ab+b^2x^n)}+\frac{a^3(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m
)*(a + b*x^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^
n + b^2*x^(2*n)])/((1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 +
 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 2*n)
*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n
 + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rubi in Sympy [A] time = 42.7706, size = 248, normalized size = 1.04

$$\frac{a^3b\,(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d\,(m+1)\,(ab+b^2x^n)}+\frac{3a^2b^2x^n\,(dx)^{-n}\,(dx)^{m+n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d\,(ab+b^2x^n)\,(m+n+1)}+\frac{3ab^3x^{2n}\,(dx)^{-2n}\,(dx)^{m+2n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d\,(ab+b^2x^n)\,(m+2n+1)}+\frac{b^4x^{-m}x^{m+3n+1}\,(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(ab+b^2x^n)\,(m+3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] a**3*b*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(d*
(m + 1)*(a*b + b**2*x**n)) + 3*a**2*b**2*x**n*(d*x)**(-n)*(d*x)**
(m + n + 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(d*(a*b + b**
2*x**n)*(m + n + 1)) + 3*a*b**3*x**(2*n)*(d*x)**(-2*n)*(d*x)**(m
+ 2*n + 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(d*(a*b + b**2
*x**n)*(m + 2*n + 1)) + b**4*x**(-m)*x**(m + 3*n + 1)*(d*x)**m*sq
rt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/((a*b + b**2*x**n)*(m + 3*n
 + 1))
```

Mathematica [A] time = 0.153519, size = 90, normalized size = 0.38

$$\frac{x(dx)^m \left((a+bx^n)^2\right)^{3/2} \left(\frac{a^3}{m+1}+\frac{3a^2bx^n}{m+n+1}+\frac{3ab^2x^{2n}}{m+2n+1}+\frac{b^3x^{3n}}{m+3n+1}\right)}{(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1
+ m + n) + (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m
 + 3*n)))/(a + b*x^n)^3
```

Maple [C] time = 0.086, size = 532, normalized size = 2.2

$$x\left(3\,ma^3+6\,b^3mn\,(x^n)^3+3\,a^2bm^3x^n+9\,ab^2m^2\,(x^n)^2+9\,ab^2n^2\,(x^n)^2+9\,a^2bm^2x^n+18\,a^2bn^2x^n+9\,mab^2\,(x^n)^2+12\,ab^2\,(x^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(3*m*a^3+6*b^3*m*n*(x^n)^3+3*a^2*
b*m^3*x^n+9*a*b^2*m^2*(x^n)^2+9*a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n
+18*a^2*b*n^2*x^n+9*m*a*b^2*(x^n)^2+12*a*b^2*(x^n)^2*n+9*m*a^2*b*
x^n+a^3+b^3*(x^n)^3+a^3*m^3+3*a^3*m^2+11*a^3*n^2+6*a^3*n+6*a^3*n^
3+6*a^3*m^2*n+11*a^3*m*n^2+12*a^3*m*n+3*a*b^2*m^3*(x^n)^2+b^3*m^3
*(x^n)^3+3*b^3*m^2*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^
3*(x^n)^3*n+3*a*b^2*(x^n)^2+3*a^2*b*x^n+15*a^2*b*n*x^n+12*a*b^2*m
```

```
^2*n*(x^n)^2+9*a*b^2*m*n^2*(x^n)^2+15*a^2*b*m^2*n*x^n+18*a^2*b*m*
n^2*x^n+3*b^3*m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^3+24*a*b^2*m*n*(x^n
)^2+30*a^2*b*m*n*x^n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*exp(-1/2*
m*(I*Pi*csgn(I*d*x)^3-I*Pi*csgn(I*d*x)^2*csgn(I*d)-I*Pi*csgn(I*d*
x)^2*csgn(I*x)+I*Pi*csgn(I*d*x)*csgn(I*d)*csgn(I*x)-2*ln(x)-2*ln(
d)))
```

Maxima [A] time = 0.767178, size = 373, normalized size = 1.57

$$\frac{\left(m^3+3\,m^2(2\,n+1)+6\,n^3+\left(11\,n^2+12\,n+3\right)m+11\,n^2+6\,n+1\right)a^3d^mxx^m+\left(m^3+3\,m^2(n+1)+\left(2\,n^2+6\,n+3\right)m+2\,n^2+\cdots\right.}{m^4+2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] ((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2
+ 6*n + 1)*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n +
3)*m + 2*n^2 + 3*n + 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(
m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^
2*d^m*x*e^(m*log(x) + 2*n*log(x)) + 3*(m^3 + m^2*(5*n + 3) + (6*n
^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m*x*e^(m*log(x) + n*l
og(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6*n^3
+ 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

Fricas [A] time = 0.285579, size = 527, normalized size = 2.21

$$\frac{\left(b^3m^3+3\,b^3m^2+3\,b^3m+b^3+2\left(b^3m+b^3\right)n^2+3\left(b^3m^2+2\,b^3m+b^3\right)n\right)xx^{3\,n}e^{(m\log(d)+m\log(x))}+3\left(ab^2m^3+3\,ab^2m^2+3\,a\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(
b^3*m^2 + 2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3
*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^
2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(
d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b +
 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x
*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*m^2 +
 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^
3)*n)*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 1
1*(m^2 + 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m
```

```
+ 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.380374, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*(d*x)^m,x, algorithm="giac")

[Out] Done
```

3.523 $\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=212

$$\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (
b^4*x^(3*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)
*(a*b + b^2*x^n)) + (3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b
^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + 2*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.160002, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (
b^4*x^(3*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)
*(a*b + b^2*x^n)) + (3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b
^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + 2*n)*(a*b + b^2*x^n))
```

Rubi in Sympy [A] time = 29.5476, size = 197, normalized size = 0.93

$$\frac{a^3bx^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(n+1\right)\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] a**3*b*x**3*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(3*(a*b + b**
2*x**n)) + 3*a**2*b**2*x**(n + 3)*sqrt(a**2 + 2*a*b*x**n + b**2*x
**(2*n))/((n + 3)*(a*b + b**2*x**n)) + 3*a*b**3*x**(2*n + 3)*sqrt
(a**2 + 2*a*b*x**n + b**2*x**(2*n))/((2*n + 3)*(a*b + b**2*x**n))
 + b**4*x**(3*n + 3)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(3*(
n + 1)*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.106807, size = 123, normalized size = 0.58

$$\frac{x^3\sqrt{(a+bx^n)^2}\left(a^3\left(2n^3+11n^2+18n+9\right)+9a^2b\left(2n^2+5n+3\right)x^n+9ab^2\left(n^2+4n+3\right)x^{2n}+b^3\left(2n^2+9n+9\right)x^{3n}\right)}{3(n+1)(n+3)(2n+3)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2
*b*(3 + 5*n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*
(9 + 9*n + 2*n^2)*x^(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x
^n))
```

Maple [A] time = 0.025, size = 146, normalized size = 0.7

$$\frac{x^3a^3}{3\,a+3\,bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3x^3\,(x^n)^3}{(3\,a+3\,bx^n)\,(1+n)}\sqrt{(a+bx^n)^2}+3\,\frac{\sqrt{(a+bx^n)^2}ab^2x^3\,(x^n)^2}{(a+bx^n)\,(3+2\,n)}+3\,\frac{\sqrt{(a+bx^n)^2}a^2bx^3x^n}{(a+bx^n)\,(3+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x^3*a^3+1/3*((a+b*x^n)^2)^(1/2)
/(a+b*x^n)*b^3*x^3/(1+n)*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*
a*b^2/(3+2*n)*x^3*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(
3+n)*x^3*x^n
```

Maxima [A] time = 0.754999, size = 146, normalized size = 0.69

$$\frac{\left(2\,n^2+9\,n+9\right)b^3x^3x^{3\,n}+9\left(n^2+4\,n+3\right)ab^2x^3x^{2\,n}+9\left(2\,n^2+5\,n+3\right)a^2bx^3x^n+\left(2\,n^3+11\,n^2+18\,n+9\right)a^3x^3}{3\left(2\,n^3+11\,n^2+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*
x^3*x^(2*n) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2
 + 18*n + 9)*a^3*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Fricas [A] time = 0.271024, size = 194, normalized size = 0.92

$$\frac{\left(2\,b^3n^2+9\,b^3n+9\,b^3\right)x^3x^{3\,n}+9\left(ab^2n^2+4\,ab^2n+3\,ab^2\right)x^3x^{2\,n}+9\left(2\,a^2bn^2+5\,a^2bn+3\,a^2b\right)x^3x^n+\left(2\,a^3n^3+11\,a^3n^2+18\right.}{3\left(2\,n^3+11\,n^2+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4
*a*b^2*n + 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*
a^2*b)*x^3*x^n + (2*a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)
/(2*n^3 + 11*n^2 + 18*n + 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.290506, size = 410, normalized size = 1.93

$$\frac{2\,a^3n^3x^3\text{sign}\left(bx^n+a\right)+2\,b^3n^2x^3e^{(3\,n\ln(x))}\text{sign}\left(bx^n+a\right)+9\,ab^2n^2x^3e^{(2\,n\ln(x))}\text{sign}\left(bx^n+a\right)+18\,a^2bn^2x^3e^{(n\ln(x))}\text{sign}\left(bx^n+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*a^3*n^3*x^3*sign(b*x^n + a) + 2*b^3*n^2*x^3*e^(3*n*ln(x))*
sign(b*x^n + a) + 9*a*b^2*n^2*x^3*e^(2*n*ln(x))*sign(b*x^n + a) +
 18*a^2*b*n^2*x^3*e^(n*ln(x))*sign(b*x^n + a) + 11*a^3*n^2*x^3*si
gn(b*x^n + a) + 9*b^3*n*x^3*e^(3*n*ln(x))*sign(b*x^n + a) + 36*a*
b^2*n*x^3*e^(2*n*ln(x))*sign(b*x^n + a) + 45*a^2*b*n*x^3*e^(n*ln(
x))*sign(b*x^n + a) + 18*a^3*n*x^3*sign(b*x^n + a) + 9*b^3*x^3*e^
(3*n*ln(x))*sign(b*x^n + a) + 27*a*b^2*x^3*e^(2*n*ln(x))*sign(b*x
^n + a) + 27*a^2*b*x^3*e^(n*ln(x))*sign(b*x^n + a) + 9*a^3*x^3*si
gn(b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

3.524 $\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=211

$$\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (
3*a*b^3*x^(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1
+ n)*(a*b + b^2*x^n)) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n
 + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + 3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.139376, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (
3*a*b^3*x^(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1
+ n)*(a*b + b^2*x^n)) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n
 + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + 3*n)*(a*b + b^2*x^n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3b\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\,dx}{ab+b^2x^n}+\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(n+1\right)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] a**3*b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*Integral(x, x)/(a*
b + b**2*x**n) + 3*a**2*b**2*x**(n + 2)*sqrt(a**2 + 2*a*b*x**n +
b**2*x**(2*n))/((n + 2)*(a*b + b**2*x**n)) + 3*a*b**3*x**(2*n + 2
)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(2*(n + 1)*(a*b + b**2*
x**n)) + b**4*x**(3*n + 2)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)
)/((3*n + 2)*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.106605, size = 124, normalized size = 0.59

$$\frac{x^2\sqrt{(a+bx^n)^2}\left(a^3\left(3n^3+11n^2+12n+4\right)+6a^2b\left(3n^2+5n+2\right)x^n+3ab^2\left(3n^2+8n+4\right)x^{2n}+2b^3\left(n^2+3n+2\right)x^{3n}\right)}{2(n+1)(n+2)(3n+2)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2
*b*(2 + 5*n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*
b^3*(2 + 3*n + n^2)*x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b
*x^n))
```

Maple [A] time = 0.025, size = 145, normalized size = 0.7

$$\frac{x^2a^3}{2\,a+2\,bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3x^2\,(x^n)^3}{(a+bx^n)\,(2+3\,n)}\sqrt{(a+bx^n)^2}+\frac{3\,ab^2x^2\,(x^n)^2}{(2\,a+2\,bx^n)\,(1+n)}\sqrt{(a+bx^n)^2}+3\,\frac{\sqrt{(a+bx^n)^2}a^2bx^2x^n}{(a+bx^n)\,(2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x^2*a^3+((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*b^3/(2+3*n)*x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*
a*b^2*x^2/(1+n)*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(2+
n)*x^2*x^n
```

Maxima [A] time = 0.754804, size = 147, normalized size = 0.7

$$\frac{2\left(n^2+3\,n+2\right)b^3x^2x^{3\,n}+3\left(3\,n^2+8\,n+4\right)ab^2x^2x^{2\,n}+6\left(3\,n^2+5\,n+2\right)a^2bx^2x^n+\left(3\,n^3+11\,n^2+12\,n+4\right)a^3x^2}{2\left(3\,n^3+11\,n^2+12\,n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^
2*x^2*x^(2*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n
^2 + 12*n + 4)*a^3*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Fricas [A] time = 0.283687, size = 196, normalized size = 0.93

$$\frac{2\left(b^3n^2+3\,b^3n+2\,b^3\right)x^2x^{3\,n}+3\left(3\,ab^2n^2+8\,ab^2n+4\,ab^2\right)x^2x^{2\,n}+6\left(3\,a^2bn^2+5\,a^2bn+2\,a^2b\right)x^2x^n+\left(3\,a^3n^3+11\,a^3n^2+\right.}{2\left(3\,n^3+11\,n^2+12\,n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 +
 8*a*b^2*n + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n +
2*a^2*b)*x^2*x^n + (3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^
2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.290692, size = 410, normalized size = 1.94

$$\frac{3\,a^3n^3x^2\mathrm{sign}\left(bx^n+a\right)+2\,b^3n^2x^2e^{(3\,n\ln(x))}\mathrm{sign}\left(bx^n+a\right)+9\,ab^2n^2x^2e^{(2\,n\ln(x))}\mathrm{sign}\left(bx^n+a\right)+18\,a^2bn^2x^2e^{(n\ln(x))}\mathrm{sign}\left(bx^n+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^3*n^3*x^2*sign(b*x^n + a) + 2*b^3*n^2*x^2*e^(3*n*ln(x))*
sign(b*x^n + a) + 9*a*b^2*n^2*x^2*e^(2*n*ln(x))*sign(b*x^n + a) +
 18*a^2*b*n^2*x^2*e^(n*ln(x))*sign(b*x^n + a) + 11*a^3*n^2*x^2*si
gn(b*x^n + a) + 6*b^3*n*x^2*e^(3*n*ln(x))*sign(b*x^n + a) + 24*a*
b^2*n*x^2*e^(2*n*ln(x))*sign(b*x^n + a) + 30*a^2*b*n*x^2*e^(n*ln(
x))*sign(b*x^n + a) + 12*a^3*n*x^2*sign(b*x^n + a) + 4*b^3*x^2*e^
(3*n*ln(x))*sign(b*x^n + a) + 12*a*b^2*x^2*e^(2*n*ln(x))*sign(b*x
^n + a) + 12*a^2*b*x^2*e^(n*ln(x))*sign(b*x^n + a) + 4*a^3*x^2*si
gn(b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

3.525 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=206

$$\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(a+bx^n)^3}$$

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*
a^2*b^4*x^(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)
*(a*b + b^2*x^n)^3) + (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2
*x^(2*n))^(3/2))/((1 + 2*n)*(a*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)
*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 3*n)*(a*b + b^2*x^n
)^3)
```

Rubi [A] time = 0.120441, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*
a^2*b^4*x^(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)
*(a*b + b^2*x^n)^3) + (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2
*x^(2*n))^(3/2))/((1 + 2*n)*(a*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)
*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 3*n)*(a*b + b^2*x^n
)^3)
```

Rubi in Sympy [A] time = 18.1789, size = 194, normalized size = 0.94

$$\frac{12a^3bn^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)(2n+1)(3n+1)\left(2ab+2b^2x^n\right)}+\frac{6a^2n^2x\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)(2n+1)(3n+1)}+\frac{3nx\left(2a^2+2abx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(2n+1)(3n+1)}+\frac{x\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{3}{2}}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] 12*a**3*b*n**3*x*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/((n + 1)
*(2*n + 1)*(3*n + 1)*(2*a*b + 2*b**2*x**n)) + 6*a**2*n**2*x*sqrt(
a**2 + 2*a*b*x**n + b**2*x**(2*n))/((n + 1)*(2*n + 1)*(3*n + 1))
+ 3*n*x*(2*a**2 + 2*a*b*x**n)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2
*n))/(2*(2*n + 1)*(3*n + 1)) + x*(a**2 + 2*a*b*x**n + b**2*x**(2*
n))**(3/2)/(3*n + 1)
```

Mathematica [A] time = 0.0978857, size = 122, normalized size = 0.59

$$\frac{x\sqrt{(a+bx^n)^2}\left(a^3\left(6n^3+11n^2+6n+1\right)+3a^2b\left(6n^2+5n+1\right)x^n+3ab^2\left(3n^2+4n+1\right)x^{2n}+b^3\left(2n^2+3n+1\right)x^{3n}\right)}{(n+1)(2n+1)(3n+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*
(1 + 5*n + 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(
1 + 3*n + 2*n^2)*x^(3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^
n))
```

Maple [A] time = 0.024, size = 138, normalized size = 0.7

$$\frac{a^3x}{a+bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3x\,(x^n)^3}{(a+bx^n)\,(1+3\,n)}\sqrt{(a+bx^n)^2}+3\,\frac{\sqrt{(a+bx^n)^2}ab^2x\,(x^n)^2}{(a+bx^n)\,(1+2\,n)}+3\,\frac{\sqrt{(a+bx^n)^2}a^2bxx^n}{(a+bx^n)\,(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)
*b^3/(1+3*n)*x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/(1+2
*n)*x*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(1+n)*x*x^n
```

Maxima [A] time = 0.755722, size = 136, normalized size = 0.66

$$\frac{\left(2\,n^2+3\,n+1\right)b^3xx^{3\,n}+3\,\left(3\,n^2+4\,n+1\right)ab^2xx^{2\,n}+3\,\left(6\,n^2+5\,n+1\right)a^2bxx^n+\left(6\,n^3+11\,n^2+6\,n+1\right)a^3x}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^
(2*n) + 3*(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n +
 1)*a^3*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Fricas [A] time = 0.275877, size = 176, normalized size = 0.85

$$\frac{\left(2\,b^{3}n^{2}+3\,b^{3}n+b^{3}\right)xx^{3\,n}+3\,\left(3\,ab^{2}n^{2}+4\,ab^{2}n+ab^{2}\right)xx^{2\,n}+3\,\left(6\,a^{2}bn^{2}+5\,a^{2}bn+a^{2}b\right)xx^{n}+\left(6\,a^{3}n^{3}+11\,a^{3}n^{2}+6\,a^{3}n+a\right.}{6\,n^{3}+11\,n^{2}+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2
*n + a*b^2)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n
 + (6*a^3*n^3 + 11*a^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 +
6*n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.288559, size = 371, normalized size = 1.8

$$\frac{6\,a^{3}n^{3}x\mathrm{sign}\left(bx^{n}+a\right)+2\,b^{3}n^{2}xe^{(3\,n\ln(x))}\mathrm{sign}\left(bx^{n}+a\right)+9\,ab^{2}n^{2}xe^{(2\,n\ln(x))}\mathrm{sign}\left(bx^{n}+a\right)+18\,a^{2}bn^{2}xe^{(n\ln(x))}\mathrm{sign}\left(bx^{n}+a\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] (6*a^3*n^3*x*sign(b*x^n + a) + 2*b^3*n^2*x*e^(3*n*ln(x))*sign(b*x
^n + a) + 9*a*b^2*n^2*x*e^(2*n*ln(x))*sign(b*x^n + a) + 18*a^2*b*
n^2*x*e^(n*ln(x))*sign(b*x^n + a) + 11*a^3*n^2*x*sign(b*x^n + a)
+ 3*b^3*n*x*e^(3*n*ln(x))*sign(b*x^n + a) + 12*a*b^2*n*x*e^(2*n*l
n(x))*sign(b*x^n + a) + 15*a^2*b*n*x*e^(n*ln(x))*sign(b*x^n + a)
+ 6*a^3*n*x*sign(b*x^n + a) + b^3*x*e^(3*n*ln(x))*sign(b*x^n + a)
 + 3*a*b^2*x*e^(2*n*ln(x))*sign(b*x^n + a) + 3*a^2*b*x*e^(n*ln(x)
)*sign(b*x^n + a) + a^3*x*sign(b*x^n + a))/(6*n^3 + 11*n^2 + 6*n
+ 1)
```

3.526 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx$

Optimal. Leaf size=196

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(ab+b^2x^n)}+\frac{a^3\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2
*x^n)) + (3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2
*n*(a*b + b^2*x^n)) + (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(
2*n)])/(3*n*(a*b + b^2*x^n)) + (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^
(2*n)]*Log[x])/(a + b*x^n)
```

Rubi [A] time = 0.139127, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(ab+b^2x^n)}+\frac{a^3\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```


```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2
*x^n)) + (3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2
*n*(a*b + b^2*x^n)) + (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(
2*n)])/(3*n*(a*b + b^2*x^n)) + (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^
(2*n)]*Log[x])/(a + b*x^n)
```

Rubi in Sympy [A] time = 18.0133, size = 143, normalized size = 0.73

$$\frac{2a^3b\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{2ab+2b^2x^n}+\frac{a^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{n}+\frac{\left(2a^2+2abx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n}+\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
```

```
[Out] 2*a**3*b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*log(x)/(2*a*b +
2*b**2*x**n) + a**2*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/n + (
2*a**2 + 2*a*b*x**n)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(4*n
) + (a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2)/(3*n)
```

Mathematica [A] time = 0.0640078, size = 66, normalized size = 0.34

$$\frac{\sqrt{(a+bx^n)^2}\left(6a^3n\log(x)+bx^n\left(18a^2+9abx^n+2b^2x^{2n}\right)\right)}{6n\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n))
+ 6*a^3*n*Log[x]))/(6*n*(a + b*x^n))
```

Maple [A] time = 0.031, size = 127, normalized size = 0.7

$$\frac{a^3\ln(x)}{a+bx^n}\sqrt{(a+bx^n)^2}+\frac{b^3\,(x^n)^3}{(3\,a+3\,bx^n)\,n}\sqrt{(a+bx^n)^2}+\frac{3\,ab^2\,(x^n)^2}{(2\,a+2\,bx^n)\,n}\sqrt{(a+bx^n)^2}+3\,\frac{\sqrt{(a+bx^n)^2}a^2bx^n}{(a+bx^n)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*ln(x)+1/3*((a+b*x^n)^2)^(1/2)/(
a+b*x^n)*b^3/n*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/n*
(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/n*x^n
```

Maxima [A] time = 0.753302, size = 58, normalized size = 0.3

$$a^3\log(x)+\frac{2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)
/n
```

Fricas [A] time = 0.271802, size = 59, normalized size = 0.3

$$\frac{6\,a^3n\log\left(x\right)+2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*
x^n)/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

3.527 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2} dx$

Optimal. Leaf size=212

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3
*a^2*b^2*x^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)
*(a*b + b^2*x^n)) - (3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n +
b^2*x^(2*n)])/((1 - 2*n)*(a*b + b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.169799, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3
*a^2*b^2*x^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)
*(a*b + b^2*x^n)) - (3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n +
b^2*x^(2*n)])/((1 - 2*n)*(a*b + b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 3*n)*(a*b + b^2*x^n))
```

Rubi in Sympy [A] time = 30.2446, size = 194, normalized size = 0.92

$$-\frac{a^3b\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(-n+1)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(-2n+1)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(-3n+1)\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] -a**3*b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(x*(a*b + b**2*x*
*n)) - 3*a**2*b**2*x**(n - 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2
*n))/((-n + 1)*(a*b + b**2*x**n)) - 3*a*b**3*x**(2*n - 1)*sqrt(a*
*2 + 2*a*b*x**n + b**2*x**(2*n))/((-2*n + 1)*(a*b + b**2*x**n)) -
 b**4*x**(3*n - 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/((-3*n
 + 1)*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.147203, size = 124, normalized size = 0.58

$$\frac{\sqrt{(a+bx^n)^2}\left(a^3\left(-6n^3+11n^2-6n+1\right)+3a^2b\left(6n^2-5n+1\right)x^n+3ab^2\left(3n^2-4n+1\right)x^{2n}+b^3\left(2n^2-3n+1\right)x^{3n}\right)}{(n-1)(2n-1)(3n-1)x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1
 - 5*n + 6*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1
- 3*n + 2*n^2)*x^(3*n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b
*x^n))
```

Maple [A] time = 0.038, size = 147, normalized size = 0.7

$$-\frac{a^3}{(a+bx^n)\,x}\sqrt{(a+bx^n)^2}+\frac{b^3\,(x^n)^3}{(a+bx^n)\,(-1+3\,n)\,x}\sqrt{(a+bx^n)^2}+3\,\frac{\sqrt{(a+bx^n)^2}ab^2\,(x^n)^2}{(a+bx^n)\,(-1+2\,n)\,x}+3\,\frac{\sqrt{(a+bx^n)^2}a^2bx^n}{(a+bx^n)\,(-1+n)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n
)/(-1+3*n)*b^3/x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+2*n)
*a*b^2/x*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*a^2*b/x*x
^n
```

Maxima [A] time = 0.756412, size = 136, normalized size = 0.64

$$\frac{\left(2\,n^2-3\,n+1\right)b^3x^{3\,n}+3\,\left(3\,n^2-4\,n+1\right)ab^2x^{2\,n}+3\,\left(6\,n^2-5\,n+1\right)a^2bx^n-\left(6\,n^3-11\,n^2+6\,n-1\right)a^3}{(6\,n^3-11\,n^2+6\,n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n
) + 3*(6*n^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^
3)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

Fricas [A] time = 0.288842, size = 177, normalized size = 0.83

$$-\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-\left(2\,b^3n^2-3\,b^3n+b^3\right)x^{3\,n}-3\,\left(3\,ab^2n^2-4\,ab^2n+ab^2\right)x^{2\,n}-3\,\left(6\,a^2bn^2-5\,a^2bn+a^2b\right)x^n}{(6\,n^3-11\,n^2+6\,n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n +
 b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(
6*a^2*b*n^2 - 5*a^2*b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)
*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

3.528 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=218

$$-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\,(ab+b^2x^n)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\,(ab+b^2x^n)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\,(ab+b^2x^n)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) -
(3*a*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1
- n))*(a*b + b^2*x^n)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x
^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*
Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - 3*n)*(a*b + b^2*x^n))
```

Rubi [A] time = 0.168925, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\,(ab+b^2x^n)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\,(ab+b^2x^n)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\,(ab+b^2x^n)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) -
(3*a*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1
- n))*(a*b + b^2*x^n)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x
^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*
Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - 3*n)*(a*b + b^2*x^n))
```

Rubi in Sympy [A] time = 29.8733, size = 197, normalized size = 0.9

$$-\frac{a^3b\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(ab+b^2x^n)}-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(-n+2)\,(ab+b^2x^n)}-\frac{3ab^3x^{2n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\,(-n+1)\,(ab+b^2x^n)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(-3n+2)\,(ab+b^2x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] -a**3*b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(2*x**2*(a*b + b*
*2*x**n)) - 3*a**2*b**2*x**(n - 2)*sqrt(a**2 + 2*a*b*x**n + b**2*
x**(2*n))/((-n + 2)*(a*b + b**2*x**n)) - 3*a*b**3*x**(2*n - 2)*sq
rt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(2*(-n + 1)*(a*b + b**2*x**
n)) - b**4*x**(3*n - 2)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))/(
(-3*n + 2)*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.143147, size = 124, normalized size = 0.57

$$\frac{\sqrt{(a+bx^n)^2}\left(a^3\left(-3n^3+11n^2-12n+4\right)+6a^2b\left(3n^2-5n+2\right)x^n+3ab^2\left(3n^2-8n+4\right)x^{2n}+2b^3\left(n^2-3n+2\right)x^{3n}\right)}{2(n-2)(n-1)(3n-2)x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(
2 - 5*n + 3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*
(2 - 3*n + n^2)*x^(3*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a
+ b*x^n))
```

Maple [A] time = 0.039, size = 145, normalized size = 0.7

$$-\frac{a^3}{(2a+2bx^n)x^2}\sqrt{(a+bx^n)^2}+\frac{b^3(x^n)^3}{(a+bx^n)(-2+3n)x^2}\sqrt{(a+bx^n)^2}+\frac{3ab^2(x^n)^2}{(2a+2bx^n)(-1+n)x^2}\sqrt{(a+bx^n)^2}+3\frac{\sqrt{(a+bx^n)^2}a^2bx^n}{(a+bx^n)(-2+n)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x^2+((a+b*x^n)^2)^(1/2)/(a
+b*x^n)/(-2+3*n)*b^3/x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n
)/(-1+n)*a*b^2/x^2*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+n)
*a^2*b/x^2*x^n
```

Maxima [A] time = 0.751542, size = 136, normalized size = 0.62

$$\frac{2\left(n^{2}-3\,n+2\right)b^{3}x^{3\,n}+3\left(3\,n^{2}-8\,n+4\right)ab^{2}x^{2\,n}+6\left(3\,n^{2}-5\,n+2\right)a^{2}bx^{n}-\left(3\,n^{3}-11\,n^{2}+12\,n-4\right)a^{3}}{2\,(3\,n^{3}-11\,n^{2}+12\,n-4)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^
(2*n) + 6*(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n -
4)*a^3)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

Fricas [A] time = 0.272873, size = 181, normalized size = 0.83

$$-\frac{3\,a^{3}n^{3}-11\,a^{3}n^{2}+12\,a^{3}n-4\,a^{3}-2\left(b^{3}n^{2}-3\,b^{3}n+2\,b^{3}\right)x^{3\,n}-3\left(3\,ab^{2}n^{2}-8\,ab^{2}n+4\,ab^{2}\right)x^{2\,n}-6\left(3\,a^{2}bn^{2}-5\,a^{2}bn+2\right.}{2\,(3\,n^{3}-11\,n^{2}+12\,n-4)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*
b^3*n + 2*b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^
(2*n) - 6*(3*a^2*b*n^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n
^2 + 12*n - 4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

3.529 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{m+1}\left(a+bx^n\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x
^(2*n)])
```

Rubi [A] time = 0.098685, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(dx)^{m+1}\left(a+bx^n\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x
^(2*n)])
```

Rubi in Sympy [A] time = 16.8283, size = 66, normalized size = 0.87

$$\frac{b\,(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}1,\frac{m+1}{n}\\ \frac{m+n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ad\,(m+1)\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] b*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((1
, (m + 1)/n), ((m + n + 1)/n,), -b*x**n/a)/(a*d*(m + 1)*(a*b + b*
*2*x**n))
```

Mathematica [A] time = 0.0478835, size = 62, normalized size = 0.82

$$\frac{x(dx)^m (a + bx^n) \,_2F_1\left(1, \frac{m+1}{n}; \frac{m+1}{n} + 1; -\frac{bx^n}{a}\right)}{a(m + 1)\sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m
)/n, -((b*x^n)/a)])/(a*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{a^2 + 2\,abx^n + b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**n)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.530 $\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\left(a+bx^n\right)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a
)])/(3*a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0784323, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{x^3\left(a+bx^n\right)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a
)])/(3*a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 15.3922, size = 56, normalized size = 0.88

$$\frac{bx^3\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}1,\frac{3}{n}\\\frac{n+3}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{3a\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] b*x**3*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((1, 3/n), ((
n + 3)/n,), -b*x**n/a)/(3*a*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.0357715, size = 53, normalized size = 0.83

$$\frac{x^3(a+bx^n)\, {}_2F_1\left(1,\frac{3}{n};1+\frac{3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*x^n)/a)]
)/(3*a*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt{a^2+2\,abx^n+b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a + b*x**n)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.531 $\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0596151, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^2(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 13.8874, size = 56, normalized size = 0.88

$$\frac{bx^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}1,\frac{2}{n}\\ \frac{n+2}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{2a\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] b*x**2*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((1, 2/n), ((
n + 2)/n,), -b*x**n/a)/(2*a*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.032477, size = 53, normalized size = 0.83

$$\frac{x^2(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)]
)/(2*a*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a^2+2\,abx^n+b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a + b*x**n)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.532 $\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=55

$$\frac{x\,(a+bx^n)\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0383224, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x\,(a+bx^n)\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 4.94657, size = 53, normalized size = 0.96

$$\frac{bx\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a\,(ab+b^2x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] b*x*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((1, 1/n), (1 +
1/n,), -b*x**n/a)/(a*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.0202204, size = 44, normalized size = 0.8

$$\frac{x\left(a+bx^{n}\right)\,{}_{2}F_{1}\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^{n}}{a}\right)}{a\sqrt{\left(a+bx^{n}\right)^{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2}+2\,abx^{n}+b^{2}x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.533 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=85

$$\frac{\log(x)\,(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((
a + b*x^n)*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n
)])
```

Rubi [A] time = 0.101774, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{\log(x)\,(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((
a + b*x^n)*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n
)])
```

Rubi in Sympy [A] time = 20.457, size = 90, normalized size = 1.06

$$\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\log\left(x^n\right)}{an\left(ab+b^2x^n\right)}-\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\log\left(a+bx^n\right)}{an\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*log(x**n)/(a*n*(a*b + b
**2*x**n)) - b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*log(a + b*
x**n)/(a*n*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.0352295, size = 42, normalized size = 0.49

$$\frac{(a+bx^n)\,(n\log(x)-\log{(a+bx^n)})}{an\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*(n*Log[x] - Log[a + b*x^n]))/(a*n*Sqrt[(a + b*x^n)^2
])
```

Maple [A] time = 0.029, size = 66, normalized size = 0.8

$$\frac{\ln{(x)}}{(a+bx^n)\,a}\sqrt{(a+bx^n)^2}-\frac{1}{(a+bx^n)\,an}\sqrt{(a+bx^n)^2}\ln{\left(x^n+\frac{a}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a-((a+b*x^n)^2)^(1/2)/(a+b*x^
n)/a/n*ln(x^n+a/b)
```

Maxima [A] time = 0.763262, size = 36, normalized size = 0.42

$$\frac{\log{(x)}}{a}-\frac{\log{\left(\frac{bx^n+a}{b}\right)}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Fricas [A] time = 0.281927, size = 30, normalized size = 0.35

$$\frac{n\log{(x)}-\log{(bx^n+a)}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

3.534 $\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rubi [A] time = 0.0740505, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rubi in Sympy [A] time = 14.8806, size = 56, normalized size = 0.86

$$-\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}1,-\frac{1}{n}\\ \frac{n-1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ax\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] -b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((1, -1/n), ((n -
 1)/n,), -b*x**n/a)/(a*x*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.0291856, size = 51, normalized size = 0.78

$$-\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n
)/a)])/(a*x*Sqrt[(a + b*x^n)^2]))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a^2+2\,abx^n+b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{(a + bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a + b*x**n)**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

3.535 $\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -((a + b*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0728333, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 15.0622, size = 60, normalized size = 0.9

$$-\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}1,-\frac{2}{n}\\ \frac{n-2}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{2ax^2\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] -b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((1, -2/n), ((n -
 2)/n,), -b*x**n/a)/(2*a*x**2*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.029274, size = 53, normalized size = 0.79

$$-\frac{(a+bx^n)\,_2F_1\left(1,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)])/
(2*a*x^2*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt{a^2+2\,abx^n+b^2x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a + b*x**n)**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

3.536 $\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{m+1}\,(a+bx^n)\;{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2
*x^(2*n)])
```

Rubi [A] time = 0.0952113, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{(dx)^{m+1}\,(a+bx^n)\;{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2
*x^(2*n)])
```

Rubi in Sympy [A] time = 16.3555, size = 68, normalized size = 0.89

$$\frac{b\,(dx)^{m+1}\,\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}3,\frac{m+1}{n}\\ \frac{m+n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a^3d\,(m+1)\,(ab+b^2x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] b*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((3
, (m + 1)/n), ((m + n + 1)/n,), -b*x**n/a)/(a**3*d*(m + 1)*(a*b +
 b**2*x**n))
```

Mathematica [A] time = 0.19964, size = 119, normalized size = 1.57

$$\frac{x(dx)^m\left(a+bx^n\right)\left(a^2n+\frac{\left(m^2+m(2-3n)+2n^2-3n+1\right)\left(a+bx^n\right)^2{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{m+1}-a(m-2n+1)\left(a+bx^n\right)\right)}{2a^3n^2\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*(a^2*n - a*(1 + m - 2*n)*(a + b*x^n) + ((1
 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2)*(a + b*x^n)^2*Hypergeometric2
F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(1 + m)))/(2*a^3*n
^2*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int (dx)^m\left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(m^2-m(3\,n-2)+2\,n^2-3\,n+1\right)d^m\int\frac{x^m}{2\left(a^2bn^2x^n+a^3n^2\right)}\,dx-\frac{ad^m(m-3\,n+1)xx^m+bd^m(m-2\,n+1)xe^{(m\log(x)+n\log(x))}}{2\left(a^2b^2n^2x^{2\,n}+2\,a^3bn^2x^n+a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*d^m*integrate(1/2*x^m/(a^2*
b*n^2*x^n + a^3*n^2), x) - 1/2*(a*d^m*(m - 3*n + 1)*x*x^m + b*d^m
```

```
*(m - 2*n + 1)*x*e^(m*log(x) + n*log(x)))/(a^2*b^2*n^2*x^(2*n) +
2*a^3*b*n^2*x^n + a^4*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.537 $\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\left(a+bx^n\right)\,_2F_1\left(3,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a
)])/(3*a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0770577, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{x^3\left(a+bx^n\right)\,_2F_1\left(3,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a
)])/(3*a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 14.6689, size = 58, normalized size = 0.91

$$\frac{bx^3\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}3,\ \frac{3}{n}\\ \frac{n+3}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{3a^3\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] b*x**3*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((3, 3/n), ((
n + 3)/n,), -b*x**n/a)/(3*a**3*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.102641, size = 98, normalized size = 1.53

$$\frac{x^3 (a+bx^n) \left(3a^2n+\left(2n^2-9n+9\right)(a+bx^n)^2\ {}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)+3a(2n-3)(a+bx^n)\right)}{6a^3n^2\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)*(3*a^2*n + 3*a*(-3 + 2*n)*(a + b*x^n) + (9 - 9*n
 + 2*n^2)*(a + b*x^n)^2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b
*x^n)/a)]))/(6*a^3*n^2*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^2\left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,n^2-9\,n+9\right)\int\frac{x^2}{2\,(a^2bn^2x^n+a^3n^2)}\,dx+\frac{b(2\,n-3)x^3x^n+3\,a(n-1)x^3}{2\,(a^2b^2n^2x^{2\,n}+2\,a^3bn^2x^n+a^4n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 9*n + 9)*integrate(1/2*x^2/(a^2*b*n^2*x^n + a^3*n^2), x)
 + 1/2*(b*(2*n - 3)*x^3*x^n + 3*a*(n - 1)*x^3)/(a^2*b^2*n^2*x^(2*
n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**n)**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.538 $\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2(a+bx^n)\,{}_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.0596573, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^2(a+bx^n)\,{}_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 13.1949, size = 58, normalized size = 0.91

$$\frac{bx^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}3,\frac{2}{n}\\ \frac{n+2}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{2a^3\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] b*x**2*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((3, 2/n), ((
n + 2)/n,), -b*x**n/a)/(2*a**3*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.0936577, size = 93, normalized size = 1.45

$$\frac{x^2 (a+bx^n) \left(a^2 n+\left(n^2-3n+2\right) (a+bx^n)^2 \, {}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)+2a(n-1) (a+bx^n)\right)}{2a^3 n^2 \left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)*(a^2*n + 2*a*(-1 + n)*(a + b*x^n) + (2 - 3*n + n
^2)*(a + b*x^n)^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/
a)]))/(2*a^3*n^2*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x \left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(n^2-3\,n+2\right)\int \frac{x}{a^2bn^2x^n+a^3n^2}\,dx+\frac{2\,b(n-1)x^2x^n+a(3\,n-2)x^2}{2\,(a^2b^2n^2x^{2\,n}+2\,a^3bn^2x^n+a^4n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(
2*b*(n - 1)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a
^3*b*n^2*x^n + a^4*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**n)**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.539 $\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{x\,(a+bx^n)^3\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\,(a^2+2abx^n+b^2x^{2n})^{3/2}}$$

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^
n)/a)])/(a^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Rubi [A] time = 0.0372614, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x\,(a+bx^n)^3\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\,(a^2+2abx^n+b^2x^{2n})^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^
n)/a)])/(a^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Rubi in Sympy [A] time = 5.08792, size = 54, normalized size = 0.95

$$\frac{bx\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}3,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a^3\,(ab+b^2x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] b*x*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((3, 1/n), (1 +
1/n,), -b*x**n/a)/(a**3*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.124598, size = 93, normalized size = 1.63

$$\frac{x\left(\left(2n^2-3n+1\right)\left(a+bx^n\right)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+a\left(a(3n-1)+b(2n-1)x^n\right)\right)}{2a^3n^2\left(a+bx^n\right)\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a*(a*(-1 + 3*n) + b*(-1 + 2*n)*x^n) + (1 - 3*n + 2*n^2)*(a +
b*x^n)^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))
/(2*a^3*n^2*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,n^2-3\,n+1\right)\int \frac{1}{2\left(a^2bn^2x^n+a^3n^2\right)}\,dx+\frac{b(2\,n-1)xx^n+a(3\,n-1)x}{2\left(a^2b^2n^2x^{2\,n}+2\,a^3bn^2x^n+a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1
/2*(b*(2*n - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a
^3*b*n^2*x^n + a^4*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(b^{2}x^{2\,n} + 2\,abx^{n} + a^{2}\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} + 2abx^{n} + b^{2}x^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2}x^{2\,n} + 2\,abx^{n} + a^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

3.540 $\int \frac{1}{x(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\,(a+bx^n)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x
^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(
a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b
*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.184502, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\,(a+bx^n)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x
^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(
a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b
*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 31.2413, size = 163, normalized size = 1.03

$$\frac{2a+2bx^n}{4an\,(a^2+2abx^n+b^2x^{2n})^{\frac{3}{2}}}+\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x^n)}{a^3n\,(ab+b^2x^n)}-\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\log(a+bx^n)}{a^3n\,(ab+b^2x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] (2*a + 2*b*x**n)/(4*a*n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2
)) + 1/(a**2*n*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))) + b*sqrt(
a**2 + 2*a*b*x**n + b**2*x**(2*n))*log(x**n)/(a**3*n*(a*b + b**2*
x**n)) - b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*log(a + b*x**n
)/(a**3*n*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.104482, size = 78, normalized size = 0.49

$$\frac{a\left(3a+2bx^{n}\right)+2n\log(x)\left(a+bx^{n}\right)^{2}-2\left(a+bx^{n}\right)^{2}\log\left(a+bx^{n}\right)}{2a^{3}n\left(a+bx^{n}\right)\sqrt{\left(a+bx^{n}\right)^{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^n) + 2*n*(a + b*x^n)^2*Log[x] - 2*(a + b*x^n)^2*L
og[a + b*x^n])/(2*a^3*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Maple [A] time = 0.031, size = 104, normalized size = 0.7

$$\frac{\ln(x)}{(a+bx^{n})a^{3}}\sqrt{(a+bx^{n})^{2}}+\frac{2\,bx^{n}+3\,a}{2\,(a+bx^{n})^{3}a^{2}n}\sqrt{(a+bx^{n})^{2}}-\frac{1}{(a+bx^{n})a^{3}n}\sqrt{(a+bx^{n})^{2}}\ln\left(x^{n}+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a^3+1/2*((a+b*x^n)^2)^(1/2)/(
a+b*x^n)^3*(2*b*x^n+3*a)/a^2/n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a^3/
n*ln(x^n+a/b)
```

Maxima [A] time = 0.756369, size = 95, normalized size = 0.6

$$\frac{2\,bx^{n}+3\,a}{2\,(a^{2}b^{2}nx^{2\,n}+2\,a^{3}bnx^{n}+a^{4}n)}+\frac{\log(x)}{a^{3}}-\frac{\log\left(\frac{bx^{n}+a}{b}\right)}{a^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) +
 log(x)/a^3 - log((b*x^n + a)/b)/(a^3*n)
```

Fricas [A] time = 0.289337, size = 143, normalized size = 0.9

$$\frac{2\,b^2nx^{2\,n}\log\left(x\right)+2\,a^2n\log\left(x\right)+3\,a^2+2\,(2\,abn\log\left(x\right)+ab)x^n-2\,\left(b^2x^{2\,n}+2\,abx^n+a^2\right)\log\left(bx^n+a\right)}{2\,(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n
*log(x) + a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n
+ a))/(a^3*b^2*n*x^(2*n) + 2*a^4*b*n*x^n + a^5*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

3.541 $\int \frac{1}{x^2(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(a^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rubi [A] time = 0.0738639, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(a^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rubi in Sympy [A] time = 13.9669, size = 58, normalized size = 0.89

$$-\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix} 3,-\frac{1}{n} \\ \frac{n-1}{n} \end{matrix}\middle| -\frac{bx^n}{a}\right)}{a^3x\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] -b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((3, -1/n), ((n -
 1)/n,), -b*x**n/a)/(a**3*x*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.126837, size = 98, normalized size = 1.51

$$\frac{a\left(3an+a+b(2n+1)x^n\right)-\left(2n^2+3n+1\right)\left(a+bx^n\right)^2\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{bx^n}{a}\right)}{2a^3n^2x\left(a+bx^n\right)\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] (a*(a + 3*a*n + b*(1 + 2*n)*x^n) - (1 + 3*n + 2*n^2)*(a + b*x^n)^
2*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)])/(2*a^3
*n^2*x*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,n^2+3\,n+1\right)\int \frac{1}{2\left(a^2bn^2x^2x^n+a^3n^2x^2\right)}\,dx+\frac{b(2\,n+1)x^n+a(3\,n+1)}{2\left(a^2b^2n^2xx^{2\,n}+2\,a^3bn^2xx^n+a^4n^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] (2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2)
, x) + 1/2*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n)
 + 2*a^3*b*n^2*x*x^n + a^4*n^2*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(b^{2}x^{2}x^{2\,n}+2\,abx^{2}x^{n}+a^{2}x^{2}\right)\sqrt{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*x^2*x^(2*n) + 2*a*b*x^2*x^n + a^2*x^2)*sqrt(b^2*
x^(2*n) + 2*a*b*x^n + a^2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2), x)
```

3.542 $\int \frac{1}{x^3(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -((a + b*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi [A] time = 0.074149, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rubi in Sympy [A] time = 14.2043, size = 61, normalized size = 0.91

$$-\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\,{}_2F_1\left(\begin{matrix}3,-\frac{2}{n}\\ \frac{n-2}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{2a^3x^2\left(ab+b^2x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] -b*sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n))*hyper((3, -2/n), ((n -
 2)/n,), -b*x**n/a)/(2*a**3*x**2*(a*b + b**2*x**n))
```

Mathematica [A] time = 0.110042, size = 94, normalized size = 1.4

$$\frac{(a+bx^n)\left(a^2n-(n^2+3n+2)\,(a+bx^n)^2\ {}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{bx^n}{a}\right)+2a(n+1)\,(a+bx^n)\right)}{2a^3n^2x^2\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] ((a + b*x^n)*(a^2*n + 2*a*(1 + n)*(a + b*x^n) - (2 + 3*n + n^2)*(
a + b*x^n)^2*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)]
))/(2*a^3*n^2*x^2*((a + b*x^n)^2)^(3/2))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(n^2+3\,n+2\right)\int \frac{1}{a^2bn^2x^3x^n+a^3n^2x^3}\,dx+\frac{2\,b(n+1)x^n+a(3\,n+2)}{2\,(a^2b^2n^2x^2x^{2\,n}+2\,a^3bn^2x^2x^n+a^4n^2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] (n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x)
 + 1/2*(2*b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) +
 2*a^3*b*n^2*x^2*x^n + a^4*n^2*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(b^2x^3x^{2\,n}+2\,abx^3x^n+a^2x^3\right)\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*x^3*x^(2*n) + 2*a*b*x^3*x^n + a^2*x^3)*sqrt(b^2*
x^(2*n) + 2*a*b*x^n + a^2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3), x)
```

3.543 $\int \left(a^2+b^2 x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx$

Optimal. Leaf size=52

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2
/x^(2/(1 + 2*p)))^p)/a
```

Rubi [A] time = 0.0385906, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2
/x^(2/(1 + 2*p)))^p)/a
```

Rubi in Sympy [A] time = 2.5979, size = 49, normalized size = 0.94

$$\frac{x\left(2a+2bx^{-\frac{1}{2p+1}}\right)\left(a^2+2abx^{-\frac{1}{2p+1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
```

```
[Out] x*(2*a + 2*b*x**(-1/(2*p + 1)))*(a**2 + 2*a*b*x**(-1/(2*p + 1)) +
 b**2*x**(-2/(2*p + 1)))**p/(2*a)
```

Mathematica [B] time = 0.165466, size = 121, normalized size = 2.33

$$\frac{x^{\frac{2p}{2p+1}}\left(x^{-\frac{2}{2p+1}}\left(ax^{\frac{1}{2p+1}}+b\right)^2\right)^p\left(\frac{ax^{\frac{1}{2p+1}}}{b}+1\right)^{-2p}\left(ax^{\frac{1}{2p+1}}\left(\frac{ax^{\frac{1}{2p+1}}}{b}+1\right)^{2p}+b\left(\left(\frac{ax^{\frac{1}{2p+1}}}{b}+1\right)^{2p}-1\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x^((2*p)/(1 + 2*p))*((b + a*x^(1 + 2*p)^(-1))^2/x^(2/(1 + 2*p)))
^p*(a*x^(1 + 2*p)^(-1)*(1 + (a*x^(1 + 2*p)^(-1))/b)^(2*p) + b*(-1
 + (1 + (a*x^(1 + 2*p)^(-1))/b)^(2*p))))/(a*(1 + (a*x^(1 + 2*p)^(
-1))/b)^(2*p))
```

Maple [F] time = 0.354, size = 0, normalized size = 0.

$$\int\left(a^2+b^2\left(x^{2(1+2p)^{-1}}\right)^{-1}+2\frac{ab}{x^{(1+2p)^{-1}}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(2\,abx^{-\frac{1}{2p+1}}+b^2x^{-\frac{2}{2p+1}}+a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p,x, algorithm="max
```

```
[Out] integrate((2*a*b*x^(-1/(2*p + 1)) + b^2*x^(-2/(2*p + 1)) + a^2)^p
, x)
```

Fricas [A] time = 0.29391, size = 107, normalized size = 2.06

$$\frac{\left(axx^{\left(\frac{1}{2p+1}\right)}+bx\right)\left(\frac{a^2x^{\frac{2}{2p+1}}+2abx^{\left(\frac{1}{2p+1}\right)}+b^2}{x^{\frac{2}{2p+1}}}\right)^p}{ax^{\left(\frac{1}{2p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p,x, algorithm="fri
```

```
[Out] (a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2
*p + 1)) + b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+\frac{b^2}{x^{\frac{2}{2p+1}}}+\frac{2ab}{x^{\left(\frac{1}{2p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p,x, algorithm="gia
```

```
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p,
x)
```

3.544 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\,dx$

Optimal. Leaf size=43

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

```
[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)
))
```

Rubi [A] time = 0.0291258, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + n)/(2*n)),x]

[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)
))
```

Rubi in Sympy [A] time = 2.73376, size = 41, normalized size = 0.95

$$\frac{x\left(2a+2bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)

[Out] x*(2*a + 2*b*x**n)*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-(n + 1)
/(2*n))/(2*a)
```

Mathematica [A] time = 0.0954756, size = 32, normalized size = 0.74

$$\frac{x\left(a+bx^n\right)\left(\left(a+bx^n\right)^2\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))
```

Maple [A] time = 0.053, size = 51, normalized size = 1.2

$$1\left(x+\frac{bx\mathrm{e}^{n\ln(x)}}{a}\right)\left(\mathrm{e}^{\frac{(1+n)\ln\left(a^2+2ab\mathrm{e}^{n\ln(x)}+b^2\left(\mathrm{e}^{n\ln(x)}\right)^2\right)}{2n}}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x)
```

```
[Out] (x+b/a*x*exp(n*ln(x)))/exp(1/2*(1+n)/n*ln(a^2+2*a*b*exp(n*ln(x))+
b^2*exp(n*ln(x))^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b^2x^{2n}+2abx^n+a^2\right)^{-\frac{n+1}{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)),x, algorithm="maxi
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-1/2*(n + 1)/n), x)
```

Fricas [A] time = 0.28077, size = 61, normalized size = 1.42

$$\frac{bxx^n+ax}{\left(b^2x^{2n}+2abx^n+a^2\right)^{\frac{n+1}{2n}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)),x, algorithm="frica
```

```
[Out] (b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*
a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2 x^{2n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)),x, algorithm="giac
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

3.545 $\int \left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}}\right)^p dx$

Optimal. Leaf size=130

$$\frac{2(p+1)x\left(a + bx^{-\frac{1}{2(p+1)}}\right)\left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)} - \frac{x\left(a + bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1)
+ (2*a*b)/x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2
*(1 + p))))^2*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p)
)))^p)/(a^2*(1 + 2*p))
```

Rubi [A] time = 0.109703, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$

$$\frac{2(p+1)x\left(a + bx^{-\frac{1}{2(p+1)}}\right)\left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)} - \frac{x\left(a + bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1)
+ (2*a*b)/x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2
*(1 + p))))^2*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p)
)))^p)/(a^2*(1 + 2*p))
```

Rubi in Sympy [A] time = 4.68432, size = 95, normalized size = 0.73

$$\frac{x\left(2a + 2bx^{-\frac{1}{2(p+1)}}\right)(p+1)\left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)} - \frac{x\left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}}\right)^{p+1}}{a^2(2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
```

```
[Out] x*(2*a + 2*b*x**(-1/(2*(p + 1))))*(p + 1)*(a**2 + 2*a*b*x**(-1/(2
*(p + 1))) + b**2*x**(-1/(p + 1)))**p/(a*(2*p + 1)) - x*(a**2 + 2
```

```
*a*b*x**(-1/(2*(p + 1))) + b**2*x**(-1/(p + 1)))**(p + 1)/(a**2*(
2*p + 1))
```

Mathematica [A] time = 0.308144, size = 165, normalized size = 1.27

$$\frac{x^{\frac{p}{p+1}}\left(x^{-\frac{1}{p+1}}\left(ax^{\frac{1}{2p+2}}+b\right)^2\right)^p\left(\frac{ax^{\frac{1}{2p+2}}}{b}+1\right)^{-2p}\left(a^2(2p+1)x^{\frac{1}{p+1}}\left(\frac{ax^{\frac{1}{2p+2}}}{b}+1\right)^{2p}-b^2\left(\left(\frac{ax^{\frac{1}{2p+2}}}{b}+1\right)^{2p}-1\right)+2abpx^{\frac{1}{2p+2}}\left(\frac{ax^{\frac{1}{2p+}}}{b}\right.\right.}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (x^(p/(1 + p))*((b + a*x^(2 + 2*p)^(-1))^2/x^(1 + p)^(-1))^p*(a^2
*(1 + 2*p)*x^(1 + p)^(-1)*(1 + (a*x^(2 + 2*p)^(-1))/b)^(2*p) + 2*
a*b*p*x^(2 + 2*p)^(-1)*(1 + (a*x^(2 + 2*p)^(-1))/b)^(2*p) - b^2*(
-1 + (1 + (a*x^(2 + 2*p)^(-1))/b)^(2*p))))/(a^2*(1 + 2*p)*(1 + (a
*x^(2 + 2*p)^(-1))/b)^(2*p))
```

Maple [F] time = 0.391, size = 0, normalized size = 0.

$$\int \left(a^2+\frac{b^2}{x^{(1+p)^{-1}}}+2\,ab\left(x^{1/2\,(1+p)^{-1}}\right)^{-1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2\,abx^{-\frac{1}{2\,(p+1)}}+b^2x^{-\frac{1}{p+1}}+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p,x, algorithm="maxim
```

```
[Out] integrate((2*a*b*x^(-1/2/(p + 1)) + b^2*x^(-1/(p + 1)) + a^2)^p,
x)
```

Fricas [A] time = 0.287701, size = 139, normalized size = 1.07

$$\frac{\left(2\,abpxx^{\frac{1}{2\,(p+1)}}-b^2x+\left(2\,a^2p+a^2\right)xx^{\left(\frac{1}{p+1}\right)}\right)\left(\frac{2\,abx^{\frac{1}{2\,(p+1)}}+a^2x^{\left(\frac{1}{p+1}\right)}+b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p}{(2\,a^2p+a^2)x^{\left(\frac{1}{p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p,x, algorithm="frica
```

```
[Out] (2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p +
1)))*((2*a*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p +
 1)))^p/((2*a^2*p + a^2)*x^(1/(p + 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p,x, algorithm="giac"
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

3.546 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\,dx$

Optimal. Leaf size=102

$$\frac{nx\left(a+bx^n\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))
/(a*(1 + n)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))
^((-2 - n^(-1))/2))/(a^2*(1 + n))
```

Rubi [A] time = 0.0873922, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{nx\left(a+bx^n\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + 2*n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))
/(a*(1 + n)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))
^((-2 - n^(-1))/2))/(a^2*(1 + n))
```

Rubi in Sympy [A] time = 4.31453, size = 80, normalized size = 0.78

$$\frac{x\left(2a+2bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+\frac{1}{2}}{n}}}{2a\left(n+1\right)}+\frac{nx\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1}{2n}}}{a^2\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)
```

```
[Out] x*(2*a + 2*b*x**n)*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-(n + 1/
2)/n)/(2*a*(n + 1)) + n*x*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-
1/(2*n))/(a**2*(n + 1))
```

Mathematica [C] time = 0.0725443, size = 59, normalized size = 0.58

$$\frac{x\left((a+bx^n)^2\right)^{-\frac{1}{2}/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\ {}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + 2*n)/(2*n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
 + n^(-1), -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int\left(\left(a^2+2\,abx^n+b^2x^{2\,n}\right)^{\frac{1+2\,n}{2\,n}}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

```
[Out] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{-\frac{2\,n+1}{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-1/2*(2*n + 1)/n), x)
```

Fricas [A] time = 0.286189, size = 111, normalized size = 1.09

$$\frac{b^2nxx^{2\,n}+(2\,abn+ab)xx^n+\left(a^2n+a^2\right)x}{(a^2n+a^2)(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{2\,n+1}{2\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)),x, algorithm="fr
```

```
[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2
*n + a^2)*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{2\,n+1}{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)),x, algorithm="gi
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)),
x)
```

3.547 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=117

$$\frac{(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2a^2dn(p+1)(2p+1)}-\frac{(a+bx^n)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{adn(2p+1)}$$

```
[Out] -(((a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(a*d*n*(1 + 2*p
)*(d*x)^(2*n*(1 + p)))) + (a^2 + 2*a*b*x^n + b^2*x^(2*n))^(1 + p)
/(2*a^2*d*n*(1 + p)*(1 + 2*p)*(d*x)^(2*n*(1 + p)))
```

Rubi [A] time = 0.146146, antiderivative size = 124, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086

$$\frac{\left(\frac{bx^n}{a}+1\right)^2(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn\left(2p^2+3p+1\right)}-\frac{\left(\frac{bx^n}{a}+1\right)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(((1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(d*n*(1 + 2
*p)*(d*x)^(2*n*(1 + p)))) + ((1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n +
 b^2*x^(2*n))^p)/(2*d*n*(1 + 3*p + 2*p^2)*(d*x)^(2*n*(1 + p)))
```

Rubi in Sympy [A] time = 13.515, size = 104, normalized size = 0.89

$$-\frac{(dx)^{-2n(p+1)}\left(2a+2bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2adn\left(2p+1\right)}+\frac{(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2a^2dn\left(p+1\right)\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] -(d*x)**(-2*n*(p + 1))*(2*a + 2*b*x**n)*(a**2 + 2*a*b*x**n + b**2
*x**(2*n))**p/(2*a*d*n*(2*p + 1)) + (d*x)**(-2*n*(p + 1))*(a**2 +
 2*a*b*x**n + b**2*x**(2*n))**(p + 1)/(2*a**2*d*n*(p + 1)*(2*p +
1))
```

Mathematica [C] time = 0.103026, size = 75, normalized size = 0.64

$$-\frac{x(dx)^{-2n(p+1)-1}\left((a+bx^n)^2\right)^p\left(\frac{bx^n}{a}+1\right)^{-2p}{}_2F_1\left(-2p,-2(p+1);1-2(p+1);-\frac{bx^n}{a}\right)}{2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[
-2*p, -2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(2*n*(1 + p)*(1 +
 (b*x^n)/a)^(2*p))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

```
[Out] int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^{2n}+2abx^n+a^2\right)^p (dx)^{-2n(p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1),x, algorithm
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) -
 1), x)
```

Fricas [A] time = 0.277254, size = 223, normalized size = 1.91

$$-\frac{\left(2\,abpxx^ne^{(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x))}-b^2xx^{2\,n}e^{(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x))}+\left(2\,a^2p+a^2\right)xe^{(-(2\,np+2\,n+1)\log(}\right.}{2\left(2\,a^2np^2+3\,a^2np+a^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1),x, algorithm
```

```
[Out] -1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n +
 1)*log(x)) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p
 + 2*n + 1)*log(x)) + (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log
(d) - (2*n*p + 2*n + 1)*log(x)))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^
p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2\,n} + 2\,abx^n + a^2\right)^p (dx)^{-2\,n(p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1),x, algorithm
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) -
 1), x)
```

3.548 $\int x^{-1+2n} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=103

$$\frac{a^2 \left(\frac{bx^n}{a} + 1\right)^2 \left(a^2 + 2abx^n + b^2x^{2n}\right)^p}{2b^2n(p+1)} - \frac{a^2 \left(\frac{bx^n}{a} + 1\right) \left(a^2 + 2abx^n + b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*
(1 + 2*p))) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*
n))^p)/(2*b^2*n*(1 + p))
```

Rubi [A] time = 0.139361, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{a^2 \left(\frac{bx^n}{a} + 1\right)^2 \left(a^2 + 2abx^n + b^2x^{2n}\right)^p}{2b^2n(p+1)} - \frac{a^2 \left(\frac{bx^n}{a} + 1\right) \left(a^2 + 2abx^n + b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*
(1 + 2*p))) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*
n))^p)/(2*b^2*n*(1 + p))
```

Rubi in Sympy [A] time = 18.8872, size = 78, normalized size = 0.76

$$-\frac{a\left(2a + 2bx^n\right)\left(a^2 + 2abx^n + b^2x^{2n}\right)^p}{2b^2n\left(2p+1\right)} + \frac{\left(a^2 + 2abx^n + b^2x^{2n}\right)^{p+1}}{2b^2n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] -a*(2*a + 2*b*x**n)*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**p/(2*b**
2*n*(2*p + 1)) + (a**2 + 2*a*b*x**n + b**2*x**(2*n))**(p + 1)/(2*
b**2*n*(p + 1))
```

Mathematica [A] time = 0.0541331, size = 54, normalized size = 0.52

$$\frac{(a+bx^n)\left((a+bx^n)^2\right)^p(b(2p+1)x^n-a)}{2b^2n(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] ((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(
1 + p)*(1 + 2*p))
```

Maple [C] time = 0.109, size = 148, normalized size = 1.4

$$-\frac{-2\,b^2p\,(x^n)^2-2\,apx^nb-b^2\,(x^n)^2+a^2}{(2+4\,p)\,(1+p)\,nb^2}\mathrm{e}^{-\frac{p\left(i\pi\,\left(csgn\left(i(a+bx^n)^2\right)\right)^3-2\,i\pi\,\left(csgn\left(i(a+bx^n)^2\right)\right)^2csgn(i(a+bx^n))+i\pi\,csgn\left(i(a+bx^n)^2\right)(csgn(i(a+bx^n)))^2-4\,\ln(a+bx}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

```
[Out] -1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-b^2*(x^n)^2+a^2)/(1+2*p)/(1+p)
/n/b^2*exp(-1/2*p*(I*Pi*csgn(I*(a+b*x^n)^2)^3-2*I*Pi*csgn(I*(a+b*
x^n)^2)^2*csgn(I*(a+b*x^n))+I*Pi*csgn(I*(a+b*x^n)^2)*csgn(I*(a+b*
x^n))^2-4*ln(a+b*x^n)))
```

Maxima [A] time = 0.775058, size = 80, normalized size = 0.78

$$\frac{\left(b^2(2\,p+1)x^{2\,n}+2\,abpx^n-a^2\right)(bx^n+a)^{2\,p}}{2\,(2\,p^2+3\,p+1)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)
/((2*p^2 + 3*p + 1)*b^2*n)
```

Fricas [A] time = 0.271336, size = 105, normalized size = 1.02

$$\frac{\left(2\,abpx^{n} - a^{2} + \left(2\,b^{2}p + b^{2}\right)x^{2\,n}\right)\left(b^{2}x^{2\,n} + 2\,abx^{n} + a^{2}\right)^{p}}{2\,\left(2\,b^{2}np^{2} + 3\,b^{2}np + b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) +
2*a*b*x^n + a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^{2}x^{2\,n} + 2\,abx^{n} + a^{2}\right)^{p} x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)
```

3.549 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=111

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(
b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^
2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*c^3*n)
```

Rubi [A] time = 0.239469, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(
b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^
2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*c^3*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(-3ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{c^3n\sqrt{-4ac+b^2}}+\frac{\int^{x^n}x\,dx}{cn}-\frac{\int^{x^n}b\,dx}{c^2n}+\frac{\left(-ac+b^2\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] b*(-3*a*c + b**2)*atanh((b + 2*c*x**n)/sqrt(-4*a*c + b**2))/(c**3
*n*sqrt(-4*a*c + b**2)) + Integral(x, (x, x**n))/(c*n) - Integral
(b, (x, x**n))/(c**2*n) + (-a*c + b**2)*log(a + b*x**n + c*x**(2*
n))/(2*c**3*n)
```

Mathematica [A] time = 0.260004, size = 97, normalized size = 0.87

$$\frac{\left(b^2-ac\right)\log\left(a+x^n\left(b+cx^n\right)\right)-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx^n\left(cx^n-2b\right)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x^n*
(b + c*x^n)])/(2*c^3*n)
```

Maple [B] time = 0.269, size = 973, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a
*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^
2)*n^2*ln(x)*a*b^2*c+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/
c/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2
*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a
*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c
^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*a*c-b
^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-1
0*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*
ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^
6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-
10*a*b^6*c+b^8)^(1/2)-2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^
4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a
*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-3
6*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^
2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a
^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))
*b^4-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^
2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36
*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2-ac\right)\log\left(x\right)}{c^3}+\frac{cx^{2n}-2\,bx^n}{2\,c^2n}+\int-\frac{ab^2-a^2c+\left(b^3-2\,abc\right)x^n}{c^4xx^{2n}+bc^3xx^n+ac^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] (b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + inte
grate(-(a*b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c
^3*x*x^n + a*c^3*x), x)
```

Fricas [A] time = 0.295891, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{b^2-4\,ac}c^2x^{2n}-2\,\sqrt{b^2-4\,ac}bcx^n+\left(b^2-ac\right)\sqrt{b^2-4\,ac}\log\left(cx^{2n}+bx^n+a\right)-\left(b^3-3\,abc\right)\log\left(\frac{2\,\sqrt{b^2-4\,ac}c^2x^{2n}-b^3+4\,a}{\ }\right.}{2\,\sqrt{b^2-4\,ac}c^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*c^2*x^(2*n) - 2*sqrt(b^2 - 4*a*c)*b*c*x^n
 + (b^2 - a*c)*sqrt(b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a) - (b^
3 - 3*a*b*c)*log((2*sqrt(b^2 - 4*a*c)*c^2*x^(2*n) - b^3 + 4*a*b*c
 - 2*(b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*x^n + (b^2 - 2*a*c
)*sqrt(b^2 - 4*a*c))/(c*x^(2*n) + b*x^n + a)))/(sqrt(b^2 - 4*a*c)
*c^3*n), 1/2*(sqrt(-b^2 + 4*a*c)*c^2*x^(2*n) - 2*sqrt(-b^2 + 4*a*
c)*b*c*x^n + (b^2 - a*c)*sqrt(-b^2 + 4*a*c)*log(c*x^(2*n) + b*x^n
 + a) - 2*(b^3 - 3*a*b*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + s
qrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)))/(sqrt(-b^2 + 4*a*c)*c^3*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.550 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=87

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2
*c^2*n)
```

Rubi [A] time = 0.151965, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2
*c^2*n)
```

Rubi in Sympy [A] time = 30.8082, size = 76, normalized size = 0.87

$$-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}-\frac{(-2ac+b^2)\,\mathrm{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{c^2n\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -b*log(a + b*x**n + c*x**(2*n))/(2*c**2*n) + x**n/(c*n) - (-2*a*c
 + b**2)*atanh((b + 2*c*x**n)/sqrt(-4*a*c + b**2))/(c**2*n*sqrt(-
4*a*c + b**2))
```

Mathematica [A] time = 0.152454, size = 82, normalized size = 0.94

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+x^n\left(b+cx^n\right)\right)+2cx^n}{2c^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + x^n*(b + c*x^n)])/(2*c^2*n)
```

Maple [B] time = 0.148, size = 664, normalized size = 7.6

$$-\frac{b\ln(x)}{c^2}+\frac{x^n}{cn}+4\frac{n^2\ln(x)abc}{4ac^3n^2-b^2c^2n^2}-\frac{n^2\ln(x)b^3}{4ac^3n^2-b^2c^2n^2}$$
$$-2\frac{ab}{(4ac-b^2)cn}\ln\left(x^n-1/2\frac{-2abc+b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8ab^4c+b^6}}{c(2ac-b^2)}\right)$$
$$+\frac{b^3}{(8ac-2b^2)c^2n}\ln\left(x^n-\frac{1}{2c(2ac-b^2)}\left(-2abc+b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8ab^4c+b^6}\right)\right)$$
$$+\frac{1}{(8ac-2b^2)c^2n}\ln\left(x^n-\frac{1}{2c(2ac-b^2)}\left(-2abc+b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8ab^4c+b^6}\right)\right)\sqrt{-16a^3c^3+20a^2b^2c^2-8}$$
$$-2\frac{ab}{(4ac-b^2)cn}\ln\left(x^n+1/2\frac{2abc-b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8ab^4c+b^6}}{c(2ac-b^2)}\right)$$
$$+\frac{b^3}{(8ac-2b^2)c^2n}\ln\left(x^n+\frac{1}{2c(2ac-b^2)}\left(2abc-b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8ab^4c+b^6}\right)\right)$$
$$-\frac{1}{(8ac-2b^2)c^2n}\ln\left(x^n+\frac{1}{2c(2ac-b^2)}\left(2abc-b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8ab^4c+b^6}\right)\right)\sqrt{-16a^3c^3+20a^2b^2c^2-8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-
1/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/(4*a*c-b^2)/c/n*ln(x^
n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1
/2))/c/(2*a*c-b^2))*a*b+1/2/(4*a*c-b^2)/c^2/n*ln(x^n-1/2*(-2*a*b*
c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-
b^2))*b^3+1/2/(4*a*c-b^2)/c^2/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3
*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*a^3
*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/(4*a*c-b^2)/c/n*ln(x^n
+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2
```

```
))/c/(2*a*c-b^2))*a*b+1/2/(4*a*c-b^2)/c^2/n*ln(x^n+1/2*(2*a*b*c-b
^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*b^3-1/2/(4*a*c-b^2)/c^2/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3
+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*a^3*c^3
+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\log(x)}{c^2}+\frac{x^n}{cn}-\int -\frac{ab+(b^2-ac)x^n}{c^3xx^{2n}+bc^2xx^n+ac^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] -b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c
^3*x*x^(2*n) + b*c^2*x*x^n + a*c^2*x), x)
```

Fricas [A] time = 0.285053, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{b^2-4ac}cx^n-\sqrt{b^2-4ac}b\log\left(cx^{2n}+bx^n+a\right)-\left(b^2-2ac\right)\log\left(\frac{2\sqrt{b^2-4ac}c^2x^{2n}+b^3-4abc+2\left(b^2c-4ac^2+\sqrt{b^2-4ac}bc\right)x^n+(b}{cx^{2n}+bx^n+a}\right.}{2\sqrt{b^2-4ac}c^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(b^2 - 4*a*c)*c*x^n - sqrt(b^2 - 4*a*c)*b*log(c*x^(2*
n) + b*x^n + a) - (b^2 - 2*a*c)*log((2*sqrt(b^2 - 4*a*c)*c^2*x^(2
*n) + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*c)*b*c)
*x^n + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^(2*n) + b*x^n + a)))
/(sqrt(b^2 - 4*a*c)*c^2*n), 1/2*(2*sqrt(-b^2 + 4*a*c)*c*x^n - sqr
t(-b^2 + 4*a*c)*b*log(c*x^(2*n) + b*x^n + a) + 2*(b^2 - 2*a*c)*ar
ctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 -
4*a*c)))/(sqrt(-b^2 + 4*a*c)*c^2*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3\,n-1}}{cx^{2\,n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.551 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]
*n) + Log[a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rubi [A] time = 0.10408, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]
*n) + Log[a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rubi in Sympy [A] time = 19.8149, size = 58, normalized size = 0.85

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{cn\sqrt{-4ac+b^2}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] b*atanh((b + 2*c*x**n)/sqrt(-4*a*c + b**2))/(c*n*sqrt(-4*a*c + b*
*2)) + log(a + b*x**n + c*x**(2*n))/(2*c*n)
```

Mathematica [A] time = 0.0857897, size = 66, normalized size = 0.97

$$\frac{\log\left(a+x^n\left(b+cx^n\right)\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2cn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c] + Log[a + x^n*(b + c*x^n)])/(2*c*n)
```

Maple [B] time = 0.125, size = 402, normalized size = 5.9

$$\frac{\ln(x)}{c}-4\,\frac{n^2\ln(x)\,ac}{4\,ac^2n^2-b^2cn^2}+\frac{n^2\ln(x)\,b^2}{4\,ac^2n^2-b^2cn^2}+2\,\frac{a}{(4\,ac-b^2)\,n}\ln\left(x^n-1/2\,\frac{-b^2+\sqrt{-4\,ab^2c+b^4}}{bc}\right)$$
$$-\frac{b^2}{(8\,ac-2\,b^2)\,cn}\ln\left(x^n-\frac{1}{2\,bc}\left(-b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)$$
$$+\frac{1}{(8\,ac-2\,b^2)\,cn}\ln\left(x^n-\frac{1}{2\,bc}\left(-b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)\sqrt{-4\,ab^2c+b^4}$$
$$+2\,\frac{a}{(4\,ac-b^2)\,n}\ln\left(x^n+1/2\,\frac{b^2+\sqrt{-4\,ab^2c+b^4}}{bc}\right)$$
$$-\frac{b^2}{(8\,ac-2\,b^2)\,cn}\ln\left(x^n+\frac{1}{2\,bc}\left(b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)$$
$$-\frac{1}{(8\,ac-2\,b^2)\,cn}\ln\left(x^n+\frac{1}{2\,bc}\left(b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)\sqrt{-4\,ab^2c+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-
b^2*c*n^2)*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b
^2*c+b^4)^(1/2))/b/c)*a-1/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2+(-4*
a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2
+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)+2/(4*a*c-b^2
)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*a-1/2/(4*a*c-b^2
)/c/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/(4*a*c
-b^2)/c/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*
c+b^4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(x)}{c} - \int \frac{bx^n + a}{c^2xx^{2n} + bcxx^n + acx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c
*x), x)
```

Fricas [A] time = 0.275514, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(\frac{2\sqrt{b^2-4ac}c^2x^{2n}+b^3-4abc+2\left(b^2c-4ac^2+\sqrt{b^2-4ac}bc\right)x^n+(b^2-2ac)\sqrt{b^2-4ac}}{cx^{2n}+bx^n+a}\right)+\sqrt{b^2-4ac}\log\left(cx^{2n}+bx^n+a\right)}{2\sqrt{b^2-4ac}cn}, -\frac{2b\arctan\left(-\frac{2\sqrt{-b^2+4ac}cx^n+\sqrt{-b^2+4ac}b}{b^2-4ac}\right)-\sqrt{-b^2+4ac}\log\left(cx^{2n}+bx^n+a\right)}{2\sqrt{-b^2+4ac}cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log((2*sqrt(b^2 - 4*a*c)*c^2*x^(2*n) + b^3 - 4*a*b*c + 2*
(b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*c)*b*c)*x^n + (b^2 - 2*a*c)*sqr
t(b^2 - 4*a*c))/(c*x^(2*n) + b*x^n + a)) + sqrt(b^2 - 4*a*c)*log(
c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*c*n), -1/2*(2*b*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*
c)) - sqrt(-b^2 + 4*a*c)*log(c*x^(2*n) + b*x^n + a))/(sqrt(-b^2 +
 4*a*c)*c*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.552 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=39

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
n)
```

Rubi [A] time = 0.0661677, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
n)
```

Rubi in Sympy [A] time = 11.0421, size = 37, normalized size = 0.95

$$-\frac{2\operatorname{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{n\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*atanh((b + 2*c*x**n)/sqrt(-4*a*c + b**2))/(n*sqrt(-4*a*c + b**
2))
```

Mathematica [A] time = 0.0374889, size = 43, normalized size = 1.1

$$\frac{2\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*
n)
```

Maple [B] time = 0.074, size = 113, normalized size = 2.9

$$-\frac{1}{n}\ln\left(x^n+\frac{1}{2c}\left(b\sqrt{-4ac+b^2}-4ac+b^2\right)\frac{1}{\sqrt{-4ac+b^2}}\right)\frac{1}{\sqrt{-4ac+b^2}}+\frac{1}{n}\ln\left(x^n+\frac{1}{2c}\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)\frac{1}{\sqrt{-4ac+b^2}}\right)\frac{1}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^
2)/c/(-4*a*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b*(-4
*a*c+b^2)^(1/2)+4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [A] time = 0.284568, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{2\sqrt{b^2-4ac}c^2x^{2n}-b^3+4abc-2\left(b^2c-4ac^2-\sqrt{b^2-4ac}bc\right)x^n+\left(b^2-2ac\right)\sqrt{b^2-4ac}}{cx^{2n}+bx^n+a}\right)}{\sqrt{b^2-4ac}n}, \frac{2\arctan\left(-\frac{2\sqrt{-b^2+4ac}cx^n+\sqrt{-b^2+4ac}b}{b^2-4ac}\right)}{\sqrt{-b^2+4ac}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] [log((2*sqrt(b^2 - 4*a*c)*c^2*x^(2*n) - b^3 + 4*a*b*c - 2*(b^2*c
- 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*x^n + (b^2 - 2*a*c)*sqrt(b^2 -
 4*a*c))/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), 2*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*
c))/(sqrt(-b^2 + 4*a*c)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270978, size = 53, normalized size = 1.36

$$\frac{2 \arctan\left(\frac{2 \, c x^{n} + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{\sqrt{-b^{2} + 4 \, a c} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

3.553 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=98

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 -
4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a +
b*x^n + c*x^(2*n)])/(2*a^2*n)
```

Rubi [A] time = 0.269766, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 -
4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a +
b*x^n + c*x^(2*n)])/(2*a^2*n)
```

Rubi in Sympy [A] time = 44.3469, size = 90, normalized size = 0.92

$$-\frac{x^{-n}}{an}-\frac{b\log\left(x^n\right)}{a^2n}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{a^2n\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-n)/(a*n) - b*log(x**n)/(a**2*n) + b*log(a + b*x**n + c*x**(
2*n))/(2*a**2*n) - (-2*a*c + b**2)*atanh((b + 2*c*x**n)/sqrt(-4*a
*c + b**2))/(a**2*n*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.232763, size = 87, normalized size = 0.89

$$\frac{-\frac{2(b^2-2ac)\tan^{-1}\left(\frac{2ax^{-n}+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log\left(x^{-2n}\left(a+bx^n\right)+c\right)-2ax^{-n}}{2a^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*a)/x^n - (2*(b^2 - 2*a*c)*ArcTan[(b + (2*a)/x^n)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + b*Log[c + (a + b*x^n)/x^(2*n)])/(2
*a^2*n)
```

Maple [B] time = 0.173, size = 658, normalized size = 6.7

$$-\frac{1}{anx^n}-4\,\frac{n^2\ln(x)\,abc}{4\,a^3cn^2-a^2b^2n^2}+\frac{n^2\ln(x)\,b^3}{4\,a^3cn^2-a^2b^2n^2}$$
$$+2\,\frac{bc}{a\left(4\,ac-b^2\right)n}\ln\left(x^n-1/2\,\frac{-2\,abc+b^3+\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8\,ab^4c+b^6}}{c\left(2\,ac-b^2\right)}\right)$$
$$-\frac{b^3}{\left(8\,ac-2\,b^2\right)a^2n}\ln\left(x^n-\frac{1}{2\,c\left(2\,ac-b^2\right)}\left(-2\,abc+b^3+\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8\,ab^4c+b^6}\right)\right)$$
$$+\frac{1}{\left(8\,ac-2\,b^2\right)a^2n}\ln\left(x^n-\frac{1}{2\,c\left(2\,ac-b^2\right)}\left(-2\,abc+b^3+\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8\,ab^4c+b^6}\right)\right)\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8}$$
$$+2\,\frac{bc}{a\left(4\,ac-b^2\right)n}\ln\left(x^n+1/2\,\frac{2\,abc-b^3+\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8\,ab^4c+b^6}}{c\left(2\,ac-b^2\right)}\right)$$
$$-\frac{b^3}{\left(8\,ac-2\,b^2\right)a^2n}\ln\left(x^n+\frac{1}{2\,c\left(2\,ac-b^2\right)}\left(2\,abc-b^3+\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8\,ab^4c+b^6}\right)\right)$$
$$-\frac{1}{\left(8\,ac-2\,b^2\right)a^2n}\ln\left(x^n+\frac{1}{2\,c\left(2\,ac-b^2\right)}\left(2\,abc-b^3+\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8\,ab^4c+b^6}\right)\right)\sqrt{-16\,a^3c^3+20\,a^2b^2c^2-8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3
*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*b^3+2/(4*a*c-b^2)/a/n*ln(x^n-1/2*(-
2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(
2*a*c-b^2))*b*c-1/2/(4*a*c-b^2)/a^2/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-
16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^
3+1/2/(4*a*c-b^2)/a^2/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*
a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*a^3*c^3+20*
a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/(4*a*c-b^2)/a/n*ln(x^n+1/2*(2*
```

```
a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*
a*c-b^2))*b*c-1/2/(4*a*c-b^2)/a^2/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*
a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1
/2/(4*a*c-b^2)/a^2/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*
b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*a^3*c^3+20*a^2*
b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x^{-n}}{an} - \int \frac{cx^n + b}{acxx^{2n} + abxx^n + a^2x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] -x^(-n)/(a*n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n
+ a^2*x), x)
```

Fricas [A] time = 0.284065, size = 1, normalized size = 0.01

$$\left[-\frac{2\sqrt{b^2-4ac}\,bnx^n\log(x) - \sqrt{b^2-4ac}\,bx^n\log\left(cx^{2n}+bx^n+a\right) + \left(b^2-2ac\right)x^n\log\left(\frac{2\sqrt{b^2-4ac}\,c^2x^{2n}+b^3-4abc+2\left(b^2c-4ac^2+\cdots\right.}{cx^{2n}+bx^n\cdots}\right.}{2\sqrt{b^2-4ac}\,a^2nx^n}\right.$$

$$-\frac{2\sqrt{-b^2+4ac}\,bnx^n\log(x) - \sqrt{-b^2+4ac}\,bx^n\log\left(cx^{2n}+bx^n+a\right) - 2\left(b^2-2ac\right)x^n\arctan\left(-\frac{2\sqrt{-b^2+4ac}\,cx^n+\sqrt{-b^2+4ac}\,b}{b^2-4ac}\right) \cdots}{2\sqrt{-b^2+4ac}\,a^2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(b^2 - 4*a*c)*b*n*x^n*log(x) - sqrt(b^2 - 4*a*c)*b*x
^n*log(c*x^(2*n) + b*x^n + a) + (b^2 - 2*a*c)*x^n*log((2*sqrt(b^2
 - 4*a*c)*c^2*x^(2*n) + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2 + sqrt
(b^2 - 4*a*c)*b*c)*x^n + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^(2
*n) + b*x^n + a)) + 2*sqrt(b^2 - 4*a*c)*a)/(sqrt(b^2 - 4*a*c)*a^2
*n*x^n), -1/2*(2*sqrt(-b^2 + 4*a*c)*b*n*x^n*log(x) - sqrt(-b^2 +
4*a*c)*b*x^n*log(c*x^(2*n) + b*x^n + a) - 2*(b^2 - 2*a*c)*x^n*arc
```

```
tan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4
*a*c)) + 2*sqrt(-b^2 + 4*a*c)*a)/(sqrt(-b^2 + 4*a*c)*a^2*n*x^n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-n-1}}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.554 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=126

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}-\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b
+ 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2
- a*c)*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*
a^3*n)
```

Rubi [A] time = 0.379473, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}-\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b
+ 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2
- a*c)*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*
a^3*n)
```

Rubi in Sympy [A] time = 53.6837, size = 116, normalized size = 0.92

$$-\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b\left(-3ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{a^3n\sqrt{-4ac+b^2}}+\frac{\left(-ac+b^2\right)\log\left(x^n\right)}{a^3n}-\frac{\left(-ac+b^2\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-2*n)/(2*a*n) + b*x**(-n)/(a**2*n) + b*(-3*a*c + b**2)*atanh
((b + 2*c*x**n)/sqrt(-4*a*c + b**2))/(a**3*n*sqrt(-4*a*c + b**2))
 + (-a*c + b**2)*log(x**n)/(a**3*n) - (-a*c + b**2)*log(a + b*x**
```

```
n + c*x**(2*n))/(2*a**3*n)
```

Mathematica [A] time = 0.338692, size = 105, normalized size = 0.83

$$\frac{\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{2ax^{-n}+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(ac-b^2)\log\left(x^{-2n}(a+bx^n)+c\right)+ax^{-2n}(2bx^n-a)}{2a^3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((a*(-a + 2*b*x^n))/x^(2*n) + (2*b*(b^2 - 3*a*c)*ArcTan[(b + (2*a
)/x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-b^2 + a*c)*Log
[c + (a + b*x^n)/x^(2*n)])/(2*a^3*n)
```

Maple [B] time = 0.204, size = 958, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(
x)*a^2*c^2+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4
*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3
*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2
))/c/b/(3*a*c-b^2))*c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2
*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b
/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b
^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*
a*c-b^2))*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-3
6*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^
2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*
a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4
*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*c^2-5/2/a^2/(4*a*c-b
^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-
10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)
/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*
a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/a^3/(4*a*c-b^2)/n*ln
(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6
*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-1
0*a*b^6*c+b^8)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(2\,bx^n - a)x^{-2\,n}}{2\,a^2n} + \int \frac{bcx^n + b^2 - ac}{a^2cxx^{2\,n} + a^2bxx^n + a^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n - a)*x^(-2*n)/(a^2*n) + integrate((b*c*x^n + b^2 - a
*c)/(a^2*c*x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

Fricas [A] time = 0.28598, size = 1, normalized size = 0.01

$$\left[\frac{2\left(b^2-ac\right)\sqrt{b^2-4\,ac}nx^{2\,n}\log\left(x\right)+2\,\sqrt{b^2-4\,ac}abx^n-\left(b^2-ac\right)\sqrt{b^2-4\,ac}x^{2\,n}\log\left(cx^{2\,n}+bx^n+a\right)-\left(b^3-3\,abc\right)x^{2\,n}}{2\,\sqrt{b^2-4\,ac}a^3nx^{2\,n}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2 - a*c)*sqrt(b^2 - 4*a*c)*n*x^(2*n)*log(x) + 2*sqrt(b
^2 - 4*a*c)*a*b*x^n - (b^2 - a*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log(c
*x^(2*n) + b*x^n + a) - (b^3 - 3*a*b*c)*x^(2*n)*log((2*sqrt(b^2 -
 4*a*c)*c^2*x^(2*n) - b^3 + 4*a*b*c - 2*(b^2*c - 4*a*c^2 - sqrt(b
^2 - 4*a*c)*b*c)*x^n + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^(2*n
) + b*x^n + a)) - sqrt(b^2 - 4*a*c)*a^2)/(sqrt(b^2 - 4*a*c)*a^3*n
*x^(2*n)), 1/2*(2*(b^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*x^(2*n)*log(x)
 + 2*sqrt(-b^2 + 4*a*c)*a*b*x^n - (b^2 - a*c)*sqrt(-b^2 + 4*a*c)*
x^(2*n)*log(c*x^(2*n) + b*x^n + a) - 2*(b^3 - 3*a*b*c)*x^(2*n)*ar
ctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 -
4*a*c)) - sqrt(-b^2 + 4*a*c)*a^2)/(sqrt(-b^2 + 4*a*c)*a^3*n*x^(2*
n))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-2\,n-1}}{cx^{2\,n}+bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.555 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=164

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-3n}}{3an}$$

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n
) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2
 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/
a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rubi [A] time = 0.474117, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-3n}}{3an}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n
) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2
 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/
a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rubi in Sympy [A] time = 70.4593, size = 151, normalized size = 0.92

$$-\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{x^{-n}\left(-ac+b^2\right)}{a^3n}-\frac{b\left(-2ac+b^2\right)\log\left(x^n\right)}{a^4n}+\frac{b\left(-2ac+b^2\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{a^4n\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-3*n)/(3*a*n) + b*x**(-2*n)/(2*a**2*n) - x**(-n)*(-a*c + b**
2)/(a**3*n) - b*(-2*a*c + b**2)*log(x**n)/(a**4*n) + b*(-2*a*c +
b**2)*log(a + b*x**n + c*x**(2*n))/(2*a**4*n) - (2*a**2*c**2 - 4*
a*b**2*c + b**4)*atanh((b + 2*c*x**n)/sqrt(-4*a*c + b**2))/(a**4*
n*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.527123, size = 136, normalized size = 0.83

$$\frac{ax^{-3n}\left(-2a^2+3ax^n\left(b+2cx^n\right)-6b^2x^{2n}\right)-\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{2ax^{-n}+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+3\left(b^3-2abc\right)\log\left(x^{-2n}\left(a+bx^n\right)+c\right)}{6a^4n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((a*(-2*a^2 - 6*b^2*x^(2*n) + 3*a*x^n*(b + 2*c*x^n)))/x^(3*n) - (
6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + (2*a)/x^n)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(b^3 - 2*a*b*c)*Log[c + (a + b*
x^n)/x^(2*n)])/(6*a^4*n)
```

Maple [B] time = 0.256, size = 1300, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^
n)^3+8/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n
^2-a^4*b^2*n^2)*n^2*ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2
*ln(x)*b^5-4/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+
b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*
a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*
a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*
a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))
/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+
1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3
*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*
b^2*c+b^4))*b^5+1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a
```

```
*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*
c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5
*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10
)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^
5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*
b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*
c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a
^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/
c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1
/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3
*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*
b^2*c+b^4))*b^5-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*
a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6
*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^
5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^1
0)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,abx^{n}-2\,a^{2}-6\,\left(b^{2}-ac\right)x^{2\,n}\right)x^{-3\,n}}{6\,a^{3}n}+\int-\frac{b^{3}-2\,abc+\left(b^{2}c-ac^{2}\right)x^{n}}{a^{3}cxx^{2\,n}+a^{3}bxx^{n}+a^{4}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))*x^(-3*n)/(a^3*n)
+ integrate(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*
n) + a^3*b*x*x^n + a^4*x), x)
```

Fricas [A] time = 0.295659, size = 1, normalized size = 0.01

$$\left[\frac{3\,\sqrt{b^{2}-4\,ac}a^{2}bx^{n}-6\,\left(b^{3}-2\,abc\right)\sqrt{b^{2}-4\,ac}nx^{3\,n}\log\left(x\right)-2\,\sqrt{b^{2}-4\,ac}a^{3}+3\,\left(b^{3}-2\,abc\right)\sqrt{b^{2}-4\,ac}x^{3\,n}\log\left(cx^{2\,n}+bx\right.}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(b^2 - 4*a*c)*a^2*b*x^n - 6*(b^3 - 2*a*b*c)*sqrt(b^2
- 4*a*c)*n*x^(3*n)*log(x) - 2*sqrt(b^2 - 4*a*c)*a^3 + 3*(b^3 - 2*
```

```
a*b*c)*sqrt(b^2 - 4*a*c)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 3*(
b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^(3*n)*log((2*sqrt(b^2 - 4*a*c)*c^2
*x^(2*n) - b^3 + 4*a*b*c - 2*(b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)
*b*c)*x^n + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^(2*n) + b*x^n +
 a)) - 6*(a*b^2 - a^2*c)*sqrt(b^2 - 4*a*c)*x^(2*n))/(sqrt(b^2 - 4
*a*c)*a^4*n*x^(3*n)), 1/6*(3*sqrt(-b^2 + 4*a*c)*a^2*b*x^n - 6*(b^
3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*x^(3*n)*log(x) - 2*sqrt(-b^2 +
4*a*c)*a^3 + 3*(b^3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(3*n)*log(c*x
^(2*n) + b*x^n + a) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^(3*n)*arc
tan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4
*a*c)) - 6*(a*b^2 - a^2*c)*sqrt(-b^2 + 4*a*c)*x^(2*n))/(sqrt(-b^2
 + 4*a*c)*a^4*n*x^(3*n))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-3n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.556 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=353

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b
 + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 -
 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x
^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b -
 Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4
)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4
*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rubi [A] time = 1.12749, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b
 + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 -
 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x
^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b -
 Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4
)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4
*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rubi in Sympy [A] time = 111.414, size = 311, normalized size = 0.88

$$-\frac{2\cdot 2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{\frac{n}{4}}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{n\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2\cdot 2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{\frac{n}{4}}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{n\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2\cdot 2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{\frac{n}{4}}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{n\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2\cdot 2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{\frac{n}{4}}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{n\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*2**(3/4)*c**(3/4)*atan(2**(1/4)*c**(1/4)*x**(n/4)/(-b + sqrt(-
4*a*c + b**2))**(1/4))/(n*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(
-4*a*c + b**2)) - 2*2**(3/4)*c**(3/4)*atanh(2**(1/4)*c**(1/4)*x**
(n/4)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(n*(-b + sqrt(-4*a*c + b
**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2*2**(3/4)*c**(3/4)*atan(2**(
1/4)*c**(1/4)*x**(n/4)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(n*(-b
- sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2*2**(3/4)*c
**(3/4)*atanh(2**(1/4)*c**(1/4)*x**(n/4)/(-b - sqrt(-4*a*c + b**2
))**(1/4))/(n*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**
2))
```

Mathematica [C] time = 0.0714122, size = 62, normalized size = 0.18

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{4\log\left(x^{n/4}-\#1\right)-n\log(x)}{2\#1^7c+\#1^3b}\&\right]}{4n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (-(n*Log[x]) + 4*Log[x^(n/4) - #1
])/(b*#1^3 + 2*c*#1^7) & ]/(4*n)
```

Maple [C] time = 0.391, size = 280, normalized size = 0.8

$$\sum_{\_R = RootOf\left(\left(256 a^7 c^4 n^8 - 256 a^6 b^2 c^3 n^8 + 96 a^5 b^4 c^2 n^8 - 16 a^4 b^6 c n^8 + a^3 b^8 n^8\right)\_Z^8 + \left(-48 a^3 b c^3 n^4 + 40 a^2 b^3 c^2 n^4 - 11 a b^5 c n^4 + b^7 n^4\right)\_Z^4 + c^3\right)} \_R \ln\left(x^{\frac{n}{4}} + \left(16 \frac{n^5 b a^5 c^2}{a c^2 - b^2 c} - 8 \frac{n^5 b^3 a^4 c}{a c^2 - b^2 c} + \frac{n^5 b^5 a^3}{a c^2 - b^2 c}\right) \_R^5 + \left(2 \frac{a^2 c^2 n}{a c^2 - b^2 c} - 4 \frac{a b^2 c n}{a c^2 - b^2 c} + \frac{b^4 n}{a c^2 - b^2 c}\right) \_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*
c)*n^5*b^3*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*
c)*n*a^2*c^2-4/(a*c^2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R)
,_R=RootOf((256*a^7*c^4*n^8-256*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^
8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8+(-48*a^3*b*c^3*n^4+40*a^2*b^
3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [A] time = 0.47247, size = 4443, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2
)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*arctan(-1/2*sqrt(2)*((a^3*b^5 -
8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (
b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 -
 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))/(2*(b^2*c
- a*c^2)*x*x^(1/4*n - 1) - x*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2
*c^4)*x^2*x^(1/2*n - 2) - sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a
^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a
^9*c^3)*n^8)) - (b^8 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^
3 + 8*a^4*c^4)*n^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n
^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48
*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*n^4)))/x^2))) - 2*sqrt(2)*sqrt(sqrt(2)*sqr
t(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c
 + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n
^4)))*arctan(-1/2*sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)
*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)*n^4)))/(2*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - x
*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^(1/2*n - 2) + sq
rt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3
 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^8 - 8*a*b
^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(((a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^
8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))
/x^2))) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c
^2)*x*x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*
c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8
*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sq
rt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2
```

```
*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - sqrt(2)*((a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)
) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) +
b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) -
 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1
/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8
*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*s
qrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) -
 b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(
-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b
^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^4 - 5
*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c
)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.557 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=610

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$-\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

```
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3)
)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[
(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) + (2
^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3
)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) -
 (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(
1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n
) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/
3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n)
+ (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b
 + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)
])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rubi [A] time = 2.16576, antiderivative size = 610, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$-\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3)
)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[
(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) + (2
^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3
)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) -
 (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(
1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n
) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/
3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n)
+ (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b
 + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)
])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rubi in Sympy [A] time = 170.553, size = 559, normalized size = 0.92

$$-\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x^{\frac{n}{3}}+\sqrt[3]{b+\sqrt{-4ac+b^2}}\right)}{n\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}x^{\frac{2n}{3}}-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x^{\frac{n}{3}}\sqrt[3]{b+\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{2n\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{2}{3}}\sqrt{3}c^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{\frac{n}{3}}}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{n\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(\sqrt[3]{2}\sqrt[3]{c}x^{\frac{n}{3}}+\sqrt[3]{b-\sqrt{-4ac+b^2}}\right)}{n\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{2}{3}}c^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}x^{\frac{2n}{3}}-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x^{\frac{n}{3}}\sqrt[3]{b-\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{2n\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{2}{3}}\sqrt{3}c^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{\frac{n}{3}}}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{n\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2**(2/3)*c**(2/3)*log(2**(1/3)*c**(1/3)*x**(n/3) + (b + sqrt(-4*
a*c + b**2))**(1/3))/(n*(b + sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*
a*c + b**2)) + 2**(2/3)*c**(2/3)*log(c**(2/3)*x**(2*n/3) - 2**(2/
3)*c**(1/3)*x**(n/3)*(b + sqrt(-4*a*c + b**2))**(1/3)/2 + 2**(1/3
)*(b + sqrt(-4*a*c + b**2))**(2/3)/2)/(2*n*(b + sqrt(-4*a*c + b**
2))**(2/3)*sqrt(-4*a*c + b**2)) + 2**(2/3)*sqrt(3)*c**(2/3)*atan(
sqrt(3)*(-2*2**(1/3)*c**(1/3)*x**(n/3)/(3*(b + sqrt(-4*a*c + b**2
))**(1/3)) + 1/3))/(n*(b + sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*
c + b**2)) + 2**(2/3)*c**(2/3)*log(2**(1/3)*c**(1/3)*x**(n/3) + (
b - sqrt(-4*a*c + b**2))**(1/3))/(n*(b - sqrt(-4*a*c + b**2))**(2
/3)*sqrt(-4*a*c + b**2)) - 2**(2/3)*c**(2/3)*log(c**(2/3)*x**(2*n
/3) - 2**(2/3)*c**(1/3)*x**(n/3)*(b - sqrt(-4*a*c + b**2))**(1/3)
/2 + 2**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)/2)/(2*n*(b - sqrt(
-4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b**2)) - 2**(2/3)*sqrt(3)*c*
*(2/3)*atan(sqrt(3)*(-2*2**(1/3)*c**(1/3)*x**(n/3)/(3*(b - sqrt(-
```

```
4*a*c + b**2))**(1/3)) + 1/3))/(n*(b - sqrt(-4*a*c + b**2))**(2/3
)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0679407, size = 62, normalized size = 0.1

$$\frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{3\log\left(x^{n/3}-\#1\right)-n\log(x)}{2\#1^5c+\#1^2b}\&\right]}{3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (-(n*Log[x]) + 3*Log[x^(n/3) - #1
])/(b*#1^2 + 2*c*#1^5) & ]/(3*n)
```

Maple [C] time = 0.348, size = 260, normalized size = 0.4

$$\sum_{\_R=RootOf\left((64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4cn^6-a^2b^6n^6)\_Z^6+(16a^2bc^2n^3-8ab^3cn^3+b^5n^3)\_Z^3+c^2\right)}\_R\ln\left(x^{\frac{n}{3}}+\left(-16\frac{n^4ba^4c^2}{2ac^2-b^2c}+8\frac{n^4b^3a^3c}{2ac^2-b^2c}-\frac{n^4b^5a^2}{2ac^2-b^2c}\right)\_R^4+\left(4\frac{a^2c^2n}{2ac^2-b^2c}-5\frac{ab^2cn}{2ac^2-b^2c}+\frac{b^4n}{2ac^2-b^2c}\right)\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2
-b^2*c)*n^4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a
*c^2-b^2*c)*n*a^2*c^2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*
c)*n*b^4)*_R),_R=RootOf((64*a^5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3
*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2*b*c^2*n^3-8*a*b^3*c*n^3+b^5*
n^3)*_Z^3+c^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [A] time = 0.429887, size = 6064, normalized size = 9.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*
a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-(1/2
)^(1/3)*(sqrt(3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt(
(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2
)*n)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) +
b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)/(4*(b^2*c - 2*a*c^2)*x*x^(1/3
*n - 1) - 2*sqrt(2)*x*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)
*x^2*x^(2/3*n - 2) - (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 3
2*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6
)) - (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(
1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^
6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) - (1/2)^(2/3)*((a^2*b^9
 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^
4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*
c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^8 - 10*a*b^6*c + 36*a
^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(((a^2*b^2 - 4*a^3
*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4
*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)
*n^3))^(2/3))/x^2) - (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4
*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5
*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^4 - 6*a*b^2*c +
8*a^2*c^2)*n)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4
*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)
*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))) - 2*sqrt(3)*(1/2)^
(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^
2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
- b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-(1/2)^(1/3)*(sqrt(3
)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)*n^6)) + sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(-((a^2*b^
2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 1
2*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 -
```

```
 4*a^3*c)*n^3))^(1/3)/(4*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) - 2*sq
rt(2)*x*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n
- 2) + (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3
- 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^
6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^6*c -
8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(-(
(a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*
b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^
2*b^2 - 4*a^3*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7
*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((
b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^
2*c^2 - 64*a^7*c^3)*n^6)) + (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 -
56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt
((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3)
)/x^2) + (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*
a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n
)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(
(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)
/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))) + (1/2)^(1/3)*(((a^2*b^2 - 4*
a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*
b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3
*c)*n^3))^(1/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) + (1/2)
^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 -
64*a^7*c^3)*n^6)) - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(((a^2*b^2 -
 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*
a^3*c)*n^3))^(1/3))/x) + (1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*s
qrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a
^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1
/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) - (1/2)^(1/3)*((a^2
*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*
n^6)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(-((a^2*b^2 - 4*a^3*c)*n
^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3)
)^(1/3))/x) - 1/2*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4
 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(
8*(2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1/2
)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*
c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*
b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^6*c - 8*a*b^4*c^2
 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4
*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5
*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^
3*c)*n^3))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*
b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a
^7*c^3)*n^6)) - (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c
^3 + 32*a^4*c^4)*n^2)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*
```

```
a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2) - 1/2*
(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*
n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(8*(2*(b^4*c^2 - 4
*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3)*((a^2*b^7
*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((
b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^
2*c^2 - 64*a^7*c^3)*n^6)) + (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3
 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt
((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)
 + (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^
5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) +
(b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)
*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^
2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
- b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.558 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n
) - (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*n)
```

Rubi [A] time = 0.367823, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n
) - (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*n)
```

Rubi in Sympy [A] time = 34.2183, size = 151, normalized size = 0.89

$$-\frac{2\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x^{\frac{n}{2}}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{n\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{2\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x^{\frac{n}{2}}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{n\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*sqrt(2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x**(n/2)/sqrt(b + sqrt(-4
*a*c + b**2)))/(n*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**
2)) + 2*sqrt(2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x**(n/2)/sqrt(b - sq
rt(-4*a*c + b**2)))/(n*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c
+ b**2))
```

Mathematica [C] time = 0.0558754, size = 60, normalized size = 0.36

$$\frac{\text{RootSum}\left[\#1^4 c+\#1^2 b+a\&,\frac{2\log\left(x^{n/2}-\#1\right)-n\log(x)}{2\#1^3 c+\#1 b}\&\right]}{2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] RootSum[a + b*#1^2 + c*#1^4 & , (-(n*Log[x]) + 2*Log[x^(n/2) - #1
])/(b*#1 + 2*c*#1^3) & ]/(2*n)
```

Maple [C] time = 0.179, size = 114, normalized size = 0.7

$$\sum_{\_R=RootOf\left((16\,a^3c^2n^4-8\,a^2b^2cn^4+ab^4n^4)\_Z^4+(-4\,abcn^2+b^3n^2)\_Z^2+c\right)}\_R\ln\left(x^{\frac{n}{2}}+\left(4\,n^3ba^2-\frac{n^3b^3a}{c}\right)\_R^3+\left(2\,an-\frac{b^2n}{c}\right)\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n
*b^2)*_R),_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z
^4+(-4*a*b*c*n^2+b^3*n^2)*_Z^2+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [A]   time = 0.295488, size = 1081, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3
*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1)
+ sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^
4)) - (b^2 - 4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*
b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt
(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)
) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - sqrt(2
)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b
^2 - 4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*
a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt
(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((
a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3
 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*
c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4
)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*sqrt(((a*b^2 -
 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*
a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b
*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt
(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((
a*b^2 - 4*a^2*c)*n^2)))/x)
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.559 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])
/(a^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2
*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rubi [A] time = 0.794926, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])
/(a^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2
*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rubi in Sympy [A] time = 77.2929, size = 202, normalized size = 0.99

$$-\frac{2x^{-\frac{n}{2}}}{an}+\frac{\sqrt{2}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{a^{\frac{3}{2}}n\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{a^{\frac{3}{2}}n\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*x**(-n/2)/(a*n) + sqrt(2)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**
2))*atan(sqrt(2)*sqrt(a)*x**(-n/2)/sqrt(b + sqrt(-4*a*c + b**2)))
/(a**(3/2)*n*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) -
 sqrt(2)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqr
t(a)*x**(-n/2)/sqrt(b - sqrt(-4*a*c + b**2)))/(a**(3/2)*n*sqrt(b
- sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0975331, size = 105, normalized size = 0.51

$$-\frac{4x^{-n/2}-\text{RootSum}\left[\#1^4a+\#1^2b+c\&,\frac{2\#1^2b\log\left(x^{-n/2}-\#1\right)+\#1^2bn\log(x)+2c\log\left(x^{-n/2}-\#1\right)+cn\log(x)}{2\#1^3a+\#1b}\&\right]}{2an}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(4/x^(n/2) - RootSum[c + b*#1^2 + a*#1^4 & , (c*n*Log[x] + 2*c*L
og[x^(-n/2) - #1] + b*n*Log[x]*#1^2 + 2*b*Log[x^(-n/2) - #1]*#1^2
)/(b*#1 + 2*a*#1^3) & ])/(2*a*n)
```

Maple [C] time = 0.307, size = 268, normalized size = 1.3

$$-2\,\frac{1}{anx^{n/2}}+\sum_{\_R=RootOf\left((16\,a^5c^2n^4-8\,a^4b^2cn^4+a^3b^4n^4)\_Z^4+(12\,a^2bc^2n^2-7\,ab^3cn^2+b^5n^2)\_Z^2+c^3\right)}\_R\ln\left(x^{\frac{n}{2}}+\left(-8\,\frac{a^5n^3c^2}{ac^3-b^2c^2}+6\,\frac{n^3b^2a^4c}{ac^3-b^2c^2}-\frac{n^3b^4a^3}{ac^3-b^2c^2}\right)\_R^3+\left(-5\,\frac{a^2bc^2n}{ac^3-b^2c^2}+5\,\frac{ab^3cn}{ac^3-b^2c^2}-\frac{b^5n}{ac^3-b^2c^2}\right)\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^
5*c^2+6/(a*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a
^3)*_R^3+(-5/(a*c^3-b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*
a*c-1/(a*c^3-b^2*c^2)*n*b^5)*_R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*
b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*b*c^2*n^2-7*a*b^3*c*n^2+b^5*n
^2)*_Z^2+c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,x^{-\frac{1}{2}\,n}}{an}-\int\frac{cx^{\frac{3}{2}\,n}+bx^{\frac{1}{2}\,n}}{acxx^{2\,n}+abxx^{n}+a^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] -2*x^(-1/2*n)/(a*n) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*
x*x^(2*n) + a*b*x*x^n + a^2*x), x)
```

Fricas [A] time = 0.311095, size = 1659, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3
*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) +
sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*
sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
(a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n
^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)
/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n -
 1) - sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^
2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^
4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a
*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/
2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^3*b^2 -
4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 +
 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^
(-1/2*n - 1) - sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c +
 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*
```

```
c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^3*b^
2 - 4*a^4*c)*n^2)))/x) - 4*x*x^(-1/2*n - 1))/(a*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n} + bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.560 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=699

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{2\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}\right)}{2\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\sqrt{3}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{3x^{-n/3}}{an}$$

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x
^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (Sqrt[3]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
(2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt
[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) + ((b - (
b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3)
 + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4
*a*c])^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b +
 Sqrt[b^2 - 4*a*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*
a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sq
```

```
rt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) + (2^(2/3)*a^(
2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n)
- ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b +
 Sqrt[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt
[b^2 - 4*a*c])^(2/3)*n)
```

Rubi [A] time = 3.15106, antiderivative size = 699, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{2\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}\right)}{2\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\sqrt{3}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{3x^{-n/3}}{an}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x
^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (Sqrt[3]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
(2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt
[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) + ((b - (
b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3)
 + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4
*a*c])^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b +
 Sqrt[b^2 - 4*a*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*
a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sq
rt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) + (2^(2/3)*a^(
2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n)
- ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b +
 Sqrt[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt
[b^2 - 4*a*c])^(2/3)*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.115379, size = 107, normalized size = 0.15

$$-\frac{9x^{-n/3}-\text{RootSum}\left[\#1^6a+\#1^3b+c\&,\frac{3\#1^3b\log\left(x^{-n/3}-\#1\right)+\#1^3bn\log(x)+3c\log\left(x^{-n/3}-\#1\right)+cn\log(x)}{2\#1^5a+\#1^2b}\&\right]}{3an}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(9/x^(n/3) - RootSum[c + b*#1^3 + a*#1^6 & , (c*n*Log[x] + 3*c*L
og[x^(-n/3) - #1] + b*n*Log[x]*#1^3 + 3*b*Log[x^(-n/3) - #1]*#1^3
```

```
)/(b*#1^2 + 2*a*#1^5) & ])/(3*a*n)
```

Maple [C] time = 0.585, size = 534, normalized size = 0.8

$$-3\frac{1}{anx^{n/3}} + \sum_{\_R=RootOf\left((64\,a^7c^3n^6-48\,a^6b^2c^2n^6+12\,a^5b^4cn^6-a^4b^6n^6)\_Z^6+(-32\,a^3bc^3n^3+32\,a^2b^3c^2n^3-10\,ab^5cn^3+b^7n^3)\_Z^3+c^4\right)} \_R\ln\Bigg(x^{\frac{n}{3}} + \Bigg(-64\frac{a^8n^5c^4}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}+112\frac{n^5b^2a^7c^3}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}-60\frac{n^5b^4a^6c^2}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}+13\frac{n^5b^6a^5c}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}-\frac{}{2\,a^2c^5} + \Bigg(28\frac{bn^2a^4c^4}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}-63\frac{b^3n^2a^3c^3}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}+42\frac{b^5n^2a^2c^2}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}-11\frac{n^2b^7ac}{2\,a^2c^5-4\,ab^2c^4+b^4c^3}+\frac{}{2\,a^2c^5-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^
4+b^4*c^3)*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^
2*a^7*c^3-60/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(
2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2
*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^
3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^3*a^3*c
^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-11/(2*a^2*c
^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^4*
c^3)*n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+
12*a^5*b^4*c*n^6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*
c^2*n^3-10*a*b^5*c*n^3+b^7*n^3)*_Z^3+c^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\,x^{-\frac{1}{3}\,n}}{an} - \int \frac{cx^{\frac{5}{3}\,n} + bx^{\frac{2}{3}\,n}}{acxx^{2\,n} + abxx^{n} + a^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] -3*x^(-1/3*n)/(a*n) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*
x*x^(2*n) + a*b*x*x^n + a^2*x), x)
```

Fricas [A] time = 0.560027, size = 7922, normalized size = 11.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^
3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*arctan((1/2)^(1/3)*
(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6
 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - sqrt(3)*
(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(((a^4*b^2 - 4*
a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^
3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^
11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)/(
4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + 2*sqrt(2)*
x*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c^
5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*((a^4*b^9*c - 12*
a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4
*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^
4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*
n^6)) - (b^10*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4
+ 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1)*(((a^4*b^2 - 4
*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a
^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)
- (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^
7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*
b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 -
12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^12 - 14*
a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 1
52*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt(
(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) +
b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2) - (1/2)^(1/
3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12
*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^6 - 8*a*b^
4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(((a^4*b^2 - 4*a^5*c)*n^3*sq
rt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4
)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6))
 + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))) - 4*sqrt(3)*
(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9
*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((
a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*arctan((1/2)^(1/3)*(sqrt(3)*(a^4*b
^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c
```

```
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^6 - 8*a*b^4*
c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6))
- b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)/(4*(b^4*c - 4*a
*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + 2*sqrt(2)*x*sqrt((2*(b^8
*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)
*x^2*x^(-2/3*n - 2) + (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c^2 +
50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 -
 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^10*
c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c
^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^
4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)
) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) + (1/2)^(2/3)
*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 3
52*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c
+ 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a*b^10*c + 76
*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^
5 + 32*a^6*c^6)*n^2)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*
c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2) + (1/2)^(1/3)*((a^4*b^5
 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^6 - 8*a*b^4*c + 18*a^2
*b^2*c^2 - 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6
 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*
a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))) - 2*(1/2)^(1/3)*a*n*(((
a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2
*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n
^3))^(1/3)*log((2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n -
 1) + (1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6))
- (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(((a^4*b^2 -
4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*
c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*
a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)
)/x) - 2*(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6
 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*
a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) - (1/2)^(1/3)*((a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2
- 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9
*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((
```

```
a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) + (1/2)^(1/3)*a*n*(((a^4*b^2 -
 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2
*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64
*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3
)*log(8*(2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c
^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*((a^4*b^9*c - 12
*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^
4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)
*n^6)) - (b^10*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4
 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1)*(((a^4*b^2 -
4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*
c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*
a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)
 - (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a
^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 -
 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^12 - 14
*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 -
152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) +
 b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2) + (1/2)^(1
/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2
- 4*a^5*c)*n^3))^(1/3)*log(8*(2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b
^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) + (1/2)^(
1/3)*((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c
^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b
^2*c^2 - 64*a^11*c^3)*n^6)) + (b^10*c - 12*a*b^8*c^2 + 52*a^2*b^6
*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3
*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2
- 4*a^5*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c +
98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^
5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c
^3)*n^6)) + (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^
3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(-((a^4*
b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2
 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))
^(2/3))/x^2) + 6*x*x^(-1/3*n - 1))/(a*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n} + bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.561 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=414

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$
$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$
$$-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{4x^{-n/4}}{an}$$

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4)
)])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b
^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sq
rt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c
])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/
(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[
b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^
(3/4)*n)
```

Rubi [A] time = 1.56316, antiderivative size = 414, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{4x^{-n/4}}{an}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4)
)])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b
^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sq
rt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c
])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/
(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[
b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^
(3/4)*n)
```

Rubi in Sympy [A] time = 179.699, size = 403, normalized size = 0.97

$$-\frac{4x^{-\frac{n}{4}}}{an}+\frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-\frac{n}{4}}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{a^{\frac{5}{4}}n\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-\frac{n}{4}}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{a^{\frac{5}{4}}n\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-\frac{n}{4}}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{a^{\frac{5}{4}}n\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-\frac{n}{4}}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{a^{\frac{5}{4}}n\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -4*x**(-n/4)/(a*n) + 2**(3/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b*
*2))*atan(2**(1/4)*a**(1/4)*x**(-n/4)/(-b + sqrt(-4*a*c + b**2))*
*(1/4))/(a**(5/4)*n*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c
 + b**2)) + 2**(3/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan
h(2**(1/4)*a**(1/4)*x**(-n/4)/(-b + sqrt(-4*a*c + b**2))**(1/4))/
(a**(5/4)*n*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)
) - 2**(3/4)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(2**(1/4
)*a**(1/4)*x**(-n/4)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(a**(5/4)
*n*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) - 2**(3
/4)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*a**(1/
4)*x**(-n/4)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(a**(5/4)*n*(-b -
 sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.109415, size = 105, normalized size = 0.25

$$\frac{\text{RootSum}\left[\#1^8a+\#1^4b+c\&,\frac{4\#1^4b\log\left(x^{-n/4}-\#1\right)+\#1^4bn\log(x)+4c\log\left(x^{-n/4}-\#1\right)+cn\log(x)}{2\#1^7a+\#1^3b}\&\right]-16x^{-n/4}}{4an}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-16/x^(n/4) + RootSum[c + b*#1^4 + a*#1^8 & , (c*n*Log[x] + 4*c*
Log[x^(-n/4) - #1] + b*n*Log[x]*#1^4 + 4*b*Log[x^(-n/4) - #1]*#1^
4)/(b*#1^3 + 2*a*#1^7) & ])/(4*a*n)
```

Maple [C] time = 0.841, size = 630, normalized size = 1.5

$$-4\,\frac{1}{anx^{n/4}}+\sum_{\_R=RootOf\left(\left(256\,a^9c^4n^8-256\,a^8b^2c^3n^8+96\,a^7b^4c^2n^8-16\,a^6b^6cn^8+a^5b^8n^8\right)\_Z^8+\left(80\,a^4bc^4n^4-120\,a^3b^3c^3n^4+61\,a^2b^5c^2n^4-13\,ab^7cn^4+b^9n^4\right)\_Z^4+c^5\right)}\_R\ln\Bigg(x$$
$$+\left(-128\,\frac{a^{10}n^7c^5}{a^2c^6-3\,ab^2c^5+b^4c^4}+352\,\frac{n^7b^2a^9c^4}{a^2c^6-3\,ab^2c^5+b^4c^4}-280\,\frac{n^7b^4a^8c^3}{a^2c^6-3\,ab^2c^5+b^4c^4}+98\,\frac{n^7b^6a^7c^2}{a^2c^6-3\,ab^2c^5+b^4c^4}-16\,\frac{n}{a^2c^6-3}\right.$$
$$+\left(-36\,\frac{n^3ba^5c^5}{a^2c^6-3\,ab^2c^5+b^4c^4}+129\,\frac{n^3b^3a^4c^4}{a^2c^6-3\,ab^2c^5+b^4c^4}-138\,\frac{n^3b^5a^3c^3}{a^2c^6-3\,ab^2c^5+b^4c^4}+63\,\frac{n^3b^7a^2c^2}{a^2c^6-3\,ab^2c^5+b^4c^4}-13\,\frac{n^3}{a^2c^6-3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5
+b^4*c^4)*n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*
a^9*c^4-280/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2
*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5
+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^10*
a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b*a^5*c^5+129/(a
^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c^6-3*a*b^2*
c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3
*b^7*a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*
c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n
^8-256*a^8*b^2*c^3*n^8+96*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^
8*n^8)*_Z^8+(80*a^4*b*c^4*n^4-120*a^3*b^3*c^3*n^4+61*a^2*b^5*c^2*
n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c^5))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,x^{-\frac{1}{4}\,n}}{an}-\int\frac{cx^{\frac{7}{4}\,n}+bx^{\frac{3}{4}\,n}}{acxx^{2\,n}+abxx^{n}+a^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] -4*x^(-1/4*n)/(a*n) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*
x*x^(2*n) + a*b*x*x^n + a^2*x), x)
```

Fricas [A] time = 0.626246, size = 5652, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*
a^6*b^2*c + 16*a^7*c^2)*n^4)))*arctan(1/2*sqrt(2)*((a^5*b^5 - 8*a
^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^
2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16
*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)*n^4)))/(2*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x
*x^(-1/4*n - 1) + x*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c
^4 - 6*a^3*b^2*c^5 + a^4*c^6)*x^2*x^(-1/2*n - 2) - sqrt(2)*((a^5*
b^11 - 15*a^6*b^9*c + 85*a^7*b^7*c^2 - 220*a^8*b^5*c^3 + 240*a^9*
b^3*c^4 - 64*a^10*b*c^5)*n^6*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^12 - 12*a*b^10*c + 55*a^2*b^8
*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6
*c^6)*n^2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^1
0*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^
5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*n^4)))/x^2))) - 4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
```

```
b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a
^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*arctan(1/2*sqrt(2)*((a^
5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4*c + 13
*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/(2*(b^4*c - 3*a*b^2*c^2 +
a^2*c^3)*x*x^(-1/4*n - 1) + x*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11
*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*x^2*x^(-1/2*n - 2) + sqrt
(2)*((a^5*b^11 - 15*a^6*b^9*c + 85*a^7*b^7*c^2 - 220*a^8*b^5*c^3
+ 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^12 - 12*a*b^10*c +
55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c
^5 + 8*a^6*c^6)*n^2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n
^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n
^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)*n^4)))/x^2))) - sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b
^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*
c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c -
 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*
a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c
^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*n^4))))/x) + sqrt(2)*a*n*sqrt(sqrt(2)*sqrt
(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4
*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5
*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) + sqrt(2)*a*n*sqrt(sqr
t(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a
*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))
)*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(
2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^
```

```
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4
*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)
/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*s
qrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^1
0*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^
5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1)
- sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*
a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2
*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) + 8*x*x^
(-1/4*n - 1))/(a*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.562 $\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=140

$$-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (-2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c
*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(3*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]))
```

Rubi [A] time = 0.28012, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c
*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(3*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]))
```

Rubi in Sympy [A] time = 24.611, size = 122, normalized size = 0.87

$$-\frac{2cx^3{}_2F_1\left(\begin{matrix}1,\frac{3}{n}\\\frac{n+3}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{3\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)}-\frac{2cx^3{}_2F_1\left(\begin{matrix}1,\frac{3}{n}\\\frac{n+3}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{3\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*x**3*hyper((1, 3/n), ((n + 3)/n,), -2*c*x**n/(b + sqrt(-4*a*
c + b**2)))/(3*(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2))) - 2*c*x**
3*hyper((1, 3/n), ((n + 3)/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2
)))/(3*(-4*a*c + b**2 - b*sqrt(-4*a*c + b**2)))
```

Mathematica [A] time = 1.16769, size = 265, normalized size = 1.89

$$-\frac{2}{3}cx^3\left(\frac{1-\left(\frac{x^n}{x^n-\frac{\sqrt{b^2-4ac}-b}{2c}}\right)^{-3/n}{}_2F_1\left(-\frac{3}{n},-\frac{3}{n};\frac{n-3}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{1-8^{-1/n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-3/n}{}_2F_1\left(-\frac{3}{n},-\frac{3}{n};\frac{n-3}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*((1 - Hypergeometric2F1[-3/n, -3/n, (-3 + n)/n, (b - Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-(-b +
Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(3/n))/(b^2 - 4*a*c - b*Sqrt[b^2
 - 4*a*c]) + (1 - Hypergeometric2F1[-3/n, -3/n, (-3 + n)/n, (b +
Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(8^n^(-1)*(
(c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/n)))/(Sqrt[b^2 - 4*
a*c]*(b + Sqrt[b^2 - 4*a*c]))))/3
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")

[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")

[Out] integral(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.563 $\int \frac{x}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=136

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -((c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2*H
ypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.115744, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2*H
ypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 22.1616, size = 112, normalized size = 0.82

$$-\frac{cx^2\,{}_2F_1\left(\begin{matrix}1,\frac{2}{n}\\ \frac{n+2}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{-4ac+b^2+b\sqrt{-4ac+b^2}}-\frac{cx^2\,{}_2F_1\left(\begin{matrix}1,\frac{2}{n}\\ \frac{n+2}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{-4ac+b^2-b\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -c*x**2*hyper((1, 2/n), ((n + 2)/n,), -2*c*x**n/(b + sqrt(-4*a*c
+ b**2)))/(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2)) - c*x**2*hyper(
```

```
(1, 2/n), ((n + 2)/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/(-4*
a*c + b**2 - b*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 1.43682, size = 263, normalized size = 1.93

$$-cx^2\left(\frac{1-\left(\frac{x^n}{x^n-\frac{\sqrt{b^2-4ac}-b}{2c}}\right)^{-2/n}{}_2F_1\left(-\frac{2}{n},-\frac{2}{n};\frac{n-2}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{1-4^{-1/n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-2/n}{}_2F_1\left(-\frac{2}{n},-\frac{2}{n};\frac{n-2}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(c*x^2*((1 - Hypergeometric2F1[-2/n, -2/n, (-2 + n)/n, (b - Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-(-b + Sq
rt[b^2 - 4*a*c])/(2*c) + x^n))^(2/n))/(b^2 - 4*a*c - b*Sqrt[b^2 -
 4*a*c]) + (1 - Hypergeometric2F1[-2/n, -2/n, (-2 + n)/n, (b + Sq
rt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(4^n^(-1)*((c
*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)))/(Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c]))))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral(x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

3.564 $\int \frac{1}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=124

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (-2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (2*c*x
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.124857, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] (-2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (2*c*x
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 11.7729, size = 112, normalized size = 0.9

$$-\frac{2cx{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{-4ac+b^2+b\sqrt{-4ac+b^2}}-\frac{2cx{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{-4ac+b^2-b\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*x*hyper((1, 1/n), (1 + 1/n,), -2*c*x**n/(b + sqrt(-4*a*c + b
**2)))/(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2)) - 2*c*x*hyper((1,
```

```
1/n), (1 + 1/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/(-4*a*c +
b**2 - b*sqrt(-4*a*c + b**2))
```

Mathematica [B] time = 1.08581, size = 261, normalized size = 2.1

$$-2cx\left(\frac{1-\left(\frac{x^n}{x^n-\frac{\sqrt{b^2-4ac}-b}{2c}}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{n-1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{1-2^{-1/n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{n-1}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] -2*c*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b -
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-(-b
 + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/(b^2 - 4*a*c - b*Sqrt
[b^2 - 4*a*c]) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n
)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(
2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/(Sq
rt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(a+bx^n+cx^{2\,n}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

3.565 $\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=74

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]
*n) + Log[x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rubi [A] time = 0.151955, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]
*n) + Log[x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rubi in Sympy [A] time = 28.3556, size = 66, normalized size = 0.89

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{an\sqrt{-4ac+b^2}}+\frac{\log\left(x^n\right)}{an}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] b*atanh((b + 2*c*x**n)/sqrt(-4*a*c + b**2))/(a*n*sqrt(-4*a*c + b*
*2)) + log(x**n)/(a*n) - log(a + b*x**n + c*x**(2*n))/(2*a*n)
```

Mathematica [A] time = 0.176913, size = 74, normalized size = 1.

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}}+\frac{\log(a+x^n(b+cx^n))}{n}-2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a
*c]*n) - 2*Log[x] + Log[a + x^n*(b + c*x^n)]/n)/(2*a)
```

Maple [B] time = 0.112, size = 397, normalized size = 5.4

$$4\,\frac{n^2\ln(x)\,ac}{4\,a^2cn^2-ab^2n^2}-\frac{n^2\ln(x)\,b^2}{4\,a^2cn^2-ab^2n^2}-2\,\frac{c}{(4\,ac-b^2)\,n}\ln\left(x^n-1/2\,\frac{-b^2+\sqrt{-4\,ab^2c+b^4}}{bc}\right)$$
$$+\frac{b^2}{2\,a\,(4\,ac-b^2)\,n}\ln\left(x^n-\frac{1}{2\,bc}\left(-b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)$$
$$+\frac{1}{2\,a\,(4\,ac-b^2)\,n}\ln\left(x^n-\frac{1}{2\,bc}\left(-b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)\sqrt{-4\,ab^2c+b^4}$$
$$-2\,\frac{c}{(4\,ac-b^2)\,n}\ln\left(x^n+1/2\,\frac{b^2+\sqrt{-4\,ab^2c+b^4}}{bc}\right)$$
$$+\frac{b^2}{2\,a\,(4\,ac-b^2)\,n}\ln\left(x^n+\frac{1}{2\,bc}\left(b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)$$
$$-\frac{1}{2\,a\,(4\,ac-b^2)\,n}\ln\left(x^n+\frac{1}{2\,bc}\left(b^2+\sqrt{-4\,ab^2c+b^4}\right)\right)\sqrt{-4\,ab^2c+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)
*n^2*ln(x)*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^
(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^
4)^(1/2))/b/c)*b^2+1/2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2
*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)-2/(4*a*c-b^2)/n*ln(x^n
+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/n*ln(x
^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^2)/n*
ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [A] time = 0.297817, size = 1, normalized size = 0.01

$$\Bigg[\frac{2\sqrt{b^2-4ac}\,n\log(x)+b\log\left(\frac{2\sqrt{b^2-4ac}\,c^2x^{2n}+b^3-4abc+2\left(b^2c-4ac^2+\sqrt{b^2-4ac}\,bc\right)x^n+\left(b^2-2ac\right)\sqrt{b^2-4ac}}{cx^{2n}+bx^n+a}\right)-\sqrt{b^2-4ac}\log\big(cx^{2n}+}{2\sqrt{b^2-4ac}\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(b^2 - 4*a*c)*n*log(x) + b*log((2*sqrt(b^2 - 4*a*c)*c
^2*x^(2*n) + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*
c)*b*c)*x^n + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^(2*n) + b*x^n
 + a)) - sqrt(b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/(sqrt(b^2
- 4*a*c)*a*n), 1/2*(2*sqrt(-b^2 + 4*a*c)*n*log(x) - 2*b*arctan(-(
2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c))
 - sqrt(-b^2 + 4*a*c)*log(c*x^(2*n) + b*x^n + a))/(sqrt(-b^2 + 4*
a*c)*a*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x),x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.566 $\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=142

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x) + (2
*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x)
```

Rubi [A] time = 0.139895, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x) + (2
*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x)
```

Rubi in Sympy [A] time = 24.3419, size = 114, normalized size = 0.8

$$\frac{2c{}_2F_1\left(\begin{matrix}1,-\frac{1}{n}\\ \frac{n-1}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{x\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)}+\frac{2c{}_2F_1\left(\begin{matrix}1,-\frac{1}{n}\\ \frac{n-1}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{x\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] 2*c*hyper((1, -1/n), ((n - 1)/n,), -2*c*x**n/(b + sqrt(-4*a*c + b
**2)))/(x*(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2))) + 2*c*hyper((1
, -1/n), ((n - 1)/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/(x*(-
4*a*c + b**2 - b*sqrt(-4*a*c + b**2)))
```

Mathematica [A] time = 0.726893, size = 240, normalized size = 1.69

$$\frac{c2^{\frac{1}{n}+1}\left(\frac{\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{\frac{1}{n}}{}_2F_1\left(1+\frac{1}{n},1+\frac{1}{n};2+\frac{1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}-b-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{\frac{1}{n}+1}{}_2F_1\left(1+\frac{1}{n},1+\frac{1}{n};2+\frac{1}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)}{c}\right)}{(n+1)x\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^(1 + n^(-1))*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n
^(-1)*Hypergeometric2F1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b -
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x^n) + (((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n))^(1 + n^(-1))*Hypergeometric2F1[1 + n^(-1), 1 + n^(-1), 2 +
 n^(-1), (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(1 + n)*x)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n}+bx^n+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{cx^2x^{2\,n}+bx^2x^n+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2\,n}+bx^n+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.567 $\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=140

$$\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2) + (c*Hy
pergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x^2)
```

Rubi [A] time = 0.125386, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2) + (c*Hy
pergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x^2)
```

Rubi in Sympy [A] time = 24.295, size = 114, normalized size = 0.81

$$\frac{c_2F_1\left(\begin{matrix}1,-\frac{2}{n}\\ \frac{n-2}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{x^2\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)}+\frac{c_2F_1\left(\begin{matrix}1,-\frac{2}{n}\\ \frac{n-2}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{x^2\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] c*hyper((1, -2/n), ((n - 2)/n,), -2*c*x**n/(b + sqrt(-4*a*c + b**
2)))/(x**2*(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2))) + c*hyper((1,
 -2/n), ((n - 2)/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/(x**2*
(-4*a*c + b**2 - b*sqrt(-4*a*c + b**2)))
```

Mathematica [A] time = 0.750385, size = 258, normalized size = 1.84

$$\frac{c2^{\frac{n+2}{n}}\left(\frac{\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{2/n}{}_2F_1\left(\frac{n+2}{n},\frac{n+2}{n};2+\frac{2}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}-b-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{\frac{n+2}{n}}{}_2F_1\left(\frac{n+2}{n},\frac{n+2}{n};2+\frac{2}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)}{c}\right)}{(n+2)x^2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^((2 + n)/n)*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2
/n)*Hypergeometric2F1[(2 + n)/n, (2 + n)/n, 2 + 2/n, (b - Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 -
4*a*c] - 2*c*x^n) + (((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^
((2 + n)/n)*Hypergeometric2F1[(2 + n)/n, (2 + n)/n, 2 + 2/n, (b +
 Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(c*x^n)))
/(Sqrt[b^2 - 4*a*c]*(2 + n)*x^2)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="maxima")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{cx^3x^{2n}+bx^3x^n+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="fricas")

[Out] integral(1/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(cx^{2n}+bx^n+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.568 $\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n
)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.50697, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n
)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 34.1241, size = 128, normalized size = 0.86

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{4}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x**4*sqrt(a + b*x**n + c*x**(2*n))*appellf1(4/n, -1/2, -1/2, (n +
 4)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-
```

```
4*a*c + b**2)))/(4*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 7.54789, size = 820, normalized size = 5.54

$$x^4\left(\frac{4a^2bn(n+2)\left(2cx^n+b-\sqrt{b^2-4ac}\right)\left(2cx^n+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{n+4}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac-}}\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(b+\sqrt{b^2-4ac}\right)(n+4)^2\left(\left(b+\sqrt{b^2-4ac}\right)nF_1\left(2+\frac{4}{n};\frac{1}{2},\frac{3}{2};3+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)x^n-\left(\sqrt{b^2-4ac}-b\right)nF_1\left(2+\frac{4}{n};\frac{3}{2},\frac{1}{2};3+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2}{\sqrt{b^2}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*((a + x^n*(b + c*x^n))^2/(4 + n) + (4*a^2*b*n*(2 + n)*x^n*(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]])/((-b + Sqrt[b^2 -
 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(4 + n)^2*((b + Sqrt[b^2 - 4*a*c
])*n*x^n*AppellF1[2 + 4/n, 1/2, 3/2, 3 + 4/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt
[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 4/n, 3/2, 1/2, 3 + 4/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
- 8*a*(2 + n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (
a^2*n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(4*a*(4
+ n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt
[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 1/2, 3/2, 2 + 4/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 3/2, 1/2, 2 + 4/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)])))))/(a + x^n*(b + c*x^n))^(3/2)
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2\,n}+bx^n+a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2\,n}+bx^n+a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

3.569 $\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n
)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.469974, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n
)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 33.5349, size = 128, normalized size = 0.86

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf_1}\left(\frac{3}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x**3*sqrt(a + b*x**n + c*x**(2*n))*appellf1(3/n, -1/2, -1/2, (n +
 3)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-
```

```
4*a*c + b**2)))/(3*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 8.24288, size = 825, normalized size = 5.57

$$x^3\left(\frac{6a^2bn(2n+3)\left(2cx^n+b-\sqrt{b^2-4ac}\right)\left(2cx^n+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{n+3}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}}\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(b+\sqrt{b^2-4ac}\right)(n+3)^2\left(\left(b+\sqrt{b^2-4ac}\right)nF_1\left(2+\frac{3}{n};\frac{1}{2},\frac{3}{2};3+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)x^n-\left(\sqrt{b^2-4ac}-b\right)nF_1\left(2+\frac{3}{n};\frac{3}{2},\frac{1}{2};3+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2}{\sqrt{b^2}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*((3*(a + x^n*(b + c*x^n))^2)/(3 + n) + (6*a^2*b*n*(3 + 2*n)*
x^n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt
[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(3 + n)^2*((b + Sqrt[b^2 -
 4*a*c])*n*x^n*AppellF1[2 + 3/n, 1/2, 3/2, 3 + 3/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b
+ Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 3/n, 3/2, 1/2, 3 + 3/n, (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])] - 4*a*(3 + 2*n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)])) + (a^2*n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(4
*a*(3 + n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b
+ Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 1/2, 3/2, 2 + 3/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 3/2, 1/2, 2 + 3/n
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])])))))/(3*(a + x^n*(b + c*x^n))^(3/2))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a + bx^n + cx^{2n}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

3.570 $\int x\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n
)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.351197, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n
)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 28.4207, size = 128, normalized size = 0.86

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x**2*sqrt(a + b*x**n + c*x**(2*n))*appellf1(2/n, -1/2, -1/2, (n +
 2)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-
```

```
4*a*c + b**2)))/(2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 8.44303, size = 816, normalized size = 5.51

$$x^2\left(\frac{4a^2bn(n+1)\left(2cx^n+b-\sqrt{b^2-4ac}\right)\left(2cx^n+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{n+2}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac-}}\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(b+\sqrt{b^2-4ac}\right)(n+2)^2\left(\left(b+\sqrt{b^2-4ac}\right)nF_1\left(2+\frac{2}{n};\frac{1}{2},\frac{3}{2};3+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)x^n-\left(\sqrt{b^2-4ac}-b\right)nF_1\left(2+\frac{2}{n};\frac{3}{2},\frac{1}{2};3+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2}{\sqrt{b^2}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*((a + x^n*(b + c*x^n))^2/(2 + n) + (4*a^2*b*n*(1 + n)*x^n*(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 -
 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(2 + n)^2*((b + Sqrt[b^2 - 4*a*c
])*n*x^n*AppellF1[2 + 2/n, 1/2, 3/2, 3 + 2/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt
[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 2/n, 3/2, 1/2, 3 + 2/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
- 8*a*(1 + n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (
a^2*n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*c*(2 + n
)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*c*n*x^n*((b + Sqr
t[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 1/2, 3/2, 2 + 2/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (
b - Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 3/2, 1/2, 2 + 2/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])]))))/(a + x^n*(b + c*x^n))^(3/2)
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

3.571 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n
^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.221845, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n
^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 37.1299, size = 124, normalized size = 0.89

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,\operatorname{appellf}_1\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, -1/2, -1/2, 1 + 1/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c
*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 8.73882, size = 786, normalized size = 5.65

$$x\left(\frac{2a^2bn(2n+1)x^n\left(-\sqrt{b^2-4ac}+b+2cx^n\right)\left(\sqrt{b^2-4ac}+b+2cx^n\right)F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4}}\right.}{(n+1)^2\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b\right)\left(nx^n\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(2+\frac{1}{n};\frac{1}{2},\frac{3}{2};3+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(2+\frac{1}{n};\frac{3}{2},\frac{1}{2};3+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*((a + x^n*(b + c*x^n))^2/(1 + n) + (2*a^2*b*n*(1 + 2*n)*x^n*(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b
^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)^2*(-4*(a + 2*a*n)*Ap
pellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqr
t[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
 + (b - Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])]))) + (a^2*n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1),
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])])/(c*(-((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1),
 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (-b + Sqrt[b^2 - 4*a*c])*n*x^n*A
ppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 4*a*(1 + n)*Ap
pellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))))/(a + x^n*(b + c*x
^n))^(3/2)
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.572 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2
*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x
^n)/(2*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rubi [A] time = 0.237417, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2
*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x
^n)/(2*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rubi in Sympy [A] time = 31.0399, size = 102, normalized size = 0.86

$$-\frac{\sqrt{a}\operatorname{atanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\operatorname{atanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}+\frac{\sqrt{a+bx^n+cx^{2n}}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

```
[Out] -sqrt(a)*atanh((2*a + b*x**n)/(2*sqrt(a)*sqrt(a + b*x**n + c*x**(
2*n))))/n + b*atanh((b + 2*c*x**n)/(2*sqrt(c)*sqrt(a + b*x**n + c
*x**(2*n))))/(2*sqrt(c)*n) + sqrt(a + b*x**n + c*x**(2*n))/n
```

Mathematica [A] time = 0.283989, size = 118, normalized size = 0.99

$$\frac{2\sqrt{a+x^n\left(b+cx^n\right)}-2\sqrt{a}\log\left(2\sqrt{a}\sqrt{a+x^n\left(b+cx^n\right)}+2a+bx^n\right)+\frac{b\log\left(2\sqrt{c}\sqrt{a+x^n(b+cx^n)}+b+2cx^n\right)}{\sqrt{c}}+2\sqrt{a}n\log(x)}{2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] (2*Sqrt[a + x^n*(b + c*x^n)] + 2*Sqrt[a]*n*Log[x] - 2*Sqrt[a]*Log
[2*a + b*x^n + 2*Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)]] + (b*Log[b +
2*c*x^n + 2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]])/Sqrt[c])/(2*n)
```

Maple [A] time = 0.05, size = 125, normalized size = 1.1

$$\frac{1}{n}\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}+\frac{b}{2\,n}\ln\left(1\left(\frac{b}{2}+ce^{n\ln(x)}\right)\frac{1}{\sqrt{c}}+\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}\right)\frac{1}{\sqrt{c}}$$
$$-\frac{1}{n}\sqrt{a}\ln\left(\frac{1}{e^{n\ln(x)}}\left(2\,a+be^{n\ln(x)}+2\,\sqrt{a}\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(1/2)/x,x)
```

```
[Out] 1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c
*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))
/c^(1/2)-1/n*a^(1/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*
ln(x))+c*exp(n*ln(x))^2)^(1/2))/exp(n*ln(x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.325591, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(-8\,c^{\frac{5}{2}}x^{2\,n}-8\,bc^{\frac{3}{2}}x^{n}-4\left(2\,c^{2}x^{n}+bc\right)\sqrt{cx^{2\,n}+bx^{n}+a}-\left(b^{2}+4\,ac\right)\sqrt{c}\right)+2\,\sqrt{a}\sqrt{c}\log\left(-\frac{8\,abx^{n}+8\,a^{2}+\left(b^{2}+4\,ac\right)x^{2\,n}-4\left(\cdots\right.}{x^{2n}}\right.}{4\,\sqrt{c}n}\right.$$

$$-\frac{4\,\sqrt{-a}\sqrt{c}\arctan\left(\frac{bx^{n}+2\,a}{2\,\sqrt{cx^{2\,n}+bx^{n}+a}\sqrt{-a}}\right)-b\log\left(-8\,c^{\frac{5}{2}}x^{2\,n}-8\,bc^{\frac{3}{2}}x^{n}-4\left(2\,c^{2}x^{n}+bc\right)\sqrt{cx^{2\,n}+bx^{n}+a}-\left(b^{2}+4\,ac\right)\sqrt{c}\right)-\cdots}{4\,\sqrt{c}n}$$

$$\left.-\frac{2\,\sqrt{-a}\sqrt{-c}\arctan\left(\frac{bx^{n}+2\,a}{2\,\sqrt{cx^{2\,n}+bx^{n}+a}\sqrt{-a}}\right)-b\arctan\left(\frac{2\,\sqrt{-c}cx^{n}+b\sqrt{-c}}{2\,\sqrt{cx^{2\,n}+bx^{n}+a}c}\right)-2\,\sqrt{cx^{2\,n}+bx^{n}+a}\sqrt{-c}}{2\,\sqrt{-c}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(-8*c^(5/2)*x^(2*n) - 8*b*c^(3/2)*x^n - 4*(2*c^2*x^n +
 b*c)*sqrt(c*x^(2*n) + b*x^n + a) - (b^2 + 4*a*c)*sqrt(c)) + 2*sq
rt(a)*sqrt(c)*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4
*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)
) + 4*sqrt(c*x^(2*n) + b*x^n + a)*sqrt(c))/(sqrt(c)*n), 1/2*(b*ar
ctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))/(sqrt(c*x^(2*n) + b*x^n
+ a)*c)) + sqrt(a)*sqrt(-c)*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*
c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n
 + a))/x^(2*n)) + 2*sqrt(c*x^(2*n) + b*x^n + a)*sqrt(-c))/(sqrt(-
c)*n), -1/4*(4*sqrt(-a)*sqrt(c)*arctan(1/2*(b*x^n + 2*a)/(sqrt(c*
x^(2*n) + b*x^n + a)*sqrt(-a))) - b*log(-8*c^(5/2)*x^(2*n) - 8*b*
c^(3/2)*x^n - 4*(2*c^2*x^n + b*c)*sqrt(c*x^(2*n) + b*x^n + a) - (
b^2 + 4*a*c)*sqrt(c)) - 4*sqrt(c*x^(2*n) + b*x^n + a)*sqrt(c))/(s
qrt(c)*n), -1/2*(2*sqrt(-a)*sqrt(-c)*arctan(1/2*(b*x^n + 2*a)/(sq
rt(c*x^(2*n) + b*x^n + a)*sqrt(-a))) - b*arctan(1/2*(2*sqrt(-c)*c
*x^n + b*sqrt(-c))/(sqrt(c*x^(2*n) + b*x^n + a)*c)) - 2*sqrt(c*x^
(2*n) + b*x^n + a)*sqrt(-c))/(sqrt(-c)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^{n}+cx^{2n}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

3.573 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1
 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi [A] time = 0.468941, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1
 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi in Sympy [A] time = 33.6334, size = 128, normalized size = 0.86

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf}_1\left(-\frac{1}{n},-\frac{1}{2},-\frac{1}{2},\frac{n-1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] -sqrt(a + b*x**n + c*x**(2*n))*appellf1(-1/n, -1/2, -1/2, (n - 1)
/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a
```

```
*c + b**2)))/(x*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt
(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 8.58844, size = 821, normalized size = 5.51

$$\frac{2a^2bn(2n-1)\left(2cx^n+b-\sqrt{b^2-4ac}\right)\left(2cx^n+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{n-1}{n};\frac{1}{2},\frac{1}{2};2-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{\left(\sqrt{b^2-4ac}-b\right)\left(b+\sqrt{b^2-4ac}\right)(n-1)^2\left(\left(b+\sqrt{b^2-4ac}\right)nF_1\left(2-\frac{1}{n};\frac{1}{2},\frac{3}{2};3-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)x^n-\left(\sqrt{b^2-4ac}-b\right)nF_1\left(2-\frac{1}{n};\frac{3}{2},\frac{1}{2};3-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] ((a + x^n*(b + c*x^n))^2/(-1 + n) + (2*a^2*b*n*(-1 + 2*n)*x^n*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*
AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^
2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-1 + n)^2*((b + Sqrt[b^2 - 4
*a*c])*n*x^n*AppellF1[2 - n^(-1), 1/2, 3/2, 3 - n^(-1), (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] -
(-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1), 3/2, 1/2, 3 -
 n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])] + 4*a*(1 - 2*n)*AppellF1[(-1 + n)/n, 1/2, 1/2, 2
- n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])) + (a^2*n*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 +
 n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])])/(c*(4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 +
 n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n)/
n, 1/2, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF
1[(-1 + n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))))/(x*(a + x^n*(b + c
*x^n))^(3/2))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

3.574 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n
)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(2*x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.452266, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n
)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(2*x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 33.8119, size = 131, normalized size = 0.87

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(-\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{n-2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] -sqrt(a + b*x**n + c*x**(2*n))*appellf1(-2/n, -1/2, -1/2, (n - 2)
/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a
```

```
*c + b**2)))/(2*x**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 8.92062, size = 816, normalized size = 5.4

$$\frac{4a^2b(n-1)n\left(2cx^n+b-\sqrt{b^2-4ac}\right)\left(2cx^n+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{n-2}{n};\frac{1}{2},\frac{1}{2};2-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{\left(\sqrt{b^2-4ac}-b\right)\left(b+\sqrt{b^2-4ac}\right)(n-2)^2\left(\left(b+\sqrt{b^2-4ac}\right)nF_1\left(2-\frac{2}{n};\frac{1}{2},\frac{3}{2};3-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)x^n-\left(\sqrt{b^2-4ac}-b\right)nF_1\left(2-\frac{2}{n};\frac{3}{2},\frac{1}{2};3-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] ((a + x^n*(b + c*x^n))^2/(-2 + n) + (4*a^2*b*(-1 + n)*n*x^n*(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*Ap
pellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4
*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-2 + n)^2*((b + Sqrt[b^2 - 4*a*c]
)*n*x^n*AppellF1[2 - 2/n, 1/2, 3/2, 3 - 2/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[
b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 3/2, 1/2, 3 - 2/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] -
 8*a*(-1 + n)*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) +
(a^2*n*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*c*(-
2 + n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*c*n*x^n*((
b + Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2, 2 -
 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])])))/(x^2*(a + x^n*(b + c*x^n))^(3/2))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+bx^n+cx^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

3.575 $\int x^3 \left(a + bx^n + cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=149

$$\frac{ax^4\sqrt{a + bx^n + cx^{2n}}F_1\left(\frac{4}{n}; -\frac{3}{2}, -\frac{3}{2}; \frac{n+4}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1}}$$

```
[Out] (a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.468794, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{ax^4\sqrt{a + bx^n + cx^{2n}}F_1\left(\frac{4}{n}; -\frac{3}{2}, -\frac{3}{2}; \frac{n+4}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 38.3291, size = 129, normalized size = 0.87

$$\frac{ax^4\sqrt{a + bx^n + cx^{2n}}\,\text{appellf}_1\left(\frac{4}{n}, -\frac{3}{2}, -\frac{3}{2}, \frac{n+4}{n}, -\frac{2cx^n}{b-\sqrt{-4ac+b^2}}, -\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}} + 1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] a*x**4*sqrt(a + b*x**n + c*x**(2*n))*appellf1(4/n, -3/2, -3/2, (n
 + 4)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt
```

```
(-4*a*c + b**2)))/(4*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.33341, size = 3165, normalized size = 21.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]*(((64*a*c + 96*a*c*n + 3*b^2*n^2 + 32
*a*c*n^2)*x^4)/(8*c*(2 + n)*(4 + n)*(4 + 3*n)) + (b*(8 + 7*n)*x^(
4 + n))/(4*(2 + n)*(4 + 3*n)) + (c*x^(4 + 2*n))/(4 + 3*n)) - (48*
a^3*b*n^2*x^(4 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(4 + n)^2*(
4 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x
^n*AppellF1[2 + 4/n, 1/2, 3/2, 3 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 -
 4*a*c])*n*x^n*AppellF1[2 + 4/n, 3/2, 1/2, 3 + 4/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*
(2 + n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (12*a^2
*b^3*n^2*x^(4 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)])/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(4 + n)^2*
(4 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*
x^n*AppellF1[2 + 4/n, 1/2, 3/2, 3 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2
- 4*a*c])*n*x^n*AppellF1[2 + 4/n, 3/2, 1/2, 3 + 4/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a
*(2 + n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (18*a^
3*b*n^3*x^(4 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(4 + n)^2*(4
+ 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n
*AppellF1[2 + 4/n, 1/2, 3/2, 3 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4
*a*c])*n*x^n*AppellF1[2 + 4/n, 3/2, 1/2, 3 + 4/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*(2
 + n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (3*a^2*b^
3*n^3*x^(4 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
```

```
/(2*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(4 + n)^2*(
4 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x
^n*AppellF1[2 + 4/n, 1/2, 3/2, 3 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 -
 4*a*c])*n*x^n*AppellF1[2 + 4/n, 3/2, 1/2, 3 + 4/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*
(2 + n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (6*a^4*
n^2*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[
b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(2 + n)*(4 + 3*n)*(a + x^n*
(b + c*x^n))^(3/2)*(-4*a*(4 + n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 1/
2, 3/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n
)/n, 3/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (3*a^3*b^2*n^2*x^4*(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF
1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(2*c*(b - Sqrt[b^2 - 4*a*c])*
(b + Sqrt[b^2 - 4*a*c])*(2 + n)*(4 + 3*n)*(a + x^n*(b + c*x^n))^(
3/2)*(-4*a*(4 + n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*
x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 1/2, 3/2, 2 + 4/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 3/2, 1/2,
 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])]))) - (3*a^4*n^3*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[4/n, 1/2, 1/2, (
4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c]
)*(2 + n)*(4 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(4 + n)*App
ellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4
*a*c])*AppellF1[(4 + n)/n, 1/2, 3/2, 2 + 4/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[
b^2 - 4*a*c])*AppellF1[(4 + n)/n, 3/2, 1/2, 2 + 4/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int x^{3} \left(a + b x^{n} + c x^{2 n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + bx^n + a)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3,x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + bx^n + a)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

3.576 $\int x^2 \left(a + bx^n + cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=149

$$\frac{ax^3\sqrt{a + bx^n + cx^{2n}}F_1\left(\frac{3}{n}; -\frac{3}{2}, -\frac{3}{2}; \frac{n+3}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1}}$$

```
[Out] (a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.475298, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{ax^3\sqrt{a + bx^n + cx^{2n}}F_1\left(\frac{3}{n}; -\frac{3}{2}, -\frac{3}{2}; \frac{n+3}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 38.6419, size = 129, normalized size = 0.87

$$\frac{ax^3\sqrt{a + bx^n + cx^{2n}}\,\text{appellf}_1\left(\frac{3}{n}, -\frac{3}{2}, -\frac{3}{2}, \frac{n+3}{n}, -\frac{2cx^n}{b-\sqrt{-4ac+b^2}}, -\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}} + 1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] a*x**3*sqrt(a + b*x**n + c*x**(2*n))*appellf1(3/n, -3/2, -3/2, (n
 + 3)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt
```

```
(-4*a*c + b**2)))/(3*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.4163, size = 3165, normalized size = 21.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]*(((36*a*c + 72*a*c*n + 3*b^2*n^2 + 32
*a*c*n^2)*x^3)/(12*c*(1 + n)*(3 + n)*(3 + 2*n)) + (b*(6 + 7*n)*x^
(3 + n))/(6*(1 + n)*(3 + 2*n)) + (c*x^(3 + 2*n))/(3*(1 + n))) - (
12*a^3*b*n^2*x^(3 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*
a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)*
(3 + n)^2*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*
x^n*AppellF1[2 + 3/n, 1/2, 3/2, 3 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2
- 4*a*c])*n*x^n*AppellF1[2 + 3/n, 3/2, 1/2, 3 + 3/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a
*(3 + 2*n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (3*a
^2*b^3*n^2*x^(3 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])])/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)*
(3 + n)^2*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*
x^n*AppellF1[2 + 3/n, 1/2, 3/2, 3 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2
- 4*a*c])*n*x^n*AppellF1[2 + 3/n, 3/2, 1/2, 3 + 3/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a
*(3 + 2*n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (6*a
^3*b*n^3*x^(3 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)*(3 +
 n)^2*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*
AppellF1[2 + 3/n, 1/2, 3/2, 3 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*
a*c])*n*x^n*AppellF1[2 + 3/n, 3/2, 1/2, 3 + 3/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(3
+ 2*n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (a^2*b^3
*n^3*x^(3 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/
```

```
(2*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)*(3 +
 n)^2*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*
AppellF1[2 + 3/n, 1/2, 3/2, 3 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*
a*c])*n*x^n*AppellF1[2 + 3/n, 3/2, 1/2, 3 + 3/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(3
+ 2*n)*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (4*a^4*n
^2*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b
^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)*(3 + 2*n)*(a + x^n*(
b + c*x^n))^(3/2)*(-4*a*(3 + n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 1/2
, 3/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)
/n, 3/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (a^3*b^2*n^2*x^3*(b - Sqrt[b^2
 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[3
/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])*(b +
Sqrt[b^2 - 4*a*c])*(1 + n)*(3 + 2*n)*(a + x^n*(b + c*x^n))^(3/2)*
(-4*a*(3 + n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*(
(b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 1/2, 3/2, 2 + 3/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 3/2, 1/2, 2 +
3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])]))) - (8*a^4*n^3*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[3/n, 1/2, 1/2, (3 + n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])])/(3*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(
1 + n)*(3 + 2*n)*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(3 + n)*Appell
F1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*
c])*AppellF1[(3 + n)/n, 1/2, 3/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[(3 + n)/n, 3/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int x^2 \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2,x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

3.577 $\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=149

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.351895, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 30.011, size = 129, normalized size = 0.87

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] a*x**2*sqrt(a + b*x**n + c*x**(2*n))*appellf1(2/n, -3/2, -3/2, (n
 + 2)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt
```

```
(-4*a*c + b**2)))/(2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.38399, size = 3165, normalized size = 21.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]*(((16*a*c + 48*a*c*n + 3*b^2*n^2 + 32
*a*c*n^2)*x^2)/(8*c*(1 + n)*(2 + n)*(2 + 3*n)) + (b*(4 + 7*n)*x^(
2 + n))/(4*(1 + n)*(2 + 3*n)) + (c*x^(2 + 2*n))/(2 + 3*n)) - (24*
a^3*b*n^2*x^(2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(2 + n)^2*(
2 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x
^n*AppellF1[2 + 2/n, 1/2, 3/2, 3 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 -
 4*a*c])*n*x^n*AppellF1[2 + 2/n, 3/2, 1/2, 3 + 2/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*
(1 + n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (6*a^2*
b^3*n^2*x^(2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(2 + n)^2*(
2 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x
^n*AppellF1[2 + 2/n, 1/2, 3/2, 3 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 -
 4*a*c])*n*x^n*AppellF1[2 + 2/n, 3/2, 1/2, 3 + 2/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*
(1 + n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (18*a^3
*b*n^3*x^(2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
)/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(2 + n)^2*(2 +
 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*
AppellF1[2 + 2/n, 1/2, 3/2, 3 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*
a*c])*n*x^n*AppellF1[2 + 2/n, 3/2, 1/2, 3 + 2/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*(1
+ n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (3*a^2*b^3
*n^3*x^(2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/
```

```
(2*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(2 + n)^2*(2
 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^
n*AppellF1[2 + 2/n, 1/2, 3/2, 3 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 -
4*a*c])*n*x^n*AppellF1[2 + 2/n, 3/2, 1/2, 3 + 2/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*(
1 + n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (6*a^4*n
^2*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b
^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)*(2 + 3*n)*(a + x^n*(
b + c*x^n))^(3/2)*(-4*a*(2 + n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 1/2
, 3/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)
/n, 3/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (3*a^3*b^2*n^2*x^2*(b - Sqrt[b
^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1
[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(2*c*(b - Sqrt[b^2 - 4*a*c])*(
b + Sqrt[b^2 - 4*a*c])*(1 + n)*(2 + 3*n)*(a + x^n*(b + c*x^n))^(3
/2)*(-4*a*(2 + n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x
^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 1/2, 3/2, 2 + 2/n
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 3/2, 1/2,
2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])]))) - (6*a^4*n^3*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[2/n, 1/2, 1/2, (2
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])
*(1 + n)*(2 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(2 + n)*Appe
llF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*
a*c])*AppellF1[(2 + n)/n, 1/2, 3/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b
^2 - 4*a*c])*AppellF1[(2 + n)/n, 3/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int x \left(a + b x^{n} + c x^{2 n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

3.578 $\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.224714, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 37.0902, size = 126, normalized size = 0.9

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] a*x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, -3/2, -3/2, 1 + 1
/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a
```

```
*c + b**2)))/(sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2
*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.32739, size = 3058, normalized size = 21.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]*(((4*a*c + 24*a*c*n + 3*b^2*n^2 + 32*
a*c*n^2)*x)/(4*c*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (b*(2 + 7*n)*x^(1
 + n))/(2*(1 + 2*n)*(1 + 3*n)) + (c*x^(1 + 2*n))/(1 + 3*n)) - (12
*a^3*b*n^2*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n
)^2*(1 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF
1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2
 - 4*a*c])*AppellF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*
a*c])]))) + (3*a^2*b^3*n^2*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[1 + n^(-1), 1/2,
 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^
2 - 4*a*c])*(1 + n)^2*(1 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*(-4*(
a + 2*a*n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*
x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 1/2, 3/2, 3 + n
^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 3/2,
 1/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])]))) - (18*a^3*b*n^3*x^(1 + n)*(b - Sqrt[b
^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c
])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)^2*(1 + 3*n)*(a + x^n*(b + c*x^
n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1),
1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[2
 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (3*a^2*b^3*n^3*x^(1
+ n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b
```

```
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(2*c*(
b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)^2*(1 + 3*n
)*(a + x^n*(b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-1)
, 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*A
ppellF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) -
(12*a^4*n^2*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(
(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + 2*n)*(1 + 3*
n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*App
ellF1[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 -
 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
- 4*a*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (
3*a^3*b^2*n^2*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + 2*n)*(1
+ 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n
*AppellF1[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b
^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])] - 4*a*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))
 - (24*a^4*n^3*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
)/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + 2*n)*(1 +
 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*
AppellF1[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^
2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] - 4*a*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \left(a+b x^{n}+c x^{2 n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.579 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx$

Optimal. Leaf size=173

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{\left(8ac+b^2+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) +
 (a + b*x^n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*
x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a
*c)*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])
])/(16*c^(3/2)*n)
```

Rubi [A] time = 0.398101, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{\left(8ac+b^2+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) +
 (a + b*x^n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*
x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a
*c)*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])
])/(16*c^(3/2)*n)
```

Rubi in Sympy [A] time = 48.8707, size = 151, normalized size = 0.87

$$-\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b\left(-12ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{\frac{3}{2}}n}+\frac{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}{3n}+\frac{\sqrt{a+bx^n+cx^{2n}}\left(4ac+\frac{b^2}{2}+bcx^n\right)}{4cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

```
[Out] -a**(3/2)*atanh((2*a + b*x**n)/(2*sqrt(a)*sqrt(a + b*x**n + c*x**
(2*n))))/n - b*(-12*a*c + b**2)*atanh((b + 2*c*x**n)/(2*sqrt(c)*s
qrt(a + b*x**n + c*x**(2*n))))/(16*c**(3/2)*n) + (a + b*x**n + c*
x**(2*n))**(3/2)/(3*n) + sqrt(a + b*x**n + c*x**(2*n))*(4*a*c + b
**2/2 + b*c*x**n)/(4*c*n)
```

Mathematica [A] time = 0.720881, size = 161, normalized size = 0.93

$$-\frac{a^{3/2}\log\left(2\sqrt{a}\sqrt{a+x^n\left(b+cx^n\right)}+2a+bx^n\right)}{n}+a^{3/2}\log(x)-\frac{b\left(b^2-12ac\right)\log\left(2\sqrt{c}\sqrt{a+x^n\left(b+cx^n\right)}+b+2cx^n\right)}{16c^{3/2}n}+\frac{\sqrt{a+x^n\left(b+cx^n\right)}\left(8c\left(4a+cx^{2n}\right)+3b^2+14bcx^n\right)}{24cn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] (Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^(
2*n))))/(24*c*n) + a^(3/2)*Log[x] - (a^(3/2)*Log[2*a + b*x^n + 2*
Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)]])/n - (b*(b^2 - 12*a*c)*Log[b +
 2*c*x^n + 2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]])/(16*c^(3/2)*n)
```

Maple [A] time = 0.051, size = 209, normalized size = 1.2

$$\frac{8\,c^2\left(e^{n\ln(x)}\right)^2+14\,be^{n\ln(x)}c+32\,ac+3\,b^2}{24\,cn}\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}-\frac{b^3}{16\,n}\ln\left(1\left(\frac{b}{2}+ce^{n\ln(x)}\right)\frac{1}{\sqrt{c}}+\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}\right)c^{-\frac{3}{2}}+\frac{3\,ab}{4\,n}\ln\left(1\left(\frac{b}{2}+ce^{n\ln(x)}\right)\frac{1}{\sqrt{c}}+\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}\right)\frac{1}{\sqrt{c}}-\frac{1}{n}a^{\frac{3}{2}}\ln\left(\frac{1}{e^{n\ln(x)}}\left(2\,a+be^{n\ln(x)}+2\,\sqrt{a}\sqrt{a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(3/2)/x,x)
```

```
[Out] 1/24*(8*c^2*exp(n*ln(x))^2+14*b*exp(n*ln(x))*c+32*a*c+3*b^2)*(a+b
*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)/c/n-1/16/c^(3/2)/n*b^3*ln((
1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
^(1/2))+3/4/c^(1/2)/n*a*b*ln((1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*
exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/n*a^(3/2)*ln((2*a+b*exp(n
*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/exp(
n*ln(x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.418184, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^(3/2)*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)
*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n +
 a))/x^(2*n)) - 3*(b^3 - 12*a*b*c)*log(-8*c^(5/2)*x^(2*n) - 8*b*c
^(3/2)*x^n - 4*(2*c^2*x^n + b*c)*sqrt(c*x^(2*n) + b*x^n + a) - (b
^2 + 4*a*c)*sqrt(c)) + 4*(8*c^(5/2)*x^(2*n) + 14*b*c^(3/2)*x^n +
(3*b^2 + 32*a*c)*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a))/(c^(3/2)*n
), 1/48*(24*a^(3/2)*sqrt(-c)*c*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4
*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*
x^n + a))/x^(2*n)) - 3*(b^3 - 12*a*b*c)*arctan(1/2*(2*sqrt(-c)*c*
x^n + b*sqrt(-c))/(sqrt(c*x^(2*n) + b*x^n + a)*c)) + 2*(8*sqrt(-c
)*c^2*x^(2*n) + 14*b*sqrt(-c)*c*x^n + (3*b^2 + 32*a*c)*sqrt(-c))*
sqrt(c*x^(2*n) + b*x^n + a))/(sqrt(-c)*c*n), -1/96*(96*sqrt(-a)*a
*c^(3/2)*arctan(1/2*(b*x^n + 2*a)/(sqrt(c*x^(2*n) + b*x^n + a)*sq
rt(-a))) + 3*(b^3 - 12*a*b*c)*log(-8*c^(5/2)*x^(2*n) - 8*b*c^(3/2
)*x^n - 4*(2*c^2*x^n + b*c)*sqrt(c*x^(2*n) + b*x^n + a) - (b^2 +
4*a*c)*sqrt(c)) - 4*(8*c^(5/2)*x^(2*n) + 14*b*c^(3/2)*x^n + (3*b^
```

```
2 + 32*a*c)*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a))/(c^(3/2)*n), -1
/48*(48*sqrt(-a)*a*sqrt(-c)*c*arctan(1/2*(b*x^n + 2*a)/(sqrt(c*x^
(2*n) + b*x^n + a)*sqrt(-a))) + 3*(b^3 - 12*a*b*c)*arctan(1/2*(2*
sqrt(-c)*c*x^n + b*sqrt(-c))/(sqrt(c*x^(2*n) + b*x^n + a)*c)) - 2
*(8*sqrt(-c)*c^2*x^(2*n) + 14*b*sqrt(-c)*c*x^n + (3*b^2 + 32*a*c)
*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a))/(sqrt(-c)*c*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

3.580 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -(
(1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi [A] time = 0.465627, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -(
(1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rubi in Sympy [A] time = 37.8959, size = 129, normalized size = 0.86

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(-\frac{1}{n},-\frac{3}{2},-\frac{3}{2},\frac{n-1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] -a*sqrt(a + b*x**n + c*x**(2*n))*appellf1(-1/n, -3/2, -3/2, (n -
1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4
```

```
*a*c + b**2)))/(x*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sq
rt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.34865, size = 3181, normalized size = 21.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]*((4*a*c - 24*a*c*n + 3*b^2*n^2 + 32*a
*c*n^2)/(4*c*(-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x) + (b*(-2 + 7*n)*x^
(-1 + n))/(2*(-1 + 2*n)*(-1 + 3*n)) + (c*x^(-1 + 2*n))/(-1 + 3*n)
) + (12*a^3*b*n^2*x^(-1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-1 + n)/n, 1/2, 1/2, 2
- n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c]
)*(-1 + n)^2*(-1 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^
2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1), 1/2, 3/2, 3 - n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1), 3/2, 1/2
, 3 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + 4*a*(1 - 2*n)*AppellF1[(-1 + n)/n, 1/2, 1/
2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])])) - (3*a^2*b^3*n^2*x^(-1 + n)*(b - Sqrt[b^2
 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(
-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c
])*(b + Sqrt[b^2 - 4*a*c])*(-1 + n)^2*(-1 + 3*n)*(a + x^n*(b + c*
x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1), 1
/2, 3/2, 3 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*Appe
llF1[2 - n^(-1), 3/2, 1/2, 3 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 4*a*(1 - 2*n)*App
ellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (18*a^3*b*n^3*
x^(-1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^n)*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/
((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-1 + n)^2*(-1 +
 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*
AppellF1[2 - n^(-1), 1/2, 3/2, 3 - n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^
2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1), 3/2, 1/2, 3 - n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] + 4*a*(1 - 2*n)*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])])) + (3*a^2*b^3*n^3*x^(-1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-1 + n)/n, 1/2, 1/
```

```
2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])])/(2*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2
 - 4*a*c])*(-1 + n)^2*(-1 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b
+ Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1), 1/2, 3/2, 3 - n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1),
 3/2, 1/2, 3 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])] + 4*a*(1 - 2*n)*AppellF1[(-1 + n)/n
, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (12*a^4*n^2*(-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-n^(-
1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + S
qrt[b^2 - 4*a*c])*(-1 + 2*n)*(-1 + 3*n)*x*(a + x^n*(b + c*x^n))^(
3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n)/n, 1/2, 3/2,
 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n)/n
, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (3*a^3*b^2*n^2*(-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-
n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])*
(b + Sqrt[b^2 - 4*a*c])*(-1 + 2*n)*(-1 + 3*n)*x*(a + x^n*(b + c*x
^n))^(3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n,
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n)/n, 1/2
, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(-1
+ n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (24*a^4*n^3*(-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF
1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])
*(b + Sqrt[b^2 - 4*a*c])*(-1 + 2*n)*(-1 + 3*n)*x*(a + x^n*(b + c*
x^n))^(3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n)/n, 1/
2, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(-1
 + n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}} \left(a + b x^{n} + c x^{2 n}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^{n} + a\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

3.581 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 -
 n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.448074, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 -
 n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 36.7002, size = 133, normalized size = 0.88

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(-\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{n-2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] -a*sqrt(a + b*x**n + c*x**(2*n))*appellf1(-2/n, -3/2, -3/2, (n -
2)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4
```

```
*a*c + b**2)))/(2*x**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.36221, size = 3165, normalized size = 20.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]*((16*a*c - 48*a*c*n + 3*b^2*n^2 + 32*
a*c*n^2)/(8*c*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2) + (b*(-4 + 7*n)*x
^(-2 + n))/(4*(-1 + n)*(-2 + 3*n)) + (c*x^(-2 + 2*n))/(-2 + 3*n))
 + (24*a^3*b*n^2*x^(-2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 -
 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-
2 + n)^2*(-2 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 -
4*a*c])*n*x^n*AppellF1[2 - 2/n, 1/2, 3/2, 3 - 2/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b +
 Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 3/2, 1/2, 3 - 2/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])] - 8*a*(-1 + n)*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])) - (6*a^2*b^3*n^2*x^(-2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-2 + n)/n, 1/2, 1/2,
 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*
c])*(-2 + n)^2*(-2 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[
b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 1/2, 3/2, 3 - 2/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] -
 (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 3/2, 1/2, 3 - 2
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])] - 8*a*(-1 + n)*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n,
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])])) - (18*a^3*b*n^3*x^(-2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-2 + n)/n, 1/2,
 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4
*a*c])*(-2 + n)^2*(-2 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sq
rt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 1/2, 3/2, 3 - 2/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 3/2, 1/2, 3
- 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] - 8*a*(-1 + n)*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])) + (3*a^2*b^3*n^3*x^(-2 + n)*(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-2 + n)/n,
1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
```

```
(-b + Sqrt[b^2 - 4*a*c])])/(2*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt
[b^2 - 4*a*c])*(-2 + n)^2*(-2 + 3*n)*(a + x^n*(b + c*x^n))^(3/2)*
((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 1/2, 3/2, 3 - 2/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 3/2,
 1/2, 3 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] - 8*a*(-1 + n)*AppellF1[(-2 + n)/n, 1/2, 1/
2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])) + (6*a^4*n^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x
^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-2/n, 1/2, 1/2, (-
2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c]
)*(-1 + n)*(-2 + 3*n)*x^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(-2 +
 n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqr
t[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n, (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] +
(b - Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2, 2 - 2/n, (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])]))) - (3*a^3*b^2*n^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])])/(2*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-
1 + n)*(-2 + 3*n)*x^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(-2 + n)*
AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^
2 - 4*a*c])*AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
 Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2, 2 - 2/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
]))) - (6*a^4*n^3*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
)/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-1 + n)*(-2 +
 3*n)*x^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(-2 + n)*AppellF1[-2/
n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*
AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

3.582 $\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(4*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.479022, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(4*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 33.2304, size = 126, normalized size = 0.85

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{4a\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x**4*sqrt(a + b*x**n + c*x**(2*n))*appellf1(4/n, 1/2, 1/2, (n + 4
)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*
```

```
a*c + b**2)))/(4*a*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.496931, size = 415, normalized size = 2.8

$$-\frac{a^2(n+4)x^4\left(-\sqrt{b^2-4ac}+b+2cx^n\right)\left(\sqrt{b^2-4ac}+\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+x^n\left(b+cx^n\right)\right)^{3/2}\left(nx^n\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{n+4}{n};\frac{1}{2},\frac{3}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] -((a^2*(4 + n)*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/
((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*
x^n))^(3/2)*(-4*a*(4 + n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 1/2, 3/2,
 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 3/
2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])]))))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.583 $\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(3*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.475795, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(3*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 33.1732, size = 126, normalized size = 0.85

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\operatorname{appellf}_1\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{3a\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x**3*sqrt(a + b*x**n + c*x**(2*n))*appellf1(3/n, 1/2, 1/2, (n + 3
)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*
```

```
a*c + b**2)))/(3*a*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.483157, size = 417, normalized size = 2.82

$$-\frac{4a^2(n+3)x^3\left(-\sqrt{b^2-4ac}+b+2cx^n\right)\left(\sqrt{b^2-4ac}+\right.}{3\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+x^n\left(b+cx^n\right)\right)^{3/2}\left(nx^n\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{n+3}{n};\frac{1}{2},\frac{3}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (-4*a^2*(3 + n)*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/(3*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b +
 c*x^n))^(3/2)*(-4*a*(3 + n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 1/2, 3
/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n,
 3/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.584 $\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.349566, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 28.2028, size = 126, normalized size = 0.85

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf}_1\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2a\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x**2*sqrt(a + b*x**n + c*x**(2*n))*appellf1(2/n, 1/2, 1/2, (n + 2
)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*
```

```
a*c + b**2)))/(2*a*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.490196, size = 415, normalized size = 2.8

$$-\frac{2a^2(n+2)x^2\left(-\sqrt{b^2-4ac}+b+2cx^n\right)\left(\sqrt{b^2-4ac}+\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)(a+x^n(b+cx^n))^{3/2}\left(nx^n\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{n+2}{n};\frac{1}{2},\frac{3}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (-2*a^2*(2 + n)*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c
*x^n))^(3/2)*(-4*a*(2 + n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 1/2, 3/2
, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 3
/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")

[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)

[Out] Integral(x/sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^{2n} + bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.585 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])])/Sqrt[a + b*x^n + c*x^(2*n)]
```

Rubi [A] time = 0.219431, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])])/Sqrt[a + b*x^n + c*x^(2*n)]
```

Rubi in Sympy [A] time = 40.0504, size = 122, normalized size = 0.88

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf}_1\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, 1/2, 1/2, 1 + 1/n,
-2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c +
```

```
 b**2)))/(a*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c
*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.350269, size = 400, normalized size = 2.88

$$-\frac{4a^2(n+1)x\left(-\sqrt{b^2-4ac}+b+2cx^n\right)\left(\sqrt{b^2-4ac}\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+x^n\left(b+cx^n\right)\right)^{3/2}\left(nx^n\left(\sqrt{b^2-4ac}+b\right)F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (-4*a^2*(1 + n)*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b +
 c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1)
, 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*A
ppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*Ap
pellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")

[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)

[Out] Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2\,n} + bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.586 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/
(Sqrt[a]*n))
```

Rubi [A] time = 0.0856025, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/
(Sqrt[a]*n))
```

Rubi in Sympy [A] time = 12.4458, size = 41, normalized size = 0.87

$$-\frac{\operatorname{atanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] -atanh((2*a + b*x**n)/(2*sqrt(a)*sqrt(a + b*x**n + c*x**(2*n))))/
(sqrt(a)*n)
```

Mathematica [A] time = 0.113137, size = 50, normalized size = 1.06

$$\frac{n\log(x)-\log\left(2\sqrt{a}\sqrt{a+x^n\left(b+cx^n\right)}+2a+bx^n\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]

[Out] (n*Log[x] - Log[2*a + b*x^n + 2*Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)]
])/(Sqrt[a]*n)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int\frac{1}{x}\frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)

[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285165, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{8\,a^{\frac{3}{2}}bx^n+8\,a^{\frac{5}{2}}+\left(b^2+4\,ac\right)\sqrt{a}x^{2\,n}-4\,\left(abx^n+2\,a^2\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}n},-\frac{\arctan\left(\frac{\sqrt{-a}bx^n+2\,\sqrt{-a}a}{2\,\sqrt{cx^{2\,n}+bx^n+a}a}\right)}{\sqrt{-a}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a^(3/2)*b*x^n + 8*a^(5/2) + (b^2 + 4*a*c)*sqrt(a)*x^
(2*n) - 4*(a*b*x^n + 2*a^2)*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))
/(sqrt(a)*n), -arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)/(sqrt(c
*x^(2*n) + b*x^n + a)*a))/(sqrt(-a)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2\,n} + bx^n + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

3.587 $\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(x*Sqrt[a + b*x^n + c*x^(2*n)]))
```

Rubi [A] time = 0.463284, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(x*Sqrt[a + b*x^n + c*x^(2*n)]))
```

Rubi in Sympy [A] time = 33.2611, size = 126, normalized size = 0.85

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf}_1\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{n-1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{ax\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] -sqrt(a + b*x**n + c*x**(2*n))*appellf1(-1/n, 1/2, 1/2, (n - 1)/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(a*x*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt
(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.424412, size = 415, normalized size = 2.79

$$-\frac{4a^2(n-1)\left(\sqrt{b^2-4ac}-b-2cx^n\right)\left(\sqrt{b^2-4ac}+\right.}{x\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+x^n\left(b+cx^n\right)\right)^{3/2}\left(nx^n\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{n-1}{n};\frac{1}{2},\frac{3}{2};2-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] (-4*a^2*(-1 + n)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x*(a + x^n*(
b + c*x^n))^(3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (-1
+ n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n)
/n, 1/2, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[(-1 + n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**n + c*x**(2*n))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n} + bx^n + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.588 $\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(2*x^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.460782, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(2*x^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 33.177, size = 129, normalized size = 0.85

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,\operatorname{appellf}_1\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{n-2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2ax^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] -sqrt(a + b*x**n + c*x**(2*n))*appellf1(-2/n, 1/2, 1/2, (n - 2)/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(2*a*x**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.518478, size = 415, normalized size = 2.75

$$-\frac{2a^2(n-2)\left(\sqrt{b^2-4ac}-b-2cx^n\right)\left(\sqrt{b^2-4ac}+\right.}{x^2\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)(a+x^n(b+cx^n))^{3/2}\left(nx^n\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{n-2}{n};\frac{1}{2},\frac{3}{2};2-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] (-2*a^2*(-2 + n)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x^2*(a + x^n*(b
 + c*x^n))^(3/2)*(-4*a*(-2 + n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n,
1/2, 3/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(-2
+ n)/n, 3/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt{a+bx^n+cx^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**n + c*x**(2*n))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.589 $\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(4*a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.480818, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(4*a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 41.3711, size = 128, normalized size = 0.85

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf}_1\left(\frac{4}{n},\frac{3}{2},\frac{3}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{4a^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] x**4*sqrt(a + b*x**n + c*x**(2*n))*appellf1(4/n, 3/2, 3/2, (n + 4
)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*
```

```
a*c + b**2)))/(4*a**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1
)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 5.76469, size = 1947, normalized size = 12.89

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*((-2*(b^2 - 2*a*c + b*c*x^n)*(a + x^n*(b + c*x^n)))/n + (64*
a^2*b*c*(2 + n)*x^n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])])/((-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(4 + n)*
((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 4/n, 1/2, 3/2, 3 + 4/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 4/n, 3/2,
 1/2, 3 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] - 8*a*(2 + n)*AppellF1[(4 + n)/n, 1/2, 1/2,
 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])) + (a^2*(4 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[4/n, 1/2, 1/2, (4 +
 n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])])/(4*a*(4 + n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 1/2, 3/
2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n,
3/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])])) - (a*b^2*(4 + n)*(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[4/n, 1/2
, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])])/(4*c*(4*a*(4 + n)*AppellF1[4/n, 1/2, 1/
2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(4
 + n)/n, 1/2, 3/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
llF1[(4 + n)/n, 3/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (4*a^2*(4 + n)*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*
AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(n*(4*a*(4 + n)*Appell
F1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*
c])*AppellF1[(4 + n)/n, 1/2, 3/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[(4 + n)/n, 3/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (2*
```

```
a*b^2*(4 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c*n*
(4*a*(4 + n)*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((
b + Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 1/2, 3/2, 2 + 4/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(4 + n)/n, 3/2, 1/2, 2 + 4
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])]))))/(a*(-b^2 + 4*a*c)*(a + x^n*(b + c*x^n))^(3/2))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int x^{3} \left(a+b x^{n}+c x^{2 n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(c x^{2 n}+b x^{n}+a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + bx^n + cx^{2n})^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(cx^{2n} + bx^n + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.590 $\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}} dx$

Optimal. Leaf size=151

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(3*a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.482878, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(3*a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 43.3092, size = 128, normalized size = 0.85

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{3}{n},\frac{3}{2},\frac{3}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{3a^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] x**3*sqrt(a + b*x**n + c*x**(2*n))*appellf1(3/n, 3/2, 3/2, (n + 3
)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*
```

```
a*c + b**2)))/(3*a**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1
)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.22094, size = 2229, normalized size = 14.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (2*x^3*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*Sqrt[a + b*x
^n + c*x^(2*n)]) - (24*a*b*c*(3 + 2*n)*x^(3 + n)*(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(3 +
 n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 -
 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(3 + n)*(a + x^n*(b + c*x^n))^
(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 3/n, 1/2, 3/2,
3 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 3/n
, 3/2, 1/2, 3 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(3 + 2*n)*AppellF1[(3 + n)/n, 1
/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])])) + (4*a*b^2*(3 + n)*x^3*(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[3/n
, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])])/(3*(-b^2 + 4*a*c)*(b - Sqrt[b^2 -
4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n))^(3/2)*(-4*a
*(3 + n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b +
Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 1/2, 3/2, 2 + 3/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
+ (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 3/2, 1/2, 2 + 3/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*
a*c])]))) - (16*a^2*c*(3 + n)*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[3/n, 1/2, 1/2, (3 +
 n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])])/(3*(-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqr
t[b^2 - 4*a*c])*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(3 + n)*AppellF
1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c
])*AppellF1[(3 + n)/n, 1/2, 3/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*AppellF1[(3 + n)/n, 3/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (8*a
*b^2*(3 + n)*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((
-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(
a + x^n*(b + c*x^n))^(3/2)*(-4*a*(3 + n)*AppellF1[3/n, 1/2, 1/2,
```

```
(3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 +
n)/n, 1/2, 3/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF
1[(3 + n)/n, 3/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (16*a^2*c*(3 + n)*x^3
*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b
 - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(a + x^n*(b + c*x
^n))^(3/2)*(-4*a*(3 + n)*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 1/2, 3/2,
2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + n)/n, 3/2
, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int x^2 \left(a + b x^n + c x^{2n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c x^{2n} + b x^n + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.591 $\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(2*a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.355707, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(2*a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 28.7335, size = 128, normalized size = 0.85

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf}_1\left(\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] x**2*sqrt(a + b*x**n + c*x**(2*n))*appellf1(2/n, 3/2, 3/2, (n + 2
)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*
```

```
a*c + b**2)))/(2*a**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1
)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 5.43996, size = 1947, normalized size = 12.89

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (2*x^2*(-(((b^2 - 2*a*c + b*c*x^n)*(a + x^n*(b + c*x^n)))/n) + (1
6*a^2*b*c*(1 + n)*x^n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])])/((-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(2 + n
)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 2/n, 1/2, 3/2, 3 +
2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 + 2/n, 3/
2, 1/2, 3 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])] - 8*a*(1 + n)*AppellF1[(2 + n)/n, 1/2, 1/
2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])) + (a^2*(2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[2/n, 1/2, 1/2, (2
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])])/(4*a*(2 + n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*
a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 1/2,
3/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n
, 3/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])])) - (a*b^2*(2 + n)*(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[2/n, 1
/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(4*a*(2 + n)*AppellF1[2/n, 1/2,
1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
(2 + n)/n, 1/2, 3/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Ap
pellF1[(2 + n)/n, 3/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (2*a^2*(2 + n)*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(n*(4*a*(2 + n)*Appe
llF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*
a*c])*AppellF1[(2 + n)/n, 1/2, 3/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b
^2 - 4*a*c])*AppellF1[(2 + n)/n, 3/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (
```

```
a*b^2*(2 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c*n*
(4*a*(2 + n)*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((
b + Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 1/2, 3/2, 2 + 2/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(2 + n)/n, 3/2, 1/2, 2 + 2
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])))))/(a*(-b^2 + 4*a*c)*(a + x^n*(b + c*x^n))^(3/2))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int x \left(a + b x^{n} + c x^{2 n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c x^{2 n} + b x^{n} + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.592 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])])/(a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.220972, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])])/(a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 36.35, size = 124, normalized size = 0.87

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf}_1\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, 3/2, 3/2, 1 + 1/n,
-2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c +
```

```
 b**2)))/(a**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(
2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.18428, size = 2144, normalized size = 15.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (2*x*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*Sqrt[a + b*x^n
 + c*x^(2*n)]) - (8*a*b*c*(1 + 2*n)*x^(1 + n)*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[1 + n^(
-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2
- 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(1 + n)*(a + x^n*(b + c*x^n))
^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1),
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 1/2
, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[2 +
n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (4*a*b^2*(1 + n)*x*(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*A
ppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b
- Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n)
)^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 1/2,
3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1
[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*AppellF1[
n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (16*a^2*c*(1 + n)*x*(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*A
ppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b
- Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n)
)^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 1/2,
3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1
[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*AppellF1[
n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (8*a*b^2*(1 + n)*x*(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*Ap
pellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b -
 Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(a + x^n*(b + c*x^n
```

```
))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 1/2,
 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF
1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*AppellF1
[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (16*a^2*c*(1 + n)*x*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*
AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b
 - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(a + x^n*(b + c*x
^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 1/
2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*Appel
lF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*Appell
F1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \left(a + bx^n + cx^{2n}\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)

[Out] int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(-3/2),x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(-3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n + cx^{2n})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2\,n} + bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

3.593 $\int \frac{1}{x(a+bx^n+cx^{2n})^{3/2}} \, dx$

Optimal. Leaf size=98

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c
*x^(2*n)]) - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*
x^(2*n)])]/(a^(3/2)*n)
```

Rubi [A] time = 0.166185, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c
*x^(2*n)]) - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*
x^(2*n)])]/(a^(3/2)*n)
```

Rubi in Sympy [A] time = 23.0723, size = 87, normalized size = 0.89

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(-4ac+b^2\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\operatorname{atanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x**n)/(a*n*(-4*a*c + b**2)*sqrt(a + b*x**n
 + c*x**(2*n))) - atanh((2*a + b*x**n)/(2*sqrt(a)*sqrt(a + b*x**n
 + c*x**(2*n))))/(a**(3/2)*n)
```

Mathematica [A] time = 0.884211, size = 100, normalized size = 1.02

$$\frac{\log(x)-\frac{\frac{2\sqrt{a}\left(2ac-b^2-bcx^n\right)}{\left(b^2-4ac\right)\sqrt{a+x^n(b+cx^n)}}+\log\left(2\sqrt{a}\sqrt{a+x^n(b+cx^n)}+2a+bx^n\right)}{n}}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (Log[x] - ((2*Sqrt[a]*(-b^2 + 2*a*c - b*c*x^n))/((b^2 - 4*a*c)*Sq
rt[a + x^n*(b + c*x^n)]) + Log[2*a + b*x^n + 2*Sqrt[a]*Sqrt[a + x
^n*(b + c*x^n)]])/n)/a^(3/2)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Fricas [A] time = 0.330033, size = 1, normalized size = 0.01

$$\left[\frac{\left(ab^2-4\,a^2c+\left(b^2c-4\,ac^2\right)x^{2\,n}+\left(b^3-4\,abc\right)x^n\right)\log\left(-\frac{8\,a^{\frac{3}{2}}bx^n+8\,a^{\frac{5}{2}}+\left(b^2+4\,ac\right)\sqrt{a}x^{2\,n}-4\,\left(abx^n+2\,a^2\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)+4\,\left(\sqrt{a}bcx\right.}{2\,\left(\left(ab^2c-4\,a^2c^2\right)\sqrt{a}nx^{2\,n}+\left(ab^3-4\,a^2bc\right)\sqrt{a}nx^n+\left(a^2b^2-4\,a^3c\right)\sqrt{a}n\right)}\right.$$

$$-\frac{\left(ab^2-4\,a^2c+\left(b^2c-4\,ac^2\right)x^{2\,n}+\left(b^3-4\,abc\right)x^n\right)\arctan\left(\frac{\sqrt{-a}bx^n+2\,\sqrt{-a}a}{2\,\sqrt{cx^{2\,n}+bx^n+a}a}\right)-2\,\left(\sqrt{-a}bcx^n+\left(b^2-2\,ac\right)\sqrt{-a}\right)\sqrt{cx^{2\,n}+bx}}{\left(ab^2c-4\,a^2c^2\right)\sqrt{-a}nx^{2\,n}+\left(ab^3-4\,a^2bc\right)\sqrt{-a}nx^n+\left(a^2b^2-4\,a^3c\right)\sqrt{-a}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^(2*n) + (b^3 - 4*a*b
*c)*x^n)*log(-(8*a^(3/2)*b*x^n + 8*a^(5/2) + (b^2 + 4*a*c)*sqrt(a
)*x^(2*n) - 4*(a*b*x^n + 2*a^2)*sqrt(c*x^(2*n) + b*x^n + a))/x^(2
*n)) + 4*(sqrt(a)*b*c*x^n + (b^2 - 2*a*c)*sqrt(a))*sqrt(c*x^(2*n)
 + b*x^n + a))/((a*b^2*c - 4*a^2*c^2)*sqrt(a)*n*x^(2*n) + (a*b^3
- 4*a^2*b*c)*sqrt(a)*n*x^n + (a^2*b^2 - 4*a^3*c)*sqrt(a)*n), -((a
*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^(2*n) + (b^3 - 4*a*b*c)*x^n)
*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)/(sqrt(c*x^(2*n) + b*x
^n + a)*a)) - 2*(sqrt(-a)*b*c*x^n + (b^2 - 2*a*c)*sqrt(-a))*sqrt(
c*x^(2*n) + b*x^n + a))/((a*b^2*c - 4*a^2*c^2)*sqrt(-a)*n*x^(2*n)
 + (a*b^3 - 4*a^2*b*c)*sqrt(-a)*n*x^n + (a^2*b^2 - 4*a^3*c)*sqrt(
-a)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

3.594 $\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}} dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(a*x*Sqrt[a + b*x^n + c*x^(2*n)]))
```

Rubi [A] time = 0.464031, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(a*x*Sqrt[a + b*x^n + c*x^(2*n)]))
```

Rubi in Sympy [A] time = 43.0356, size = 128, normalized size = 0.84

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(-\frac{1}{n},\frac{3}{2},\frac{3}{2},\frac{n-1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a^2x\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] -sqrt(a + b*x**n + c*x**(2*n))*appellf1(-1/n, 3/2, 3/2, (n - 1)/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(a**2*x*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.18849, size = 2225, normalized size = 14.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*x*Sqrt[a + b*x^n
 + c*x^(2*n)]) + (8*a*b*c*(-1 + 2*n)*x^(-1 + n)*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-1 +
 n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^
2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-1 + n)*n*(a + x^n*(b + c*x^
n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - n^(-1), 1/2
, 3/2, 3 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*Appell
F1[2 - n^(-1), 3/2, 1/2, 3 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 4*a*(1 - 2*n)*Appel
lF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (4*a*b^2*(-1 + n
)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*
a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x*(a + x^n*(
b + c*x^n))^(3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (-1
+ n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n)
/n, 1/2, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[(-1 + n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (16*a^2*c*(-1 +
n)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4
*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x*(a + x^n*
(b + c*x^n))^(3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (-1
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 + n
)/n, 1/2, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appel
lF1[(-1 + n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (8*a*b^2*(-1 +
n)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4
*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*x*(a + x^
```

```
n*(b + c*x^n))^(3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2, (
-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-1 +
 n)/n, 1/2, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*App
ellF1[(-1 + n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (16*a^2*c*(-1
 + n)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2
+ 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*x*(a +
 x^n*(b + c*x^n))^(3/2)*(-4*a*(-1 + n)*AppellF1[-n^(-1), 1/2, 1/2
, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-
1 + n)/n, 1/2, 3/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*
AppellF1[(-1 + n)/n, 3/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2\,n} + bx^n + a)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

3.595 $\int \frac{1}{x^3(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(2*a*x^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.460117, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(2*a*x^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 41.4012, size = 131, normalized size = 0.85

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf}_1\left(-\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{n-2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2a^2x^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] -sqrt(a + b*x**n + c*x**(2*n))*appellf1(-2/n, 3/2, 3/2, (n - 2)/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(2*a**2*x**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) +
 1)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.20746, size = 2221, normalized size = 14.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*x^2*Sqrt[a + b*x
^n + c*x^(2*n)]) + (32*a*b*c*(-1 + n)*x^(-2 + n)*(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(-2
+ n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2
- 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-2 + n)*n*(a + x^n*(b + c*x^n)
)^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2/n, 1/2, 3/2
, 3 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[2 - 2
/n, 3/2, 1/2, 3 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])] - 8*a*(-1 + n)*AppellF1[(-2 + n)/n,
 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])])) + (2*a*b^2*(-2 + n)*(-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-2/
n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 -
4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x^2*(a + x^n*(b + c*x^n))^(3/2)*(
-4*a*(-2 + n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n
*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n,
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2,
2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])]))) - (8*a^2*c*(-2 + n)*(-b + Sqrt[b^2 - 4*a*c] -
2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-2/n, 1/2, 1/
2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b
 + Sqrt[b^2 - 4*a*c])*x^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(-2 +
 n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqr
t[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n, (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] +
(b - Sqrt[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2, 2 - 2/n, (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])]))) + (8*a*b^2*(-2 + n)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^
2 - 4*a*c])*n*x^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(-2 + n)*Appe
```

```
llF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 -
4*a*c])*AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n, (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqr
t[b^2 - 4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2, 2 - 2/n, (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
 - (16*a^2*c*(-2 + n)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a
*c])*n*x^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*a*(-2 + n)*AppellF1[-2
/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])
*AppellF1[(-2 + n)/n, 1/2, 3/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
 4*a*c])*AppellF1[(-2 + n)/n, 3/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+bx^n+cx^{2n}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2\,n} + bx^{n} + a\right)^{\frac{3}{2}} x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

3.596 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3\,dx$

Optimal. Leaf size=182

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}+\frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 +
 2*n)*(d*x)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)
^m)/(1 + m + 3*n) + (3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m
+ 4*n) + (3*b*c^2*x^(1 + 5*n)*(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1
+ 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*x)^(1 + m))/(d*(1 + m))
```

Rubi [A] time = 0.301036, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}+\frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 +
 2*n)*(d*x)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)
^m)/(1 + m + 3*n) + (3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m
+ 4*n) + (3*b*c^2*x^(1 + 5*n)*(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1
+ 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*x)^(1 + m))/(d*(1 + m))
```

Rubi in Sympy [A] time = 49.4216, size = 221, normalized size = 1.21

$$\frac{a^3\,(dx)^{m+1}}{d\,(m+1)}+\frac{3a^2bx^{-m}x^{m+n+1}\,(dx)^m}{m+n+1}+\frac{3ax^{-m}x^{m+2n+1}\,(dx)^m\left(ac+b^2\right)}{m+2n+1}+\frac{3bc^2x^{5n}\,(dx)^{-5n}\,(dx)^{m+5n+1}}{d\,(m+5n+1)}+\frac{bx^{-m}x^{m+3n+1}\,(dx)^m\left(6ac+b^2\right)}{m+3n+1}+\frac{c^3x^{6n}\,(dx)^{-6n}\,(dx)^{m+6n+1}}{d\,(m+6n+1)}+\frac{3cx^{4n}\,(dx)^{-4n}\,(dx)^{m+4n+1}\left(ac+b^2\right)}{d\,(m+4n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] a**3*(d*x)**(m + 1)/(d*(m + 1)) + 3*a**2*b*x**(-m)*x**(m + n + 1)
*(d*x)**m/(m + n + 1) + 3*a*x**(-m)*x**(m + 2*n + 1)*(d*x)**m*(a*
c + b**2)/(m + 2*n + 1) + 3*b*c**2*x**(5*n)*(d*x)**(-5*n)*(d*x)**
(m + 5*n + 1)/(d*(m + 5*n + 1)) + b*x**(-m)*x**(m + 3*n + 1)*(d*x
)**m*(6*a*c + b**2)/(m + 3*n + 1) + c**3*x**(6*n)*(d*x)**(-6*n)*(
d*x)**(m + 6*n + 1)/(d*(m + 6*n + 1)) + 3*c*x**(4*n)*(d*x)**(-4*n
)*(d*x)**(m + 4*n + 1)*(a*c + b**2)/(d*(m + 4*n + 1))
```

Mathematica [A] time = 0.390508, size = 137, normalized size = 0.75

$$x(dx)^m \left(\frac{a^3}{m+1}+\frac{3a^2bx^n}{m+n+1}+\frac{3ax^{2n}\left(ac+b^2\right)}{m+2n+1}+\frac{bx^{3n}\left(6ac+b^2\right)}{m+3n+1}+\frac{3cx^{4n}\left(ac+b^2\right)}{m+4n+1}+\frac{3bc^2x^{5n}}{m+5n+1}+\frac{c^3x^{6n}}{m+6n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 +
a*c)*x^(2*n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m +
3*n) + (3*c*(b^2 + a*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n)
)/(1 + m + 5*n) + (c^3*x^(6*n))/(1 + m + 6*n))
```

Maple [C] time = 0.154, size = 3798, normalized size = 20.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] x*(45*b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c^2*m^3*(x^n)
^5+780*b*c^2*n^3*(x^n)^5+75*c^3*m*n*(x^n)^6+6*a*b*c*(x^n)^3+90*a*
b*c*m^4*(x^n)^3+3048*a*b*c*n^4*(x^n)^3+510*a*c^2*m^2*n*(x^n)^4+12
84*a*c^2*m*n^2*(x^n)^4+510*b^2*c*m^2*n*(x^n)^4+1284*b^2*c*m*n^2*(
x^n)^4+240*b*c^2*m*n*(x^n)^5+300*a^2*b*m^4*n*x^n+1860*a^2*b*m^3*n
^2*x^n+5220*a^2*b*m^2*n^3*x^n+2904*a*b*c*m*n^2*(x^n)^3+540*a*b*c*
m*n*(x^n)^3+6*m*a^3+1440*a*b*c*n^5*(x^n)^3+510*a*c^2*m^3*n*(x^n)^
4+1926*a*c^2*m^2*n^2*(x^n)^4+2763*a*c^2*m*n^3*(x^n)^4+510*b^2*c*m
^3*n*(x^n)^4+1926*b^2*c*m^2*n^2*(x^n)^4+2763*b^2*c*m*n^3*(x^n)^4+
```

480*b*c^2*m^2*n*(x^n)^5+1140*b*c^2*m*n^2*(x^n)^5+60*a^2*b*m^5*n*x
^n+465*a^2*b*m^4*n^2*x^n+90*b^3*m*n*(x^n)^3+60*a^2*b*m^3*x^n+45*a
*b^2*m^2*(x^n)^2+411*a*b^2*n^2*(x^n)^2+45*a^2*b*m^2*x^n+465*a^2*b
*n^2*x^n+18*m*a*b^2*(x^n)^2+57*a*b^2*(x^n)^2*n+18*m*a^2*b*x^n+274
*c^3*m^2*n^4*(x^n)^6+120*c^3*m*n^5*(x^n)^6+3*b*c^2*m^6*(x^n)^5+75
*c^3*m^4*n*(x^n)^6+340*c^3*m^3*n^2*(x^n)^6+675*c^3*m^2*n^3*(x^n)^
6+548*c^3*m*n^4*(x^n)^6+18*a^2*c*m^5*(x^n)^2+1080*a^2*c*n^5*(x^n)
^2+18*a*b^2*m^5*(x^n)^2+1080*a*b^2*n^5*(x^n)^2+60*a*c^2*m^3*(x^n)
^4+921*a*c^2*n^3*(x^n)^4+180*b^3*m^3*n*(x^n)^3+726*b^3*m^2*n^2*(x
^n)^3+1116*b^3*m*n^3*(x^n)^3+60*b^2*c*m^3*(x^n)^4+921*b^2*c*n^3*(
x^n)^4+45*b*c^2*m^2*(x^n)^5+255*a*c^2*m^4*n*(x^n)^4+1284*a*c^2*m^
3*n^2*(x^n)^4+2763*a*c^2*m^2*n^3*(x^n)^4+2376*a*c^2*m*n^4*(x^n)^4
+255*b^2*c*m^4*n*(x^n)^4+1284*b^2*c*m^3*n^2*(x^n)^4+2763*b^2*c*m^
2*n^3*(x^n)^4+2376*b^2*c*m*n^4*(x^n)^4+480*b*c^2*m^3*n*(x^n)^5+17
10*b*c^2*m^2*n^2*(x^n)^5+51*a*c^2*(x^n)^4*n+18*b^2*c*(x^n)^4*m+3*
a^2*c*(x^n)^2+21*a^3*m^5*n+175*a^3*m^4*n^2+735*a^3*m^3*n^3+1624*a
^3*m^2*n^4+1764*a^3*m*n^5+105*a^3*m^4*n+700*a^3*m^3*n^2+2205*a^3*
m^2*n^3+3248*a^3*m*n^4+210*a^3*m^3*n+1050*a^3*m^2*n^2+2205*a^3*m*
n^3+a^3+c^3*(x^n)^6+a^3*m^6+6*a^3*m^5+1764*a^3*n^5+15*a^3*m^4+162
4*a^3*n^4+720*a^3*n^6+b^3*(x^n)^3+20*a^3*m^3+15*a^3*m^2+175*a^3*n
^2+21*a^3*n+735*a^3*n^3+285*b*c^2*n^2*(x^n)^5+18*a^2*b*m^5*x^n+21
60*a^2*b*n^5*x^n+45*a^2*c*m^4*(x^n)^2+2106*a^2*c*n^4*(x^n)^2+45*a
*b^2*m^4*(x^n)^2+2106*a*b^2*n^4*(x^n)^2+45*a*c^2*m^2*(x^n)^4+321*
a*c^2*n^2*(x^n)^4+45*b^2*c*m^2*(x^n)^4+321*b^2*c*n^2*(x^n)^4+18*m
*c^2*b*(x^n)^5+48*c^2*b*(x^n)^5*n+45*a^2*b*m^4*x^n+3132*a^2*b*n^4
*x^n+60*a^2*c*m^3*(x^n)^2+15*c^3*m^5*n*(x^n)^6+85*c^3*m^4*n^2*(x^
n)^6+225*c^3*m^3*n^3*(x^n)^6+210*a^3*m^2*n+700*a^3*m*n^2+105*a^3*
m*n+60*a*b^2*m^3*(x^n)^2+20*b^3*m^3*(x^n)^3+15*b^3*m^2*(x^n)^3+12
1*b^3*n^2*(x^n)^3+6*m*b^3*(x^n)^3+18*b^3*(x^n)^3*n+3*a*b^2*(x^n)^
2+3*a^2*b*x^n+3048*a*b*c*m^2*n^4*(x^n)^3+6096*a*b*c*m*n^4*(x^n)^3
+1440*a*b*c*m*n^5*(x^n)^3+540*a*b*c*m^4*n*(x^n)^3+2904*a*b*c*m^3*
n^2*(x^n)^3+6696*a*b*c*m^2*n^3*(x^n)^3+1383*a^2*c*n^3*(x^n)^2+18*
a^2*c*(x^n)^2*m+57*a^2*c*(x^n)^2*n+411*a^2*c*n^2*(x^n)^2+1383*a*b
^2*n^3*(x^n)^2+18*a*c^2*(x^n)^4*m+51*b^2*c*(x^n)^4*n+1740*a^2*b*n
^3*x^n+45*a^2*c*m^2*(x^n)^2+3*a^2*b*m^6*x^n+60*a^2*b*n*x^n+570*a*
b^2*m^2*n*(x^n)^2+1644*a*b^2*m*n^2*(x^n)^2+600*a^2*b*m^2*n*x^n+18
60*a^2*b*m*n^2*x^n+c^3*m^6*(x^n)^6+2340*b*c^2*m*n^3*(x^n)^5+57*a^
2*c*m^5*n*(x^n)^2+411*a^2*c*m^4*n^2*(x^n)^2+1383*a^2*c*m^3*n^3*(x
^n)^2+2106*a^2*c*m^2*n^4*(x^n)^2+1080*a^2*c*m*n^5*(x^n)^2+57*a*b^
2*m^5*n*(x^n)^2+411*a*b^2*m^4*n^2*(x^n)^2+3*a*c^2*(x^n)^4+726*a*b
*c*m^4*n^2*(x^n)^3+2232*a*b*c*m^3*n^3*(x^n)^3+3*b^2*c*(x^n)^4+6*b
^3*m^5*(x^n)^3+240*b^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+85*c^3*n^2*
(x^n)^6+15*b^3*m^4*(x^n)^3+508*b^3*n^4*(x^n)^3+6*m*c^3*(x^n)^6+15
c^3(x^n)^6*n+372*b^3*n^3*(x^n)^3+3*c^2*b*(x^n)^5+15*c^3*m^4*(x^
n)^6+274*c^3*n^4*(x^n)^6+b^3*m^6*(x^n)^3+20*c^3*m^3*(x^n)^6+225*c
^3*n^3*(x^n)^6+120*c^3*n^5*(x^n)^6+6*c^3*m^5*(x^n)^6+36*a*b*c*(x^
n)^3*m+108*a*b*c*(x^n)^3*n+48*b*c^2*m^5*n*(x^n)^5+285*b*c^2*m^4*n
^2*(x^n)^5+780*b*c^2*m^3*n^3*(x^n)^5+972*b*c^2*m^2*n^4*(x^n)^5+43
2*b*c^2*m*n^5*(x^n)^5+51*a*c^2*m^5*n*(x^n)^4+321*a*c^2*m^4*n^2*(x
^n)^4+921*a*c^2*m^3*n^3*(x^n)^4+1188*a*c^2*m^2*n^4*(x^n)^4+540*a*
c^2*m*n^5*(x^n)^4+51*b^2*c*m^5*n*(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^
4+921*b^2*c*m^3*n^3*(x^n)^4+1188*b^2*c*m^2*n^4*(x^n)^4+540*b^2*c*
m*n^5*(x^n)^4+240*b*c^2*m^4*n*(x^n)^5+1140*b*c^2*m^3*n^2*(x^n)^5+
2340*b*c^2*m^2*n^3*(x^n)^5+1944*b*c^2*m*n^4*(x^n)^5+6*a*b*c*m^6*(

```
x^n)^3+972*b*c^2*n^4*(x^n)^5+150*c^3*m^2*n*(x^n)^6+340*c^3*m*n^2*
(x^n)^6+3*a^2*c*m^6*(x^n)^2+3*a*b^2*m^6*(x^n)^2+45*a*c^2*m^4*(x^n
)^4+1188*a*c^2*n^4*(x^n)^4+90*b^3*m^4*n*(x^n)^3+484*b^3*m^3*n^2*(
x^n)^3+1116*b^3*m^2*n^3*(x^n)^3+1016*b^3*m*n^4*(x^n)^3+18*a*c^2*m
^5*(x^n)^4+540*a*c^2*n^5*(x^n)^4+18*b^3*m^5*n*(x^n)^3+121*b^3*m^4
*n^2*(x^n)^3+372*b^3*m^3*n^3*(x^n)^3+508*b^3*m^2*n^4*(x^n)^3+240*
b^3*m*n^5*(x^n)^3+18*b^2*c*m^5*(x^n)^4+540*b^2*c*n^5*(x^n)^4+45*b
*c^2*m^4*(x^n)^5+255*a*c^2*m*n*(x^n)^4+255*b^2*c*m*n*(x^n)^4+600*
a^2*b*m^3*n*x^n+2790*a^2*b*m^2*n^2*x^n+5220*a^2*b*m*n^3*x^n+570*a
^2*c*m^2*n*(x^n)^2+1644*a^2*c*m*n^2*(x^n)^2+90*a*b*c*m^2*(x^n)^3+
726*a*b*c*n^2*(x^n)^3+285*a^2*c*m*n*(x^n)^2+1080*a*b*c*m^3*n*(x^n
)^3+1383*a*b^2*m^3*n^3*(x^n)^2+2106*a*b^2*m^2*n^4*(x^n)^2+1080*a*
b^2*m*n^5*(x^n)^2+36*a*b*c*m^5*(x^n)^3+3*a*c^2*m^6*(x^n)^4+3*b^2*
c*m^6*(x^n)^4+18*b*c^2*m^5*(x^n)^5+432*b*c^2*n^5*(x^n)^5+150*c^3*
m^3*n*(x^n)^6+510*c^3*m^2*n^2*(x^n)^6+675*c^3*m*n^3*(x^n)^6+180*b
^3*m^2*n*(x^n)^3+484*b^3*m*n^2*(x^n)^3+1740*a^2*b*m^3*n^3*x^n+313
2*a^2*b*m^2*n^4*x^n+2160*a^2*b*m*n^5*x^n+285*a^2*c*m^4*n*(x^n)^2+
1644*a^2*c*m^3*n^2*(x^n)^2+4149*a^2*c*m^2*n^3*(x^n)^2+4212*a^2*c*
m*n^4*(x^n)^2+285*a*b^2*m^4*n*(x^n)^2+1644*a*b^2*m^3*n^2*(x^n)^2+
4149*a*b^2*m^2*n^3*(x^n)^2+4212*a*b^2*m*n^4*(x^n)^2+285*a*b^2*m*n
*(x^n)^2+300*a^2*b*m*n*x^n+108*a*b*c*m^5*n*(x^n)^3+4356*a*b*c*m^2
*n^2*(x^n)^3+6696*a*b*c*m*n^3*(x^n)^3+1080*a*b*c*m^2*n*(x^n)^3+24
66*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n^3*(x^n)^2+570*a*b^2*m^3*n
*(x^n)^2+2466*a*b^2*m^2*n^2*(x^n)^2+4149*a*b^2*m*n^3*(x^n)^2+120*
a*b*c*m^3*(x^n)^3+2232*a*b*c*n^3*(x^n)^3+6264*a^2*b*m*n^4*x^n+570
*a^2*c*m^3*n*(x^n)^2)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)
/(1+m+5*n)/(1+m+6*n)*exp(-1/2*m*(I*Pi*csgn(I*d*x)^3-I*Pi*csgn(I*d
*x)^2*csgn(I*d)-I*Pi*csgn(I*d*x)^2*csgn(I*x)+I*Pi*csgn(I*d*x)*csg
n(I*d)*csgn(I*x)-2*ln(x)-2*ln(d)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^3*(d*x)^m,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305498, size = 3109, normalized size = 17.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^3*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^
3)*n^5 + 15*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m
 + 225*(c^3*m^3 + 3*c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*
m^4 + 4*c^3*m^3 + 6*c^3*m^2 + 4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 +
5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2 + 5*c^3*m + c^3)*n)*x*x^(6*n)
*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b*c^2*m^5 + 15*b*c^2*
m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*c^2*m^2 + 3
24*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2*m
^3 + 3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4
 + 4*b*c^2*m^3 + 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2
*m^5 + 5*b*c^2*m^4 + 10*b*c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*
c^2)*n)*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 3*((b^2*c + a*c^2)*m^
6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b^2*c + a*c^2 + (b^2*c + a*c^2)*
m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*c + a*c^2 + (b^2*c + a
*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a*c^2)*m^3 + 3
07*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m^2 +
 3*(b^2*c + a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^
2 + 107*((b^2*c + a*c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*
c^2 + 6*(b^2*c + a*c^2)*m^2 + 4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c
 + a*c^2)*m + 17*((b^2*c + a*c^2)*m^5 + 5*(b^2*c + a*c^2)*m^4 + 1
0*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 10*(b^2*c + a*c^2)*m^2 +
5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + ((b^3
 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(b^3 + 6*a*b*c + (b
^3 + 6*a*b*c)*m)*n^5 + 15*(b^3 + 6*a*b*c)*m^4 + 508*(b^3 + 6*a*b*
c + (b^3 + 6*a*b*c)*m^2 + 2*(b^3 + 6*a*b*c)*m)*n^4 + 20*(b^3 + 6*
a*b*c)*m^3 + 372*((b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 3*(b^3 +
6*a*b*c)*m^2 + 3*(b^3 + 6*a*b*c)*m)*n^3 + b^3 + 6*a*b*c + 15*(b^3
 + 6*a*b*c)*m^2 + 121*((b^3 + 6*a*b*c)*m^4 + 4*(b^3 + 6*a*b*c)*m^
3 + b^3 + 6*a*b*c + 6*(b^3 + 6*a*b*c)*m^2 + 4*(b^3 + 6*a*b*c)*m)*
n^2 + 6*(b^3 + 6*a*b*c)*m + 18*((b^3 + 6*a*b*c)*m^5 + 5*(b^3 + 6*
a*b*c)*m^4 + 10*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 10*(b^3 + 6
*a*b*c)*m^2 + 5*(b^3 + 6*a*b*c)*m)*n)*x*x^(3*n)*e^(m*log(d) + m*l
og(x)) + 3*((a*b^2 + a^2*c)*m^6 + 6*(a*b^2 + a^2*c)*m^5 + 360*(a*
b^2 + a^2*c + (a*b^2 + a^2*c)*m)*n^5 + 15*(a*b^2 + a^2*c)*m^4 + 7
02*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*m^2 + 2*(a*b^2 + a^2*c)*m)*n^
4 + 20*(a*b^2 + a^2*c)*m^3 + 461*((a*b^2 + a^2*c)*m^3 + a*b^2 + a
^2*c + 3*(a*b^2 + a^2*c)*m^2 + 3*(a*b^2 + a^2*c)*m)*n^3 + a*b^2 +
 a^2*c + 15*(a*b^2 + a^2*c)*m^2 + 137*((a*b^2 + a^2*c)*m^4 + 4*(a
*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 6*(a*b^2 + a^2*c)*m^2 + 4*(a*
b^2 + a^2*c)*m)*n^2 + 6*(a*b^2 + a^2*c)*m + 19*((a*b^2 + a^2*c)*m
^5 + 5*(a*b^2 + a^2*c)*m^4 + 10*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2
*c + 10*(a*b^2 + a^2*c)*m^2 + 5*(a*b^2 + a^2*c)*m)*n)*x*x^(2*n)*e
^(m*log(d) + m*log(x)) + 3*(a^2*b*m^6 + 6*a^2*b*m^5 + 15*a^2*b*m^
4 + 20*a^2*b*m^3 + 720*(a^2*b*m + a^2*b)*n^5 + 15*a^2*b*m^2 + 104
4*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n^4 + 6*a^2*b*m + 580*(a^2*b*m^
3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b)*n^3 + a^2*b + 155*(a^2*b*m^4
 + 4*a^2*b*m^3 + 6*a^2*b*m^2 + 4*a^2*b*m + a^2*b)*n^2 + 20*(a^2*b
*m^5 + 5*a^2*b*m^4 + 10*a^2*b*m^3 + 10*a^2*b*m^2 + 5*a^2*b*m + a^
2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^6 + 720*a^3*n^6 +
6*a^3*m^5 + 15*a^3*m^4 + 20*a^3*m^3 + 1764*(a^3*m + a^3)*n^5 + 15
*a^3*m^2 + 1624*(a^3*m^2 + 2*a^3*m + a^3)*n^4 + 6*a^3*m + 735*(a^
```

```
3*m^3 + 3*a^3*m^2 + 3*a^3*m + a^3)*n^3 + a^3 + 175*(a^3*m^4 + 4*a
^3*m^3 + 6*a^3*m^2 + 4*a^3*m + a^3)*n^2 + 21*(a^3*m^5 + 5*a^3*m^4
 + 10*a^3*m^3 + 10*a^3*m^2 + 5*a^3*m + a^3)*n)*x*e^(m*log(d) + m*
log(x)))/(m^7 + 720*(m + 1)*n^6 + 7*m^6 + 1764*(m^2 + 2*m + 1)*n^
5 + 21*m^5 + 1624*(m^3 + 3*m^2 + 3*m + 1)*n^4 + 35*m^4 + 735*(m^4
 + 4*m^3 + 6*m^2 + 4*m + 1)*n^3 + 35*m^3 + 175*(m^5 + 5*m^4 + 10*
m^3 + 10*m^2 + 5*m + 1)*n^2 + 21*m^2 + 21*(m^6 + 6*m^5 + 15*m^4 +
 20*m^3 + 15*m^2 + 6*m + 1)*n + 7*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.333153, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^3*(d*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.597 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx$

Optimal. Leaf size=117

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n
)*(d*x)^m)/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3
*n) + (c^2*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m
))/(d*(1 + m))
```

Rubi [A] time = 0.146697, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n
)*(d*x)^m)/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3
*n) + (c^2*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m
))/(d*(1 + m))
```

Rubi in Sympy [A] time = 27.872, size = 141, normalized size = 1.21

$$\frac{a^2\,(dx)^{m+1}}{d\,(m+1)}+\frac{2abx^{-m}x^{m+n+1}\,(dx)^m}{m+n+1}+\frac{2bcx^{3n}\,(dx)^{-3n}\,(dx)^{m+3n+1}}{d\,(m+3n+1)}+\frac{c^2x^{4n}\,(dx)^{-4n}\,(dx)^{m+4n+1}}{d\,(m+4n+1)}+\frac{x^{-m}x^{m+2n+1}\,(dx)^m\left(2ac+b^2\right)}{m+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] a**2*(d*x)**(m + 1)/(d*(m + 1)) + 2*a*b*x**(-m)*x**(m + n + 1)*(d
*x)**m/(m + n + 1) + 2*b*c*x**(3*n)*(d*x)**(-3*n)*(d*x)**(m + 3*n
 + 1)/(d*(m + 3*n + 1)) + c**2*x**(4*n)*(d*x)**(-4*n)*(d*x)**(m +
```

```
 4*n + 1)/(d*(m + 4*n + 1)) + x**(-m)*x**(m + 2*n + 1)*(d*x)**m*(
2*a*c + b**2)/(m + 2*n + 1)
```

Mathematica [A] time = 0.139337, size = 86, normalized size = 0.74

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^{2n}\left(2ac+b^2\right)}{m+2n+1}+\frac{2abx^n}{m+n+1}+\frac{2bcx^{3n}}{m+3n+1}+\frac{c^2x^{4n}}{m+4n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)
*x^(2*n))/(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^
(4*n))/(1 + m + 4*n))
```

Maple [C] time = 0.101, size = 1065, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] x*(12*a*c*m^2*(x^n)^2+38*a*c*n^2*(x^n)^2+24*b^2*m*n*(x^n)^2+8*m*b
*c*(x^n)^3+14*b*c*(x^n)^3*n+12*a*b*m^2*x^n+52*a*b*n^2*x^n+8*a*c*(
x^n)^2*m+16*a*c*(x^n)^2*n+8*a*b*x^n*m+18*a*b*x^n*n+a^2+48*a*c*m^2
*n*(x^n)^2+76*a*c*m*n^2*(x^n)^2+42*b*c*m*n*(x^n)^3+54*a*b*m^2*n*x
^n+104*a*b*m*n^2*x^n+48*a*c*m*n*(x^n)^2+54*a*b*m*n*x^n+c^2*(x^n)^
4+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^2*m^2+35*a^2*n^2+24*a^2*n^4+4*
a^2*m+10*a^2*n+b^2*(x^n)^2+10*a^2*m^3*n+35*a^2*m^2*n^2+50*a^2*m*n
^3+30*a^2*m^2*n+70*a^2*m*n^2+30*a^2*m*n+8*b*c*m^3*(x^n)^3+16*b*c*
n^3*(x^n)^3+18*c^2*m*n*(x^n)^4+2*a*b*m^4*x^n+8*a*c*m^3*(x^n)^2+24
*a*c*n^3*(x^n)^2+24*b^2*m^2*n*(x^n)^2+38*b^2*m*n^2*(x^n)^2+12*b*c
*m^2*(x^n)^3+28*b*c*n^2*(x^n)^3+8*a*b*m^3*x^n+48*a*b*n^3*x^n+38*a
*c*m^2*n^2*(x^n)^2+24*a*c*m*n^3*(x^n)^2+42*b*c*m^2*n*(x^n)^3+56*b
*c*m*n^2*(x^n)^3+18*a*b*m^3*n*x^n+52*a*b*m^2*n^2*x^n+48*a*b*m*n^3
*x^n+2*b*c*m^4*(x^n)^3+18*c^2*m^2*n*(x^n)^4+22*c^2*m*n^2*(x^n)^4+
2*a*c*m^4*(x^n)^2+8*b^2*m^3*n*(x^n)^2+19*b^2*m^2*n^2*(x^n)^2+12*b
^2*m*n^3*(x^n)^2+11*c^2*m^2*n^2*(x^n)^4+6*c^2*m*n^3*(x^n)^4+6*c^2
*m^3*n*(x^n)^4+2*a*b*x^n+4*b^2*(x^n)^2*m+8*b^2*(x^n)^2*n+2*b*c*(x
^n)^3+2*a*c*(x^n)^2+11*c^2*n^2*(x^n)^4+4*b^2*m^3*(x^n)^2+12*b^2*n
^3*(x^n)^2+4*m*c^2*(x^n)^4+6*c^2*(x^n)^4*n+6*b^2*m^2*(x^n)^2+19*b
^2*n^2*(x^n)^2+c^2*m^4*(x^n)^4+4*c^2*m^3*(x^n)^4+6*c^2*n^3*(x^n)^
```

```
4+b^2*m^4*(x^n)^2+6*c^2*m^2*(x^n)^4+14*b*c*m^3*n*(x^n)^3+28*b*c*m
^2*n^2*(x^n)^3+16*b*c*m*n^3*(x^n)^3+16*a*c*m^3*n*(x^n)^2)/(1+m)/(
1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)*exp(-1/2*m*(I*Pi*csgn(I*d*x)
^3-I*Pi*csgn(I*d*x)^2*csgn(I*d)-I*Pi*csgn(I*d*x)^2*csgn(I*x)+I*Pi
*csgn(I*d*x)*csgn(I*d)*csgn(I*x)-2*ln(x)-2*ln(d)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^2*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294903, size = 953, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^2*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m
 + 11*(c^2*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^
2 + 3*c^2*m + c^2)*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*
m^4 + 4*b*c*m^3 + 6*b*c*m^2 + 8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*
(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c + 7*(b*c*m^3 + 3*b*c*m^2 + 3*
b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + ((b^2 + 2*a*c
)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 + 2*a*c)*m)*
n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c +
 2*(b^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b
^2 + 2*a*c)*m^3 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*
a*c)*m)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b
*m^3 + 6*a*b*m^2 + 24*(a*b*m + a*b)*n^3 + 4*a*b*m + 26*(a*b*m^2 +
 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b*m^3 + 3*a*b*m^2 + 3*a*b*m + a*
b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*m^4 + 24*a^2*n^4 + 4*a
^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m + 35*(a^2*m^2
 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2*m +
 a^2)*n)*x*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4
 + 50*(m^2 + 2*m + 1)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n
^2 + 10*m^2 + 10*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283026, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^2*(d*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.598 $\int (dx)^m \left(a+bx^n+cx^{2n}\right) \, dx$

Optimal. Leaf size=58

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 +
m + 2*n) + (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rubi [A] time = 0.0527297, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 +
m + 2*n) + (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rubi in Sympy [A] time = 10.6456, size = 63, normalized size = 1.09

$$\frac{a\,(dx)^{m+1}}{d\,(m+1)}+\frac{bx^{n}\,(dx)^{-n}\,(dx)^{m+n+1}}{d\,(m+n+1)}+\frac{cx^{-m}x^{m+2n+1}\,(dx)^{m}}{m+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] a*(d*x)**(m + 1)/(d*(m + 1)) + b*x**n*(d*x)**(-n)*(d*x)**(m + n +
 1)/(d*(m + n + 1)) + c*x**(-m)*x**(m + 2*n + 1)*(d*x)**m/(m + 2*
n + 1)
```

Mathematica [A] time = 0.100575, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a}{m+1}+x^n\left(\frac{b}{m+n+1}+\frac{cx^n}{m+2n+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)
))
```

Maple [C] time = 0.07, size = 205, normalized size = 3.5

$$\frac{x\left(cm^{2}\left(x^{n}\right)^{2}+cmn\left(x^{n}\right)^{2}+bm^{2}x^{n}+2\,bmnx^{n}+2\,mc\left(x^{n}\right)^{2}+c\left(x^{n}\right)^{2}n+am^{2}+3\,amn+2\,an^{2}+2\,mbx^{n}+2\,bx^{n}n+c\left(x^{n}\right)^{2}+\right.}{(1+m)\,(1+m+n)\,(1+m+2\,n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+b*m^2*x^n+2*b*m*n*x^n+2*m*c*(x^n)^
2+c*(x^n)^2*n+a*m^2+3*a*m*n+2*a*n^2+2*m*b*x^n+2*b*x^n*n+c*(x^n)^2
+2*a*m+3*a*n+b*x^n+a)/(1+m)/(1+m+n)/(1+m+2*n)*exp(-1/2*m*(I*Pi*cs
gn(I*d*x)^3-I*Pi*csgn(I*d*x)^2*csgn(I*d)-I*Pi*csgn(I*d*x)^2*csgn(
I*x)+I*Pi*csgn(I*d*x)*csgn(I*d)*csgn(I*x)-2*ln(x)-2*ln(d)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276881, size = 192, normalized size = 3.31

$$\frac{\left(cm^{2}+2\,cm+(cm+c)n+c\right)xx^{2\,n}e^{\left(m\log(d)+m\log(x)\right)}+\left(bm^{2}+2\,bm+2\,(bm+b)n+b\right)xx^{n}e^{\left(m\log(d)+m\log(x)\right)}+\left(am^{2}+2\,an^{2}+\right.}{m^{3}+2\,(m+1)n^{2}+3\,m^{2}+3\,(m^{2}+2\,m+1)n+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + (b*m^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*l
og(x)) + (a*m^2 + 2*a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log
(d) + m*log(x)))/(m^3 + 2*(m + 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)
*n + 3*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287805, size = 765, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (c*m^2*x*e^(m*ln(d) + m*ln(x) + 2*n*ln(x)) + c*m*n*x*e^(m*ln(d) +
 m*ln(x) + 2*n*ln(x)) + b*m^2*x*e^(m*ln(d) + m*ln(x) + n*ln(x)) +
 c*m^2*x*e^(m*ln(d) + m*ln(x) + n*ln(x)) + 2*b*m*n*x*e^(m*ln(d) +
 m*ln(x) + n*ln(x)) + c*m*n*x*e^(m*ln(d) + m*ln(x) + n*ln(x)) + a
*m^2*x*e^(m*ln(d) + m*ln(x)) + b*m^2*x*e^(m*ln(d) + m*ln(x)) + c*
m^2*x*e^(m*ln(d) + m*ln(x)) + 3*a*m*n*x*e^(m*ln(d) + m*ln(x)) + 2
*b*m*n*x*e^(m*ln(d) + m*ln(x)) + c*m*n*x*e^(m*ln(d) + m*ln(x)) +
2*a*n^2*x*e^(m*ln(d) + m*ln(x)) + 2*c*m*x*e^(m*ln(d) + m*ln(x) +
2*n*ln(x)) + c*n*x*e^(m*ln(d) + m*ln(x) + 2*n*ln(x)) + 2*b*m*x*e^
(m*ln(d) + m*ln(x) + n*ln(x)) + 2*c*m*x*e^(m*ln(d) + m*ln(x) + n*
ln(x)) + 2*b*n*x*e^(m*ln(d) + m*ln(x) + n*ln(x)) + c*n*x*e^(m*ln(
d) + m*ln(x) + n*ln(x)) + 2*a*m*x*e^(m*ln(d) + m*ln(x)) + 2*b*m*x
*e^(m*ln(d) + m*ln(x)) + 2*c*m*x*e^(m*ln(d) + m*ln(x)) + 3*a*n*x*
e^(m*ln(d) + m*ln(x)) + 2*b*n*x*e^(m*ln(d) + m*ln(x)) + c*n*x*e^(
m*ln(d) + m*ln(x)) + c*x*e^(m*ln(d) + m*ln(x) + 2*n*ln(x)) + b*x*
e^(m*ln(d) + m*ln(x) + n*ln(x)) + c*x*e^(m*ln(d) + m*ln(x) + n*ln
(x)) + a*x*e^(m*ln(d) + m*ln(x)) + b*x*e^(m*ln(d) + m*ln(x)) + c*
x*e^(m*ln(d) + m*ln(x)))/(m^3 + 3*m^2*n + 2*m*n^2 + 3*m^2 + 6*m*n
```

```
+ 2*n^2 + 3*m + 3*n + 1)
```

3.599 $\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=175

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqr
t[b^2 - 4*a*c])*d*(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1
[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi [A] time = 0.406217, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqr
t[b^2 - 4*a*c])*d*(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1
[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi in Sympy [A] time = 29.5365, size = 144, normalized size = 0.82

$$-\frac{2c\,(dx)^{m+1}\,{}_2F_1\left(\left.\begin{matrix}1,\frac{m+1}{n}\\\frac{m+n+1}{n}\end{matrix}\right|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{d\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}+\frac{2c\,(dx)^{m+1}\,{}_2F_1\left(\left.\begin{matrix}1,\frac{m+1}{n}\\\frac{m+n+1}{n}\end{matrix}\right|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{d\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*(d*x)**(m + 1)*hyper((1, (m + 1)/n), ((m + n + 1)/n,), -2*c*
x**n/(b + sqrt(-4*a*c + b**2)))/(d*(b + sqrt(-4*a*c + b**2))*(m +
 1)*sqrt(-4*a*c + b**2)) + 2*c*(d*x)**(m + 1)*hyper((1, (m + 1)/n
), ((m + n + 1)/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/(d*(b -
 sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 1.91469, size = 307, normalized size = 1.75

$$-\frac{x(dx)^m\left(\frac{2c\left(1-2^{-\frac{m+1}{n}}\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{-\frac{m+1}{n}}{}_2F_1\left(-\frac{m+1}{n},-\frac{m+1}{n};1-\frac{m+1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{2c\left(1-2^{-\frac{m+1}{n}}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-\frac{m+1}{n}}{}_2F_1\left(-\frac{m+1}{n},-\frac{m+1}{n};1-\frac{m+1}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((x*(d*x)^m*((2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -((1 + m)
/n), 1 - (1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)]/(2^((1 + m)/n)*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^n))^((1 + m)/n))))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c
*(1 - Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), (-1 - m + n)/
n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^
((1 + m)/n)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/
n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/(1 + m))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.600 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=328

$$\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4ac}+4ac(m-2n+1)+b^2(-(m-n+1))\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a +
 b*x^n + c*x^(2*n))) + (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n)
)/Sqrt[b^2 - 4*a*c] - b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)*n) - (c*(
4*a*c*(1 + m - 2*n) - b^2*(1 + m - n) + b*Sqrt[b^2 - 4*a*c]*(1 +
m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)^(3/2)*(
b + Sqrt[b^2 - 4*a*c])*d*(1 + m)*n)
```

Rubi [A] time = 1.88159, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4ac}+4ac(m-2n+1)+b^2(-(m-n+1))\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```


```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a +
 b*x^n + c*x^(2*n))) + (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n)
)/Sqrt[b^2 - 4*a*c] - b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
```

```
])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)*n) - (c*(
4*a*c*(1 + m - 2*n) - b^2*(1 + m - n) + b*Sqrt[b^2 - 4*a*c]*(1 +
m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)^(3/2)*(
b + Sqrt[b^2 - 4*a*c])*d*(1 + m)*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)

[Out] Timed out
```

Mathematica [B] time = 6.45158, size = 3515, normalized size = 10.72

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]

[Out] (x*(d*x)^m*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*(a + b*x
^n + c*x^(2*n))) - (b*c*x^(1 + n)*(d*x)^m*(x^n)^((1 + m)/n - (1 +
 m + n)/n)*(-(((x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^
(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1
+ m)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])
/(2*c) + x^n))])/Sqrt[b^2 - 4*a*c]) + ((x^n/(-(-b + Sqrt[b^2 - 4*
a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n
), -((1 + m)/n), 1 - (1 + m)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-
(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])/Sqrt[b^2 - 4*a*c]))/(a*(
-b^2 + 4*a*c)*(1 + m)) + (b*c*x^(1 + n)*(d*x)^m*(x^n)^((1 + m)/n
- (1 + m + n)/n)*(-(((x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)
)^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1
 - (1 + m)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4
*a*c])/(2*c) + x^n))])/Sqrt[b^2 - 4*a*c]) + ((x^n/(-(-b + Sqrt[b^
2 - 4*a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1
+ m)/n), -((1 + m)/n), 1 - (1 + m)/n, -(-b + Sqrt[b^2 - 4*a*c])/(
2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])/Sqrt[b^2 - 4*a*c])
)/(a*(-b^2 + 4*a*c)*(1 + m)*n) + (b*c*m*x^(1 + n)*(d*x)^m*(x^n)^(
(1 + m)/n - (1 + m + n)/n)*(-(((x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2
*c) + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), 1 - (1 + m)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sq
```

```
rt[b^2 - 4*a*c])/(2*c) + x^n))])/Sqrt[b^2 - 4*a*c]) + ((x^n/(-(-b
 + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hypergeometri
c2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -(-b + Sqrt[b^2 -
 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])/Sqrt[b^2
 - 4*a*c]))/(a*(-b^2 + 4*a*c)*(1 + m)*n) + (b^2*x*(d*x)^m*((1 - (
x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hype
rgeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -(-b - S
qrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])
/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2
/(2*c)) + (1 - (x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^
(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1
+ m)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])
/(2*c) + x^n))])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt
[b^2 - 4*a*c])^2/(2*c))))/(a*(-b^2 + 4*a*c)*(1 + m)) - (4*c*x*(d*
x)^m*((1 - (x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^(-1)
 - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)
/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*
c) + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2
 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c)
+ x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)
/n), 1 - (1 + m)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b
^2 - 4*a*c])/(2*c) + x^n))])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c)
+ (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/((-b^2 + 4*a*c)*(1 + m)) -
(b^2*x*(d*x)^m*((1 - (x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)
)^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1
 - (1 + m)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4
*a*c])/(2*c) + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b
- Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-(-b + Sqrt[b^2 - 4*a*
c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n),
 -((1 + m)/n), 1 - (1 + m)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-
b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])/((b*(-b + Sqrt[b^2 - 4*a*c
]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(a*(-b^2 + 4*a*c)
*(1 + m)*n) + (2*c*x*(d*x)^m*((1 - (x^n/(-(-b - Sqrt[b^2 - 4*a*c]
)/(2*c) + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -
((1 + m)/n), 1 - (1 + m)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b
- Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c])
)/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-(-b + S
qrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1
[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -(-b + Sqrt[b^2 - 4*a
*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])/((b*(-b + Sq
rt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/((-
b^2 + 4*a*c)*(1 + m)*n) - (b^2*m*x*(d*x)^m*((1 - (x^n/(-(-b - Sqr
t[b^2 - 4*a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-
((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -(-b - Sqrt[b^2 - 4*a*c
])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))])/((b*(-b - Sqrt
[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 -
(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hyp
ergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -(-b +
Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]
)/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^
2/(2*c))))/(a*(-b^2 + 4*a*c)*(1 + m)*n) + (2*c*m*x*(d*x)^m*((1 -
(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n^(-1) - m/n)*Hyp
ergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -(-b -
Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]
```

```
)/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^
2/(2*c)) + (1 - (x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^(-n
^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1
 + m)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c]
)/(2*c) + x^n))])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqr
t[b^2 - 4*a*c])^2/(2*c))))/((-b^2 + 4*a*c)*(1 + m)*n)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcd^m x e^{(m\log(x)+n\log(x))} + \left(b^2 d^m - 2\,acd^m\right) x x^m}{a^2b^2n - 4\,a^3cn + (ab^2cn - 4\,a^2c^2n)x^{2\,n} + (ab^3n - 4\,a^2bcn)x^n} + \int -\frac{bcd^m(m-n+1)e^{(m\log(x)+n\log(x))} + \left(b^2d^m(m-n+1) - 2\,acd^m(m-2\,n+1)\right)x^m}{a^2b^2n - 4\,a^3cn + (ab^2cn - 4\,a^2c^2n)x^{2\,n} + (ab^3n - 4\,a^2bcn)x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] (b*c*d^m*x*e^(m*log(x) + n*log(x)) + (b^2*d^m - 2*a*c*d^m)*x*x^m)
/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*
b^3*n - 4*a^2*b*c*n)*x^n) + integrate(-(b*c*d^m*(m - n + 1)*e^(m*
log(x) + n*log(x)) + (b^2*d^m*(m - n + 1) - 2*a*c*d^m*(m - 2*n +
1))*x^m)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*
n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^2x^{4\,n} + 2\,abx^n + a^2 + (2\,bcx^n + b^2 + 2\,ac)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^(4*n) + 2*a*b*x^n + a^2 + (2*b*c*x^n + b^
2 + 2*a*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^{2n} + bx^n + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.601 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=615

$$-\frac{c(dx)^{m+1}\left(-8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)+6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b^2-4ac}\,(2a}{2a^2d(m+1)n^2\left(b^2-4ac\right)^{5/2}}$$

$$-\frac{c(dx)^{m+1}\left(8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)-6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b^2-4ac}\,(2ac}{2a^2d(m+1)n^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{\right.}}$$

$$-\frac{(dx)^{m+1}\left(4a^2c^2(m-4n+1)-bcx^n\left(2ac(2m-7n+2)-b^2(m-2n+1)\right)-5ab^2c(m-3n+1)+b^4(m-2n+1)\right)}{2a^2dn^2\left(b^2-4ac\right)^2\left(a+bx^n+cx^{2n}\right)}$$

$$+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a
 + b*x^n + c*x^(2*n))^2) - ((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n
) - 5*a*b^2*c*(1 + m - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 +
 2*m - 7*n) - b^2*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*d*n
^2*(a + b*x^n + c*x^(2*n))) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2 +
 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) - b^4*(1 + m^2 + m*(
2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) - 4*n
+ 3*n^2) - 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x)
^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b - Sqrt[
b^2 - 4*a*c])*d*(1 + m)*n^2) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2
+ 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) + b^4*(1 + m^2 + m*
(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) - 4*n
 + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x
)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b + Sqrt
[b^2 - 4*a*c])*d*(1 + m)*n^2)
```

Rubi [A] time = 19.0273, antiderivative size = 637, normalized size of antiderivative = 1.04, number

of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{c(dx)^{m+1}\left(-8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)+6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b^2-4ac}\,(2a}{2a^2d(m+1)n^2\left(b^2-4ac\right)^{5/2}}$$

$$-\frac{c(dx)^{m+1}\left(8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)-6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b^2-4ac}\,(2ac}{2a^2d(m+1)n^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{}\right.}$$

$$-\frac{(dx)^{m+1}\left(4a^2c^2(m-4n+1)-bcx^n\left(2ac(2m-7n+2)-b^2(m-2n+1)\right)-5ab^2c(m-3n+1)+b^4(m-2n+1)\right)}{2a^2dn^2\left(b^2-4ac\right)^2\left(a+bx^n+cx^{2n}\right)}$$

$$+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a
 + b*x^n + c*x^(2*n))^2) - ((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n
) - 5*a*b^2*c*(1 + m - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 +
 2*m - 7*n) - b^2*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*d*n
^2*(a + b*x^n + c*x^(2*n))) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2 +
 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) - b^4*(1 + m^2 + m*(
2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) - 4*n
+ 3*n^2) - 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x)
^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b - Sqrt[
b^2 - 4*a*c])*d*(1 + m)*n^2) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2
+ 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) + b^4*(1 + m^2 + m*
(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) - 4*n
 + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x
)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b + Sqrt
[b^2 - 4*a*c])*d*(1 + m)*n^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Mathematica [B] time = 7.10444, size = 12289, normalized size = 19.98

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*b^2*c*d^m*(5*m - 21*n + 5) - a*b^4*d^m*(m - 3*n + 1) -
4*a^3*c^2*d^m*(m - 6*n + 1))*x*x^m + (2*a*b*c^3*d^m*(2*m - 7*n +
2) - b^3*c^2*d^m*(m - 2*n + 1))*x*e^(m*log(x) + 3*n*log(x)) + (a*
b^2*c^2*d^m*(9*m - 29*n + 9) - 2*b^4*c*d^m*(m - 2*n + 1) - 4*a^2*
c^3*d^m*(m - 4*n + 1))*x*e^(m*log(x) + 2*n*log(x)) - (b^5*d^m*(m
- 2*n + 1) - 4*a*b^3*c*d^m*(m - 3*n + 1) + 2*a^2*b*c^2*d^m*n)*x*e
^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c
^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x
^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)
*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*
n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) -
```

```
integrate(-1/2*(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^4*d^m -
(5*m^2 - m*(21*n - 10) + 16*n^2 - 21*n + 5)*a*b^2*c*d^m + 4*(m^2
- 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*a^2*c^2*d^m)*x^m + ((m^2 - m*(
3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d^m - 2*(2*m^2 - m*(9*n - 4) +
7*n^2 - 9*n + 2)*a*b*c^2*d^m)*e^(m*log(x) + n*log(x)))/(a^3*b^4*n
^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^
2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*
n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^3x^{6\,n} + b^3x^{3\,n} + 3\,a^2bx^n + a^3 + 3\,(bc^2x^n + b^2c + ac^2)x^{4\,n} + 3\,(2\,abcx^n + ab^2 + a^2c)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a^2*b*x^n + a^3 +
 3*(b*c^2*x^n + b^2*c + a*c^2)*x^(4*n) + 3*(2*a*b*c*x^n + a*b^2 +
 a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^{2\,n} + bx^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.602 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{3}{2},-\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n,
-3/2, -3/2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])])
```

Rubi [A] time = 0.485225, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{3}{2},-\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n,
-3/2, -3/2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])])
```

Rubi in Sympy [A] time = 39.9347, size = 139, normalized size = 0.86

$$\frac{a\left(dx\right)^{m+1}\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(\frac{m+1}{n},-\frac{3}{2},-\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{d\left(m+1\right)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] a*(d*x)**(m + 1)*sqrt(a + b*x**n + c*x**(2*n))*appellf1((m + 1)/n
, -3/2, -3/2, (m + n + 1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)),
 -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1)*sqrt(2*c*x**n/(b
 - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**
2)) + 1))
```

Mathematica [B] time = 11.5381, size = 5259, normalized size = 32.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2\,n} + bx^{n} + a\right)^{\frac{3}{2}} (dx)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

3.603 $\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{1}{2},-\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1
/2, -1/2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
])
```

Rubi [A] time = 0.471221, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{m+1}{n};-\frac{1}{2},-\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1
/2, -1/2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
])
```

Rubi in Sympy [A] time = 35.7589, size = 138, normalized size = 0.86

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(\frac{m+1}{n},-\frac{1}{2},-\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{d\left(m+1\right)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**n + c*x**(2*n))*appellf1((m + 1)/n,
-1/2, -1/2, (m + n + 1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -
2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1)*sqrt(2*c*x**n/(b -
 sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)
) + 1))
```

Mathematica [B] time = 6.09769, size = 930, normalized size = 5.81

$$\frac{x\sqrt{bx^n+cx^{2n}+a}(dx)^m}{m+n+1} + \frac{4a^3nx\left(2cx^n+b-\sqrt{b^2-4ac}\right)\left(2cx^n+b\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(m+1)\left((cx^n+b)x^n+a\right)^{3/2}\left(4a(m+n+1)F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)\right.} + \frac{2a^2bn(m+2n+1)x^{n+1}\left(2cx^n+b-\sqrt{b^2-4ac}\right.}{\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(m+n+1)^2\left((cx^n+b)x^n+a\right)^{3/2}\left(4a(m+2n+1)F_1\left(\frac{m+n+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+2n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)])/(1 + m + n) + (4*a^3*n*x*
(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(
(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*(a + x^n*
(b + c*x^n))^(3/2)*(4*a*(1 + m + n)*AppellF1[(1 + m)/n, 1/2, 1/2,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
(1 + m + n)/n, 1/2, 3/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*AppellF1[(1 + m + n)/n, 3/2, 1/2, (1 + m + 2*n)/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)]))) + (2*a^2*b*n*(1 + m + 2*n)*x^(1 + n)*(d*x)^m*(b - Sqrt[b^2
- 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(1
 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + m + n)^2*(a + x^n*(b + c*x^n
))^(3/2)*(4*a*(1 + m + 2*n)*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1
+ m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(1
 + m + 2*n)/n, 1/2, 3/2, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*AppellF1[(1 + m + 2*n)/n, 3/2, 1/2, (1 + m + 3*n)/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])])))
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int (dx)^{m} \sqrt{a+b x^{n}+c x^{2 n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)

[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c x^{2 n}+b x^{n}+a}\,(dx)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m,x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{m} \sqrt{a+b x^{n}+c x^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)

[Out] Integral((d*x)**m*sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2\,n} + bx^n + a}\,(dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m,x, algorithm="giac")

[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

3.604 $\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/
2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.469492, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/
2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 36.6309, size = 136, normalized size = 0.85

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf}_1\left(\frac{m+1}{n},\frac{1}{2},\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{ad\,(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**n + c*x**(2*n))*appellf1((m + 1)/n,
1/2, 1/2, (m + n + 1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*
```

```
c*x**n/(b + sqrt(-4*a*c + b**2)))/(a*d*(m + 1)*sqrt(2*c*x**n/(b -
 sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)
) + 1))
```

Mathematica [B] time = 0.502606, size = 440, normalized size = 2.75

$$\frac{4a^2x(m+n+1)(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^n\right)}{(m+1)\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)(a+x^n(b+cx^n))^{3/2}\left(4a(m+n+1)F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (4*a^2*(1 + m + n)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(1 + m)/n, 1/2, 1/2, (1
+ m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*
c])*(1 + m)*(a + x^n*(b + c*x^n))^(3/2)*(4*a*(1 + m + n)*AppellF1
[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^
2 - 4*a*c])*AppellF1[(1 + m + n)/n, 1/2, 3/2, (1 + m + 2*n)/n, (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 3/2, 1/2,
(1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.605 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}} \, dx$

Optimal. Leaf size=163

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{3}{2},\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/
2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)
])
```

Rubi [A] time = 0.4882, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{n};\frac{3}{2},\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/
2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)
])
```

Rubi in Sympy [A] time = 45.2513, size = 138, normalized size = 0.85

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf}_1\left(\frac{m+1}{n},\frac{3}{2},\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a^2d\,(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**n + c*x**(2*n))*appellf1((m + 1)/n,
3/2, 3/2, (m + n + 1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*
c*x**n/(b + sqrt(-4*a*c + b**2)))/(a**2*d*(m + 1)*sqrt(2*c*x**n/(
b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**n/(b + sqrt(-4*a*c + b*
*2)) + 1))
```

Mathematica [B] time = 6.2686, size = 3743, normalized size = 22.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (2*x*(d*x)^m*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*Sqrt[a
 + b*x^n + c*x^(2*n)]) - (4*a*b^2*(1 + m + n)*x*(d*x)^m*(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*Appell
F1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b
 - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*(a + x^n*(b
 + c*x^n))^(3/2)*(4*a*(1 + m + n)*AppellF1[(1 + m)/n, 1/2, 1/2, (
1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(1
 + m + n)/n, 1/2, 3/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
4*a*c])*AppellF1[(1 + m + n)/n, 3/2, 1/2, (1 + m + 2*n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
))) + (16*a^2*c*(1 + m + n)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(1 + m)/n, 1/2,
 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*
c])*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*(a + x^n*(b + c*x^n))^(3/2)*(
4*a*(1 + m + n)*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 1/2,
3/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(
1 + m + n)/n, 3/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (8*a*b^2*(1
+ m + n)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 -
 4*a*c])*(1 + m)*n*(a + x^n*(b + c*x^n))^(3/2)*(4*a*(1 + m + n)*A
ppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b +
Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 1/2, 3/2, (1 + m + 2*n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 3/2
, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
```

x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (16*a^2*c*(1 + m + n)*x*(d*x)
^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n)*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2
 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + m)
n(a + x^n*(b + c*x^n))^(3/2)*(4*a*(1 + m + n)*AppellF1[(1 + m)/
n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c
])*AppellF1[(1 + m + n)/n, 1/2, 3/2, (1 + m + 2*n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 3/2, 1/2, (1 + m +
2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])]))) + (8*a*b^2*m*(1 + m + n)*x*(d*x)^m*(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[
(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b -
Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n*(a + x^n*(b
+ c*x^n))^(3/2)*(4*a*(1 + m + n)*AppellF1[(1 + m)/n, 1/2, 1/2, (1
 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(1
+ m + n)/n, 1/2, 3/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4
*a*c])*AppellF1[(1 + m + n)/n, 3/2, 1/2, (1 + m + 2*n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
)) - (16*a^2*c*m*(1 + m + n)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(1 + m)/n, 1/2
, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a
c])(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n*(a + x^n*(b + c*x^n))^(3/2
)*(4*a*(1 + m + n)*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 1/
2, 3/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF
1[(1 + m + n)/n, 3/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (8*a*b*c*
(1 + m + 2*n)*x^(1 + n)*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(1 + m + n)/n, 1/2, 1
/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*
c])*(b + Sqrt[b^2 - 4*a*c])*n*(1 + m + n)*(a + x^n*(b + c*x^n))^(
3/2)*(4*a*(1 + m + 2*n)*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m
+ 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m
 + 2*n)/n, 1/2, 3/2, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*
a*c])*AppellF1[(1 + m + 2*n)/n, 3/2, 1/2, (1 + m + 3*n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
))) + (8*a*b*c*m*(1 + m + 2*n)*x^(1 + n)*(d*x)^m*(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[(1 +
 m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b
- Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(1 + m + n)*(a + x
^n*(b + c*x^n))^(3/2)*(4*a*(1 + m + 2*n)*AppellF1[(1 + m + n)/n,

```
1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - n*x^n*((b + Sqrt[b^2 - 4*a*c]
)*AppellF1[(1 + m + 2*n)/n, 1/2, 3/2, (1 + m + 3*n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (
b - Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + 2*n)/n, 3/2, 1/2, (1 + m
 + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int (dx)^{m} \left(a+b x^{n}+c x^{2 n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{m}}{\left(c x^{2 n}+b x^{n}+a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.606 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p,
-p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.320689, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p,
-p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 35.0149, size = 129, normalized size = 0.82

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] (d*x)**(m + 1)*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*
c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2
*n))**p*appellf1((m + 1)/n, -p, -p, (m + n + 1)/n, -2*c*x**n/(b -
 sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(d*(m
```

```
+ 1))
```

Mathematica [B] time = 5.6303, size = 534, normalized size = 3.38

$$-\frac{2^{-p-1}x(m+n+1)\left(\sqrt{b^2-4ac}+b\right)(dx)^m\left(\sqrt{b^2-4ac}-b-2cx^n\right)\left(x^n\left(\sqrt{b^2-4ac}-b\right)-\cdots\right.}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^n\right)\left(npx^n\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{m+n+1}{n};1-p,-p;\frac{m+2n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2c\cdots}{\sqrt{b^2-\cdots}}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -((2^(-1 - p)*(b + Sqrt[b^2 - 4*a*c])*(1 + m + n)*x*(d*x)^m*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x^n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
c)^p*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n*(b + c*x^n)
)^(-1 + p)*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-
b + Sqrt[b^2 - 4*a*c])*(1 + m)*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x
^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(-2*a*(1 + m + n)*AppellF
1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p*x^n*((-b + Sqrt[
b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 1 - p, -p, (1 + m + 2*n)/n,
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, -p, 1 -
 p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])]))))
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+bx^n+a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2n} + bx^n + a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

3.607 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$

Optimal. Leaf size=46

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(
8*e)
```

Rubi [A] time = 0.10966, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(
8*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int^{(d+ex)^2} x\,dx}{2e}+\frac{b\,(d+ex)^6}{6e}+\frac{c\,(d+ex)^8}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] a*Integral(x, (x, (d + e*x)**2))/(2*e) + b*(d + e*x)**6/(6*e) + c
*(d + e*x)**8/(8*e)
```

Mathematica [B] time = 0.0647406, size = 150, normalized size = 3.26

$$\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)+d^3x\left(a+bd^2+cd^4\right)+\frac{1}{6}e^5x^6\left(b+21cd^2\right)+de^4x^5\left(b+7cd^2\right)+cde^6x^7+\frac{1}{8}ce^7x^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)
/2 + (d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 +
 35*c*d^4)*e^3*x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)
*e^5*x^6)/6 + c*d*e^6*x^7 + (c*e^7*x^8)/8
```

Maple [B] time = 0.002, size = 298, normalized size = 6.5

$$\frac{e^7cx^8}{8}+e^6dcx^7+\frac{\left(15\,d^2e^5c+e^3\left(6\,cd^2e^2+be^2\right)\right)x^6}{6}+\frac{\left(13\,d^3ce^4+3\,e^2d\left(6\,cd^2e^2+be^2\right)+e^3\left(4\,cd^3e+2\,bde\right)\right)x^5}{5}+\frac{\left(4\,d^4ce^3+3\,d^2e\left(6\,cd^2e^2+be^2\right)+3\,e^2d\left(4\,cd^3e+2\,bde\right)+e^3\left(cd^4+bd^2+a\right)\right)x^4}{4}+\frac{\left(d^3\left(6\,cd^2e^2+be^2\right)+3\,d^2e\left(4\,cd^3e+2\,bde\right)+3\,e^2d\left(cd^4+bd^2+a\right)\right)x^3}{3}+\frac{\left(d^3\left(4\,cd^3e+2\,bde\right)+3\,d^2e\left(cd^4+bd^2+a\right)\right)x^2}{2}+d^3\left(cd^4+bd^2+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*c*x^8+e^6*d*c*x^7+1/6*(15*d^2*e^5*c+e^3*(6*c*d^2*e^2+b*e^
2))*x^6+1/5*(13*d^3*c*e^4+3*e^2*d*(6*c*d^2*e^2+b*e^2)+e^3*(4*c*d^
3*e+2*b*d*e))*x^5+1/4*(4*d^4*c*e^3+3*d^2*e*(6*c*d^2*e^2+b*e^2)+3*
e^2*d*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d^2+a))*x^4+1/3*(d^3*(6*c*
d^2*e^2+b*e^2)+3*d^2*e*(4*c*d^3*e+2*b*d*e)+3*e^2*d*(c*d^4+b*d^2+a
))*x^3+1/2*(d^3*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a))*x^2+
d^3*(c*d^4+b*d^2+a)*x
```

Maxima [A] time = 0.752987, size = 192, normalized size = 4.17

$$\frac{1}{8}\,ce^7x^8+cde^6x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5x^6+\left(7\,cd^3+bd\right)e^4x^5+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2x^3+\frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ex^2+\left(cd^7+bd^5+ad^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d
^3 + b*d)*e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(
21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3
*a*d^2)*e*x^2 + (c*d^7 + b*d^5 + a*d^3)*x
```

Fricas [A] time = 0.25692, size = 1, normalized size = 0.02

$$\frac{1}{8}x^8e^7c + x^7e^6dc + \frac{7}{2}x^6e^5d^2c + 7x^5e^4d^3c + \frac{35}{4}x^4e^3d^4c + \frac{1}{6}x^6e^5b + 7x^3e^2d^5c + x^5e^4db + \frac{7}{2}x^2ed^6c + \frac{5}{2}x^4e^3d^2b + xd^7c + \frac{10}{3}x^3e^2d^3b + \frac{5}{2}x^2ed^4b + \frac{1}{4}x^4e^3a + xd^5b + x^3e^2da + \frac{3}{2}x^2ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^7*c + x^7*e^6*d*c + 7/2*x^6*e^5*d^2*c + 7*x^5*e^4*d^3*c
 + 35/4*x^4*e^3*d^4*c + 1/6*x^6*e^5*b + 7*x^3*e^2*d^5*c + x^5*e^4
*d*b + 7/2*x^2*e*d^6*c + 5/2*x^4*e^3*d^2*b + x*d^7*c + 10/3*x^3*e
^2*d^3*b + 5/2*x^2*e*d^4*b + 1/4*x^4*e^3*a + x*d^5*b + x^3*e^2*d*
a + 3/2*x^2*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.202125, size = 178, normalized size = 3.87

$$cde^6x^7 + \frac{ce^7x^8}{8} + x^6\left(\frac{be^5}{6} + \frac{7cd^2e^5}{2}\right) + x^5\left(bde^4 + 7cd^3e^4\right) + x^4\left(\frac{ae^3}{4} + \frac{5bd^2e^3}{2} + \frac{35cd^4e^3}{4}\right) + x^3\left(ade^2 + \frac{10bd^3e^2}{3} + 7cd^5e^2\right) + x^2\left(\frac{3ad^2e}{2} + \frac{5bd^4e}{2} + \frac{7cd^6e}{2}\right) + x\left(ad^3 + bd^5 + cd^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2)
 + x**5*(b*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e*
*3/2 + 35*c*d**4*e**3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*
c*d**5*e**2) + x**2*(3*a*d**2*e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2)
+ x*(a*d**3 + b*d**5 + c*d**7)
```

GIAC/XCAS [A] time = 0.266209, size = 224, normalized size = 4.87

$$\frac{1}{8}\,cx^{8}e^{7} + cdx^{7}e^{6} + \frac{7}{2}\,cd^{2}x^{6}e^{5} + 7\,cd^{3}x^{5}e^{4} + \frac{35}{4}\,cd^{4}x^{4}e^{3} + 7\,cd^{5}x^{3}e^{2} + \frac{7}{2}\,cd^{6}x^{2}e + cd^{7}x + \frac{1}{6}\,bx^{6}e^{5} + bdx^{5}e^{4} + \frac{5}{2}\,bd^{2}x^{4}e^{3} + \frac{10}{3}\,bd^{3}x^{3}e^{2} + \frac{5}{2}\,bd^{4}x^{2}e + bd^{5}x + \frac{1}{4}\,ax^{4}e^{3} + adx^{3}e^{2} + \frac{3}{2}\,ad^{2}x^{2}e + ad^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^3,x, algorithm="giac")
```

```
[Out] 1/8*c*x^8*e^7 + c*d*x^7*e^6 + 7/2*c*d^2*x^6*e^5 + 7*c*d^3*x^5*e^4
 + 35/4*c*d^4*x^4*e^3 + 7*c*d^5*x^3*e^2 + 7/2*c*d^6*x^2*e + c*d^7
*x + 1/6*b*x^6*e^5 + b*d*x^5*e^4 + 5/2*b*d^2*x^4*e^3 + 10/3*b*d^3
*x^3*e^2 + 5/2*b*d^4*x^2*e + b*d^5*x + 1/4*a*x^4*e^3 + a*d*x^3*e^
2 + 3/2*a*d^2*x^2*e + a*d^3*x
```

3.608 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=89

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c
)*(d + e*x)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^
12)/(12*e)
```

Rubi [A] time = 0.396266, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c
)*(d + e*x)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^
12)/(12*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int^{(d+ex)^2} x\,dx}{2e}+\frac{ab\,(d+ex)^6}{3e}+\frac{bc\,(d+ex)^{10}}{5e}+\frac{c^2\,(d+ex)^{12}}{12e}+\frac{(d+ex)^8\left(2ac+b^2\right)}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] a**2*Integral(x, (x, (d + e*x)**2))/(2*e) + a*b*(d + e*x)**6/(3*e
) + b*c*(d + e*x)**10/(5*e) + c**2*(d + e*x)**12/(12*e) + (d + e*
x)**8*(2*a*c + b**2)/(8*e)
```

Mathematica [B] time = 0.21506, size = 401, normalized size = 4.51

$$\frac{1}{4}e^3x^4\left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3\left(3a^2+20abd^2+42acd^4+21b^2d^4+72bcd^6+55c^2d^8\right)+\frac{1}{2}d^2ex^2\left(3a^2+10abd^2+14acd^4+7b^2d^4+18bcd^6+11c^2d^8\right)+\frac{1}{8}e^7x^8\left(2ac+b^2+72bcd^2+330c^2d^4\right)+de^6x^7\left(2ac+b^2+24bcd^2+66c^2d^4\right)+\frac{1}{6}e^5x^6\left(2ab+42acd^2+21b^2d^2+252bcd^4+462c^2d^6\right)+\frac{1}{5}de^4x^5\left(10ab+70acd^2+35b^2d^2+252bcd^4+330c^2d^6\right)+d^3x\left(a+bd^2+cd^4\right)^2+\frac{1}{10}ce^9x^{10}\left(2b+55cd^2\right)+\frac{1}{3}cde^8x^9\left(6b+55cd^2\right)+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^
4 + 14*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 +
20*a*b*d^2 + 21*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e
^2*x^3)/3 + ((a^2 + 20*a*b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*
c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d*(10*a*b + 35*b^2*d^2 + 70*a*
c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5 + ((2*a*b + 21*b^2*
d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/6 + d*(b^2
 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*
b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9
)/3 + (c*(2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e
^11*x^12)/12
```

Maple [B] time = 0.002, size = 1314, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*c^2*x^12+e^10*d*c^2*x^11+1/10*(27*d^2*e^9*c^2+e^3*(2*(6
*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6))*x^10+1/9*(25*d^3*c^2*e^8
+3*e^2*d*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+e^3*(2*(4*c
*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3))*x^9+1/8*(8*
d^4*c^2*e^7+3*d^2*e*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+
3*e^2*d*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^
3)+e^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*
c*d^2*e^2+b*e^2)^2))*x^8+1/7*(d^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16
```

```
*c^2*d^2*e^6)+3*d^2*e*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2
+b*e^2)*c*d*e^3)+3*e^2*d*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*
b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+e^3*(8*(c*d^4+b*d^2+a)*c*d*
e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2)))*x^7+1/6*(d^3*(2*(
4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+3*d^2*e*(
2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^
2+b*e^2)^2)+3*e^2*d*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d
*e)*(6*c*d^2*e^2+b*e^2))+e^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^
2)+(4*c*d^3*e+2*b*d*e)^2))*x^6+1/5*(d^3*(2*(c*d^4+b*d^2+a)*c*e^4+
8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*e*(8*(
c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))
+3*e^2*d*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*
e)^2)+2*e^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e))*x^5+1/4*(d^3*(8*
(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2)
)+3*d^2*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d
*e)^2)+6*e^2*d*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d
^2+a)^2)*x^4+1/3*(d^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c
*d^3*e+2*b*d*e)^2)+6*d^2*e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*
e^2*d*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2*d^3*(c*d^4+b*d^2+a)*(4*c*d^3*
e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a)^2)*x^2+d^3*(c*d^4+b*d^2+a)^2*x
```

Maxima [A] time = 0.752595, size = 544, normalized size = 6.11

$$\begin{aligned}&\frac{1}{12}c^2e^{11}x^{12}+c^2de^{10}x^{11}+\frac{1}{10}\left(55c^2d^2+2bc\right)e^9x^{10}+\frac{1}{3}\left(55c^2d^3+6bcd\right)e^8x^9\\&+\frac{1}{8}\left(330c^2d^4+72bcd^2+b^2+2ac\right)e^7x^8+\left(66c^2d^5+24bcd^3+\left(b^2+2ac\right)d\right)e^6x^7\\&+\frac{1}{6}\left(462c^2d^6+252bcd^4+21\left(b^2+2ac\right)d^2+2ab\right)e^5x^6\\&+\frac{1}{5}\left(330c^2d^7+252bcd^5+35\left(b^2+2ac\right)d^3+10abd\right)e^4x^5\\&+\frac{1}{4}\left(165c^2d^8+168bcd^6+35\left(b^2+2ac\right)d^4+20abd^2+a^2\right)e^3x^4\\&+\frac{1}{3}\left(55c^2d^9+72bcd^7+21\left(b^2+2ac\right)d^5+20abd^3+3a^2d\right)e^2x^3\\&+\frac{1}{2}\left(11c^2d^{10}+18bcd^8+7\left(b^2+2ac\right)d^6+10abd^4+3a^2d^2\right)ex^2\\&+\left(c^2d^{11}+2bcd^9+\left(b^2+2ac\right)d^7+2abd^5+a^2d^3\right)x\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^3,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*
e^9*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4
+ 72*b*c*d^2 + b^2 + 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 +
(b^2 + 2*a*c)*d)*e^6*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b
^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5
 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 + 1/4*(165*c^2*d^8 +
```

```
168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*x^4 +
1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3
+ 3*a^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a
*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 +
 (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*x
```

Fricas [A] time = 0.266645, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}e^{11}c^2 + x^{11}e^{10}dc^2 + \frac{11}{2}x^{10}e^9d^2c^2 + \frac{55}{3}x^9e^8d^3c^2 + \frac{165}{4}x^8e^7d^4c^2 + \frac{1}{5}x^{10}e^9cb + 66x^7e^6d^5c^2 + 2x^9e^8dcb + 77x^6e^5d^6c^2 + 9x^8e^7d^2cb + 66x^5e^4d^7c^2 + 24x^7e^6d^3cb + \frac{165}{4}x^4e^3d^8c^2 + 42x^6e^5d^4cb + \frac{1}{8}x^8e^7b^2 + \frac{1}{4}x^8e^7ca + \frac{55}{3}x^3e^2d^9c^2 + \frac{252}{5}x^5e^4d^5cb + x^7e^6db^2 + 2x^7e^6dca + \frac{11}{2}x^2ed^{10}c^2 + 42x^4e^3d^6cb + \frac{7}{2}x^6e^5d^2b^2 + 7x^6e^5d^2ca + xd^{11}c^2 + 24x^3e^2d^7cb + 7x^5e^4d^3b^2 + 14x^5e^4d^3ca + 9x^2ed^8cb + \frac{35}{4}x^4e^3d^4b^2 + \frac{35}{2}x^4e^3d^4ca + \frac{1}{3}x^6e^5ba + 2xd^9cb + 7x^3e^2d^5b^2 + 14x^3e^2d^5ca + 2x^5e^4dba + \frac{7}{2}x^2ed^6b^2 + 7x^2ed^6ca + 5x^4e^3d^2ba + xd^7b^2 + 2xd^7ca + \frac{20}{3}x^3e^2d^3ba + 5x^2ed^4ba + \frac{1}{4}x^4e^3a^2 + 2xd^5ba + x^3e^2da^2 + \frac{3}{2}x^2ed^2a^2 + xd^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^11*c^2 + x^11*e^10*d*c^2 + 11/2*x^10*e^9*d^2*c^2 + 55
/3*x^9*e^8*d^3*c^2 + 165/4*x^8*e^7*d^4*c^2 + 1/5*x^10*e^9*c*b + 6
6*x^7*e^6*d^5*c^2 + 2*x^9*e^8*d*c*b + 77*x^6*e^5*d^6*c^2 + 9*x^8*
e^7*d^2*c*b + 66*x^5*e^4*d^7*c^2 + 24*x^7*e^6*d^3*c*b + 165/4*x^4
*e^3*d^8*c^2 + 42*x^6*e^5*d^4*c*b + 1/8*x^8*e^7*b^2 + 1/4*x^8*e^7
*c*a + 55/3*x^3*e^2*d^9*c^2 + 252/5*x^5*e^4*d^5*c*b + x^7*e^6*d*b
^2 + 2*x^7*e^6*d*c*a + 11/2*x^2*e*d^10*c^2 + 42*x^4*e^3*d^6*c*b +
 7/2*x^6*e^5*d^2*b^2 + 7*x^6*e^5*d^2*c*a + x*d^11*c^2 + 24*x^3*e^
2*d^7*c*b + 7*x^5*e^4*d^3*b^2 + 14*x^5*e^4*d^3*c*a + 9*x^2*e*d^8*
c*b + 35/4*x^4*e^3*d^4*b^2 + 35/2*x^4*e^3*d^4*c*a + 1/3*x^6*e^5*b
*a + 2*x*d^9*c*b + 7*x^3*e^2*d^5*b^2 + 14*x^3*e^2*d^5*c*a + 2*x^5
*e^4*d*b*a + 7/2*x^2*e*d^6*b^2 + 7*x^2*e*d^6*c*a + 5*x^4*e^3*d^2*
b*a + x*d^7*b^2 + 2*x*d^7*c*a + 20/3*x^3*e^2*d^3*b*a + 5*x^2*e*d^
4*b*a + 1/4*x^4*e^3*a^2 + 2*x*d^5*b*a + x^3*e^2*d*a^2 + 3/2*x^2*e
*d^2*a^2 + x*d^3*a^2
```

Sympy [A] time = 0.469583, size = 559, normalized size = 6.28

$$c^2de^{10}x^{11} + \frac{c^2e^{11}x^{12}}{12} + x^{10}\left(\frac{bce^9}{5} + \frac{11c^2d^2e^9}{2}\right) + x^9\left(2bcde^8 + \frac{55c^2d^3e^8}{3}\right) + x^8\left(\frac{ace^7}{4} + \frac{b^2e^7}{8} + 9bcd^2e^7 + \frac{165c^2d^4e^7}{4}\right) + x^7\left(2acde^6 + b^2de^6 + 24bcd^3e^6 + 66c^2d^5e^6\right) + x^6\left(\frac{abe^5}{3} + 7acd^2e^5 + \frac{7b^2d^2e^5}{2} + 42bcd^4e^5 + 77c^2d^6e^5\right) + x^5\left(2abde^4 + 14acd^3e^4 + 7b^2d^3e^4 + \frac{252bcd^5e^4}{5} + 66c^2d^7e^4\right) + x^4\left(\frac{a^2e^3}{4} + 5abd^2e^3 + \frac{35acd^4e^3}{2} + \frac{35b^2d^4e^3}{4} + 42bcd^6e^3 + \frac{165c^2d^8e^3}{4}\right) + x^3\left(a^2de^2 + \frac{20abd^3e^2}{3} + 14acd^5e^2 + 7b^2d^5e^2 + 24bcd^7e^2 + \frac{55c^2d^9e^2}{3}\right) + x^2\left(\frac{3a^2d^2e}{2} + 5abd^4e + 7acd^6e + \frac{7b^2d^6e}{2} + 9bcd^8e + \frac{11c^2d^{10}e}{2}\right) + x\left(a^2d^3 + 2abd^5 + 2acd^7 + b^2d^7 + 2bcd^9 + c^2d^{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11
*c**2*d**2*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) +
x**8*(a*c*e**7/4 + b**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*
e**7/4) + x**7*(2*a*c*d*e**6 + b**2*d*e**6 + 24*b*c*d**3*e**6 + 6
6*c**2*d**5*e**6) + x**6*(a*b*e**5/3 + 7*a*c*d**2*e**5 + 7*b**2*d
**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*e**5) + x**5*(2*a*b*
d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c*d**5*e**4/
5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 + 35
*a*c*d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c
**2*d**8*e**3/4) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*
c*d**5*e**2 + 7*b**2*d**5*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*
e**2/3) + x**2*(3*a**2*d**2*e/2 + 5*a*b*d**4*e + 7*a*c*d**6*e + 7
*b**2*d**6*e/2 + 9*b*c*d**8*e + 11*c**2*d**10*e/2) + x*(a**2*d**3
 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*b*c*d**9 + c**2*d**11)
```

GIAC/XCAS [A] time = 0.268788, size = 730, normalized size = 8.2

$$\frac{1}{12}\,c^2x^{12}e^{11} + c^2dx^{11}e^{10} + \frac{11}{2}\,c^2d^2x^{10}e^9 + \frac{55}{3}\,c^2d^3x^9e^8 + \frac{165}{4}\,c^2d^4x^8e^7 + 66\,c^2d^5x^7e^6 + 77\,c^2d^6x^6e^5 + 66\,c^2d^7x^5e^4 + \frac{165}{4}\,c^2d^8x^4e^3 + \frac{55}{3}\,c^2d^9x^3e^2 + \frac{11}{2}\,c^2d^{10}x^2e + c^2d^{11}x + \frac{1}{5}\,bcx^{10}e^9 + 2\,bcdx^9e^8 + 9\,bcd^2x^8e^7 + 24\,bcd^3x^7e^6 + 42\,bcd^4x^6e^5 + \frac{252}{5}\,bcd^5x^5e^4 + 42\,bcd^6x^4e^3 + 24\,bcd^7x^3e^2 + 9\,bcd^8x^2e + 2\,bcd^9x + \frac{1}{8}\,b^2x^8e^7 + \frac{1}{4}\,acx^8e^7 + b^2dx^7e^6 + 2\,acdx^7e^6 + \frac{7}{2}\,b^2d^2x^6e^5 + 7\,acd^2x^6e^5 + 7\,b^2d^3x^5e^4 + 14\,acd^3x^5e^4 + \frac{35}{4}\,b^2d^4x^4e^3 + \frac{35}{2}\,acd^4x^4e^3 + 7\,b^2d^5x^3e^2 + 14\,acd^5x^3e^2 + \frac{7}{2}\,b^2d^6x^2e + 7\,acd^6x^2e + b^2d^7x + 2\,acd^7x + \frac{1}{3}\,abx^6e^5 + 2\,abdx^5e^4 + 5\,abd^2x^4e^3 + \frac{20}{3}\,abd^3x^3e^2 + 5\,abd^4x^2e + 2\,abd^5x + \frac{1}{4}\,a^2x^4e^3 + a^2dx^3e^2 + \frac{3}{2}\,a^2d^2x^2e + a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^3,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12*e^11 + c^2*d*x^11*e^10 + 11/2*c^2*d^2*x^10*e^9 + 55
/3*c^2*d^3*x^9*e^8 + 165/4*c^2*d^4*x^8*e^7 + 66*c^2*d^5*x^7*e^6 +
 77*c^2*d^6*x^6*e^5 + 66*c^2*d^7*x^5*e^4 + 165/4*c^2*d^8*x^4*e^3
+ 55/3*c^2*d^9*x^3*e^2 + 11/2*c^2*d^10*x^2*e + c^2*d^11*x + 1/5*b
*c*x^10*e^9 + 2*b*c*d*x^9*e^8 + 9*b*c*d^2*x^8*e^7 + 24*b*c*d^3*x^
7*e^6 + 42*b*c*d^4*x^6*e^5 + 252/5*b*c*d^5*x^5*e^4 + 42*b*c*d^6*x
^4*e^3 + 24*b*c*d^7*x^3*e^2 + 9*b*c*d^8*x^2*e + 2*b*c*d^9*x + 1/8
*b^2*x^8*e^7 + 1/4*a*c*x^8*e^7 + b^2*d*x^7*e^6 + 2*a*c*d*x^7*e^6
+ 7/2*b^2*d^2*x^6*e^5 + 7*a*c*d^2*x^6*e^5 + 7*b^2*d^3*x^5*e^4 + 1
4*a*c*d^3*x^5*e^4 + 35/4*b^2*d^4*x^4*e^3 + 35/2*a*c*d^4*x^4*e^3 +
 7*b^2*d^5*x^3*e^2 + 14*a*c*d^5*x^3*e^2 + 7/2*b^2*d^6*x^2*e + 7*a
*c*d^6*x^2*e + b^2*d^7*x + 2*a*c*d^7*x + 1/3*a*b*x^6*e^5 + 2*a*b*
d*x^5*e^4 + 5*a*b*d^2*x^4*e^3 + 20/3*a*b*d^3*x^3*e^2 + 5*a*b*d^4*
x^2*e + 2*a*b*d^5*x + 1/4*a^2*x^4*e^3 + a^2*d*x^3*e^2 + 3/2*a^2*d
^2*x^2*e + a^2*d^3*x
```

3.609 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal. Leaf size=138

$$\frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}$$

```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 +
 a*c)*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e)
+ (c*(b^2 + a*c)*(d + e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14
*e) + (c^3*(d + e*x)^16)/(16*e)
```

Rubi [A] time = 0.754898, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 +
 a*c)*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e)
+ (c*(b^2 + a*c)*(d + e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14
*e) + (c^3*(d + e*x)^16)/(16*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\int^{(d+ex)^2} x\,dx}{2e}+\frac{a^2b\,(d+ex)^6}{2e}+\frac{3a\,(d+ex)^8\left(ac+b^2\right)}{8e}+\frac{3bc^2\,(d+ex)^{14}}{14e}+\frac{b\,(d+ex)^{10}\left(6ac+b^2\right)}{10e}+\frac{c^3\,(d+ex)^{16}}{16e}+\frac{c\,(d+ex)^{12}\left(ac+b^2\right)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] a**3*Integral(x, (x, (d + e*x)**2))/(2*e) + a**2*b*(d + e*x)**6/(
2*e) + 3*a*(d + e*x)**8*(a*c + b**2)/(8*e) + 3*b*c**2*(d + e*x)**
14/(14*e) + b*(d + e*x)**10*(6*a*c + b**2)/(10*e) + c**3*(d + e*x
)**16/(16*e) + c*(d + e*x)**12*(a*c + b**2)/(4*e)
```

Mathematica [B] time = 0.635026, size = 797, normalized size = 5.78

$$\frac{1}{16}c^3e^{15}x^{16} + c^3de^{14}x^{15} + \frac{3}{14}c^2\left(35cd^2 + b\right)e^{13}x^{14} + c^2d\left(35cd^2 + 3b\right)e^{12}x^{13} + \frac{1}{4}c\left(455c^2d^4 + 78bcd^2 + b^2 + ac\right)e^{11}x^{12} + 3cd\left(91c^2d^4 + 26bcd^2 + b^2 + ac\right)e^{10}x^{11} + \frac{1}{10}\left(5005c^3d^6 + 2145bc^2d^4 + 165ac^2d^2 + 165b^2cd^2 + b^3 + 6abc\right)e^9x^{10} + d\left(715c^3d^6 + 429bc^2d^4 + 55ac^2d^2 + 55b^2cd^2 + b^3 + 6abc\right)e^8x^9 + \frac{3}{8}\left(2145c^3d^8 + 1716bc^2d^6 + 330ac^2d^4 + 330b^2cd^4 + 12b^3d^2 + 72abcd^2 + ab^2 + a^2c\right)e^7x^8 + \frac{1}{7}d\left(5005c^3d^8 + 5148bc^2d^6 + 1386ac^2d^4 + 1386b^2cd^4 + 84b^3d^2 + 504abcd^2 + 21ab^2 + 21a^2c\right)e^6x^7 + \frac{1}{2}\left(1001c^3d^{10} + 1287bc^2d^8 + 462ac^2d^6 + 462b^2cd^6 + 42b^3d^4 + 252abcd^4 + 21ab^2d^2 + 21a^2cd^2 + a^2b\right)e^5x^6 + \frac{3}{5}d\left(455c^3d^{10} + 715bc^2d^8 + 330ac^2d^6 + 330b^2cd^6 + 42b^3d^4 + 252abcd^4 + 35ab^2d^2 + 35a^2cd^2 + 5a^2b\right)e^4x^5 + \frac{1}{4}\left(455c^3d^{12} + 858bc^2d^{10} + 495ac^2d^8 + 495b^2cd^8 + 84b^3d^6 + 504abcd^6 + 105ab^2d^4 + 105a^2cd^4 + 30a^2bd^2 + a^3\right)e^3x^4 + d\left(35c^3d^{12} + 78bc^2d^{10} + 55ac^2d^8 + 55b^2cd^8 + 12b^3d^6 + 72abcd^6 + 21ab^2d^4 + 21a^2cd^4 + 10a^2bd^2 + a^3\right)e^2x^3 + \frac{3}{2}d^2\left(cd^4 + bd^2 + a\right)^2\left(5cd^4 + 3bd^2 + a\right)ex^2 + d^3\left(cd^4 + bd^2 + a\right)^3x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3
*b*d^2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4
 + 21*a^2*c*d^4 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a
*c^2*d^8 + 78*b*c^2*d^10 + 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*
b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 84*b^3*d^6 + 504*a*b*c*d^
6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10 + 455*c^3*d^12
)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 42*b
^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^
2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^
2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*
d^6 + 1287*b*c^2*d^8 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 +
 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*
c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*x^7)/7 + (3*(a*b^2 +
 a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 330*a*c^2*d^
4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c
```

```
+ 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*
x^9 + ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^
2*d^4 + 5005*c^3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^
2 + 91*c^2*d^4)*e^10*x^11 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*
d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^2)*e^12*x^13 + (3*c^2*(b
+ 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3*e^15*x^16)/16
```

Maple [B] time = 0.003, size = 7550, normalized size = 54.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.753657, size = 1177, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*
e^13*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4
 + 78*b*c^2*d^2 + b^2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b
*c^2*d^3 + (b^2*c + a*c^2)*d)*e^10*x^11 + 1/10*(5005*c^3*d^6 + 21
45*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*x^10
+ (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 +
6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b
^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*x
^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^
5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*
(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b
^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*x^6 + 3/5
*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^
3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*x^5 +
1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a
^3)*e^3*x^4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d
^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d
^5 + a^3*d)*e^2*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c
 + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a*b^2 +
```

```
 a^2*c)*d^6 + a^3*d^2)*e*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*
c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 +
a^2*c)*d^7 + a^3*d^3)*x
```

Fricas [A] time = 0.278862, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^15*c^3 + x^15*e^14*d*c^3 + 15/2*x^14*e^13*d^2*c^3 + 3
5*x^13*e^12*d^3*c^3 + 455/4*x^12*e^11*d^4*c^3 + 3/14*x^14*e^13*c^
2*b + 273*x^11*e^10*d^5*c^3 + 3*x^13*e^12*d*c^2*b + 1001/2*x^10*e
^9*d^6*c^3 + 39/2*x^12*e^11*d^2*c^2*b + 715*x^9*e^8*d^7*c^3 + 78*
x^11*e^10*d^3*c^2*b + 6435/8*x^8*e^7*d^8*c^3 + 429/2*x^10*e^9*d^4
*c^2*b + 1/4*x^12*e^11*c*b^2 + 1/4*x^12*e^11*c^2*a + 715*x^7*e^6*
d^9*c^3 + 429*x^9*e^8*d^5*c^2*b + 3*x^11*e^10*d*c*b^2 + 3*x^11*e^
10*d*c^2*a + 1001/2*x^6*e^5*d^10*c^3 + 1287/2*x^8*e^7*d^6*c^2*b +
 33/2*x^10*e^9*d^2*c*b^2 + 33/2*x^10*e^9*d^2*c^2*a + 273*x^5*e^4*
d^11*c^3 + 5148/7*x^7*e^6*d^7*c^2*b + 55*x^9*e^8*d^3*c*b^2 + 55*x
^9*e^8*d^3*c^2*a + 455/4*x^4*e^3*d^12*c^3 + 1287/2*x^6*e^5*d^8*c^
2*b + 495/4*x^8*e^7*d^4*c*b^2 + 1/10*x^10*e^9*b^3 + 495/4*x^8*e^7
*d^4*c^2*a + 3/5*x^10*e^9*c*b*a + 35*x^3*e^2*d^13*c^3 + 429*x^5*e
^4*d^9*c^2*b + 198*x^7*e^6*d^5*c*b^2 + x^9*e^8*d*b^3 + 198*x^7*e^
6*d^5*c^2*a + 6*x^9*e^8*d*c*b*a + 15/2*x^2*e*d^14*c^3 + 429/2*x^4
*e^3*d^10*c^2*b + 231*x^6*e^5*d^6*c*b^2 + 9/2*x^8*e^7*d^2*b^3 + 2
31*x^6*e^5*d^6*c^2*a + 27*x^8*e^7*d^2*c*b*a + x*d^15*c^3 + 78*x^3
*e^2*d^11*c^2*b + 198*x^5*e^4*d^7*c*b^2 + 12*x^7*e^6*d^3*b^3 + 19
8*x^5*e^4*d^7*c^2*a + 72*x^7*e^6*d^3*c*b*a + 39/2*x^2*e*d^12*c^2*
b + 495/4*x^4*e^3*d^8*c*b^2 + 21*x^6*e^5*d^4*b^3 + 495/4*x^4*e^3*
d^8*c^2*a + 126*x^6*e^5*d^4*c*b*a + 3/8*x^8*e^7*b^2*a + 3/8*x^8*e
^7*c*a^2 + 3*x*d^13*c^2*b + 55*x^3*e^2*d^9*c*b^2 + 126/5*x^5*e^4*
d^5*b^3 + 55*x^3*e^2*d^9*c^2*a + 756/5*x^5*e^4*d^5*c*b*a + 3*x^7*
e^6*d*b^2*a + 3*x^7*e^6*d*c*a^2 + 33/2*x^2*e*d^10*c*b^2 + 21*x^4*
e^3*d^6*b^3 + 33/2*x^2*e*d^10*c^2*a + 126*x^4*e^3*d^6*c*b*a + 21/
2*x^6*e^5*d^2*b^2*a + 21/2*x^6*e^5*d^2*c*a^2 + 3*x*d^11*c*b^2 + 1
2*x^3*e^2*d^7*b^3 + 3*x*d^11*c^2*a + 72*x^3*e^2*d^7*c*b*a + 21*x^
5*e^4*d^3*b^2*a + 21*x^5*e^4*d^3*c*a^2 + 9/2*x^2*e*d^8*b^3 + 27*x
^2*e*d^8*c*b*a + 105/4*x^4*e^3*d^4*b^2*a + 105/4*x^4*e^3*d^4*c*a^
2 + 1/2*x^6*e^5*b*a^2 + x*d^9*b^3 + 6*x*d^9*c*b*a + 21*x^3*e^2*d^
5*b^2*a + 21*x^3*e^2*d^5*c*a^2 + 3*x^5*e^4*d*b*a^2 + 21/2*x^2*e*d
^6*b^2*a + 21/2*x^2*e*d^6*c*a^2 + 15/2*x^4*e^3*d^2*b*a^2 + 3*x*d^
7*b^2*a + 3*x*d^7*c*a^2 + 10*x^3*e^2*d^3*b*a^2 + 15/2*x^2*e*d^4*b
*a^2 + 1/4*x^4*e^3*a^3 + 3*x*d^5*b*a^2 + x^3*e^2*d*a^3 + 3/2*x^2*
e*d^2*a^3 + x*d^3*a^3
```

Sympy [A] time = 0.946342, size = 1314, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*x**15 + c**3*e**15*x**16/16 + x**14*(3*b*c**2*e**13/
14 + 15*c**3*d**2*e**13/2) + x**13*(3*b*c**2*d*e**12 + 35*c**3*d*
*3*e**12) + x**12*(a*c**2*e**11/4 + b**2*c*e**11/4 + 39*b*c**2*d*
*2*e**11/2 + 455*c**3*d**4*e**11/4) + x**11*(3*a*c**2*d*e**10 + 3
*b**2*c*d*e**10 + 78*b*c**2*d**3*e**10 + 273*c**3*d**5*e**10) + x
**10*(3*a*b*c*e**9/5 + 33*a*c**2*d**2*e**9/2 + b**3*e**9/10 + 33*
b**2*c*d**2*e**9/2 + 429*b*c**2*d**4*e**9/2 + 1001*c**3*d**6*e**9
/2) + x**9*(6*a*b*c*d*e**8 + 55*a*c**2*d**3*e**8 + b**3*d*e**8 +
55*b**2*c*d**3*e**8 + 429*b*c**2*d**5*e**8 + 715*c**3*d**7*e**8)
+ x**8*(3*a**2*c*e**7/8 + 3*a*b**2*e**7/8 + 27*a*b*c*d**2*e**7 +
495*a*c**2*d**4*e**7/4 + 9*b**3*d**2*e**7/2 + 495*b**2*c*d**4*e**
7/4 + 1287*b*c**2*d**6*e**7/2 + 6435*c**3*d**8*e**7/8) + x**7*(3*
a**2*c*d*e**6 + 3*a*b**2*d*e**6 + 72*a*b*c*d**3*e**6 + 198*a*c**2
*d**5*e**6 + 12*b**3*d**3*e**6 + 198*b**2*c*d**5*e**6 + 5148*b*c*
*2*d**7*e**6/7 + 715*c**3*d**9*e**6) + x**6*(a**2*b*e**5/2 + 21*a
**2*c*d**2*e**5/2 + 21*a*b**2*d**2*e**5/2 + 126*a*b*c*d**4*e**5 +
 231*a*c**2*d**6*e**5 + 21*b**3*d**4*e**5 + 231*b**2*c*d**6*e**5
+ 1287*b*c**2*d**8*e**5/2 + 1001*c**3*d**10*e**5/2) + x**5*(3*a**
2*b*d*e**4 + 21*a**2*c*d**3*e**4 + 21*a*b**2*d**3*e**4 + 756*a*b*
c*d**5*e**4/5 + 198*a*c**2*d**7*e**4 + 126*b**3*d**5*e**4/5 + 198
*b**2*c*d**7*e**4 + 429*b*c**2*d**9*e**4 + 273*c**3*d**11*e**4) +
 x**4*(a**3*e**3/4 + 15*a**2*b*d**2*e**3/2 + 105*a**2*c*d**4*e**3
/4 + 105*a*b**2*d**4*e**3/4 + 126*a*b*c*d**6*e**3 + 495*a*c**2*d*
*8*e**3/4 + 21*b**3*d**6*e**3 + 495*b**2*c*d**8*e**3/4 + 429*b*c*
*2*d**10*e**3/2 + 455*c**3*d**12*e**3/4) + x**3*(a**3*d*e**2 + 10
*a**2*b*d**3*e**2 + 21*a**2*c*d**5*e**2 + 21*a*b**2*d**5*e**2 + 7
2*a*b*c*d**7*e**2 + 55*a*c**2*d**9*e**2 + 12*b**3*d**7*e**2 + 55*
b**2*c*d**9*e**2 + 78*b*c**2*d**11*e**2 + 35*c**3*d**13*e**2) + x
**2*(3*a**3*d**2*e/2 + 15*a**2*b*d**4*e/2 + 21*a**2*c*d**6*e/2 +
21*a*b**2*d**6*e/2 + 27*a*b*c*d**8*e + 33*a*c**2*d**10*e/2 + 9*b*
*3*d**8*e/2 + 33*b**2*c*d**10*e/2 + 39*b*c**2*d**12*e/2 + 15*c**3
*d**14*e/2) + x*(a**3*d**3 + 3*a**2*b*d**5 + 3*a**2*c*d**7 + 3*a*
b**2*d**7 + 6*a*b*c*d**9 + 3*a*c**2*d**11 + b**3*d**9 + 3*b**2*c*
d**11 + 3*b*c**2*d**13 + c**3*d**15)
```

GIAC/XCAS [A] time = 0.269519, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^3,x, algorithm="giac")

[Out] Done
```

3.610 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$

Optimal. Leaf size=55

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

```
[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d
 + e*x)^8)/(8*e)
```

Rubi [A] time = 0.122093, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d
 + e*x)^8)/(8*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{af^3\int^{(d+ex)^2} x\,dx}{2e}+\frac{bf^3\left(d+ex\right)^6}{6e}+\frac{cf^3\left(d+ex\right)^8}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] a*f**3*Integral(x, (x, (d + e*x)**2))/(2*e) + b*f**3*(d + e*x)**6
/(6*e) + c*f**3*(d + e*x)**8/(8*e)
```

Mathematica [B] time = 0.0611145, size = 154, normalized size = 2.8

$$f^3\left(\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)\right.$$
$$\left.+d^3x\left(a+bd^2+cd^4\right)+\frac{1}{6}e^5x^6\left(b+21cd^2\right)+de^4x^5\left(b+7cd^2\right)+cde^6x^7+\frac{1}{8}ce^7x^8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e
*x^2)/2 + (d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*
d^2 + 35*c*d^4)*e^3*x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c
*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c*e^7*x^8)/8)
```

Maple [B] time = 0.002, size = 349, normalized size = 6.4

$$\frac{e^7 f^3 c x^8}{8} + d f^3 e^6 c x^7 + \frac{\left(15 d^2 f^3 e^5 c + e^3 f^3 \left(6 c d^2 e^2 + b e^2\right)\right) x^6}{6} + \frac{\left(13 d^3 f^3 c e^4 + 3 d f^3 e^2 \left(6 c d^2 e^2 + b e^2\right) + e^3 f^3 \left(4 c d^3 e + 2 b d e\right)\right) x^5}{5} + \frac{\left(4 d^4 f^3 c e^3 + 3 d^2 f^3 e \left(6 c d^2 e^2 + b e^2\right) + 3 d f^3 e^2 \left(4 c d^3 e + 2 b d e\right) + e^3 f^3 \left(c d^4 + b d^2 + a\right)\right) x^4}{4} + \frac{\left(d^3 f^3 \left(6 c d^2 e^2 + b e^2\right) + 3 d^2 f^3 e \left(4 c d^3 e + 2 b d e\right) + 3 d f^3 e^2 \left(c d^4 + b d^2 + a\right)\right) x^3}{3} + \frac{\left(d^3 f^3 \left(4 c d^3 e + 2 b d e\right) + 3 d^2 f^3 e \left(c d^4 + b d^2 + a\right)\right) x^2}{2} + d^3 f^3 \left(c d^4 + b d^2 + a\right) x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*f^3*c*x^8+d*f^3*e^6*c*x^7+1/6*(15*d^2*f^3*e^5*c+e^3*f^3*(
6*c*d^2*e^2+b*e^2))*x^6+1/5*(13*d^3*f^3*c*e^4+3*d*f^3*e^2*(6*c*d^
2*e^2+b*e^2)+e^3*f^3*(4*c*d^3*e+2*b*d*e))*x^5+1/4*(4*d^4*f^3*c*e^
3+3*d^2*f^3*e*(6*c*d^2*e^2+b*e^2)+3*d*f^3*e^2*(4*c*d^3*e+2*b*d*e)
+e^3*f^3*(c*d^4+b*d^2+a))*x^4+1/3*(d^3*f^3*(6*c*d^2*e^2+b*e^2)+3*
d^2*f^3*e*(4*c*d^3*e+2*b*d*e)+3*d*f^3*e^2*(c*d^4+b*d^2+a))*x^3+1/
2*(d^3*f^3*(4*c*d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a))*x^2+d
^3*f^3*(c*d^4+b*d^2+a)*x
```

Maxima [A] time = 0.747397, size = 224, normalized size = 4.07

$$\frac{1}{8} c e^7 f^3 x^8 + c d e^6 f^3 x^7 + \frac{1}{6} \left(21 c d^2 + b\right) e^5 f^3 x^6 + \left(7 c d^3 + b d\right) e^4 f^3 x^5 + \frac{1}{4} \left(35 c d^4 + 10 b d^2 + a\right) e^3 f^3 x^4 + \frac{1}{3} \left(21 c d^5 + 10 b d^3 + 3 a d\right) e^2 f^3 x^3 + \frac{1}{2} \left(7 c d^6 + 5 b d^4 + 3 a d^2\right) e f^3 x^2 + \left(c d^7 + b d^5 + a d^3\right) f^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^3,x, algorithm="maxima
```

```
[Out] 1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*
x^6 + (7*c*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)
*e^3*f^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/
2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^
3)*f^3*x
```

Fricas [A] time = 0.242006, size = 1, normalized size = 0.02

$$\frac{1}{8}x^8f^3e^7c + x^7f^3e^6dc + \frac{7}{2}x^6f^3e^5d^2c + 7x^5f^3e^4d^3c + \frac{35}{4}x^4f^3e^3d^4c + \frac{1}{6}x^6f^3e^5b + 7x^3f^3e^2d^5c + x^5f^3e^4db + \frac{7}{2}x^2f^3ed^6c + \frac{5}{2}x^4f^3e^3d^2b + xf^3d^7c + \frac{10}{3}x^3f^3e^2d^3b + \frac{5}{2}x^2f^3ed^4b + \frac{1}{4}x^4f^3e^3a + xf^3d^5b + x^3f^3e^2da + \frac{3}{2}x^2f^3ed^2a + xf^3d^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^3,x, algorithm="fricas
```

```
[Out] 1/8*x^8*f^3*e^7*c + x^7*f^3*e^6*d*c + 7/2*x^6*f^3*e^5*d^2*c + 7*x
^5*f^3*e^4*d^3*c + 35/4*x^4*f^3*e^3*d^4*c + 1/6*x^6*f^3*e^5*b + 7
*x^3*f^3*e^2*d^5*c + x^5*f^3*e^4*d*b + 7/2*x^2*f^3*e*d^6*c + 5/2*
x^4*f^3*e^3*d^2*b + x*f^3*d^7*c + 10/3*x^3*f^3*e^2*d^3*b + 5/2*x^
2*f^3*e*d^4*b + 1/4*x^4*f^3*e^3*a + x*f^3*d^5*b + x^3*f^3*e^2*d*a
 + 3/2*x^2*f^3*e*d^2*a + x*f^3*d^3*a
```

Sympy [A] time = 0.231997, size = 240, normalized size = 4.36

$$cde^6f^3x^7 + \frac{ce^7f^3x^8}{8} + x^6\left(\frac{be^5f^3}{6} + \frac{7cd^2e^5f^3}{2}\right) + x^5\left(bde^4f^3 + 7cd^3e^4f^3\right) + x^4\left(\frac{ae^3f^3}{4} + \frac{5bd^2e^3f^3}{2} + \frac{35cd^4e^3f^3}{4}\right) + x^3\left(ade^2f^3 + \frac{10bd^3e^2f^3}{3} + 7cd^5e^2f^3\right) + x^2\left(\frac{3ad^2ef^3}{2} + \frac{5bd^4ef^3}{2} + \frac{7cd^6ef^3}{2}\right) + x\left(ad^3f^3 + bd^5f^3 + cd^7f^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7
*c*d**2*e**5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3)
```

```
+ x**4*(a*e**3*f**3/4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**
3/4) + x**3*(a*d*e**2*f**3 + 10*b*d**3*e**2*f**3/3 + 7*c*d**5*e**
2*f**3) + x**2*(3*a*d**2*e*f**3/2 + 5*b*d**4*e*f**3/2 + 7*c*d**6*
e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*d**7*f**3)
```

GIAC/XCAS [A] time = 0.270184, size = 297, normalized size = 5.4

$$\frac{1}{8}\,cf^3x^8e^7 + cdf^3x^7e^6 + \frac{7}{2}\,cd^2f^3x^6e^5 + 7\,cd^3f^3x^5e^4 + \frac{35}{4}\,cd^4f^3x^4e^3 + 7\,cd^5f^3x^3e^2 + \frac{7}{2}\,cd^6f^3x^2e + cd^7f^3x + \frac{1}{6}\,bf^3x^6e^5 + bdf^3x^5e^4 + \frac{5}{2}\,bd^2f^3x^4e^3 + \frac{10}{3}\,bd^3f^3x^3e^2 + \frac{5}{2}\,bd^4f^3x^2e + bd^5f^3x + \frac{1}{4}\,af^3x^4e^3 + adf^3x^3e^2 + \frac{3}{2}\,ad^2f^3x^2e + ad^3f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^3,x, algorithm="giac")
```

```
[Out] 1/8*c*f^3*x^8*e^7 + c*d*f^3*x^7*e^6 + 7/2*c*d^2*f^3*x^6*e^5 + 7*c
*d^3*f^3*x^5*e^4 + 35/4*c*d^4*f^3*x^4*e^3 + 7*c*d^5*f^3*x^3*e^2 +
 7/2*c*d^6*f^3*x^2*e + c*d^7*f^3*x + 1/6*b*f^3*x^6*e^5 + b*d*f^3*
x^5*e^4 + 5/2*b*d^2*f^3*x^4*e^3 + 10/3*b*d^3*f^3*x^3*e^2 + 5/2*b*
d^4*f^3*x^2*e + b*d^5*f^3*x + 1/4*a*f^3*x^4*e^3 + a*d*f^3*x^3*e^2
 + 3/2*a*d^2*f^3*x^2*e + a*d^3*f^3*x
```

3.611 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=104

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2
 + 2*a*c)*f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) +
 (c^2*f^3*(d + e*x)^12)/(12*e)
```

Rubi [A] time = 0.40664, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2
 + 2*a*c)*f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) +
 (c^2*f^3*(d + e*x)^12)/(12*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2f^3\int^{(d+ex)^2} x\,dx}{2e}+\frac{abf^3\,(d+ex)^6}{3e}+\frac{bcf^3\,(d+ex)^{10}}{5e}+\frac{c^2f^3\,(d+ex)^{12}}{12e}+\frac{f^3\,(d+ex)^8\left(2ac+b^2\right)}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] a**2*f**3*Integral(x, (x, (d + e*x)**2))/(2*e) + a*b*f**3*(d + e*
x)**6/(3*e) + b*c*f**3*(d + e*x)**10/(5*e) + c**2*f**3*(d + e*x)*
*12/(12*e) + f**3*(d + e*x)**8*(2*a*c + b**2)/(8*e)
```

Mathematica [B] time = 0.213757, size = 405, normalized size = 3.89

$$f^3\left(\frac{1}{4}e^3x^4\left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3\left(3a^2+20abd^2+42acd^4+21b^2d^4+72bcd^6+55c^2d^8\right)+\frac{1}{2}d^2ex^2\left(3a^2+10abd^2+14acd^4+7b^2d^4+18bcd^6+11c^2d^8\right)+\frac{1}{8}e^7x^8\left(2ac+b^2+72bcd^2+330c^2d^4\right)+de^6x^7\left(2ac+b^2+24bcd^2+66c^2d^4\right)+\frac{1}{6}e^5x^6\left(2ab+42acd^2+21b^2d^2+252bcd^4+462c^2d^6\right)+\frac{1}{5}de^4x^5\left(10ab+70acd^2+35b^2d^2+252bcd^4+330c^2d^6\right)+d^3x\left(a+bd^2+cd^4\right)^2+\frac{1}{10}ce^9x^{10}\left(2b+55cd^2\right)+\frac{1}{3}cde^8x^9\left(6b+55cd^2\right)+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b
^2*d^4 + 14*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a
^2 + 20*a*b*d^2 + 21*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d
^8)*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 1
68*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d*(10*a*b + 35*b^2*d^2 +
70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5 + ((2*a*b + 21
*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/6 + d
*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c
+ 72*b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^
8*x^9)/3 + (c*(2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (
c^2*e^11*x^12)/12)
```

Maple [B] time = 0.002, size = 1413, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*f^3*c^2*x^12+d*f^3*e^10*c^2*x^11+1/10*(27*d^2*f^3*e^9*c
^2+e^3*f^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6))*x^10+1/9
*(25*d^3*f^3*c^2*e^8+3*d*f^3*e^2*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*
c^2*d^2*e^6)+e^3*f^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+
b*e^2)*c*d*e^3))*x^9+1/8*(8*d^4*f^3*c^2*e^7+3*d^2*f^3*e*(2*(6*c*d
^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+3*d*f^3*e^2*(2*(4*c*d^3*e+2*b
*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+e^3*f^3*(2*(c*d^4+b*d^
```

```
2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2))*
x^8+1/7*(d^3*f^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+3*d
^2*f^3*e*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e
^3)+3*d*f^3*e^2*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*
d*e^3+(6*c*d^2*e^2+b*e^2)^2)+e^3*f^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2
*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2)))*x^7+1/6*(d^3*f^3*(2*(4
*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+3*d^2*f^3*
e*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2
*e^2+b*e^2)^2)+3*d*f^3*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*
e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+e^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d
^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2))*x^6+1/5*(d^3*f^3*(2*(c*d^4+
b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^
2)+3*d^2*f^3*e*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(
6*c*d^2*e^2+b*e^2))+3*d*f^3*e^2*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b
*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*f^3*(c*d^4+b*d^2+a)*(4*c*d^3*e
+2*b*d*e))*x^5+1/4*(d^3*f^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3
*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d^2*f^3*e*(2*(c*d^4+b*d^2+a)*(
6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d*f^3*e^2*(c*d^4+b*d^
2+a)*(4*c*d^3*e+2*b*d*e)+e^3*f^3*(c*d^4+b*d^2+a)^2)*x^4+1/3*(d^3*
f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)
+6*d^2*f^3*e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d*f^3*e^2*(c*d
^4+b*d^2+a)^2)*x^3+1/2*(2*d^3*f^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*
d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a)^2)*x^2+d^3*f^3*(c*d^4+b*d^2+a)^2
*x
```

Maxima [A] time = 0.759418, size = 593, normalized size = 5.7

$$\frac{1}{12}\,c^2e^{11}f^3x^{12}+c^2de^{10}f^3x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9$$
$$+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7f^3x^8+\left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6f^3x^7$$
$$+\frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5f^3x^6$$
$$+\frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4f^3x^5$$
$$+\frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3f^3x^4$$
$$+\frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2f^3x^3$$
$$+\frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ef^3x^2$$
$$+\left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^3,x, algorithm="maxi
```

```
[Out] 1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 +
 2*b*c)*e^9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1
```

```
/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2
*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d
^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*f^3*x^6 + 1/
5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e
^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^
4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7
+ 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*
(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*
a^2*d^2)*e*f^3*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 +
2*a*b*d^5 + a^2*d^3)*f^3*x
```

Fricas [A] time = 0.245044, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}f^3e^{11}c^2 + x^{11}f^3e^{10}dc^2 + \frac{11}{2}x^{10}f^3e^9d^2c^2 + \frac{55}{3}x^9f^3e^8d^3c^2 + \frac{165}{4}x^8f^3e^7d^4c^2 + \frac{1}{5}x^{10}f^3e^9cb + 66x^7f^3e^6d^5c^2 + 2x^9f^3e^8dcb + 77x^6f^3e^5d^6c^2 + 9x^8f^3e^7d^2cb + 66x^5f^3e^4d^7c^2 + 24x^7f^3e^6d^3cb + \frac{165}{4}x^4f^3e^3d^8c^2 + 42x^6f^3e^5d^4cb + \frac{1}{8}x^8f^3e^7b^2 + \frac{1}{4}x^8f^3e^7ca + \frac{55}{3}x^3f^3e^2d^9c^2 + \frac{252}{5}x^5f^3e^4d^5cb + x^7f^3e^6db^2 + 2x^7f^3e^6dca + \frac{11}{2}x^2f^3ed^{10}c^2 + 42x^4f^3e^3d^6cb + \frac{7}{2}x^6f^3e^5d^2b^2 + 7x^6f^3e^5d^2ca + xf^3d^{11}c^2 + 24x^3f^3e^2d^7cb + 7x^5f^3e^4d^3b^2 + 14x^5f^3e^4d^3ca + 9x^2f^3ed^8cb + \frac{35}{4}x^4f^3e^3d^4b^2 + \frac{35}{2}x^4f^3e^3d^4ca + \frac{1}{3}x^6f^3e^5ba + 2xf^3d^9cb + 7x^3f^3e^2d^5b^2 + 14x^3f^3e^2d^5ca + 2x^5f^3e^4dba + \frac{7}{2}x^2f^3ed^6b^2 + 7x^2f^3ed^6ca + 5x^4f^3e^3d^2ba + xf^3d^7b^2 + 2xf^3d^7ca + \frac{20}{3}x^3f^3e^2d^3ba + 5x^2f^3ed^4ba + \frac{1}{4}x^4f^3e^3a^2 + 2xf^3d^5ba + x^3f^3e^2da^2 + \frac{3}{2}x^2f^3ed^2a^2 + xf^3d^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^3,x, algorithm="fric
```

```
[Out] 1/12*x^12*f^3*e^11*c^2 + x^11*f^3*e^10*d*c^2 + 11/2*x^10*f^3*e^9*
d^2*c^2 + 55/3*x^9*f^3*e^8*d^3*c^2 + 165/4*x^8*f^3*e^7*d^4*c^2 +
1/5*x^10*f^3*e^9*c*b + 66*x^7*f^3*e^6*d^5*c^2 + 2*x^9*f^3*e^8*d*c
*b + 77*x^6*f^3*e^5*d^6*c^2 + 9*x^8*f^3*e^7*d^2*c*b + 66*x^5*f^3*
e^4*d^7*c^2 + 24*x^7*f^3*e^6*d^3*c*b + 165/4*x^4*f^3*e^3*d^8*c^2
+ 42*x^6*f^3*e^5*d^4*c*b + 1/8*x^8*f^3*e^7*b^2 + 1/4*x^8*f^3*e^7*
c*a + 55/3*x^3*f^3*e^2*d^9*c^2 + 252/5*x^5*f^3*e^4*d^5*c*b + x^7*
f^3*e^6*d*b^2 + 2*x^7*f^3*e^6*d*c*a + 11/2*x^2*f^3*e*d^10*c^2 + 4
2*x^4*f^3*e^3*d^6*c*b + 7/2*x^6*f^3*e^5*d^2*b^2 + 7*x^6*f^3*e^5*d
^2*c*a + x*f^3*d^11*c^2 + 24*x^3*f^3*e^2*d^7*c*b + 7*x^5*f^3*e^4*
d^3*b^2 + 14*x^5*f^3*e^4*d^3*c*a + 9*x^2*f^3*e*d^8*c*b + 35/4*x^4
*f^3*e^3*d^4*b^2 + 35/2*x^4*f^3*e^3*d^4*c*a + 1/3*x^6*f^3*e^5*b*a
 + 2*x*f^3*d^9*c*b + 7*x^3*f^3*e^2*d^5*b^2 + 14*x^3*f^3*e^2*d^5*c
```

```
*a + 2*x^5*f^3*e^4*d*b*a + 7/2*x^2*f^3*e*d^6*b^2 + 7*x^2*f^3*e*d^
6*c*a + 5*x^4*f^3*e^3*d^2*b*a + x*f^3*d^7*b^2 + 2*x*f^3*d^7*c*a +
 20/3*x^3*f^3*e^2*d^3*b*a + 5*x^2*f^3*e*d^4*b*a + 1/4*x^4*f^3*e^3
*a^2 + 2*x*f^3*d^5*b*a + x^3*f^3*e^2*d*a^2 + 3/2*x^2*f^3*e*d^2*a^
2 + x*f^3*d^3*a^2
```

Sympy [A] time = 0.531626, size = 722, normalized size = 6.94

$$c^{2}de^{10}f^{3}x^{11} + \frac{c^{2}e^{11}f^{3}x^{12}}{12} + x^{10}\left(\frac{bce^{9}f^{3}}{5} + \frac{11c^{2}d^{2}e^{9}f^{3}}{2}\right) + x^{9}\left(2bcde^{8}f^{3} + \frac{55c^{2}d^{3}e^{8}f^{3}}{3}\right) + x^{8}\left(\frac{ace^{7}f^{3}}{4} + \frac{b^{2}e^{7}f^{3}}{8} + 9bcd^{2}e^{7}f^{3} + \frac{165c^{2}d^{4}e^{7}f^{3}}{4}\right) + x^{7}\left(2acde^{6}f^{3} + b^{2}de^{6}f^{3} + 24bcd^{3}e^{6}f^{3} + 66c^{2}d^{5}e^{6}f^{3}\right) + x^{6}\left(\frac{abe^{5}f^{3}}{3} + 7acd^{2}e^{5}f^{3} + \frac{7b^{2}d^{2}e^{5}f^{3}}{2} + 42bcd^{4}e^{5}f^{3} + 77c^{2}d^{6}e^{5}f^{3}\right) + x^{5}\left(2abde^{4}f^{3} + 14acd^{3}e^{4}f^{3} + 7b^{2}d^{3}e^{4}f^{3} + \frac{252bcd^{5}e^{4}f^{3}}{5} + 66c^{2}d^{7}e^{4}f^{3}\right) + x^{4}\left(\frac{a^{2}e^{3}f^{3}}{4} + 5abd^{2}e^{3}f^{3} + \frac{35acd^{4}e^{3}f^{3}}{2} + \frac{35b^{2}d^{4}e^{3}f^{3}}{4} + 42bcd^{6}e^{3}f^{3} + \frac{165c^{2}d^{8}e^{3}f^{3}}{4}\right) + x^{3}\left(a^{2}de^{2}f^{3} + \frac{20abd^{3}e^{2}f^{3}}{3} + 14acd^{5}e^{2}f^{3} + 7b^{2}d^{5}e^{2}f^{3} + 24bcd^{7}e^{2}f^{3} + \frac{55c^{2}d^{9}e^{2}f^{3}}{3}\right) + x^{2}\left(\frac{3a^{2}d^{2}ef^{3}}{2} + 5abd^{4}ef^{3} + 7acd^{6}ef^{3} + \frac{7b^{2}d^{6}ef^{3}}{2} + 9bcd^{8}ef^{3} + \frac{11c^{2}d^{10}ef^{3}}{2}\right) + x\left(a^{2}d^{3}f^{3} + 2abd^{5}f^{3} + 2acd^{7}f^{3} + b^{2}d^{7}f^{3} + 2bcd^{9}f^{3} + c^{2}d^{11}f^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e
**9*f**3/5 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3
+ 55*c**2*d**3*e**8*f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f
**3/8 + 9*b*c*d**2*e**7*f**3 + 165*c**2*d**4*e**7*f**3/4) + x**7*
(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**3 + 24*b*c*d**3*e**6*f**3 + 6
6*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/3 + 7*a*c*d**2*e**5*
f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3 + 77*c**2*
d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f**3
 + 7*b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**
7*e**4*f**3) + x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35
*a*c*d**4*e**3*f**3/2 + 35*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e*
*3*f**3 + 165*c**2*d**8*e**3*f**3/4) + x**3*(a**2*d*e**2*f**3 + 2
0*a*b*d**3*e**2*f**3/3 + 14*a*c*d**5*e**2*f**3 + 7*b**2*d**5*e**2
*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**9*e**2*f**3/3) + x**2*
(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d**6*e*f**3 + 7
*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**3/2)
```

```
 + x*(a**2*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d
**7*f**3 + 2*b*c*d**9*f**3 + c**2*d**11*f**3)
```

GIAC/XCAS [A] time = 0.269794, size = 925, normalized size = 8.89

$$\frac{1}{12}c^2f^3x^{12}e^{11} + c^2df^3x^{11}e^{10} + \frac{11}{2}c^2d^2f^3x^{10}e^9 + \frac{55}{3}c^2d^3f^3x^9e^8 + \frac{165}{4}c^2d^4f^3x^8e^7 + 66\,c^2d^5f^3x^7e^6 + 77\,c^2d^6f^3x^6e^5 + 66\,c^2d^7f^3x^5e^4 + \frac{165}{4}c^2d^8f^3x^4e^3 + \frac{55}{3}c^2d^9f^3x^3e^2 + \frac{11}{2}c^2d^{10}f^3x^2e + c^2d^{11}f^3x + \frac{1}{5}bcf^3x^{10}e^9 + 2\,bcdf^3x^9e^8 + 9\,bcd^2f^3x^8e^7 + 24\,bcd^3f^3x^7e^6 + 42\,bcd^4f^3x^6e^5 + \frac{252}{5}bcd^5f^3x^5e^4 + 42\,bcd^6f^3x^4e^3 + 24\,bcd^7f^3x^3e^2 + 9\,bcd^8f^3x^2e + 2\,bcd^9f^3x + \frac{1}{8}b^2f^3x^8e^7 + \frac{1}{4}acf^3x^8e^7 + b^2df^3x^7e^6 + 2\,acdf^3x^7e^6 + \frac{7}{2}b^2d^2f^3x^6e^5 + 7\,acd^2f^3x^6e^5 + 7\,b^2d^3f^3x^5e^4 + 14\,acd^3f^3x^5e^4 + \frac{35}{4}b^2d^4f^3x^4e^3 + \frac{35}{2}acd^4f^3x^4e^3 + 7\,b^2d^5f^3x^3e^2 + 14\,acd^5f^3x^3e^2 + \frac{7}{2}b^2d^6f^3x^2e + 7\,acd^6f^3x^2e + b^2d^7f^3x + 2\,acd^7f^3x + \frac{1}{3}abf^3x^6e^5 + 2\,abdf^3x^5e^4 + 5\,abd^2f^3x^4e^3 + \frac{20}{3}abd^3f^3x^3e^2 + 5\,abd^4f^3x^2e + 2\,abd^5f^3x + \frac{1}{4}a^2f^3x^4e^3 + a^2df^3x^3e^2 + \frac{3}{2}a^2d^2f^3x^2e + a^2d^3f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^3,x, algorithm="giac")
```

```
[Out] 1/12*c^2*f^3*x^12*e^11 + c^2*d*f^3*x^11*e^10 + 11/2*c^2*d^2*f^3*x
^10*e^9 + 55/3*c^2*d^3*f^3*x^9*e^8 + 165/4*c^2*d^4*f^3*x^8*e^7 +
66*c^2*d^5*f^3*x^7*e^6 + 77*c^2*d^6*f^3*x^6*e^5 + 66*c^2*d^7*f^3*
x^5*e^4 + 165/4*c^2*d^8*f^3*x^4*e^3 + 55/3*c^2*d^9*f^3*x^3*e^2 +
11/2*c^2*d^10*f^3*x^2*e + c^2*d^11*f^3*x + 1/5*b*c*f^3*x^10*e^9 +
 2*b*c*d*f^3*x^9*e^8 + 9*b*c*d^2*f^3*x^8*e^7 + 24*b*c*d^3*f^3*x^7
*e^6 + 42*b*c*d^4*f^3*x^6*e^5 + 252/5*b*c*d^5*f^3*x^5*e^4 + 42*b*
c*d^6*f^3*x^4*e^3 + 24*b*c*d^7*f^3*x^3*e^2 + 9*b*c*d^8*f^3*x^2*e
+ 2*b*c*d^9*f^3*x + 1/8*b^2*f^3*x^8*e^7 + 1/4*a*c*f^3*x^8*e^7 + b
^2*d*f^3*x^7*e^6 + 2*a*c*d*f^3*x^7*e^6 + 7/2*b^2*d^2*f^3*x^6*e^5
+ 7*a*c*d^2*f^3*x^6*e^5 + 7*b^2*d^3*f^3*x^5*e^4 + 14*a*c*d^3*f^3*
x^5*e^4 + 35/4*b^2*d^4*f^3*x^4*e^3 + 35/2*a*c*d^4*f^3*x^4*e^3 + 7
*b^2*d^5*f^3*x^3*e^2 + 14*a*c*d^5*f^3*x^3*e^2 + 7/2*b^2*d^6*f^3*x
^2*e + 7*a*c*d^6*f^3*x^2*e + b^2*d^7*f^3*x + 2*a*c*d^7*f^3*x + 1/
3*a*b*f^3*x^6*e^5 + 2*a*b*d*f^3*x^5*e^4 + 5*a*b*d^2*f^3*x^4*e^3 +
 20/3*a*b*d^3*f^3*x^3*e^2 + 5*a*b*d^4*f^3*x^2*e + 2*a*b*d^5*f^3*x
 + 1/4*a^2*f^3*x^4*e^3 + a^2*d*f^3*x^3*e^2 + 3/2*a^2*d^2*f^3*x^2*
e + a^2*d^3*f^3*x
```

3.612 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal. Leaf size=159

$$\frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}$$

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*
a*(b^2 + a*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d +
e*x)^10)/(10*e) + (c*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c
^2*f^3*(d + e*x)^14)/(14*e) + (c^3*f^3*(d + e*x)^16)/(16*e)
```

Rubi [A] time = 0.757957, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*
a*(b^2 + a*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d +
e*x)^10)/(10*e) + (c*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c
^2*f^3*(d + e*x)^14)/(14*e) + (c^3*f^3*(d + e*x)^16)/(16*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3f^3\int^{(d+ex)^2} x\,dx}{2e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3af^3(d+ex)^8\left(ac+b^2\right)}{8e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{bf^3(d+ex)^{10}\left(6ac+b^2\right)}{10e}+\frac{c^3f^3(d+ex)^{16}}{16e}+\frac{cf^3(d+ex)^{12}\left(ac+b^2\right)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] a**3*f**3*Integral(x, (x, (d + e*x)**2))/(2*e) + a**2*b*f**3*(d +
 e*x)**6/(2*e) + 3*a*f**3*(d + e*x)**8*(a*c + b**2)/(8*e) + 3*b*c
**2*f**3*(d + e*x)**14/(14*e) + b*f**3*(d + e*x)**10*(6*a*c + b**
2)/(10*e) + c**3*f**3*(d + e*x)**16/(16*e) + c*f**3*(d + e*x)**12
*(a*c + b**2)/(4*e)
```

Mathematica [B] time = 0.639177, size = 801, normalized size = 5.04

$$f^3\left(\frac{1}{16}c^3e^{15}x^{16}+c^3de^{14}x^{15}+\frac{3}{14}c^2\left(35cd^2+b\right)e^{13}x^{14}+c^2d\left(35cd^2+3b\right)e^{12}x^{13}+\frac{1}{4}c\left(455c^2d^4+78bcd^2+b^2+ac\right)e^{11}x^{12}+3cd\left(91c^2d^4+26bcd^2+b^2+ac\right)e^{10}x^{11}+\frac{1}{10}\left(5005c^3d^6+2145bc^2d^4+165ac^2d^2+165b^2cd^2+b^3+6abc\right)e^9x^{10}+d\left(715c^3d^6+429bc^2d^4+55ac^2d^2+55b^2cd^2+b^3+6abc\right)e^8x^9+\frac{3}{8}\left(2145c^3d^8+1716bc^2d^6+330ac^2d^4+330b^2cd^4+12b^3d^2+72abcd^2+ab^2+a^2c\right)e^7x^8+\frac{1}{7}d\left(5005c^3d^8+5148bc^2d^6+1386ac^2d^4+1386b^2cd^4+84b^3d^2+504abcd^2+21ab^2+21a^2c\right)e^6x^7+\frac{1}{2}\left(1001c^3d^{10}+1287bc^2d^8+462ac^2d^6+462b^2cd^6+42b^3d^4+252abcd^4+21ab^2d^2+21a^2cd^2+a^2b\right)e^5x^6+\frac{3}{5}d\left(455c^3d^{10}+715bc^2d^8+330ac^2d^6+330b^2cd^6+42b^3d^4+252abcd^4+35ab^2d^2+35a^2cd^2+5a^2b\right)e^4x^5+\frac{1}{4}\left(455c^3d^{12}+858bc^2d^{10}+495ac^2d^8+495b^2cd^8+84b^3d^6+504abcd^6+105ab^2d^4+105a^2cd^4+30a^2bd^2+a^3\right)e^3x^4+d\left(35c^3d^{12}+78bc^2d^{10}+55ac^2d^8+55b^2cd^8+12b^3d^6+72abcd^6+21ab^2d^4+21a^2cd^4+10a^2bd^2+a^3\right)e^2x^3+\frac{3}{2}d^2\left(cd^4+bd^2+a\right)^2\left(5cd^4+3bd^2+a\right)ex^2+d^3\left(cd^4+bd^2+a\right)^3x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(
a + 3*b*d^2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^
2*d^4 + 21*a^2*c*d^4 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 +
 55*a*c^2*d^8 + 78*b*c^2*d^10 + 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30
*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 84*b^3*d^6 + 504*a*b
*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10 + 455*c^3
*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 +
 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715
*b*c^2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 +
21*a^2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a
*c^2*d^6 + 1287*b*c^2*d^8 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*
b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*d^2 + 1386*b^2*c*d^4 + 13
86*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*x^7)/7 + (3*(a*
```

```
b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 330*a*c
^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a
*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)
*e^8*x^9 + ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145
*b*c^2*d^4 + 5005*c^3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b
*c*d^2 + 91*c^2*d^4)*e^10*x^11 + (c*(b^2 + a*c + 78*b*c*d^2 + 455
*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^2)*e^12*x^13 + (3*c^
2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3*e^15*x^16
)/16)
```

Maple [B] time = 0.002, size = 7697, normalized size = 48.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.75888, size = 1242, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^3,x, algorithm="maxi
```

```
[Out] 1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 +
 b*c^2)*e^13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 +
1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 +
3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*e^10*f^3*x^11 +
 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*c
 + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(
b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 + 3/8*(2145*c
^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c
 + 12*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148
*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 +
21*(a*b^2 + a^2*c)*d)*e^6*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b
 + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3*x^6 + 3/5*(455*c^3*d^11 + 715*
b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*
a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*(455*c^3*d^12
 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*
d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 +
```

```
 (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3
+ 6*a*b*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e
^2*f^3*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)
*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d
^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*c + a
*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c
)*d^7 + a^3*d^3)*f^3*x
```

Fricas [A] time = 0.25855, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^3,x, algorithm="fric
```

```
[Out] 1/16*x^16*f^3*e^15*c^3 + x^15*f^3*e^14*d*c^3 + 15/2*x^14*f^3*e^13
*d^2*c^3 + 35*x^13*f^3*e^12*d^3*c^3 + 455/4*x^12*f^3*e^11*d^4*c^3
 + 3/14*x^14*f^3*e^13*c^2*b + 273*x^11*f^3*e^10*d^5*c^3 + 3*x^13*
f^3*e^12*d*c^2*b + 1001/2*x^10*f^3*e^9*d^6*c^3 + 39/2*x^12*f^3*e^
11*d^2*c^2*b + 715*x^9*f^3*e^8*d^7*c^3 + 78*x^11*f^3*e^10*d^3*c^2
*b + 6435/8*x^8*f^3*e^7*d^8*c^3 + 429/2*x^10*f^3*e^9*d^4*c^2*b +
1/4*x^12*f^3*e^11*c*b^2 + 1/4*x^12*f^3*e^11*c^2*a + 715*x^7*f^3*e
^6*d^9*c^3 + 429*x^9*f^3*e^8*d^5*c^2*b + 3*x^11*f^3*e^10*d*c*b^2
+ 3*x^11*f^3*e^10*d*c^2*a + 1001/2*x^6*f^3*e^5*d^10*c^3 + 1287/2*
x^8*f^3*e^7*d^6*c^2*b + 33/2*x^10*f^3*e^9*d^2*c*b^2 + 33/2*x^10*f
^3*e^9*d^2*c^2*a + 273*x^5*f^3*e^4*d^11*c^3 + 5148/7*x^7*f^3*e^6*
d^7*c^2*b + 55*x^9*f^3*e^8*d^3*c*b^2 + 55*x^9*f^3*e^8*d^3*c^2*a +
 455/4*x^4*f^3*e^3*d^12*c^3 + 1287/2*x^6*f^3*e^5*d^8*c^2*b + 495/
4*x^8*f^3*e^7*d^4*c*b^2 + 1/10*x^10*f^3*e^9*b^3 + 495/4*x^8*f^3*e
^7*d^4*c^2*a + 3/5*x^10*f^3*e^9*c*b*a + 35*x^3*f^3*e^2*d^13*c^3 +
 429*x^5*f^3*e^4*d^9*c^2*b + 198*x^7*f^3*e^6*d^5*c*b^2 + x^9*f^3*
e^8*d*b^3 + 198*x^7*f^3*e^6*d^5*c^2*a + 6*x^9*f^3*e^8*d*c*b*a + 1
5/2*x^2*f^3*e*d^14*c^3 + 429/2*x^4*f^3*e^3*d^10*c^2*b + 231*x^6*f
^3*e^5*d^6*c*b^2 + 9/2*x^8*f^3*e^7*d^2*b^3 + 231*x^6*f^3*e^5*d^6*
c^2*a + 27*x^8*f^3*e^7*d^2*c*b*a + x*f^3*d^15*c^3 + 78*x^3*f^3*e^
2*d^11*c^2*b + 198*x^5*f^3*e^4*d^7*c*b^2 + 12*x^7*f^3*e^6*d^3*b^3
 + 198*x^5*f^3*e^4*d^7*c^2*a + 72*x^7*f^3*e^6*d^3*c*b*a + 39/2*x^
2*f^3*e*d^12*c^2*b + 495/4*x^4*f^3*e^3*d^8*c*b^2 + 21*x^6*f^3*e^5
*d^4*b^3 + 495/4*x^4*f^3*e^3*d^8*c^2*a + 126*x^6*f^3*e^5*d^4*c*b*
a + 3/8*x^8*f^3*e^7*b^2*a + 3/8*x^8*f^3*e^7*c*a^2 + 3*x*f^3*d^13*
c^2*b + 55*x^3*f^3*e^2*d^9*c*b^2 + 126/5*x^5*f^3*e^4*d^5*b^3 + 55
*x^3*f^3*e^2*d^9*c^2*a + 756/5*x^5*f^3*e^4*d^5*c*b*a + 3*x^7*f^3*
e^6*d*b^2*a + 3*x^7*f^3*e^6*d*c*a^2 + 33/2*x^2*f^3*e*d^10*c*b^2 +
 21*x^4*f^3*e^3*d^6*b^3 + 33/2*x^2*f^3*e*d^10*c^2*a + 126*x^4*f^3
*e^3*d^6*c*b*a + 21/2*x^6*f^3*e^5*d^2*b^2*a + 21/2*x^6*f^3*e^5*d^
2*c*a^2 + 3*x*f^3*d^11*c*b^2 + 12*x^3*f^3*e^2*d^7*b^3 + 3*x*f^3*d
^11*c^2*a + 72*x^3*f^3*e^2*d^7*c*b*a + 21*x^5*f^3*e^4*d^3*b^2*a +
 21*x^5*f^3*e^4*d^3*c*a^2 + 9/2*x^2*f^3*e*d^8*b^3 + 27*x^2*f^3*e*
d^8*c*b*a + 105/4*x^4*f^3*e^3*d^4*b^2*a + 105/4*x^4*f^3*e^3*d^4*c
```

```
*a^2 + 1/2*x^6*f^3*e^5*b*a^2 + x*f^3*d^9*b^3 + 6*x*f^3*d^9*c*b*a
+ 21*x^3*f^3*e^2*d^5*b^2*a + 21*x^3*f^3*e^2*d^5*c*a^2 + 3*x^5*f^3
*e^4*d*b*a^2 + 21/2*x^2*f^3*e*d^6*b^2*a + 21/2*x^2*f^3*e*d^6*c*a^
2 + 15/2*x^4*f^3*e^3*d^2*b*a^2 + 3*x*f^3*d^7*b^2*a + 3*x*f^3*d^7*
c*a^2 + 10*x^3*f^3*e^2*d^3*b*a^2 + 15/2*x^2*f^3*e*d^4*b*a^2 + 1/4
*x^4*f^3*e^3*a^3 + 3*x*f^3*d^5*b*a^2 + x^3*f^3*e^2*d*a^3 + 3/2*x^
2*f^3*e*d^2*a^3 + x*f^3*d^3*a^3
```

Sympy [A] time = 1.05511, size = 1654, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c
**2*e**13*f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*
d*e**12*f**3 + 35*c**3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**
3/4 + b**2*c*e**11*f**3/4 + 39*b*c**2*d**2*e**11*f**3/2 + 455*c**
3*d**4*e**11*f**3/4) + x**11*(3*a*c**2*d*e**10*f**3 + 3*b**2*c*d*
e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*c**3*d**5*e**10*f**3
) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f**3/2 + b**
3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e**
9*f**3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**
3 + 55*a*c**2*d**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*
e**8*f**3 + 429*b*c**2*d**5*e**8*f**3 + 715*c**3*d**7*e**8*f**3)
+ x**8*(3*a**2*c*e**7*f**3/8 + 3*a*b**2*e**7*f**3/8 + 27*a*b*c*d*
*2*e**7*f**3 + 495*a*c**2*d**4*e**7*f**3/4 + 9*b**3*d**2*e**7*f**
3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*c**2*d**6*e**7*f**3/2
+ 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**6*f**3 + 3*a*
b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*e**6
*f**3 + 12*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148
*b*c**2*d**7*e**6*f**3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*
b*e**5*f**3/2 + 21*a**2*c*d**2*e**5*f**3/2 + 21*a*b**2*d**2*e**5*
f**3/2 + 126*a*b*c*d**4*e**5*f**3 + 231*a*c**2*d**6*e**5*f**3 + 2
1*b**3*d**4*e**5*f**3 + 231*b**2*c*d**6*e**5*f**3 + 1287*b*c**2*d
**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f**3/2) + x**5*(3*a**2*b*d
*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b**2*d**3*e**4*f**3
+ 756*a*b*c*d**5*e**4*f**3/5 + 198*a*c**2*d**7*e**4*f**3 + 126*b*
*3*d**5*e**4*f**3/5 + 198*b**2*c*d**7*e**4*f**3 + 429*b*c**2*d**9
*e**4*f**3 + 273*c**3*d**11*e**4*f**3) + x**4*(a**3*e**3*f**3/4 +
 15*a**2*b*d**2*e**3*f**3/2 + 105*a**2*c*d**4*e**3*f**3/4 + 105*a
*b**2*d**4*e**3*f**3/4 + 126*a*b*c*d**6*e**3*f**3 + 495*a*c**2*d*
*8*e**3*f**3/4 + 21*b**3*d**6*e**3*f**3 + 495*b**2*c*d**8*e**3*f*
*3/4 + 429*b*c**2*d**10*e**3*f**3/2 + 455*c**3*d**12*e**3*f**3/4)
 + x**3*(a**3*d*e**2*f**3 + 10*a**2*b*d**3*e**2*f**3 + 21*a**2*c*
d**5*e**2*f**3 + 21*a*b**2*d**5*e**2*f**3 + 72*a*b*c*d**7*e**2*f*
*3 + 55*a*c**2*d**9*e**2*f**3 + 12*b**3*d**7*e**2*f**3 + 55*b**2*
```

```
c*d**9*e**2*f**3 + 78*b*c**2*d**11*e**2*f**3 + 35*c**3*d**13*e**2
*f**3) + x**2*(3*a**3*d**2*e*f**3/2 + 15*a**2*b*d**4*e*f**3/2 + 2
1*a**2*c*d**6*e*f**3/2 + 21*a*b**2*d**6*e*f**3/2 + 27*a*b*c*d**8*
e*f**3 + 33*a*c**2*d**10*e*f**3/2 + 9*b**3*d**8*e*f**3/2 + 33*b**
2*c*d**10*e*f**3/2 + 39*b*c**2*d**12*e*f**3/2 + 15*c**3*d**14*e*f
**3/2) + x*(a**3*d**3*f**3 + 3*a**2*b*d**5*f**3 + 3*a**2*c*d**7*f
**3 + 3*a*b**2*d**7*f**3 + 6*a*b*c*d**9*f**3 + 3*a*c**2*d**11*f**
3 + b**3*d**9*f**3 + 3*b**2*c*d**11*f**3 + 3*b*c**2*d**13*f**3 +
c**3*d**15*f**3)
```

GIAC/XCAS [A] time = 0.27078, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^3,x, algorithm="giac
```

```
[Out] Done
```

3.613 $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.874143, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 85.4182, size = 204, normalized size = 1.06

$$\frac{d+ex}{ce}-\frac{\sqrt{2}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}e\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}e\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (d + e*x)/(c*e) - sqrt(2)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))
*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2
*c**(3/2)*e*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) +
sqrt(2)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt
(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(3/2)*e*sqrt(b
 - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.256574, size = 219, normalized size = 1.13

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+2\sqrt{c}(d+ex)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b
^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/(2*c^(3/2)*e)
```

Maple [C] time = 0.095, size = 158, normalized size = 0.8

$$\frac{x}{c}+\frac{1}{2\,ce}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(-\_R^2be^2-2\,\_R\,bde-bd^2-a\right)\ln\left(x-\_R\right)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3+be\_R+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*
_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(
c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b
*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c} - \frac{\int \frac{be^2x^2+2\,bdex+bd^2+a}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)/(c*e^4*x^4 +
4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^
3 + b*d)*e*x + a), x)/c
```

Fricas [A] time = 0.297898, size = 1662, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2
*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2
*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) - (b^4 - 5*a*b^2*c + 4*a^
2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c^3 -
4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3
*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2
*(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)
/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(((b^2*c^3 - 4*a*c
^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4
)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^
2*c)*e*x - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)
*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4))
+ (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(((b^2*c^3 - 4*a*c^4)*e^2*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) - b^3
 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt(((b^2*
c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4
*a*c^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*
```

```
(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 -
 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*
c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(((b^2*c^3 - 4*
a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*
e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + 2*x)/c
```

Sympy [A] time = 9.92905, size = 178, normalized size = 0.92

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5e^4 - 128ab^2c^4e^4 + 16b^4c^3e^4\right) + t^2\left(48a^2bc^2e^2 - 28ab^3ce^2 + 4b^5e^2\right) + a^3, \left(t \mapsto t\log\left(x + \frac{32t^3abc^4e^3 -\right.\right.\right.$$

$$+ \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**
4*c**3*e**4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*
b**5*e**2) + a**3, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3
- 8*_t**3*b**3*c**3*e**3 - 4*_t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2
*_t*b**4*e + a**2*c*d - a*b**2*d)/(a**2*c*e - a*b**2*e)))) + x/c
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^4}{(ex+d)^4c + (ex+d)^2b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.614 $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=81

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^
2 - 4*a*c]*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rubi [A] time = 0.26502, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^
2 - 4*a*c]*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rubi in Sympy [A] time = 40.1254, size = 68, normalized size = 0.84

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2ce\sqrt{-4ac+b^2}}+\frac{\log\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}{4ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] b*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*c*e*sqrt(-
4*a*c + b**2)) + log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(4*c*e)
```

Mathematica [A] time = 0.0693669, size = 77, normalized size = 0.95

$$\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^
2 + 4*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Maple [C] time = 0.007, size = 151, normalized size = 1.9

$$\frac{1}{2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^3e^3+3\_R^2de^2+3\_Rd^2e+d^3\right)\ln\left(x-\_R\right)}{2ce^3\_R^3+6cde^2\_R^2+6d^2ec\_R+2cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*
_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(
c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b
*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A] time = 0.273404, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(\frac{2\left(b^2c-4ac^2\right)e^2x^2+4\left(b^2c-4ac^2\right)dex+b^3-4abc+2\left(b^2c-4ac^2\right)d^2+\left(2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-\cdots}{ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a}\right)\cdots}{4\sqrt{b^2-4ac}ce}\right.$$

$$-\frac{2b\arctan\left(-\frac{(2ce^2x^2+4cdex+2cd^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\sqrt{-b^2+4ac}\log\left(ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3\cdots\right.\right.}{4\sqrt{-b^2+4ac}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*
e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 +
8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c
*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + sqrt(b^2 - 4*a*c)*log(c*e^4*x
^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2
*c*d^3 + b*d)*e*x + a))/(sqrt(b^2 - 4*a*c)*c*e), -1/4*(2*b*arctan
(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c)) - sqrt(-b^2 + 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c
*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)
)/(sqrt(-b^2 + 4*a*c)*c*e)]
```

Sympy [A] time = 6.71315, size = 280, normalized size = 3.46

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2
*d*x/e + x**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c -
 b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*
c*e*(4*a*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2)) + (b*sqrt(-4
*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e + x*
*2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/
(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c -
b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.615 $\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal. Leaf size=164

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]
*e)) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4
*a*c]*e)
```

Rubi [A] time = 0.390917, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]
*e)) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4
*a*c]*e)
```

Rubi in Sympy [A] time = 50.9583, size = 151, normalized size = 0.92

$$-\frac{\sqrt{2}\sqrt{b-\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}e\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}e\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(2)*sqrt(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d +
e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*e*sqrt(-4*a*c + b*
*2)) + sqrt(2)*sqrt(b + sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)
*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*e*sqrt(-4*a*
c + b**2))
```

Mathematica [A] time = 0.163467, size = 175, normalized size = 1.07

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt
[b - Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + S
qrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.006, size = 140, normalized size = 0.9

$$\frac{1}{2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2ce^3\_R^3+6cde^2\_R^2+6d^2ec\_R+2cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*
_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+
b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A] time = 0.274828, size = 949, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a
*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c -
4*a*c^2)*e^3*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*
c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c
^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2
*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*
log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-((b^2*c - 4*a*c^2)*e^2
*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*
sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt
(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((
b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt((
(b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^
2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d)
 + 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4
*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c
 - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*
a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a
*c^3)*e^4)) + e*x + d)
```

Sympy [A] time = 5.85086, size = 104, normalized size = 0.63

$$\text{RootSum}\left(t^4\left(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4\right) + t^2\left(-16abce^2 + 4b^3e^2\right) + a, \left(t \mapsto t\log\left(x + \frac{64t^3ac^2e^3 - 16t^3b^2ce^3 - 2t}{e}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**
4*c*e**4) + _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t,
```

```
_t*log(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*
e + d)/e)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.616 $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*
a*c]*e))
```

Rubi [A] time = 0.123995, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*
a*c]*e))
```

Rubi in Sympy [A] time = 27.0955, size = 39, normalized size = 0.91

$$-\frac{\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*sqrt(-4*a*c
 + b**2))
```

Mathematica [A] time = 0.0234109, size = 46, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a
*c]*e)
```

Maple [C]   time = 0.008, size = 129, normalized size = 3.

$$\frac{1}{2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{(\_R\,e+d)\ln(x-\_R)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*
d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int\frac{ex+d}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A]   time = 0.277257, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{2\left(b^2c-4\,ac^2\right)e^2x^2+4\left(b^2c-4\,ac^2\right)dex+b^3-4\,abc+2\left(b^2c-4\,ac^2\right)d^2-\left(2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2\right.}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}}{2\sqrt{b^2-4\,ace}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*
x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - (2*c^2*e^4*x^4 + 8*
c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d
^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^
2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqrt(b^2 - 4*a*c)*e), arctan(-(2
*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c))/(sqrt(-b^2 + 4*a*c)*e)]
```

Sympy [A] time = 3.57723, size = 168, normalized size = 3.91

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*
a*c - b**2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*
e**2))/(2*e) + sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*
c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*
c*d**2)/(2*c*e**2))/(2*e)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.617 $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=94

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^
2 - 4*a*c]*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d
 + e*x)^4]/(4*a*e)
```

Rubi [A] time = 0.260266, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^
2 - 4*a*c]*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d
 + e*x)^4]/(4*a*e)
```

Rubi in Sympy [A] time = 47.5775, size = 82, normalized size = 0.87

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2ae\sqrt{-4ac+b^2}}+\frac{\log\left((d+ex)^2\right)}{2ae}-\frac{\log\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}{4ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] b*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*a*e*sqrt(-
4*a*c + b**2)) + log((d + e*x)**2)/(2*a*e) - log(a + b*(d + e*x)*
*2 + c*(d + e*x)**4)/(4*a*e)
```

Mathematica [A] time = 0.136992, size = 128, normalized size = 1.36

$$\frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2c(\right.}{4ae\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b
 - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*
a*c]*e)
```

Maple [C] time = 0.013, size = 184, normalized size = 2.

$$\frac{1}{2\,ae}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-ce^3\_R^3-3\,cde^2\_R^2+e\left(-3\,cd^2-b\right)\_R-cd^3-b\right.}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3+be\_}+\frac{\ln\left(ex+d\right)}{ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a/e*sum((-c*e^3*_R^3-3*c*d*e^2*_R^2+e*(-3*c*d^2-b)*_R-c*d^3-b
*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)
*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+ln(e*x+d)/a/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{ce^3x^3+3\,cde^2x^2+cd^3+\left(3\,cd^2+b\right)ex+bd}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\,dx}{a}+\frac{\log\left(ex+d\right)}{ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] -integrate((c*e^3*x^3 + 3*c*d*e^2*x^2 + c*d^3 + (3*c*d^2 + b)*e*x
 + b*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^
```

```
2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/a + log(e*x + d)/(a*e)
```

Fricas [A] time = 0.277624, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(\frac{2\left(b^{2}c-4\,ac^{2}\right)e^{2}x^{2}+4\left(b^{2}c-4\,ac^{2}\right)dex+b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)d^{2}+\left(2\,c^{2}e^{4}x^{4}+8\,c^{2}de^{3}x^{3}+2\,c^{2}d^{4}+2\left(6\,c^{2}d^{2}+bc\right)e^{2}x^{2}+2\,bcd^{2}+4\left(2\,c^{2}d^{3}+bcd\right)ex+b^{2}-}{ce^{4}x^{4}+4\,cde^{3}x^{3}+cd^{4}+(6\,cd^{2}+b)e^{2}x^{2}+bd^{2}+2\,(2\,cd^{3}+bd)ex+a}\right.}{4\sqrt{b^{2}-}}\right.$$

$$-\frac{2\,b\arctan\left(-\frac{\left(2\,ce^{2}x^{2}+4\,cdex+2\,cd^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)+\sqrt{-b^{2}+4\,ac}\left(\log\left(ce^{4}x^{4}+4\,cde^{3}x^{3}+cd^{4}+\left(6\,cd^{2}+b\right)e^{2}x^{2}+bd^{2}+2\left(2\,cd\right.\right.\right.}{4\sqrt{-b^{2}+4\,ac}ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*
e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 +
8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c
*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - sqrt(b^2 - 4*a*c)*(log(c*e^4*
x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a) - 4*log(e*x + d)))/(sqrt(b^2 - 4*a*c)*a*e
), -1/4*(2*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*e^4*x^4
 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a) - 4*log(e*x + d)))/(sqrt(-b^2 + 4*a*c)*a*e)]
```

Sympy [A] time = 16.2277, size = 320, normalized size = 3.4

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\frac{\log\left(\frac{d}{e}+x\right)}{ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*log(2
*d*x/e + x**2 + (-8*a**2*c*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*
c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*a
*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e
**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)
)*log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*sqrt(-4*a*c + b**2)/(4*a*e
*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*sqrt(-4*a*c + b**2)
/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(
b*c*e**2)) + log(d/e + x)/(a*e)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)), x)
```

3.618 $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.616366, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 73.6325, size = 192, normalized size = 0.98

$$\frac{\sqrt{2}\sqrt{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2ae\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2ae\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{1}{ae\,(d+ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d
 + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*e*sqrt(b + sqrt(-4*a*
c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*sqrt(c)*(b + sqrt(-4*a*
c + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c +
b**2)))/(2*a*e*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
 - 1/(a*e*(d + e*x))
```

Mathematica [A] time = 0.638187, size = 206, normalized size = 1.06

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2ae}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b +
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
))/(2*a*e)
```

Maple [C] time = 0.012, size = 168, normalized size = 0.9

$$-\frac{1}{ae\,(ex+d)}+\frac{1}{2\,ae}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(-\_R^2ce^2-2\,\_R\,cde-cd^2-b\right)\ln(x-\_R)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] -1/a/e/(e*x+d)+1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R
^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R)
,_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*
c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ce^2x^2+2\,cdex+cd^2+b}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\,dx}{a}-\frac{1}{ae^2x+ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^2),x, algorithm="maxima
```

```
[Out] -integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2 + b)/(c*e^4*x^4 + 4*c*d
*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b
*d)*e*x + a), x)/a - 1/(a*e^2*x + a*d*e)
```

Fricas [A] time = 0.28116, size = 1808, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^2),x, algorithm="fricas
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3
- 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*
x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8
*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*
c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-((a^3*b^2 - 4
*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)
*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(
a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3
*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 -
 a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - (b^5
- 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3
*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2*x + a*d*e)
*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
(a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e
^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(
1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 - 5*a*b^3*c + 4*a
^2*b*c^2)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2
- 4*a^4*c)*e^2))) + sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^3*b^2 -
4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c
)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c
```

```
^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6
*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^
6*b^2 - 4*a^7*c)*e^4)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(
((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2)))
- 2)/(a*e^2*x + a*d*e)
```

Sympy [A] time = 13.5289, size = 211, normalized size = 1.08

$$\text{RootSum}\left(t^4\left(256a^5c^2e^4-128a^4b^2ce^4+16a^3b^4e^4\right)+t^2\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2e^3\right.\right.\right.$$

$$-\frac{1}{ade+ae^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**
3*b**4*e**4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*
b**5*e**2) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**
3 + 48*_t**3*a**4*b**2*c*e**3 - 8*_t**3*a**3*b**4*e**3 - 10*_t*a*
*2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_t*b**5*e + a*c**3*d - b**2*c*
*2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e + a*e**2*x)
```

GIAC/XCAS [A] time = 1.00024, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] Done
```

3.619 $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=121

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

```
[Out] -1/(2*a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x
)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d +
 e*x])/(a^2*e) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^
2*e)
```

Rubi [A] time = 0.401787, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x
)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d +
 e*x])/(a^2*e) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^
2*e)
```

Rubi in Sympy [A] time = 64.7209, size = 110, normalized size = 0.91

$$-\frac{1}{2ae\left(d+ex\right)^2}-\frac{b\log\left((d+ex)^2\right)}{2a^2e}+\frac{b\log\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}{4a^2e}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^2e\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -1/(2*a*e*(d + e*x)**2) - b*log((d + e*x)**2)/(2*a**2*e) + b*log(
a + b*(d + e*x)**2 + c*(d + e*x)**4)/(4*a**2*e) - (-2*a*c + b**2)
*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*a**2*e*sqrt
```

```
(-4*a*c + b**2))
```

Mathematica [A] time = 0.249792, size = 154, normalized size = 1.27

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{(d+ex)^2}-4b\log(d+ex)}{4a^2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b
^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b
^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[
b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)
```

Maple [C] time = 0.018, size = 213, normalized size = 1.8

$$\frac{1}{2\,a^2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^3bce^3+3\,\_R^2bcde^2+e\left(3\,bcd^2-ac+b^2\right)\_R+b\right.}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,c}$$
$$-\frac{1}{2\,ae\,(ex+d)^2}-\frac{b\ln(ex+d)}{a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a^2/e*sum((_R^3*b*c*e^3+3*_R^2*b*c*d*e^2+e*(3*b*c*d^2-a*c+b^2
)*_R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2
*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)
)-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\,(ae^3x^2+2\,ade^2x+ad^2e)}+\frac{\int\frac{bce^3x^3+3\,bcde^2x^2+bcd^3+\left(3\,bcd^2+b^2-ac\right)ex+\left(b^2-ac\right)d}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\,dx}{a^2}-\frac{b\log(ex+d)}{a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^3),x, algorithm="maxima
```

```
[Out] -1/2/(a*e^3*x^2 + 2*a*d*e^2*x + a*d^2*e) + integrate((b*c*e^3*x^3
 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - a*c)*e*x + (b^2
 - a*c)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2
*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/a^2 - b*log(e*x + d
)/(a^2*e)
```

Fricas [A] time = 0.30177, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(b^2-2\,ac\right)e^2x^2+2\left(b^2-2\,ac\right)dex+\left(b^2-2\,ac\right)d^2\right)\log\left(\frac{2\left(b^2c-4\,ac^2\right)e^2x^2+4\left(b^2c-4\,ac^2\right)dex+b^3-4\,abc+2\left(b^2c-4\,ac^2\right)d^2+\left(2\,c^2e^4x^4+\right.}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^3),x, algorithm="fricas
```

```
[Out] [-1/4*(((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*
a*c)*d^2)*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*
d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4
+ 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b
*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*
c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - sqrt(b^2 - 4*a*c)*((b*e^2*x
^2 + 2*b*d*e*x + b*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (
6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - 4*(b*
e^2*x^2 + 2*b*d*e*x + b*d^2)*log(e*x + d) - 2*a))/((a^2*e^3*x^2 +
 2*a^2*d*e^2*x + a^2*d^2*e)*sqrt(b^2 - 4*a*c)), 1/4*(2*((b^2 - 2*
a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*arctan(
-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)) + sqrt(-b^2 + 4*a*c)*((b*e^2*x^2 + 2*b*d*e*x + b*d^2)*l
og(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - 4*(b*e^2*x^2 + 2*b*d*e*x + b*d
^2)*log(e*x + d) - 2*a))/((a^2*e^3*x^2 + 2*a^2*d*e^2*x + a^2*d^2*
e)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 46.3686, size = 464, normalized size = 3.83

$$\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2e\,(4ac-b^2)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3ce\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)}\right) + 2a^2b^2e\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)}\right) + 3abc + \cdots}{2ac^2e^2 - b^2ce^2}\right.$$

$$+\left(\frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2e\,(4ac-b^2)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3ce\left(\frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)}\right) + 2a^2b^2e\left(\frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)}\right) + 3abc + 2\cdots}{2ac^2e^2 - b^2ce^2}\right.$$

$$-\frac{1}{2ad^2e + 4ade^2x + 2ae^3x^2} - \frac{b\log\left(\frac{d}{e} + x\right)}{a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a
*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) - sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 2*a
**2*b**2*e*(b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
a**2*e*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c
*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2*e) + sqrt(-4*a
*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))*log(2*d*x/e
+ x**2 + (-8*a**3*c*e*(b/(4*a**2*e) + sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*a**2*e*(4*a*c - b**2))) + 2*a**2*b**2*e*(b/(4*a**2*e)
+ sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) +
 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b
**2*c*e**2)) - 1/(2*a*d**2*e + 4*a*d*e**2*x + 2*a*e**3*x**2) - b*
log(d/e + x)/(a**2*e)
```

GIAC/XCAS [A] time = 0.296676, size = 138, normalized size = 1.14

$$\frac{be^{(-1)}\ln\left(c + \frac{b}{(xe+d)^2} + \frac{a}{(xe+d)^4}\right)}{4\,a^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\,\sqrt{-b^2+4\,ac}a^2} - \frac{e^{(-1)}}{2\,(xe+d)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^3),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*ln(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^2 + 1/2*(b^2
 - 2*a*c)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1
)/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*e^(-1)/((x*e + d)^2*a)
```

3.620 $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

```
[Out] -1/(3*a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.04058, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```


```
[Out] -1/(3*a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 122.508, size = 228, normalized size = 1.02

$$-\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}-\frac{\sqrt{2}\sqrt{c}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a^2e\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a^2e\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -1/(3*a*e*(d + e*x)**3) + b/(a**2*e*(d + e*x)) - sqrt(2)*sqrt(c)*
(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d +
 e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a**2*e*sqrt(b + sqrt(-4*a
*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*sqrt(c)*(-2*a*c + b**2
 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b -
 sqrt(-4*a*c + b**2)))/(2*a**2*e*sqrt(b - sqrt(-4*a*c + b**2))*sq
rt(-4*a*c + b**2))
```

Mathematica [A] time = 0.377827, size = 235, normalized size = 1.05

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 -
 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2*e)
```

Maple [C] time = 0.015, size = 188, normalized size = 0.8

$$\frac{1}{2a^2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^2bce^2+2\_Rbcde+bcd^2-ac+b^2\right)\ln\left(x-\_R\right)}{2ce^3\_R^3+6cde^2\_R^2+6d^2ec\_R+2cd^3+be\_R+bc}$$
$$-\frac{1}{3ae\left(ex+d\right)^3}+\frac{b}{a^2e\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3
*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_
R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*
d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+
d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,be^2x^2+6\,bdex+3\,bd^2-a}{3\,\left(a^2e^4x^3+3\,a^2de^3x^2+3\,a^2d^2e^2x+a^2d^3e\right)}+\frac{\int\frac{bce^2x^2+2\,bcdex+bcd^2+b^2-ac}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^4),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*e^2*x^2 + 6*b*d*e*x + 3*b*d^2 - a)/(a^2*e^4*x^3 + 3*a^2*
d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e) + integrate((b*c*e^2*x^2
 + 2*b*c*d*e*x + b*c*d^2 + b^2 - a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3
+ c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x +
 a), x)/a^2
```

Fricas [A] time = 0.295854, size = 2759, normalized size = 12.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^4),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*x^
3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5
```

```
*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*
a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^
2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sq
rt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 -
 4*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2
*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*
b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a
^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2
*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a
^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 -
4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b
^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b
^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)) - (b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*
b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*
(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt(
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^
10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))*log(2*(b^4*c
^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*
c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
(a^10*b^2 - 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*
c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^
5*b^2 - 4*a^6*c)*e^2))) + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*d*e^3*
x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/(
(a^5*b^2 - 4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)
*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^
5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4))
 + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4
)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e
^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 2
*a)/(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)
```

Sympy [A] time = 47.0414, size = 347, normalized size = 1.55

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3e+9a^2d^2e^2x+9a^2de^3x^2+3a^2e^4x^3} + \text{RootSum}\left(t^4\left(256a^7c^2e^4-128a^6b^2ce^4+16a^5b^4e^4\right)+t^2\left(-80a^3bc^3e^2+100a^2b^3c^2e^2-36ab^5ce^2+4b^7e^2\right)+c^5, \left(t\mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a*
*2*d**2*e**2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + RootSum
(_t**4*(256*a**7*c**2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*
e**4) + _t**2*(-80*a**3*b*c**3*e**2 + 100*a**2*b**3*c**2*e**2 - 3
6*a*b**5*c*e**2 + 4*b**7*e**2) + c**5, Lambda(_t, _t*log(x + (-96
*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*c*e**3 - 8*_t**3*a**
5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*e - 40*_t*a
**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d -
3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b
**4*c**3*e))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*x + d)^4), x)
```

3.621 $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=270

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
+ ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.1085, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```


```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
+ ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 81.9446, size = 243, normalized size = 0.9

$$\frac{\left(2a+b\left(d+ex\right)^{2}\right)\left(d+ex\right)}{2e\left(-4ac+b^{2}\right)\left(a+b\left(d+ex\right)^{2}+c\left(d+ex\right)^{4}\right)}+\frac{\sqrt{2}\left(4ac+b^{2}+b\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^{2}}}}\right)}{4\sqrt{c}e\sqrt{b+\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(4ac+b^{2}-b\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^{2}}}}\right)}{4\sqrt{c}e\sqrt{b-\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (2*a + b*(d + e*x)**2)*(d + e*x)/(2*e*(-4*a*c + b**2)*(a + b*(d +
 e*x)**2 + c*(d + e*x)**4)) + sqrt(2)*(4*a*c + b**2 + b*sqrt(-4*a
*c + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c +
 b**2)))/(4*sqrt(c)*e*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**
2)**(3/2)) - sqrt(2)*(4*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(
sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(4*sqrt(
c)*e*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.887542, size = 263, normalized size = 0.97

$$\frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^{3}\right)}{\left(b^{2}-4ac\right)\left(a+b(d+ex)^{2}+c(d+ex)^{4}\right)}+\frac{\sqrt{2}\left(b\sqrt{b^{2}-4ac}-4ac-b^{2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^{2}-4ac}}}\right)}{\sqrt{c}\left(b^{2}-4ac\right)^{3/2}\sqrt{b-\sqrt{b^{2}-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^{2}-4ac}+4ac+b^{2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^{2}-4ac}+b}}\right)}{\sqrt{c}\left(b^{2}-4ac\right)^{3/2}\sqrt{\sqrt{b^{2}-4ac}+b}}}{4e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)
```

Maple [C] time = 0.058, size = 323, normalized size = 1.2

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(-\frac{be^2x^3}{8\,ac-2\,b^2}-\frac{3\,x^2bde}{8\,ac-2\,b^2}-\frac{(3\,bd^2+2\,a)\,x}{8\,ac-2\,b^2}-\frac{d}{2}\right.$$
$$+\frac{1}{4\,e}\sum_{\_R=RootOf(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a)}\frac{(-\_R^2be^2-2\,\_R\,bde-bd^2+2\,a)\ln(x-\_}{(4\,ac-b^2)(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2*d*b*e/(4*a*c-b^2)*x^2-1/2*(3*b*d^
2+2*a)/(4*a*c-b^2)*x-1/2/e*d*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)+1/4/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(4*a*c-b
^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)
*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{be^3x^3+3\,bde^2x^2+bd^3+(3\,bd^2+2\,a)\,ex+2\,ad}{2\,((b^2c-4\,ac^2)e^5x^4+4\,(b^2c-4\,ac^2)de^4x^3+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^3x^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)e^2x+}$$
$$-\frac{1}{2}\int-\frac{be^2x^2+2\,bdex+bd^2-2\,a}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*e^3*x^3 + 3*b*d*e^2*x^2 + b*d^3 + (3*b*d^2 + 2*a)*e*x + 2*
a*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
 (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
- 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^
4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e) - 1/2*integrate(-(b
*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*c^2)*e^4*x^4 +
4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)
*e*x), x)
```

Fricas [A] time = 0.328069, size = 3313, normalized size = 12.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^3*x^3 + 6*b*d*e^2*x^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*e*
x + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*
e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(
2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*
a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^
6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 +
 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2))*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1
/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) +
(b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*c -
12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((
b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c
^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b
^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c
^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6
*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2
 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)
*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^
2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 -
 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*d^2)*e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*x + (
3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)
*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*s
qrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4
)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64
*a^3*c^4)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*
c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2
```

```
)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x
 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2
)*e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e
^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
- 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d -
 sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c - 12
*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*
c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) +
 4*a*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^
3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2
*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)
*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.622 $\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=97

$$\frac{2a+b(d+ex)^2}{2e\,(b^2-4ac)\,(a+b(d+ex)^2+c(d+ex)^4)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\,(b^2-4ac)^{3/2}}$$

```
[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]
])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi [A] time = 0.269304, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2a+b(d+ex)^2}{2e\,(b^2-4ac)\,(a+b(d+ex)^2+c(d+ex)^4)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\,(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]
])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi in Sympy [A] time = 29.0064, size = 82, normalized size = 0.85

$$-\frac{b\,\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{2a+b\left(d+ex\right)^2}{2e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -b*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(-4*a*c +
 b**2)**(3/2)) + (2*a + b*(d + e*x)**2)/(2*e*(-4*a*c + b**2)*(a +
 b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 0.228051, size = 100, normalized size = 1.03

$$\frac{\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d
+ e*x)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c
]])/(-b^2 + 4*a*c)^(3/2))/(2*e)
```

Maple [C] time = 0.031, size = 276, normalized size = 2.9

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(-\frac{x^2be}{8\,ac-2\,b^2}-\frac{bdx}{4\,ac-b^2}-\frac{bd^2+2\,a}{2\,e\,(4\,ac-b^2)}\right)+\frac{b}{2\,e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{(-e\_R-d)\ln(x-\_R)}{(4\,ac-b^2)\left(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e/(4*a*c-b^2)*x^2-b*d/(4*a*c-b^2)*x-1/2/e*(b*d^2+2*a)/(4*
a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2*b/e*sum((-_R*e-d)/(4*a*c-b^2
)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*l
n(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_
Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-b\int -\frac{ex+d}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc}+\frac{be^2x^2+2\,bdex+bd^2+2\,a}{2\,((b^2c-4\,ac^2)e^5x^4+4\,(b^2c-4\,ac^2)de^4x^3+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^3x^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)e^2x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxima")
```

```
[Out] -b*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4
*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b
^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*
d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) +
1/2*(b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a)/((b^2*c - 4*a*c^2)*e^5*
x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c -
 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*
b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2)*e)
```

Fricas [A] time = 0.318688, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*d^2 + b^
2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*log((2*(b
^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b
*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d
^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a)*sqrt(b^2
 - 4*a*c))/(((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^
4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*
(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*
c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(b^2 - 4
*a*c)), 1/2*(2*(b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*
d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*a
rctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)) + (b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a)*sqrt(-b^2
 + 4*a*c))/(((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^
4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*
(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*
c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-b^2 +
4*a*c))]
```

Sympy [A] time = 164.745, size = 493, normalized size = 5.08

$$\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e} - \frac{2a+bd^2+2bdex+be^2x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4(8ac^2e^5-2b^2ce^5)+x^3(32ac^2de^4-8b^2cde^4)+x^2(8abce^3+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)*
*3) - b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2 + 2*b*c*d**2)/(2*b*c
*e**2))/(2*e) - b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 +
 (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/
(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2 + 2*b
*c*d**2)/(2*b*c*e**2))/(2*e) - (2*a + b*d**2 + 2*b*d*e*x + b*e**2
*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*
e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**
2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a
*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e*
*3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 -
8*b**2*c*d**3*e**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.623 $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} \, dx$

Optimal. Leaf size=254

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt
[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.869961, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```


```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt
[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 68.0459, size = 226, normalized size = 0.89

$$-\frac{\sqrt{2}\sqrt{c}\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{e\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(b-\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{e\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\left(b+2c\left(d+ex\right)^2\right)\left(d+ex\right)}{2e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(b + sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sqrt(c)
*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(e*sqrt(b + sqrt(-4*a*c
 + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(c)*(b - sqrt(-4*
a*c + b**2)/2)*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*
c + b**2)))/(e*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/
2)) - (b + 2*c*(d + e*x)**2)*(d + e*x)/(2*e*(-4*a*c + b**2)*(a +
b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 1.74921, size = 247, normalized size = 0.97

$$-\frac{\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sq
rt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x)
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/(2*e)
```

Maple [C] time = 0.028, size = 319, normalized size = 1.3

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(\frac{e^2cx^3}{4\,ac-b^2}+3\,\frac{cdex^2}{4\,ac-b^2}+\frac{(6\,cd^2+b)\,x}{8\,ac-2\,b^2}+\frac{d\,(2\,cd^2}{2\,e\,(4\,ac}\right.$$
$$+\frac{1}{4\,e}\sum_{\_R=RootOf(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a)}\frac{(2\,\_R^2ce^2+4\,\_Rcde+2\,cd^2-b)\ln(x-\_}{(4\,ac-b^2)\,(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (e^2*c/(4*a*c-b^2)*x^3+3*d*e*c/(4*a*c-b^2)*x^2+1/2*(6*c*d^2+b)/(4
*a*c-b^2)*x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)+1/4/e*sum((2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(4*a*c-b^2)/(2*_
R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R
),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4
*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,ce^3x^3+6\,cde^2x^2+2\,cd^3+(6\,cd^2+b)\,ex+bd}{2\,((b^2c-4\,ac^2)e^5x^4+4\,(b^2c-4\,ac^2)de^4x^3+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^3x^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)e^2x}$$
$$+\frac{1}{2}\int-\frac{2\,ce^2x^2+4\,cdex+2\,cd^2-b}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 2*c*d^3 + (6*c*d^2 + b)*e*x +
 b*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
+ (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
 - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d
^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e) + 1/2*integrate(-(
2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c^2)*e^4*x^4
 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)
*d)*e*x), x)
```

Fricas [A] time = 0.33667, size = 3340, normalized size = 13.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*e^3*x^3 + 12*c*d*e^2*x^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*e*
x - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*
e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(
2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*
a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((a*
b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 +
 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*s
qrt(1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e
^3*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*e^4)) - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 + 12*a*b*c)
/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + s
qrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x
^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^
2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2
)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((a*b^6 -
 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 + 12*a
*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt(1
/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sq
rt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)
) - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 + 12*a*b*c)/((a*
b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c -
 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^
2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - b^3 - 12*a*b*c)/
((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((
3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2)*((
a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/(
(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b
^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 4
8*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 1
2*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b
^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
```

```
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^
2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c
 + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6
 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c
 + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt(1/2)*((a*b^8 -
 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*
a*b^3*c + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2 - 64*a^5*c^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b
^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + 2*b*d)/((b^2*c - 4*a
*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c +
6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (
b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [A] time = 151.335, size = 578, normalized size = 2.28

$$\frac{bd + 2cd^3 + 6cde^2x^2 + 2ce^3x^3 + x\left(be + 6cd^2e\right.}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3 + 48\right.}$$
$$+\mathrm{RootSum}\left(t^4\left(1048576a^7c^6e^4 - 1572864a^6b^2c^5e^4 + 983040a^5b^4c^4e^4 - 327680a^4b^6c^3e^4 + 61440a^3b^8c^2e^4 - 6144a^2b^{10}ce^4 + 2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d + 2*c*d**3 + 6*c*d*e**2*x**2 + 2*c*e**3*x**3 + x*(b*e + 6*c*
d**2*e))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**
4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b
**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8
*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*
e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2
- 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**7*c**6*e**4 -
1572864*a**6*b**2*c**5*e**4 + 983040*a**5*b**4*c**4*e**4 - 327680
*a**4*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 - 6144*a**2*b**1
0*c*e**4 + 256*a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8
192*a**3*b**3*c**3*e**2 - 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2
) + 16*a**2*c**3 + 24*a*b**2*c**2 + 9*b**4*c, Lambda(_t, _t*log(x
 + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a**4*b**2*c**3*e**3 +
 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 128*_t*a**2*
b*c**2*e - 16*_t*a*b**3*c*e - 4*_t*b**5*e + 4*a*c**2*d + 3*b**2*c
*d)/(4*a*c**2*e + 3*b**2*c*e))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.624 $\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=96

$$\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\,(b^2-4ac)^{3/2}}-\frac{b+2c(d+ex)^2}{2e\,(b^2-4ac)\,(a+b(d+ex)^2+c(d+ex)^4)}$$

```
[Out] -(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a
*c]])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi [A] time = 0.245768, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\,(b^2-4ac)^{3/2}}-\frac{b+2c(d+ex)^2}{2e\,(b^2-4ac)\,(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a
*c]])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi in Sympy [A] time = 23.8886, size = 83, normalized size = 0.86

$$\frac{2c\,\text{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\,(-4ac+b^2)^{\frac{3}{2}}}-\frac{b+2c\,(d+ex)^2}{2e\,(-4ac+b^2)\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] 2*c*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(-4*a*c
+ b**2)**(3/2)) - (b + 2*c*(d + e*x)**2)/(2*e*(-4*a*c + b**2)*(a
+ b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 0.218578, size = 98, normalized size = 1.02

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*
c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c])/(2*(b^2 - 4*a*c)*e)
```

Maple [C] time = 0.033, size = 270, normalized size = 2.8

$$\frac{1}{ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a}\left(\frac{cex^2}{4\,ac-b^2}+2\,\frac{cdx}{4\,ac-b^2}+\frac{2\,cd^2+b}{2\,e\,(4\,ac-b^2)}\right)+\frac{c}{e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{(e\_R+d)\ln(x-\_R)}{(4\,ac-b^2)\left(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2ec\_R+2\,cd^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (e*c/(4*a*c-b^2)*x^2+2*c*d/(4*a*c-b^2)*x+1/2/e*(2*c*d^2+b)/(4*a*c
-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+c/e*sum((_R*e+d)/(4*a*c-b^2)/(2*_R^
3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),
_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,c\int -\frac{ex+d}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc}-\frac{2\,ce^2x^2+4\,cdex+2\,cd^2+b}{2\,((b^2c-4\,ac^2)e^5x^4+4\,(b^2c-4\,ac^2)de^4x^3+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^3x^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxima")
```

```
[Out] 2*c*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c -
4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(
b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)
*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) -
 1/2*(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)/((b^2*c - 4*a*c^2)*e
^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*
c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4
*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^
3 - 4*a*b*c)*d^2)*e)
```

Fricas [A] time = 0.295893, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 +
b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*log(-(2
*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*
a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - (2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^
3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^
2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d
^3 + b*d)*e*x + a)) + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqr
t(b^2 - 4*a*c))/(((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)
*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 +
2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c -
 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(b^
2 - 4*a*c)), -1/2*(4*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4 + (
6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x +
a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*s
qrt(-b^2 + 4*a*c))/(((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2
 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*
c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt
(-b^2 + 4*a*c))]
```

Sympy [A] time = 114.037, size = 495, normalized size = 5.16

$$-\frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{b+2cd^2+4cdex+2ce^2x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\left(8ac^2e^5-2b^2ce^5\right)+x^3\left(32ac^2de^4-8b^2cde^4\right)+x^2\left(8abce^3+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3
*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2
)**3) - b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c + 2*c**2*d**2)/(2
*c**2*e**2))/e + c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2
+ (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-
1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c +
2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*e*x + 2*c*e
**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d*
*4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*
b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(
8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2
*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2
 - 8*b**2*c*d**3*e**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.625 $\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=299

$$\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c
)*(a + b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 -
12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

Rubi [A] time = 1.46207, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{(d+ex)\left(-2ac+b^2+bc(d+ex)^2\right)}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/(2*a*(b^2 - 4*a*c)*e*
(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqr
```

```
t[b^2 - 4*a*c]]*e) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 90.5285, size = 255, normalized size = 0.85

$$-\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4ae\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4ae\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{(d+ex)\left(-2ac+b^2+bc(d+ex)^2\right)}{2ae\left(-4ac+b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(-12*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sq
rt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*e*sqr
t(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt
(c)*(-12*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)
*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(4*a*e*sqrt(b - sqrt(-4
*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + (d + e*x)*(-2*a*c + b**2
+ b*c*(d + e*x)**2)/(2*a*e*(-4*a*c + b**2)*(a + b*(d + e*x)**2 +
c*(d + e*x)**4))
```

Mathematica [A] time = 1.68425, size = 271, normalized size = 0.91

$$\frac{\frac{2(d+ex)\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2(b+c(d+ex)^2)\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4ae}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a
+ (d + e*x)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
```

```
t[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*e)
```

Maple [C] time = 0.055, size = 364, normalized size = 1.2

$$\frac{1}{ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + be^2x^2 + cd^4 + 2\,bdex + bd^2 + a}\left(-\frac{bce^2x^3}{2\,a\,(4\,ac - b^2)} - \frac{3\,bcdex^2}{2\,a\,(4\,ac - b^2)} + \frac{(-3\,bcd^2 + 2\,c}{2\,a\,(4\,ac}\right.$$
$$+\frac{1}{4\,ae}\sum_{\_R = RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(-\_R^2bce^2 - 2\,\_R\,bcde - bcd^2 + 6\,ac - b^2\right)\ln}{(4\,ac - b^2)\left(2\,ce^3\_R^3 + 6\,cde^2\_R^2 + 6\,d^2ec\_R + 2\,cd^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2
*(-3*b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b
^2)/a/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/a/e*sum((-_R^2*b*c*
e^2-2*_R*b*c*d*e-b*c*d^2+6*a*c-b^2)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c
*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*
d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bce^3x^3 + 3\,bcde^2x^2 + bcd^3 + \left(3\,bcd^2 + b^2 - 2\,ac\right)ex + \left(b^2 - 2\right.}{2\left((ab^2c - 4\,a^2c^2)e^5x^4 + 4\,(ab^2c - 4\,a^2c^2)de^4x^3 + (ab^3 - 4\,a^2bc + 6\,(ab^2c - 4\,a^2c^2)d^2)e^3x^2 + 2\,(2\,(ab^2c - 4\,a^2c^2)d^3 + (ab^3 - 4\right.}$$
$$-\frac{-\int\frac{bce^2x^2+2\,bcdex+bcd^2+b^2-6\,ac}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc)d^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)ex}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 -
 2*a*c)*e*x + (b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4
*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*
c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a
*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2
 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e) - 1/2*integrate(-(b*c*e^
```

```
2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 6*a*c)/((b^2*c - 4*a*c^2)*e
^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*
a*b*c)*d)*e*x), x)/a
```

Fricas [A] time = 0.34609, size = 4358, normalized size = 14.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*e^3*x^3 + 6*b*c*d*e^2*x^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 +
 b^2 - 2*a*c)*e*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*
(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c
 - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*
b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2
- 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6
*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b
^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c
^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2
- 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 8
1*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 +
 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^
6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 1
8*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 +
4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2
*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (
a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^
2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c
 + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a
*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2
*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^
2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c +
 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c
^3)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3
 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*
b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 -
 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c
```

```
^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2
 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4
+ 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b
^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 +
 (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3
*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81
*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a
^2*c^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*
c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c
 + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9
*c^3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c
^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^
3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4
 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2 - 64*a^6*c^3)*e^2))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^
4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a
*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3
 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^
2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b
^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12
*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 -
81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324
*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^
5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a
^9*c^3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2
*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b
^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2
*c^2 - 64*a^6*c^3)*e^2))) + 2*(b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*
c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a
^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^
2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)
```

Sympy [A] time = 95.6085, size = 740, normalized size = 2.47

$$-\frac{-2acd + b^2d + bcd^3 + 3bcde^2x^2 + bce^3x^3 + x\,(}{8a^3ce - 2a^2b^2e + 8a^2bcd^2e + 8a^2c^2d^4e - 2ab^3d^2e - 2ab^2cd^4e + x^4\left(8a^2c^2e^5 - 2ab^2ce^5\right) + x^3\left(32a^2c^2de^4 - 8ab^2cde^4\right) + x^2\,(8}$$

$$+\mathrm{RootSum}\Big(t^4\,(1048576a^9c^6e^4 - 1572864a^8b^2c^5e^4 + 983040a^7b^4c^4e^4 - 327680a^6b^6c^3e^4 + 61440a^5b^8c^2e^4 - 6144a^4b^{10}ce^4 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -(-2*a*c*d + b**2*d + b*c*d**3 + 3*b*c*d*e**2*x**2 + b*c*e**3*x**
3 + x*(-2*a*c*e + b**2*e + 3*b*c*d**2*e))/(8*a**3*c*e - 2*a**2*b*
*2*e + 8*a**2*b*c*d**2*e + 8*a**2*c**2*d**4*e - 2*a*b**3*d**2*e -
 2*a*b**2*c*d**4*e + x**4*(8*a**2*c**2*e**5 - 2*a*b**2*c*e**5) +
x**3*(32*a**2*c**2*d*e**4 - 8*a*b**2*c*d*e**4) + x**2*(8*a**2*b*c
*e**3 + 48*a**2*c**2*d**2*e**3 - 2*a*b**3*e**3 - 12*a*b**2*c*d**2
*e**3) + x*(16*a**2*b*c*d*e**2 + 32*a**2*c**2*d**3*e**2 - 4*a*b**
3*d*e**2 - 8*a*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**9*c
**6*e**4 - 1572864*a**8*b**2*c**5*e**4 + 983040*a**7*b**4*c**4*e*
*4 - 327680*a**6*b**6*c**3*e**4 + 61440*a**5*b**8*c**2*e**4 - 614
4*a**4*b**10*c*e**4 + 256*a**3*b**12*e**4) + _t**2*(-61440*a**5*b
*c**5*e**2 + 61440*a**4*b**3*c**4*e**2 - 24064*a**3*b**5*c**3*e**
2 + 4608*a**2*b**7*c**2*e**2 - 432*a*b**9*c*e**2 + 16*b**11*e**2)
 + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b**4*c**3, Lambda(_t, _t
*log(x + (32768*_t**3*a**7*b*c**4*e**3 - 28672*_t**3*a**6*b**3*c*
*3*e**3 + 9216*_t**3*a**5*b**5*c**2*e**3 - 1280*_t**3*a**4*b**7*c
*e**3 + 64*_t**3*a**3*b**9*e**3 + 1728*_t*a**4*c**4*e - 2304*_t*a
**3*b**2*c**3*e + 740*_t*a**2*b**4*c**2*e - 92*_t*a*b**6*c*e + 4*
_t*b**8*e + 324*a**2*c**4*d - 81*a*b**2*c**3*d + 5*b**4*c**2*d)/(
324*a**2*c**4*e - 81*a*b**2*c**3*e + 5*b**4*c**2*e))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-2),x, algorithm="giac")
```

```
[Out] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-2), x)
```

3.626 $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d +
 e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[
d + e*x]/(a^2*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*
e)
```

Rubi [A] time = 0.577985, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d +
 e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[
d + e*x]/(a^2*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*
e)
```

Rubi in Sympy [A] time = 66.2603, size = 146, normalized size = 0.9

$$\frac{-2ac+b^2+bc\left(d+ex\right)^2}{2ae\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}+\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^2e\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\log\left(\left(d+ex\right)^2\right)}{2a^2e}-\frac{\log\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}{4a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(2*a*e*(-4*a*c + b**2)*(a + b*
(d + e*x)**2 + c*(d + e*x)**4)) + b*(-6*a*c + b**2)*atanh((b + 2*
c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*a**2*e*(-4*a*c + b**2)**(
3/2)) + log((d + e*x)**2)/(2*a**2*e) - log(a + b*(d + e*x)**2 + c
*(d + e*x)**4)/(4*a**2*e)
```

Mathematica [A] time = 0.733271, size = 235, normalized size = 1.45

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log(}{(b^2-4ac)^{3/2}}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e
*x)^2*(b + c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c +
b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2
 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b
*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2*e)
```

Maple [C] time = 0.046, size = 693, normalized size = 4.3

$$-\frac{bcex^2}{2a(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)(4ac-b^2)}$$
$$-\frac{bcdx}{a(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)(4ac-b^2)}$$
$$-\frac{bcd^2}{2a(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)e(4ac-b^2)}$$
$$+\frac{c}{(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)e(4ac-b^2)}$$
$$-\frac{b^2}{2a(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)e(4ac-b^2)}$$
$$-\frac{1}{2a^2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(ce^3\left(4ac-b^2\right)\_R^3+3cde^2\left(4ac-b^2\right)\_R^2+e\left(\right.\right.}{(4ac-b^2)\left(2ce^3\right.}$$
$$+\frac{\ln(ex+d)}{a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*e*c/(4*a*c-b^2)*x^2-1/a/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)*b*c*d/(4*a*c-b^2)*x-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/
(4*a*c-b^2)*b*c*d^2+1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c-1/2/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/a^2/e*sum((c*e^3*(
4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*
c*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(4*a
*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+
b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*
e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+ln(e*x+d)/a^2/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bce^2x^2 + 2\,bcdex + bcd^2 + b^2 - 2\,ac}{2\left(\left(ab^2c - 4\,a^2c^2\right)e^5x^4 + 4\left(ab^2c - 4\,a^2c^2\right)de^4x^3 + \left(ab^3 - 4\,a^2bc + 6\left(ab^2c - 4\,a^2c^2\right)d^2\right)e^3x^2 + 2\left(2\left(ab^2c - 4\,a^2c^2\right)d^3 + \left(ab^3 - 4 \cdots\right)\right)\right)} - \frac{\int \frac{\left(b^2c-4\,ac^2\right)e^3x^3+3\left(b^2c-4\,ac^2\right)de^2x^2+\left(\left(b^2c-4\,ac^2\right)d^3+\left(b^3-5\,abc+3\left(b^2c-4\,ac^2\right)d^2\right)\right)ex+\left(b^3-5\,abc\right)d}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc)d^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)ex}\,dx}{a^2} + \frac{\log\left(ex + d\right)}{a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)),x, algorithm="maxima
```

```
[Out] 1/2*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c
 - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^
3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^
2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)
 - integrate(((b^2*c - 4*a*c^2)*e^3*x^3 + 3*(b^2*c - 4*a*c^2)*d*e
^2*x^2 + (b^2*c - 4*a*c^2)*d^3 + (b^3 - 5*a*b*c + 3*(b^2*c - 4*a*
c^2)*d^2)*e*x + (b^3 - 5*a*b*c)*d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4
*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*
e*x), x)/a^2 + log(e*x + d)/(a^2*e)
```

Fricas [A] time = 0.489128, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)),x, algorithm="fricas
```

```
[Out] [1/4*(((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*
x^3 + (b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a
*b*c^2)*d^2)*e^2*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2
+ 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*log((2
*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*
a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^
3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^
2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d
^3 + b*d)*e*x + a)) + (2*a*b*c*e^2*x^2 + 4*a*b*c*d*e*x + 2*a*b*c*
d^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c -
 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x)*log(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2
 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^2*c - 4*a*c^2)*e^4*x^4 + 4*
(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*
b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e
*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/(((a^2*b^2*c - 4*a^3*c^2)*e^
5*x^4 + 4*(a^2*b^2*c - 4*a^3*c^2)*d*e^4*x^3 + (a^2*b^3 - 4*a^3*b*
c + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^3*x^2 + 2*(2*(a^2*b^2*c - 4*
a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)*d)*e^2*x + (a^3*b^2 - 4*a^4*
c + (a^2*b^2*c - 4*a^3*c^2)*d^4 + (a^2*b^3 - 4*a^3*b*c)*d^2)*e)*s
qrt(b^2 - 4*a*c)), -1/4*(2*((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*
c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b
^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 - 6*a*
b^2*c)*d)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*a*b*c*e^2*x^2 + 4*a*b*c*d*e*
x + 2*a*b*c*d^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2)*e^4*x^4
+ 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*
d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2
*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^2*c - 4*a*c^2)*
e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 +
 (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^
2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4
*a*b*c)*d)*e*x)*log(e*x + d))*sqrt(-b^2 + 4*a*c))/(((a^2*b^2*c -
4*a^3*c^2)*e^5*x^4 + 4*(a^2*b^2*c - 4*a^3*c^2)*d*e^4*x^3 + (a^2*b
^3 - 4*a^3*b*c + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^3*x^2 + 2*(2*(a
^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)*d)*e^2*x + (a^3
*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*d^4 + (a^2*b^3 - 4*a^3*b
```

```
*c)*d^2)*e)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)),x, algorithm="giac")

[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)), x)
```

3.627 $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=348

$$-\frac{3b^2-10ac}{2a^2e\,(b^2-4ac)\,(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2e\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2e\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\,(b^2-4ac)\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*
c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 1
0*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2
- 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 3.19164, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{3b^2-10ac}{2a^2e\,(b^2-4ac)\,(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2e\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2e\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\,(b^2-4ac)\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*
c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 1
```

```
0*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2
- 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 172.702, size = 313, normalized size = 0.9

$$\frac{-2ac+b^2+bc(d+ex)^2}{2ae(d+ex)(-4ac+b^2)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3-(-10ac+3b^2)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a^2e\sqrt{b+\sqrt{-4ac+b^2}}(-4ac+b^2)^{\frac{3}{2}}} - \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3+(-10ac+3b^2)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a^2e\sqrt{b-\sqrt{-4ac+b^2}}(-4ac+b^2)^{\frac{3}{2}}} - \frac{-10ac+3b^2}{2a^2e(d+ex)(-4ac+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(2*a*e*(d + e*x)*(-4*a*c + b**
2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) + sqrt(2)*sqrt(c)*(-16*
a*b*c + 3*b**3 - (-10*a*c + 3*b**2)*sqrt(-4*a*c + b**2))*atan(sqr
t(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a**2*e*s
qrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*sq
rt(c)*(-16*a*b*c + 3*b**3 + (-10*a*c + 3*b**2)*sqrt(-4*a*c + b**2
))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/
(4*a**2*e*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) -
 (-10*a*c + 3*b**2)/(2*a**2*e*(d + e*x)*(-4*a*c + b**2))
```

Mathematica [A] time = 2.99049, size = 339, normalized size = 0.97

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b\right.}{(b^2-4ac}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 -
 2*a*c^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d
+ e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2
 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
+ e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b
^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)
```

Maple [C] time = 0.041, size = 1304, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)*e^2*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)*e^2*c/(4*a*c-b^2)*x^3*b^2-3/a/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)*d*e*c^2/(4*a*c-b^2)*x^2+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*e*
c/(4*a*c-b^2)*x^2*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^
2*d^2+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c*d^2-3/2/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)*b*c/(4*a*c-b^2)*x+1/2/a^2/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)/(4*a*c-b^2)*x*b^3-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*
c-b^2)*c^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^2*c
-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^3-1/4/a^2/e*sum((e^2*c*(10*a*c-
3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3*b^2*c*d^2+13
*a*b*c-3*b^3)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2
*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)
)-1/a^2/e/(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\,b^2c-10\,ac^2\right)e^4x^4+4\left(3\,b^2c-10\,ac^2\right)de^3x^3+\left(3\,b^2c-10\,ac^2\right)d^4+\left(3\,b^3-11\,abc+\right.}{2\left((a^2b^2c-4\,a^3c^2)e^6x^5+5\,(a^2b^2c-4\,a^3c^2)de^5x^4+(a^2b^3-4\,a^3bc+10\,(a^2b^2c-4\,a^3c^2)d^2)e^4x^3+(10\,(a^2b^2c-4\,a^3c^2)d^3+3\right.}$$
$$-\frac{\int\frac{\left(3\,b^2c-10\,ac^2\right)e^2x^2+2\left(3\,b^2c-10\,ac^2\right)dex+3\,b^3-13\,abc+\left(3\,b^2c-10\,ac^2\right)d^2}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc)d^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)ex}\,dx}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^2),x, algorithm="maxi
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*e^4*x^4 + 4*(3*b^2*c - 10*a*c^2)*d*e^3
*x^3 + (3*b^2*c - 10*a*c^2)*d^4 + (3*b^3 - 11*a*b*c + 6*(3*b^2*c
- 10*a*c^2)*d^2)*e^2*x^2 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)
*d^2 + 2*(2*(3*b^2*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x)
/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e
^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e
^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c
)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*
d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e) - 1/2
*integrate(((3*b^2*c - 10*a*c^2)*e^2*x^2 + 2*(3*b^2*c - 10*a*c^2)
*d*e*x + 3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*d^2)/((b^2*c - 4
*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^
2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^
2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
(b^3 - 4*a*b*c)*d)*e*x), x)/a^2
```

Fricas [A] time = 0.416641, size = 5846, normalized size = 16.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^2),x, algorithm="fric
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e
^3*x^3 + 2*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*
b^2*c - 10*a*c^2)*d^2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 -
11*a*b*c)*d^2 + 4*(2*(3*b^2*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c
)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^
2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c
 - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*
(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^
2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2
```

*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 -
4*a^4*c)*d)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^
2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5
625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 -
55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*
c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^
4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 520
0*a^5*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 42
0*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c
^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*x
^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c +
 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3
*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4
c + 5(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)
*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3
 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2
*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^
2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c +
48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4
*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971
*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*(
(3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 +
 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918*a*b^6*c
 + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11
 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4
*b^3*c^4 - 5200*a^5*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c
^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*
a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3
 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2
*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a
^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4
*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 -
4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7 - 105*a*b
^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12
*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 1

```
2*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^
3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*
b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1
/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344
*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^
4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^
4)) + (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c
^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*
b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c +
3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) + sqrt(1/2)*((
a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*
x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*
x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d
)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 +
3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5
 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9
*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 -
 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))
/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))*lo
g(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^
6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*
a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*
b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^
3*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^
3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1
0549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*sqrt(-(
9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6
- 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/
((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4))
)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))))
/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e
^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e
^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c
)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*
d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^2),x, algorithm="giac
```

```
[Out] Exception raised: TypeError
```

3.628 $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=213

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+ex)}{a^3e}-\frac{b^2-3ac}{a^2e\,(b^2-4ac)\,(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\,(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\,(b^2-4ac)\,(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c
 + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh
[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/
2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*Log[a + b*(d + e*x)^2 + c
*(d + e*x)^4])/(2*a^3*e)
```

Rubi [A] time = 0.724868, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+ex)}{a^3e}-\frac{b^2-3ac}{a^2e\,(b^2-4ac)\,(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\,(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\,(b^2-4ac)\,(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c
 + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh
[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/
2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*Log[a + b*(d + e*x)^2 + c
*(d + e*x)^4])/(2*a^3*e)
```

Rubi in Sympy [A] time = 100.686, size = 192, normalized size = 0.9

$$\frac{-2ac+b^2+bc\,(d+ex)^2}{2ae\,(d+ex)^2\,(-4ac+b^2)\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}-\frac{-3ac+b^2}{a^2e\,(d+ex)^2\,(-4ac+b^2)}-\frac{b\log\left((d+ex)^2\right)}{a^3e}+\frac{b\log\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}{2a^3e}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{a^3e\,(-4ac+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(2*a*e*(d + e*x)**2*(-4*a*c +
b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) - (-3*a*c + b**2)/(a
**2*e*(d + e*x)**2*(-4*a*c + b**2)) - b*log((d + e*x)**2)/(a**3*e
) + b*log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(2*a**3*e) - (6*a*
*2*c**2 - 6*a*b**2*c + b**4)*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4
*a*c + b**2))/(a**3*e*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.872726, size = 284, normalized size = 1.33

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+\right.}{(b^2-4ac)^{3/2}}}{2a^3e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c
^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - 4*b*Log[d + e*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqr
t[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c
 - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2
))/(2*a^3*e)
```

Maple [C] time = 0.054, size = 1014, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/a^2/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2*b^2-2/a/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
```

```
)*c^2*d/(4*a*c-b^2)*x+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*c-b^2
)*x*b^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2/a^2/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2*c*d^2-3/2/a/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*
x+b*d^2+a)/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*
a*c-b^2)*b^3+1/a^3/e*sum((b*e^3*c*(4*a*c-b^2)*_R^3+3*b*d*e^2*c*(4
*a*c-b^2)*_R^2+e*(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2*c^2+5*a*b^2*c-
b^4)*_R+4*a*b*c^2*d^3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d)/(4
*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*
e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+
b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/2/a^2/e/(e*x
+d)^2-2*b*ln(e*x+d)/a^3/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^3),x, algorithm="maxi
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*e^4*x^4 + 8*(b^2*c - 3*a*c^2)*d*e^3*x^3
 + 2*(b^2*c - 3*a*c^2)*d^4 + (2*b^3 - 7*a*b*c + 12*(b^2*c - 3*a*c
^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + 2*(4
*(b^2*c - 3*a*c^2)*d^3 + (2*b^3 - 7*a*b*c)*d)*e*x)/((a^2*b^2*c -
4*a^3*c^2)*e^7*x^6 + 6*(a^2*b^2*c - 4*a^3*c^2)*d*e^6*x^5 + (a^2*b
^3 - 4*a^3*b*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^5*x^4 + 4*(5*(
a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)*d)*e^4*x^3 + (
a^3*b^2 - 4*a^4*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 6*(a^2*b^3 -
 4*a^3*b*c)*d^2)*e^3*x^2 + 2*(3*(a^2*b^2*c - 4*a^3*c^2)*d^5 + 2*(
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e^2*x + ((a^2*b
^2*c - 4*a^3*c^2)*d^6 + (a^2*b^3 - 4*a^3*b*c)*d^4 + (a^3*b^2 - 4*
a^4*c)*d^2)*e) + 2*integrate(((b^3*c - 4*a*b*c^2)*e^3*x^3 + 3*(b^
3*c - 4*a*b*c^2)*d*e^2*x^2 + (b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 5*a
*b^2*c + 3*a^2*c^2 + 3*(b^3*c - 4*a*b*c^2)*d^2)*e*x + (b^4 - 5*a*
b^2*c + 3*a^2*c^2)*d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a
*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2
*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^
2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^3
- 2*b*log(e*x + d)/(a^3*e)
```

Fricas [A] time = 0.637278, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^3),x, algorithm="fric
```

```
[Out] [-1/2*(((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6
*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c
^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a
*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15
*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 +
2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c +
 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*log
((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 -
 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2
*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^
4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*
c*d^3 + b*d)*e*x + a)) + (2*(a*b^2*c - 3*a^2*c^2)*e^4*x^4 + 8*(a*
b^2*c - 3*a^2*c^2)*d*e^3*x^3 + 2*(a*b^2*c - 3*a^2*c^2)*d^4 + (2*a
*b^3 - 7*a^2*b*c + 12*(a*b^2*c - 3*a^2*c^2)*d^2)*e^2*x^2 + a^2*b^
2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*d^2 + 2*(4*(a*b^2*c - 3*a^2*c
^2)*d^3 + (2*a*b^3 - 7*a^2*b*c)*d)*e*x - ((b^3*c - 4*a*b*c^2)*e^6
*x^6 + 6*(b^3*c - 4*a*b*c^2)*d*e^5*x^5 + (b^4 - 4*a*b^2*c + 15*(b
^3*c - 4*a*b*c^2)*d^2)*e^4*x^4 + (b^3*c - 4*a*b*c^2)*d^6 + 4*(5*(
b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 4*a*b^2*c)*d)*e^3*x^3 + (b^4 - 4*
a*b^2*c)*d^4 + (15*(b^3*c - 4*a*b*c^2)*d^4 + a*b^3 - 4*a^2*b*c +
6*(b^4 - 4*a*b^2*c)*d^2)*e^2*x^2 + (a*b^3 - 4*a^2*b*c)*d^2 + 2*(3
*(b^3*c - 4*a*b*c^2)*d^5 + 2*(b^4 - 4*a*b^2*c)*d^3 + (a*b^3 - 4*a
^2*b*c)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
+ b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3*c - 4
*a*b*c^2)*e^6*x^6 + 6*(b^3*c - 4*a*b*c^2)*d*e^5*x^5 + (b^4 - 4*a*
b^2*c + 15*(b^3*c - 4*a*b*c^2)*d^2)*e^4*x^4 + (b^3*c - 4*a*b*c^2)
*d^6 + 4*(5*(b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 4*a*b^2*c)*d)*e^3*x^
3 + (b^4 - 4*a*b^2*c)*d^4 + (15*(b^3*c - 4*a*b*c^2)*d^4 + a*b^3 -
 4*a^2*b*c + 6*(b^4 - 4*a*b^2*c)*d^2)*e^2*x^2 + (a*b^3 - 4*a^2*b*
c)*d^2 + 2*(3*(b^3*c - 4*a*b*c^2)*d^5 + 2*(b^4 - 4*a*b^2*c)*d^3 +
 (a*b^3 - 4*a^2*b*c)*d)*e*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/(((
a^3*b^2*c - 4*a^4*c^2)*e^7*x^6 + 6*(a^3*b^2*c - 4*a^4*c^2)*d*e^6*
x^5 + (a^3*b^3 - 4*a^4*b*c + 15*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^5*
x^4 + 4*(5*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)
*e^4*x^3 + (a^4*b^2 - 4*a^5*c + 15*(a^3*b^2*c - 4*a^4*c^2)*d^4 +
6*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^3*x^2 + 2*(3*(a^3*b^2*c - 4*a^4*c^
2)*d^5 + 2*(a^3*b^3 - 4*a^4*b*c)*d^3 + (a^4*b^2 - 4*a^5*c)*d)*e^2
*x + ((a^3*b^2*c - 4*a^4*c^2)*d^6 + (a^3*b^3 - 4*a^4*b*c)*d^4 + (
a^4*b^2 - 4*a^5*c)*d^2)*e)*sqrt(b^2 - 4*a*c)), 1/2*(2*((b^4*c - 6
*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*
```

```
c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2
*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6
*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c
^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*
c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2
*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*
b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3
+ (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*arctan(-(2*c*e^2*x^2
+ 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2
*(a*b^2*c - 3*a^2*c^2)*e^4*x^4 + 8*(a*b^2*c - 3*a^2*c^2)*d*e^3*x^
3 + 2*(a*b^2*c - 3*a^2*c^2)*d^4 + (2*a*b^3 - 7*a^2*b*c + 12*(a*b^
2*c - 3*a^2*c^2)*d^2)*e^2*x^2 + a^2*b^2 - 4*a^3*c + (2*a*b^3 - 7*
a^2*b*c)*d^2 + 2*(4*(a*b^2*c - 3*a^2*c^2)*d^3 + (2*a*b^3 - 7*a^2*
b*c)*d)*e*x - ((b^3*c - 4*a*b*c^2)*e^6*x^6 + 6*(b^3*c - 4*a*b*c^2
)*d*e^5*x^5 + (b^4 - 4*a*b^2*c + 15*(b^3*c - 4*a*b*c^2)*d^2)*e^4*
x^4 + (b^3*c - 4*a*b*c^2)*d^6 + 4*(5*(b^3*c - 4*a*b*c^2)*d^3 + (b
^4 - 4*a*b^2*c)*d)*e^3*x^3 + (b^4 - 4*a*b^2*c)*d^4 + (15*(b^3*c -
 4*a*b*c^2)*d^4 + a*b^3 - 4*a^2*b*c + 6*(b^4 - 4*a*b^2*c)*d^2)*e^
2*x^2 + (a*b^3 - 4*a^2*b*c)*d^2 + 2*(3*(b^3*c - 4*a*b*c^2)*d^5 +
2*(b^4 - 4*a*b^2*c)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e*x)*log(c*e^4*x
^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2
*c*d^3 + b*d)*e*x + a) + 4*((b^3*c - 4*a*b*c^2)*e^6*x^6 + 6*(b^3*
c - 4*a*b*c^2)*d*e^5*x^5 + (b^4 - 4*a*b^2*c + 15*(b^3*c - 4*a*b*c
^2)*d^2)*e^4*x^4 + (b^3*c - 4*a*b*c^2)*d^6 + 4*(5*(b^3*c - 4*a*b*
c^2)*d^3 + (b^4 - 4*a*b^2*c)*d)*e^3*x^3 + (b^4 - 4*a*b^2*c)*d^4 +
 (15*(b^3*c - 4*a*b*c^2)*d^4 + a*b^3 - 4*a^2*b*c + 6*(b^4 - 4*a*b
^2*c)*d^2)*e^2*x^2 + (a*b^3 - 4*a^2*b*c)*d^2 + 2*(3*(b^3*c - 4*a*
b*c^2)*d^5 + 2*(b^4 - 4*a*b^2*c)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e*x
)*log(e*x + d))*sqrt(-b^2 + 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*e^7
*x^6 + 6*(a^3*b^2*c - 4*a^4*c^2)*d*e^6*x^5 + (a^3*b^3 - 4*a^4*b*c
 + 15*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^5*x^4 + 4*(5*(a^3*b^2*c - 4*
a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^4*x^3 + (a^4*b^2 - 4*a^
5*c + 15*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 6*(a^3*b^3 - 4*a^4*b*c)*d^
2)*e^3*x^2 + 2*(3*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 2*(a^3*b^3 - 4*a^
4*b*c)*d^3 + (a^4*b^2 - 4*a^5*c)*d)*e^2*x + ((a^3*b^2*c - 4*a^4*c
^2)*d^6 + (a^3*b^3 - 4*a^4*b*c)*d^4 + (a^4*b^2 - 4*a^5*c)*d^2)*e)
*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.288191, size = 302, normalized size = 1.42

$$\frac{\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}+\frac{be^{(-1)}\ln\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{2\,a^3}+\frac{\left(\frac{b^3c-3\,abc^2}{a}+\frac{\left(b^4e-4\,ab^2ce+2\,a^2c^2e\right)e^{(-1)}}{(xe+d)^2a}\right)e^{(-1)}}{2\left(b^2-4\,ac\right)a^2\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}-\frac{e^{(-1)}}{2\,(xe+d)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^3),x, algorithm="giac
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(
-b^2 + 4*a*c))*e^(-1)/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) +
1/2*b*e^(-1)*ln(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^3 + 1/2*((b^
3*c - 3*a*b*c^2)/a + (b^4*e - 4*a*b^2*c*e + 2*a^2*c^2*e)*e^(-1)/(
(x*e + d)^2*a))*e^(-1)/((b^2 - 4*a*c)*a^2*(c + b/(x*e + d)^2 + a/
(x*e + d)^4)) - 1/2*e^(-1)/((x*e + d)^2*a^2)
```

3.629 $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=408

$$\frac{b\left(5b^2-19ac\right)}{2a^3e\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2e\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2
 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*
c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 +
 b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4 - 29*a*
b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 6.61183, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{b\left(5b^2-19ac\right)}{2a^3e\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2e\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2
 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*
c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 +
 b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4 - 29*a*
b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.51132, size = 384, normalized size = 0.94

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2\right)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+\frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-28\right.}{}}{12a^3e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*
b^2*c + 2*a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/(
(b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]
*Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*
c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x
))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c - 28*
a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
```

```
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e)
```

Maple [C] time = 0.048, size = 1518, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c^2*e^2/(4*a*c-b^2)*x^3-1/2/a^3/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)*b^3*c*e^2/(4*a*c-b^2)*x^3+9/2/a^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)*d*b*c^2*e/(4*a*c-b^2)*x^2-3/2/a^3/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*d*b^3*c*e/(4*a*c-b^2)*x^2+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(
4*a*c-b^2)*x*b*c^2*d^2-3/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*x*b^3*c*d^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2+2/a^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)/(4*a*c-b^2)*x*b^4+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4
*a*c-b^2)*b*c^2-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*
b^3*c-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/a^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)*d/e/(4*a*c-b^2)*b^4+1/4/a^3/e*sum((b*c*e^2*(19*a*c-5*b^2)*_R
^2+2*b*c*d*e*(19*a*c-5*b^2)*_R+19*a*b*c^2*d^2-5*b^3*c*d^2-14*a^2*
c^2+24*a*b^2*c-5*b^4)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*
_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+
b*d^2+a))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^4),x, algorithm="maxi
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 18*(5*b^3*c - 19*a*b*c^2)
*d*e^5*x^5 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19
*a*b*c^2)*d^2)*e^4*x^4 + 3*(5*b^3*c - 19*a*b*c^2)*d^6 + 4*(15*(5*
b^3*c - 19*a*b*c^2)*d^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e
^3*x^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^4 + (45*(5*b^3*c -
19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(15*b^4 - 62*a*b^2*c
+ 14*a^2*c^2)*d^2)*e^2*x^2 - 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*
a^2*b*c)*d^2 + 2*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62*a
*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x)/((a^3*b
^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 +
 (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 +
 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*
x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a
^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^
5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x
^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4
 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^
7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e) + 1/2
*integrate(((5*b^3*c - 19*a*b*c^2)*e^2*x^2 + 5*b^4 - 24*a*b^2*c +
 14*a^2*c^2 + 2*(5*b^3*c - 19*a*b*c^2)*d*e*x + (5*b^3*c - 19*a*b*
c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*
x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2
)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^
2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^3
```

Fricas [A] time = 0.609197, size = 7741, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^4),x, algorithm="fric
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2
)*d*e^5*x^5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c -
 19*a*b*c^2)*d^2)*e^4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*
(5*b^3*c - 19*a*b*c^2)*d^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d
)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^4 + 2*(45*(5*b
^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(15*b^4 - 62*a
*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 20*(a*
b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^
4 - 62*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x
- 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4
*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^
4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3
 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4
*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*
```

```
b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^
2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3
*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*
b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4
*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 -
2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*
a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 3
9525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*
a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*
b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b
^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 +
 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (112
5*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c
^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c
+ 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*
a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^
2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2
401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^
17*c^3)*e^4)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 -
75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 794
08*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c +
1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12
- 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^
15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a
^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*((a
^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x
^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x
^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*
e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 1
0*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2
)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e
^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)
*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2
)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*s
qrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3
+ 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*
a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2
- 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 240
1*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17
*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*
c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^
6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325
*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8
)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2
 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*s
qrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6
*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) - (
125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3
 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 -
10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2
- 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 4
```

```
8*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c +
 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 2410
8*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*
c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4
*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^
2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^
2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^
4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b
^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*
b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2
 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3
- 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 31
5*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4
- (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqr
t((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c
^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14
*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1
125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2
*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 4341
0*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/
2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*
c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8
250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*
b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*
b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (125*b^14 - 2425*a
*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6
*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)
*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^
3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 6
4*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^
2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2
401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^
17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^1
0*c^3)*e^2))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*
(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^
3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*
d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*
(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x
^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d
^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2
)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*
e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5
+ (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*
a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a
^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 825
0*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^
4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 123
25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c
^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 5
```

```
0421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 -
 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b
^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39
525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a
^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b
^2*c^2 - 64*a^17*c^3)*e^4)) + (125*b^14 - 2425*a*b^12*c + 18940*a
^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5
*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 -
315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^
4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*s
qrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6
*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((
a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))))/((
a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*
x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*
x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)
*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 +
10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^
2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*
e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c
)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^
2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^4),x, algorithm="giac
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*x + d)^4),
x)
```

3.630 $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=341

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(
d + e*x)^2))/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) + (3*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqr
t[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt
[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a
*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.89101, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(
d + e*x)^2))/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x
```

```
)^4)) + (3*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqr
t[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt
[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a
*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 124.697, size = 314, normalized size = 0.92

$$-\frac{3\sqrt{2}\sqrt{c}\left(ac+\frac{3b^2}{4}+\frac{b\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2e\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{3\sqrt{2}\sqrt{c}\left(ac+\frac{3b^2}{4}-\frac{b\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2e\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{\left(2a+b\left(d+ex\right)^2\right)\left(d+ex\right)}{4e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)^2}-\frac{\left(d+ex\right)\left(-4ac+7b^2+12bc\left(d+ex\right)^2\right)}{8e\left(-4ac+b^2\right)^2\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*sqrt(2)*sqrt(c)*(a*c + 3*b**2/4 + b*sqrt(-4*a*c + b**2)/2)*ata
n(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2*e*s
qrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + 3*sqrt(2)*
sqrt(c)*(a*c + 3*b**2/4 - b*sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*s
qrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(2*e*sqrt(b - sqr
t(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + (2*a + b*(d + e*x)**2
)*(d + e*x)/(4*e*(-4*a*c + b**2)*(a + b*(d + e*x)**2 + c*(d + e*x
)**4)**2) - (d + e*x)*(-4*a*c + 7*b**2 + 12*b*c*(d + e*x)**2)/(8*
e*(-4*a*c + b**2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 6.24794, size = 347, normalized size = 1.02

$$-\frac{-2a(d+ex)-b(d+ex)^3}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{4ac(d+ex)-7b^2(d+ex)-12bc(d+ex)^3}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}-4ac-3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(-2*a*(d + e*x) - b*(d + e*x)^3)/(4*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (-7*b^2*(d + e*x) + 4*a*c*(d + e*x)
- 12*b*c*(d + e*x)^3)/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) - (3*Sqrt[c]*(-3*b^2 - 4*a*c + 2*b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
 - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*
(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C]   time = 0.094, size = 704, normalized size = 2.1

$$\frac{1}{(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a)^2}\left(-\frac{3\,c^2e^6bx^7}{32\,a^2c^2-16\,ab^2c+2\,b^4}-\frac{21\,c^2de^5bx^6}{32\,a^2c^2-16\,ab^2c\right.$$
$$+\frac{3}{16\,e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(-4\,\_R^2bce^2-8\,\_R\,bcde-4\,bcd^2+4\,ac\right.}{(16\,a^2c^2-8\,ab^2c+b^4)\left(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e
^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-
19*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*
a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+48*a*b*c+
15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^6-20*a*c^2
*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*
c*d^4+16*a*b*c*d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^
2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/e*sum((-4*_R^2*b*c*e^2-8*
_R*b*c*d*e-4*b*c*d^2+4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^
3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),
_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(12*b*c^2*e^7*x^7 + 84*b*c^2*d*e^6*x^6 + (252*b*c^2*d^2 + 19
*b^2*c - 4*a*c^2)*e^5*x^5 + 12*b*c^2*d^7 + 5*(84*b*c^2*d^3 + (19*
b^2*c - 4*a*c^2)*d)*e^4*x^4 + (420*b*c^2*d^4 + 5*b^3 + 16*a*b*c +
 10*(19*b^2*c - 4*a*c^2)*d^2)*e^3*x^3 + (19*b^2*c - 4*a*c^2)*d^5
+ (252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*
b*c)*d)*e^2*x^2 + (5*b^3 + 16*a*b*c)*d^3 + (84*b*c^2*d^6 + 5*(19*
b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*
d^2)*e*x + 3*(a*b^2 + 4*a^2*c)*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7
 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^
5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5
*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^
3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 +
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 +
 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e) -
 3/8*integrate((4*b*c*e^2*x^2 + 8*b*c*d*e*x + 4*b*c*d^2 - b^2 - 4
*a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*a^
2*c^2)
```

Fricas [A] time = 0.511322, size = 8955, normalized size = 26.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*e^7*x^7 + 168*b*c^2*d*e^6*x^6 + 2*(252*b*c^2*d^2
+ 19*b^2*c - 4*a*c^2)*e^5*x^5 + 24*b*c^2*d^7 + 10*(84*b*c^2*d^3 +
```

(19*b^2*c - 4*a*c^2)*d)*e^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*
a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*e^3*x^3 + 2*(19*b^2*c - 4*a*
c^2)*d^5 + 2*(252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*
b^3 + 16*a*b*c)*d)*e^2*x^2 + 2*(5*b^3 + 16*a*b*c)*d^3 + 2*(84*b*c
^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b
^3 + 16*a*b*c)*d^2)*e*x - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8
x^7 + 2(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^
6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)
*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^
3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x
^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d
^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c
 + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*
e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*
c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a
^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^1
0 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b
^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a
^2*c^3)*e*x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqr
t(1/2)*((a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^
3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*s
qrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c
^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8 - 8*a*b^6*c +
128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^
2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*
c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20
*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4
 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2
 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) + 3*
sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e
^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 3
2*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6
 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d
^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c +

16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^
5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c +
80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^
4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^1
0 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b
^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b
^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))
log(3(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*b^4*c + 4
0*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11
*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336
*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^
8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024
*a^7*c^5)*e^4)) - (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^
4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b
^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 102
4*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2
- 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*
b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^
5*b^2*c^4 - 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^
2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d
^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 -
6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20
*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4
- 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b
^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))
)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1
280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c
^2 + 16*a^2*c^3)*e*x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d
+ 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a
^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c
^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*
a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a
*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*
c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640
*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*
b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^
6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^

3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^
2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8
+ 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3
+ 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*
b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4
+ 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 +
 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4
 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*
a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2
 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/
((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1
280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c +
160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c
^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*
b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8
*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^
4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 -
20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c
^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 2
56*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^
4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^
4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3
+ 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) + 6*(a*b^2 + 4*a^2*c)*d
)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 +
 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^
3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b
^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b
*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c +
16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^
2*b^3*c + 16*a^3*b*c^2)*d^2)*e)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.631 $\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=150

$$\frac{2a+b(d+ex)^2}{4e(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}$$

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e
*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(
d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi [A] time = 0.372762, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2a+b(d+ex)^2}{4e(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e
*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(
d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi in Sympy [A] time = 36.178, size = 134, normalized size = 0.89

$$\frac{3bc\,\mathrm{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\left(-4ac+b^2\right)^{\frac{5}{2}}}-\frac{3b\left(b+2c\left(d+ex\right)^2\right)}{4e\left(-4ac+b^2\right)^2\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}+\frac{2a+b\left(d+ex\right)^2}{4e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(-4*a*
c + b**2)**(5/2)) - 3*b*(b + 2*c*(d + e*x)**2)/(4*e*(-4*a*c + b**
2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) + (2*a + b*(d + e*x)
**2)/(4*e*(-4*a*c + b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)**
2)
```

Mathematica [A] time = 0.348234, size = 146, normalized size = 0.97

$$\frac{\frac{\left(b^2-4ac\right)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2}-\frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)
 + ((b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*
(d + e*x)^2))^2 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.064, size = 544, normalized size = 3.6

$$\frac{1}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a\right)^2}\left(-\frac{3\,c^2e^5bx^6}{32\,a^2c^2-16\,ab^2c+2\,b^4}-9\,\frac{c^2de^4bx^5}{16\,a^2c^2-8\,ab^2c}\right.$$
$$+\frac{3\,bc}{2\,e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{(-e\_R-d)\ln(x-\_R)}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\left(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^4-3*c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^3-1/2*b*e*(45*c^2*d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*d^2+5*a*c+b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+10*a*b*c*d^2+2*
b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c
```

```
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2+3/2*b*c/e*sum((-_R*e-d)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*
_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_
R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(
4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxima")
```

```
[Out] -3*b*c*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (
6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^
4 - 8*a*b^2*c + 16*a^2*c^2) - 1/4*(6*b*c^2*e^6*x^6 + 36*b*c^2*d*e
^5*x^5 + 6*b*c^2*d^6 + 9*(10*b*c^2*d^2 + b^2*c)*e^4*x^4 + 9*b^2*c
*d^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)*e^3*x^3 + 2*(45*b*c^2*d^4 +
27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*x^2 + a*b^2 + 8*a^2*c + 2*(b^3
+ 5*a*b*c)*d^2 + 4*(9*b*c^2*d^5 + 9*b^2*c*d^3 + (b^3 + 5*a*b*c)*d
)*e*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 +
 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*
x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^
3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 -
6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*
a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 +
 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2
*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Fricas [A] time = 0.44027, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(b*c^3*e^8*x^8 + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^
2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x
^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a
*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a
*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*
(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*
d^2)*e^2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c
*d + (b^3*c + 2*a*b*c^2)*d^3)*e*x)*log((2*(b^2*c - 4*a*c^2)*e^2*x
^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2
*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - (6
*b*c^2*e^6*x^6 + 36*b*c^2*d*e^5*x^5 + 6*b*c^2*d^6 + 9*(10*b*c^2*d
^2 + b^2*c)*e^4*x^4 + 9*b^2*c*d^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)
*e^3*x^3 + 2*(45*b*c^2*d^4 + 27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*x^
2 + a*b^2 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2 + 4*(9*b*c^2*d^5 + 9*
b^2*c*d^3 + (b^3 + 5*a*b*c)*d)*e*x)*sqrt(b^2 - 4*a*c))/(((b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 +
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^
2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*d^2)*e)*sqrt(b^2 - 4*a*c)), -1/4*(12*(b*c^3*e^8*x^8
+ 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*
d^8 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (7
0*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14
*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*
a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*(14*b*c^3*d^6 + 15*b^2*
c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*x^2 + a^2*b*c
+ 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2
)*d^3)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (6*b*c^2*e^6*x^6 + 36*b*c^2*d*e^5*
x^5 + 6*b*c^2*d^6 + 9*(10*b*c^2*d^2 + b^2*c)*e^4*x^4 + 9*b^2*c*d^
4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)*e^3*x^3 + 2*(45*b*c^2*d^4 + 27*
b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*x^2 + a*b^2 + 8*a^2*c + 2*(b^3 + 5
*a*b*c)*d^2 + 4*(9*b*c^2*d^5 + 9*b^2*c*d^3 + (b^3 + 5*a*b*c)*d)*e
*x)*sqrt(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^
9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5
```

```
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6
- 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*
(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4
*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8
*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a
^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-b^2
+ 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac")

[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.632 $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$

Optimal. Leaf size=363

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^
2 + 20*a*c)*(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

Rubi [A] time = 2.01332, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^
```

```
2 + 20*a*c)*(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

Rubi in Sympy [A] time = 117.204, size = 332, normalized size = 0.91

$$-\frac{\left(b+2c\left(d+ex\right)^{2}\right)\left(d+ex\right)}{4e\left(-4ac+b^{2}\right)\left(a+b\left(d+ex\right)^{2}+c\left(d+ex\right)^{4}\right)^{2}} - \frac{\sqrt{2}\sqrt{c}\left(b\left(-52ac+b^{2}\right)-\sqrt{-4ac+b^{2}}\left(20ac+b^{2}\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^{2}}}}\right)}{16ae\sqrt{b+\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{5}{2}}} + \frac{\sqrt{2}\sqrt{c}\left(b\left(-52ac+b^{2}\right)+\sqrt{-4ac+b^{2}}\left(20ac+b^{2}\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^{2}}}}\right)}{16ae\sqrt{b-\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{5}{2}}} + \frac{\left(d+ex\right)\left(b\left(8ac+b^{2}\right)+c\left(d+ex\right)^{2}\left(20ac+b^{2}\right)\right)}{8ae\left(-4ac+b^{2}\right)^{2}\left(a+b\left(d+ex\right)^{2}+c\left(d+ex\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -(b + 2*c*(d + e*x)**2)*(d + e*x)/(4*e*(-4*a*c + b**2)*(a + b*(d
+ e*x)**2 + c*(d + e*x)**4)**2) - sqrt(2)*sqrt(c)*(b*(-52*a*c + b
**2) - sqrt(-4*a*c + b**2)*(20*a*c + b**2))*atan(sqrt(2)*sqrt(c)*
(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(16*a*e*sqrt(b + sqrt(-4
*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + sqrt(2)*sqrt(c)*(b*(-52*a
*c + b**2) + sqrt(-4*a*c + b**2)*(20*a*c + b**2))*atan(sqrt(2)*sq
rt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(16*a*e*sqrt(b - s
qrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + (d + e*x)*(b*(8*a*c
 + b**2) + c*(d + e*x)**2*(20*a*c + b**2))/(8*a*e*(-4*a*c + b**2)
**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 6.27244, size = 405, normalized size = 1.12

$$-\frac{b(d+ex)+2c(d+ex)^3}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{8abc(d+ex)+20ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{8ae\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(b*(d + e*x) + 2*c*(d + e*x)^3)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (b^3*(d + e*x) + 8*a*b*c*(d + e*x) +
b^2*c*(d + e*x)^3 + 20*a*c^2*(d + e*x)^3)/(8*a*(-b^2 + 4*a*c)^2*e
*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^3 - 52*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*
(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(-b
^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.094, size = 885, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^
2*d*e^5*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*
c^2*d^2+21*b^2*c*d^2+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+
b^4)/a*x^5+5/8*c*d*e^3*(140*a*c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)
/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1/8*e^2*(700*a*c^3*d^4+35*b^2*c
^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^2+5*a*b^2*c+b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21*b^2*c^2*d
^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16
```

```
*a^2*c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+14
0*a*b*c^2*d^4+10*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d
^2+16*a^2*b*c-a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a
*c^3*d^6+b^2*c^2*d^6+28*a*b*c^2*d^4+2*b^3*c*d^4+36*a^2*c^2*d^2+5*
a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/
a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/a/e*sum((e^2*c*(20*a*c+b^2)*_R^2
+2*c*d*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c*d^2-16*a*b*c+b^3)/(16
*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3
+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*e^7*x^7 + 7*(b^2*c^2 + 20*a*c^3)*d*e^6*
x^6 + (2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*x^
5 + 5*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4
*x^4 + (b^2*c^2 + 20*a*c^3)*d^7 + (35*(b^2*c^2 + 20*a*c^3)*d^4 +
b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*x
^3 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (21*(b^2*c^2 + 20*a*c^3)*d^5 +
20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)
*e^2*x^2 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + (7*(b^2*c^2 + 20*
a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c + 3
*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*x - (a*b^3 - 16*a^2*b*c)*d
)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)
*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 1
0*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b
^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4
*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5
+ (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c
 + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^
2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3
)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e) + 1/8*in
tegrate(((b^2*c + 20*a*c^2)*e^2*x^2 + 2*(b^2*c + 20*a*c^2)*d*e*x
```

```
+ b^3 - 16*a*b*c + (b^2*c + 20*a*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3
*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a), x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
```

Fricas [A] time = 0.584824, size = 10396, normalized size = 28.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*
e^6*x^6 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*
e^5*x^5 + 10*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)
*d)*e^4*x^4 + 2*(b^2*c^2 + 20*a*c^3)*d^7 + 2*(35*(b^2*c^2 + 20*a*
c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)
*d^2)*e^3*x^3 + 4*(b^3*c + 14*a*b*c^2)*d^5 + 2*(21*(b^2*c^2 + 20*
a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 3
6*a^2*c^2)*d)*e^2*x^2 + 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + 2*
(7*(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3
 + 16*a^2*b*c + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*x - sqrt(
1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4
*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d
^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5
+ 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^
2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*
a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b
^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3
 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d
^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8
*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*
c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(
-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10
 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^
2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(
(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 12
80*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c
 + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^
8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4
 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*
b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^
```

12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1
536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^
8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*
e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 +
 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c
+ 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 1
60*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^
5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*
b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1
024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2
- 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + sqr
t(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b
^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a
^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 +
 (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)
*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^
5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 1
5*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2
*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)
*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*
b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqr
t(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^
10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*
b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)
/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8
*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c
^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^
2*b^2*c^4 + 10000*a^3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*
b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 +
 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt(
(b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*
a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5
)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3
 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*
c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c +
 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*
c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^
7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 -
 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^
2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + s
qrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8

*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) + (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3

```
+ 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5
*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)
*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^
3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*
a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*
c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^
3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4
*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1
024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b
^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 1500
0*a^2*b^2*c^4 + 10000*a^3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*
a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c
^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*s
qrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c +
160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11
*c^5)*e^4)) + (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5
*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*
b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8
*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 2
0*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c
^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^
6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2)))
 - 2*(a*b^3 - 16*a^2*b*c)*d)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*
x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 -
 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*
a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*
c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2
*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3
 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*
d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d
^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6
 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*
a^4*b*c^2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.633 $\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=150

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] -(b + 2*c*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*
e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*
(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi [A] time = 0.357186, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(b + 2*c*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*
e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*
(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi in Sympy [A] time = 32.4008, size = 134, normalized size = 0.89

$$-\frac{6c^2\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{3c\left(b+2c\left(d+ex\right)^2\right)}{2e\left(-4ac+b^2\right)^2\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}-\frac{b+2c\left(d+ex\right)^2}{4e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -6*c**2*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(-4*
a*c + b**2)**(5/2)) + 3*c*(b + 2*c*(d + e*x)**2)/(2*e*(-4*a*c + b
**2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) - (b + 2*c*(d + e*
x)**2)/(4*e*(-4*a*c + b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)
**2)
```

Mathematica [A] time = 0.305838, size = 147, normalized size = 0.98

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{(b^2-4ac)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d
 + e*x)^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c
*(d + e*x)^4) + (24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.068, size = 541, normalized size = 3.6

$$\frac{1}{\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a\right)^2}\left(3\,\frac{c^3e^5x^6}{16\,a^2c^2-8\,ab^2c+b^4}+18\,\frac{c^3de^4x^5}{16\,a^2c^2-8\,ab^2c+\right.$$
$$+3\,\frac{c^2}{e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{(e\_R+d)\ln(x-\_R)}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\left(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,d^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*d*c^3/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^4+6*c^2*d*e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^3+c*e*(45*c^2*d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^
2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*c^2*d^2+4*b^2*c*d^
2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*
```

```
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2+3*c^2/e*sum((_R*e+d)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^3*c*e^3
+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=Root
Of(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+
2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxima")
```

```
[Out] 6*c^2*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4
 - 8*a*b^2*c + 16*a^2*c^2) + 1/4*(12*c^3*e^6*x^6 + 72*c^3*d*e^5*x
^5 + 12*c^3*d^6 + 18*(10*c^3*d^2 + b*c^2)*e^4*x^4 + 18*b*c^2*d^4
+ 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*x^3 + 4*(45*c^3*d^4 + 27*b*c^2*
d^2 + b^2*c + 5*a*c^2)*e^2*x^2 - b^3 + 10*a*b*c + 4*(b^2*c + 5*a*
c^2)*d^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2*c + 5*a*c^2)*d)*e*x)
/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 +
4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^
2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^
4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*
c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Fricas [A] time = 0.431268, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2 + b*c^3)*
e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*
d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4
*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*
a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*
d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*x^2 + a^2*c^2 + 4*
(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e
*x)*log(-(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x
 + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - (2*c^2*e^4*x^4 + 8*c
^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^
2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2
 + 2*(2*c*d^3 + b*d)*e*x + a)) + (12*c^3*e^6*x^6 + 72*c^3*d*e^5*x
^5 + 12*c^3*d^6 + 18*(10*c^3*d^2 + b*c^2)*e^4*x^4 + 18*b*c^2*d^4
+ 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*x^3 + 4*(45*c^3*d^4 + 27*b*c^2*
d^2 + b^2*c + 5*a*c^2)*e^2*x^2 - b^3 + 10*a*b*c + 4*(b^2*c + 5*a*
c^2)*d^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2*c + 5*a*c^2)*d)*e*x)
*sqrt(b^2 - 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^
8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*
a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3
 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d
^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a
^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b
^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^
4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(b^2 - 4*a
*c)), 1/4*(24*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2 + b*
c^3)*e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b
*c^3*d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^
4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3
 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b
*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*x^2 + a^2*c^2
 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d
^3)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^
2 + 4*a*c)/(b^2 - 4*a*c)) + (12*c^3*e^6*x^6 + 72*c^3*d*e^5*x^5 +
12*c^3*d^6 + 18*(10*c^3*d^2 + b*c^2)*e^4*x^4 + 18*b*c^2*d^4 + 24*
(10*c^3*d^3 + 3*b*c^2*d)*e^3*x^3 + 4*(45*c^3*d^4 + 27*b*c^2*d^2 +
 b^2*c + 5*a*c^2)*e^2*x^2 - b^3 + 10*a*b*c + 4*(b^2*c + 5*a*c^2)*
d^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2*c + 5*a*c^2)*d)*e*x)*sqrt
(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 +
8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

```
*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 +
3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 +
30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2
*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 +
 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 -
 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 +
2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-b^2 + 4*a*c)
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac")

[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.634 $\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=437

$$\frac{\left(\frac{d}{e}+x\right)\left(3bce^2\left(b^2-8ac\right)\left(\frac{d}{e}+x\right)^2+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2}$$

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c
)*(a + b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b
^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2
))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*e^2*(d/e + x)^2 + c*e^4*(d/e + x
)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*
c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^
2 - b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 9.91781, antiderivative size = 437, normalized size of antiderivative = 1., number of

steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{(d+ex)\left(3bc\left(b^2-8ac\right)(d+ex)^2+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(d+ex)\left(-2ac+b^2+bc(d+ex)^2\right)}{4ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/(4*a*(b^2 - 4*a*c)*e*
(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*((b^2 - 7*a*c
)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*(d + e*x)^2))/(8*a^2*(b^2
 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(
b^4 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b*(b^2 - 8*a*c)*
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.27111, size = 463, normalized size = 1.06

$$\frac{28a^2c^2(d+ex)-25ab^2c(d+ex)-24abc^2(d+ex)^3+3b^4(d+ex)+3b^3c(d+ex)^3}{8a^2e\,(4ac-b^2)^2\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2e\,(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{c}\left(-56a^2c^2+10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2e\,(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{2ac(d+ex)-b^2(d+ex)-bc(d+ex)^3}{4ae\,(4ac-b^2)\,(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] (-(b^2*(d + e*x)) + 2*a*c*(d + e*x) - b*c*(d + e*x)^3)/(4*a*(-b^2
 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4*(d +
e*x) - 25*a*b^2*c*(d + e*x) + 28*a^2*c^2*(d + e*x) + 3*b^3*c*(d +
 e*x)^3 - 24*a*b*c^2*(d + e*x)^3)/(8*a^2*(-b^2 + 4*a*c)^2*e*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c +
56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
+ (3*Sqrt[c]*(-b^4 + 10*a*b^2*c - 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a
*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x
))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.097, size = 1010, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21
/8*c^2*d*e^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8
*(-504*a*b*c^2*d^2+63*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*e^4*
c/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^
2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c
+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^3*c^2*d^4-280*a^2*c^
```

```
3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*b^3*c-3*b^5
)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-63*
b^3*c^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2
*b*c^2+60*a*b^3*c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(
-168*a*b*c^3*d^6+21*b^3*c^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4
+30*b^4*c*d^4-12*a^2*b*c^2*d^2-60*a*b^3*c*d^2+9*b^5*d^2+44*a^3*c^
2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x+1/8*d/e*
(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d^4-49*a*b^2*c^2*d^4+6*
b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+44*a^3*c^2-37*
a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2)/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2+3/16/a^2/e*sum((b*c*e^2*(-8*a*c+b^2)*_R^2+2*b*c*d*e*(-
8*a*c+b^2)*_R-8*a*b*c^2*d^2+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^4)/(
16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2
*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-3),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 21*(b^3*c^2 - 8*a*b*c^3)*d
*e^6*x^6 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8
*a*b*c^3)*d^2)*e^5*x^5 + 5*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4
*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d)*e^4*x^4 + 3*(b^3*c^2 - 8*a*b*c
^3)*d^7 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2 + 105*(b^3*c^2 - 8*a*
b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^2)*e^3*x^
3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + (63*(b^3*c^2 - 8*
a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3*(
3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + (3*b^5 - 20*a*b^3*
c - 4*a^2*b*c^2)*d^3 + (21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 -
37*a^2*b^2*c + 44*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^
3)*d^4 + 3*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d^2)*e*x + (5*a*b^4
 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^
3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 3
2*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 3
0*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14
*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*
a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 1
4*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c
```

```
- 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c +
32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16
*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5
+ (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3
*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*
a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c
- 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32
*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)
- 3/8*integrate(-((b^3*c - 8*a*b*c^2)*e^2*x^2 + b^4 - 9*a*b^2*c +
 28*a^2*c^2 + 2*(b^3*c - 8*a*b*c^2)*d*e*x + (b^3*c - 8*a*b*c^2)*d
^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2)
```

Fricas [A] time = 0.755881, size = 11548, normalized size = 26.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-3),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 42*(b^3*c^2 - 8*a*b*c^3)*
d*e^6*x^6 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2
- 8*a*b*c^3)*d^2)*e^5*x^5 + 10*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6
*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d)*e^4*x^4 + 6*(b^3*c^2 - 8*a
*b*c^3)*d^7 + 2*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2 + 105*(b^3*c^2
- 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^2)*
e^3*x^3 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + 2*(63*(b^
3*c^2 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)
*d^3 + 3*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + 2*(3*b^5
 - 20*a*b^3*c - 4*a^2*b*c^2)*d^3 + 2*(21*(b^3*c^2 - 8*a*b*c^3)*d^
6 + 5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c
^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d^2)*
e*x + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9
*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*
(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c -
8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4
*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2
)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 +
 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*
a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c
```

^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) - (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)

```
))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3
+ 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*c^3 - 44
7*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^
7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 1920
8*a^3*b^2*c^6 + 38416*a^4*c^7)*d - 27/2*sqrt(1/2)*((a^5*b^15 - 31
*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7
*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7
)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3
 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) - (b
^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a
^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c
^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^
3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 -
 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^
8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^
4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*
c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a
^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
 1024*a^10*c^5)*e^2))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^
3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4
*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 1
4*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14
*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c -
8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c
 + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4
 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4
*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5
*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c +
 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2
 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^
5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*
c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3
+ 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*
d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^
5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)
*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 +
 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 64
0*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 -
 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/
((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*
b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 10
24*a^10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*
b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3
 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a
^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b
^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c
^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*
b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10
*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 128
```

0*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) + (b^14 - 32*a*b^12*c + 464
*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*
b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*
a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a
^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280
*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^
2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^1
1*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4
 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*
c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2)))
 + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8
 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*
b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b
^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*
b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2
*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^
3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16
*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3
 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 1
6*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^
3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*
b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*
d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 1
6*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b
^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*
b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^
9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b
^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b
^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 -
1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2
 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(
27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2
*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189
*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d - 27/2*sqrt(1
/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*
c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6
 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2
 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c +
 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a
^15*c^5)*e^4)) + (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^
3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^
2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c
^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c
+ 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^1
0*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^
2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*
c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4))
)/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 +

```
 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) + 2*(5*a*b^4 - 37*a^2*b
^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)
*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7
 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 +
 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*
c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c
^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*
c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)
*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*
c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3
*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)
*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d
^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6
- 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^
8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3
*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.313712, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((e*x + d)^4*c + (e*x + d)^2*b + a)^(-3),x, algorithm="giac")
```

```
[Out] Done
```

3.635 $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=255

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}+\frac{\log(d+ex)}{a^3e}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2
*b*c*(b^2 - 7*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)
*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 -
4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b*(d + e*x)^2 +
c*(d + e*x)^4]/(4*a^3*e)
```

Rubi [A] time = 0.939089, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}+\frac{\log(d+ex)}{a^3e}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```


```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2
*b*c*(b^2 - 7*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)
*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 -
4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b*(d + e*x)^2 +
c*(d + e*x)^4]/(4*a^3*e)
```

Rubi in Sympy [A] time = 106.21, size = 238, normalized size = 0.93

$$\frac{-2ac+b^2+bc\,(d+ex)^2}{4ae\,(-4ac+b^2)\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)^2}+\frac{16a^2c^2-15ab^2c+2b^4+2bc\,(d+ex)^2\left(-7ac+b^2\right)}{4a^2e\,(-4ac+b^2)^2\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^3e\,(-4ac+b^2)^{\frac{5}{2}}}+\frac{\log\left((d+ex)^2\right)}{2a^3e}-\frac{\log\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}{4a^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(4*a*e*(-4*a*c + b**2)*(a + b*
(d + e*x)**2 + c*(d + e*x)**4)**2) + (16*a**2*c**2 - 15*a*b**2*c
+ 2*b**4 + 2*b*c*(d + e*x)**2*(-7*a*c + b**2))/(4*a**2*e*(-4*a*c
+ b**2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) + b*(30*a**2*c*
*2 - 10*a*b**2*c + b**4)*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c
 + b**2))/(2*a**3*e*(-4*a*c + b**2)**(5/2)) + log((d + e*x)**2)/(
2*a**3*e) - log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(4*a**3*e)
```

Mathematica [A] time = 6.2354, size = 421, normalized size = 1.65

$$\frac{\log(d+ex)}{a^3e}+\frac{16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^4+2b^3c(d+ex)^2}{4a^2e\,(4ac-b^2)^2\,(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\left(-16a^2c^2\sqrt{b^2-4ac}-30a^2bc^2+10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac}-b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^3e\,(b^2-4ac)^{5/2}}+\frac{\left(-16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac}+b^5\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^3e\,(b^2-4ac)^{5/2}}+\frac{2ac-b^2-bc(d+ex)^2}{4ae\,(4ac-b^2)\,(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-b^2 + 2*a*c - b*c*(d + e*x)^2)/(4*a*(-b^2 + 4*a*c)*e*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 +
 2*b^3*c*(d + e*x)^2 - 14*a*b*c^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a
*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + Log[d + e*x]/(a^3*
e) + ((-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] +
 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[
b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^3*(b^2 - 4*a*c)^(5
/2)*e) + ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c
```

```
] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*L
og[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^3*(b^2 - 4*a*c)
^(5/2)*e)
```

Maple [C] time = 0.103, size = 4477, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3
*c^2*d^6-29/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+
b^4)*b^2*c^2*d^4+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a
*b^2*c+b^4)*b^4*c*d^4-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-
8*a*b^2*c+b^4)*b^3*c*d^2-7/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5*b
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*
c^2*e^5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-29/4/a/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2+1/a^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4-3/a/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c-21
/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^3*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^5+16/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^3+24/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2*c^3*d^2-1/2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^2*b*c^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^2*b^5+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^5+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2
*c^2-8*a*b^2*c+b^4)*b^5*d^2+ln(e*x+d)/a^3/e-21/a/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^3+3/a^2/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
```

*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c^2-29/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+4/a^
2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c
-6/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*
c-7/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c
^3*d^6+3/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4
)*b^4-1/2/a^3/e*sum((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*
e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^
2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8
*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(16*a^
2*c^2-8*a*b^2*c+b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6
*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+6*a
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2+16/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3+4/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-21/4/(c
*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c*b^2-1/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^2+4/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+3/a^
2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2*b^3*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^5-105/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^4*b*d^2-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^3*b^2+4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*
e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-105/2/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+15/2/a^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c^2*d^4-8
7/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b
^2*c^2*d^2+6/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*b^4*c*d^2+15/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^3*d^2-70/a/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*c^3*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b+10/a^2/(c*e^4*x^4+4

```
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*
x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)),x, algorithm="maxima
```

```
[Out] 1/4*(2*(b^3*c^2 - 7*a*b*c^3)*e^6*x^6 + 12*(b^3*c^2 - 7*a*b*c^3)*d
*e^5*x^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3 + 30*(b^3*c^2 - 7
*a*b*c^3)*d^2)*e^4*x^4 + 2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 4*(10*(b^3
*c^2 - 7*a*b*c^3)*d^3 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d)*
e^3*x^3 + 3*a*b^4 - 21*a^2*b^2*c + 24*a^3*c^2 + (4*b^4*c - 29*a*b
^2*c^2 + 16*a^2*c^3)*d^4 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2 + 15*(b
^3*c^2 - 7*a*b*c^3)*d^4 + 3*(4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)
*d^2)*e^2*x^2 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 4*(3*(b^3*c
^2 - 7*a*b*c^3)*d^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^3 +
 (b^5 - 6*a*b^3*c - a^2*b*c^2)*d)*e*x)/((a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a
^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 +
 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(
14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c
- 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4
*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d
^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 +
 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b
^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c
 + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c
^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*
b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^
4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^
3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a
^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*
b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*
c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^
2)*e) - integrate(((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^3*x^3 + 3
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^2*x^2 + (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*d^3 + (b^5 - 9*a*b^3*c + 23*a^2*b*c^2 + 3*(b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e*x + (b^5 - 9*a*b^3*c + 23*a^
2*b*c^2)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^
2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2) + log(e*x + d)/(a^3*e)
```

Fricas [A] time = 1.29001, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)),x, algorithm="fricas
```

```
[Out] [1/4*(((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4
*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4
)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^
3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2
*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c
^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4
 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^
2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*
b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^
7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7
- 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*
a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*
b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(
b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2
*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10
*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2
*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)
*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*log((2*(b^
2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b*
c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c
*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 +
 b*d)*e*x + a)) + (2*(a*b^3*c^2 - 7*a^2*b*c^3)*e^6*x^6 + 12*(a*b^
3*c^2 - 7*a^2*b*c^3)*d*e^5*x^5 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16
*a^3*c^3 + 30*(a*b^3*c^2 - 7*a^2*b*c^3)*d^2)*e^4*x^4 + 2*(a*b^3*c
^2 - 7*a^2*b*c^3)*d^6 + 4*(10*(a*b^3*c^2 - 7*a^2*b*c^3)*d^3 + (4*
a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d)*e^3*x^3 + 3*a^2*b^4 - 2
1*a^3*b^2*c + 24*a^4*c^2 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c
^3)*d^4 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2 + 15*(a*b^3*c^2 - 7*
a^2*b*c^3)*d^4 + 3*(4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d^2)
*e^2*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*d^2 + 4*(3*(a*b^3*
c^2 - 7*a^2*b*c^3)*d^5 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3
)*d^3 + (a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*d)*e*x - ((b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*e^8*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d*e^7*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^6*x^6 + 4*(14*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*d)*e^5*x^5 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4
+ 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(14*(b^4*c^2 - 8
```

*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^3*x^3 + a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 +
2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
+ 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 8*a^
2*b^3*c + 16*a^3*b*c^2)*d^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 -
6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e
^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c^2 - 8*a*b
^2*c^3 + 16*a^2*c^4)*e^8*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d*e^7*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^6*x^6 + 4*(14*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d)*e^5*x^5 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + (b^6
- 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + 2
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(14*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^3*x^3 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3
*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)*d^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a
*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d
)*e*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/(((a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*e^9*x^8 + 8*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16
*a^5*c^4)*d*e^8*x^7 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3
+ 14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^7*x^6 + 4
(14(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + 3*(a^3*b^5*
c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^6*x^5 + (a^3*b^6 - 6*a^4*b
^4*c + 32*a^6*c^3 + 70*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
d^4 + 30(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 10*(a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + (a^3*b^6 - 6*a^4*b^4
*c + 32*a^6*c^3)*d)*e^4*x^3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b
c^2 + 14(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 15*(a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 3*(a^3*b^6 - 6*a^4*
b^4*c + 32*a^6*c^3)*d^2)*e^3*x^2 + 4*(2*(a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*d^7 + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c
^3)*d^5 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + (a^4*b^5 - 8
*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^2*x + (a^5*b^4 - 8*a^6*b^2*c + 16
*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 2*(a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + (a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*
d^2)*e)*sqrt(b^2 - 4*a*c)), -1/4*(2*((b^5*c^2 - 10*a*b^3*c^3 + 30
*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d
*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2
- 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 1
0*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^
2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d

^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2
+ 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*
b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^
5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c
^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60
*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a
^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^
5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c
^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 +
60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2
*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*
(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c +
10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a
^3*b^2*c^2)*d)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 +
b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*(a*b^3*c^2 - 7*a^2*b*c^
3)*e^6*x^6 + 12*(a*b^3*c^2 - 7*a^2*b*c^3)*d*e^5*x^5 + (4*a*b^4*c
- 29*a^2*b^2*c^2 + 16*a^3*c^3 + 30*(a*b^3*c^2 - 7*a^2*b*c^3)*d^2)
*e^4*x^4 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*d^6 + 4*(10*(a*b^3*c^2 - 7
*a^2*b*c^3)*d^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d)*e^
3*x^3 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + (4*a*b^4*c - 29*a
^2*b^2*c^2 + 16*a^3*c^3)*d^4 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2
+ 15*(a*b^3*c^2 - 7*a^2*b*c^3)*d^4 + 3*(4*a*b^4*c - 29*a^2*b^2*c
^2 + 16*a^3*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^
2)*d^2 + 4*(3*(a*b^3*c^2 - 7*a^2*b*c^3)*d^5 + (4*a*b^4*c - 29*a^2
*b^2*c^2 + 16*a^3*c^3)*d^3 + (a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*d)
*e*x - ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^8*x^8 + 8*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^7*x^7 + 2*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^6*
x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^5*x^5 + (b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*d^8 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*d^2)*e^4*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d
)*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d^4 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d
^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^2*
x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 4*(2*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)
+ 4*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^8*x^8 + 8*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d*e^7*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^6*x^6
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^5*x^5 + (b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^8 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^2)*e^4*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 +
4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*

```
b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e
^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c +
32*a^3*c^3)*d^4 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6
+ a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^2*x^2
 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 4*(2*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))*sqrt(-b^2 + 4*a*c))/(((
a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^9*x^8 + 8*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^8*x^7 + 2*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3 + 14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5
*c^4)*d^2)*e^7*x^6 + 4*(14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^3 + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^6*x^
5 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 70*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^4 + 30*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a
^5*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*
a^5*c^4)*d^5 + 10*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3
+ (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^4*x^3 + 2*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2 + 14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16
*a^5*c^4)*d^6 + 15*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4
 + 3*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^3*x^2 + 4*(2*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 3*(a^3*b^5*c - 8*a^
4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3)*d^3 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^2*x + (a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 1
6*a^5*c^4)*d^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6
 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 2*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d^2)*e)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.347916, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)),x, algorithm="giac")

[Out] Done
```

3.636 $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=484

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3e\left(b^2-4ac\right)^2(d+ex)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d +
 e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d
+ e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2
*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 -
 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*S
qrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c +
 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^
2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 2.60704, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3e\left(b^2-4ac\right)^2(d+ex)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d +
 e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d
+ e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2
*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 -
 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*S
qrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c +
 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^
2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.35516, size = 560, normalized size = 1.16

$$-\frac{1}{a^3e(d+ex)}+\frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}-5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{-84a^2bc^2(d+ex)-52a^2c^3(d+ex)^3+52ab^3c(d+ex)+47ab^2c^2(d+ex)^3-7b^5(d+ex)-7b^4c(d+ex)^3}{8a^3e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2
*c*(d + e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*(a
+ b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^
3*c*(d + e*x) - 84*a^2*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47
*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4
*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*(5*b^
5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b
^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt
[c]*(-5*b^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c
] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.093, size = 6821, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^2),x, algorithm="maxi
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 24*(5*b
^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + (30*b^5*c - 227*a
*b^3*c^2 + 392*a^2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*
c^4)*d^2)*e^6*x^6 + 6*(28*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)
*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d)*e^5*x^5 + 3*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + (15*b^6 - 91*a*b^4*
c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37*a*b^2*c^3
+ 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)
*d^2)*e^4*x^4 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 +
```

```
4*(42*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a
^2*b^2*c^2 + 324*a^3*c^3)*d)*e^3*x^3 + 8*a^2*b^4 - 64*a^3*b^2*c +
 128*a^4*c^2 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^
3)*d^4 + (84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^6 + 25*a*b
^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c - 227*a*b^3*c^2
 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 +
 324*a^3*c^3)*d^2)*e^2*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*
b*c^2)*d^2 + 2*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 +
3*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91
*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^
2*b^3*c + 364*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^
4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(
a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + 14*(6*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32
*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 4
2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*
(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32
*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 +
42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c
 - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c +
 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d
^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*
a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*
a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b
*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 -
 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^
6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e) - 3/8*i
ntegrate((5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2 + (5*b^4*c - 37*a*b^2
*c^2 + 60*a^2*c^3)*e^2*x^2 + 2*(5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c
^3)*d*e*x + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*d^2)/(c*e^4*x^4
 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a), x)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

Fricas [A] time = 1.14961, size = 13851, normalized size = 28.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^2),x, algorithm="fric
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*
```

b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 22
7*a*b^3*c^2 + 392*a^2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a
^2*c^4)*d^2)*e^6*x^6 + 12*(28*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*
c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d)*e^5*x^5
+ 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + 2*(15*b^6 - 91*
a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37*a*b^
2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*
b*c^3)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^6 + 8*(42*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30
*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*
c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d)*e^3*x^3 + 16*a^2*b^4 - 128*a
^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2
+ 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4
)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25
*a^2*b^2*c^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*
b^3*c + 364*a^3*b*c^2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60
*a^2*c^4)*d^7 + 3*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5
+ 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^3 + (2
5*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x - 3*sqrt(1/2)*((a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5
*c^4)*d^2)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6
+ (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^
5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 +
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16
*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4
+ 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*
a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a
^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2
+ (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b
^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5
*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4
*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(
a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*
c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7
*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 +
(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1
280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^
10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^
4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 10
24*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 -
640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(
-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 19573
49*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(

4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^

```
5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6
*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9
)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^1
2*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^
6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c
^8)*e^3*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351
310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^
17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)) - (125*b^17
 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 162
3534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 56
84672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*
b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4
- 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 -
640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((6
25*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3
 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^1
4*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 12
80*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c
 + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*
a^12*c^5)*e^2))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16
*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + 14*(6*(a^3
*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a
^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*
a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a
^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42
*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 3
2*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16
*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5
 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^
5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^
4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c
^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9
+ 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6
*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*
b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25
*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 2772
0*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*
a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^
5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 3513
10*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*
a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^1
7*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10
- 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b
^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(4125*b^10*c^4 - 77825*a*b^
8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^
```

2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7

```
 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4
- 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 28
35000*a^4*b^2*c^8 - 810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^
16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 6
8640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 3
23584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625*b^12 - 12250*
a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^
4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15
*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4
- 1024*a^19*c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^
13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*
b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8
*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1501
5*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 -
 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^
2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 9472
5*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*
a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^
16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5
)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b
^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))))/((a^3*b^4*c^2
 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)
*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 +
 (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d
^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5
 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*
d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*
b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2
 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^
2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^
3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 1
6*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^
6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a
^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7
*c^2)*d)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^2),x, algorithm="giac
```

```
[Out] Exception raised: TypeError
```

3.637 $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=325

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}-\frac{3b\log(d+ex)}{a^4e}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3e\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*
x)^2) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d +
 e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^
2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 -
 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3
*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*
c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)*e)
- (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d
+ e*x)^4])/(4*a^4*e)
```

Rubi [A] time = 1.16559, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}-\frac{3b\log(d+ex)}{a^4e}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3e\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*
x)^2) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d +
```

```
e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^
2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 -
 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3
*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*
c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)*e)
- (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d
+ e*x)^4])/(4*a^4*e)
```

Rubi in Sympy [A] time = 172.743, size = 313, normalized size = 0.96

$$\frac{-2ac+b^2+bc(d+ex)^2}{4ae(d+ex)^2(-4ac+b^2)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{20a^2c^2-20ab^2c+3b^4+3bc(d+ex)^2\left(-6ac+b^2\right)}{4a^2e(d+ex)^2(-4ac+b^2)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{3\left(-5ac+b^2\right)\left(-2ac+b^2\right)}{2a^3e(d+ex)^2(-4ac+b^2)^2} - \frac{3b\log\left((d+ex)^2\right)}{2a^4e} + \frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e} - \frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^4e(-4ac+b^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(4*a*e*(d + e*x)**2*(-4*a*c +
b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)**2) + (20*a**2*c**2 -
 20*a*b**2*c + 3*b**4 + 3*b*c*(d + e*x)**2*(-6*a*c + b**2))/(4*a*
*2*e*(d + e*x)**2*(-4*a*c + b**2)**2*(a + b*(d + e*x)**2 + c*(d +
 e*x)**4)) - 3*(-5*a*c + b**2)*(-2*a*c + b**2)/(2*a**3*e*(d + e*x
)**2*(-4*a*c + b**2)**2) - 3*b*log((d + e*x)**2)/(2*a**4*e) + 3*b
*log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(4*a**4*e) - 3*(-20*a**
3*c**3 + 30*a**2*b**2*c**2 - 10*a*b**4*c + b**6)*atanh((b + 2*c*(
d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*a**4*e*(-4*a*c + b**2)**(5/2
))
```

Mathematica [A] time = 6.27796, size = 491, normalized size = 1.51

$$-\frac{3b\log(d+ex)}{a^4e}-\frac{1}{2a^3e(d+ex)^2}+\frac{-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{-46a^2bc^2-28a^2c^3(d+ex)^2+29ab^3c+26ab^2c^2(d+ex)^2-4b^5-4b^4c(d+ex)^2}{4a^3e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\left(-20a^3c^3+30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+b^6\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d\right.}{4a^4e\left(b^2-4ac\right)^{5/2}}+\frac{3\left(20a^3c^3-30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}+10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}-b^6\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+e\right.}{4a^4e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/(2*a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2
*a*c^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c
*(d + e*x)^2 + 26*a*b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)
/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) -
 (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c
^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*
a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c]
+ 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10
*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] -
8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log
[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(
5/2)*e)
```

Maple [C] time = 0.111, size = 5575, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^3),x, algorithm="maxi
```

```
[Out] -1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*e^8*x^8 + 48*(b^4*c^
2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d*e^7*x^7 + 3*(4*b^5*c - 29*a*b^3*c
^2 + 46*a^2*b*c^3 + 56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^2)*
e^6*x^6 + 6*(56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^3 + 3*(4*b
^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d)*e^5*x^5 + 6*(b^4*c^2 - 7*a
*b^2*c^3 + 10*a^2*c^4)*d^8 + (6*b^6 - 36*a*b^4*c + 14*a^2*b^2*c^2
 + 100*a^3*c^3 + 420*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^4 + 4
5*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^2)*e^4*x^4 + 3*(4*b^5
*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 4*(84*(b^4*c^2 - 7*a*b^2*
c^3 + 10*a^2*c^4)*d^5 + 15*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3
)*d^3 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d)*e^
3*x^3 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 2*(3*b^6 - 18*a*b
^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^4 + (168*(b^4*c^2 - 7*a*b^2*
c^3 + 10*a^2*c^4)*d^6 + 9*a*b^5 - 68*a^2*b^3*c + 122*a^3*b*c^2 +
45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^4 + 12*(3*b^6 - 18*a
*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^2)*e^2*x^2 + (9*a*b^5 - 68
*a^2*b^3*c + 122*a^3*b*c^2)*d^2 + 2*(24*(b^4*c^2 - 7*a*b^2*c^3 +
10*a^2*c^4)*d^7 + 9*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^5 +
 4*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^3 + (9*a*b
^5 - 68*a^2*b^3*c + 122*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*e^11*x^10 + 10*(a^3*b^4*c^2 - 8*a^4*b^2*c^3
 + 16*a^5*c^4)*d*e^10*x^9 + (2*a^3*b^5*c - 16*a^4*b^3*c^2 + 32*a^
5*b*c^3 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^9*
x^8 + 8*(15*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + 2*(a
^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^8*x^7 + (a^3*b^6 -
6*a^4*b^4*c + 32*a^6*c^3 + 210*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*
a^5*c^4)*d^4 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)
*e^7*x^6 + 2*(126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5
+ 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 3*(a^3*b^6
- 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^6*x^5 + (2*a^4*b^5 - 16*a^5*b^3*
c + 32*a^6*b*c^2 + 210*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d^6 + 140*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 15*(a
^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^5*x^4 + 4*(30*(a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 28*(a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)
*d^3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^4*x^3 + (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3
 + 16*a^5*c^4)*d^8 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3
)*d^6 + 15*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 12*(a^4*b^5
 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^3*x^2 + 2*(5*(a^3*b^4*c^2 -
 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 8*(a^3*b^5*c - 8*a^4*b^3*c^2 +
 16*a^5*b*c^3)*d^7 + 3*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 +
 4*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*d)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16
*a^5*c^4)*d^10 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^8
 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^6 + 2*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d^4 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
d^2)*e) + 3*integrate(((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^3*x
^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^2*x^2 + (b^5*c -
```

```
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 9*a*b^4*c + 23*a^2*b^2*c
^2 - 10*a^3*c^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e*x
 + (b^6 - 9*a*b^4*c + 23*a^2*b^2*c^2 - 10*a^3*c^3)*d)/(c*e^4*x^4
+ 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a), x)/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2) - 3*
b*log(e*x + d)/(a^4*e)
```

Fricas [A] time = 2.39277, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^3),x, algorithm="fric
```

```
[Out] [-1/4*(3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*
e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*
c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^
3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^
5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20
*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a
^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c
^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^
2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 3
0*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a
^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*
c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b
^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^
3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2
+ 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a
^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*
(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 +
28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5
*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)
*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d
)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c
^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c
^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 +
56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 1
5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4
)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)
*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5
*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^
3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*
c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 -
 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*
```

a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^
5*c^3)*d)*e*x)*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*
c^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4
*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2
+ 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 -
4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x
^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (6*(a*b^4*c^2 - 7*a^2*
b^2*c^3 + 10*a^3*c^4)*e^8*x^8 + 48*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 1
0*a^3*c^4)*d*e^7*x^7 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c
^3 + 56*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^2)*e^6*x^6 + 6
(56(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^3 + 3*(4*a*b^5*c
- 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d)*e^5*x^5 + 6*(a*b^4*c^2 - 7*a^
2*b^2*c^3 + 10*a^3*c^4)*d^8 + (6*a*b^6 - 36*a^2*b^4*c + 14*a^3*b^
2*c^2 + 100*a^4*c^3 + 420*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4
)*d^4 + 45*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^2)*e^4*x
^4 + 2*a^3*b^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 3*(4*a*b^5*c - 29*a^
2*b^3*c^2 + 46*a^3*b*c^3)*d^6 + 4*(84*(a*b^4*c^2 - 7*a^2*b^2*c^3
+ 10*a^3*c^4)*d^5 + 15*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3
)*d^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d
)*e^3*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^
3)*d^4 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2 + 168*(a*b^4*c
^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^6 + 45*(4*a*b^5*c - 29*a^2*b^3
*c^2 + 46*a^3*b*c^3)*d^4 + 12*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2
*c^2 + 50*a^4*c^3)*d^2)*e^2*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122
*a^4*b*c^2)*d^2 + 2*(24*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*
d^7 + 9*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^5 + 4*(3*a*
b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d^3 + (9*a^2*b^5
- 68*a^3*b^3*c + 122*a^4*b*c^2)*d)*e*x - 3*((b^5*c^2 - 8*a*b^3*c
^3 + 16*a^2*b*c^4)*e^10*x^10 + 10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*d*e^9*x^9 + (2*b^6*c - 16*a*b^4*c^2 + 32*a^2*b^2*c^3 + 45
*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2)*e^8*x^8 + 8*(15*(b^5
*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3 + 2*(b^6*c - 8*a*b^4*c^2 +
16*a^2*b^2*c^3)*d)*e^7*x^7 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3 + 2
10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 + 56*(b^6*c - 8*a*b
^4*c^2 + 16*a^2*b^2*c^3)*d^2)*e^6*x^6 + (b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*d^10 + 2*(126*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4
)*d^5 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^3 + 3*(b^7 -
6*a*b^5*c + 32*a^3*b*c^3)*d)*e^5*x^5 + 2*(b^6*c - 8*a*b^4*c^2 + 1
6*a^2*b^2*c^3)*d^8 + (2*a*b^6 - 16*a^2*b^4*c + 32*a^3*b^2*c^2 + 2
10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^6 + 140*(b^6*c - 8*a*
b^4*c^2 + 16*a^2*b^2*c^3)*d^4 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^
3)*d^2)*e^4*x^4 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^6 + 4*(30*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^7 + 28*(b^6*c - 8*a*b^4*c^
2 + 16*a^2*b^2*c^3)*d^5 + 5*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^3
+ 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d)*e^3*x^3 + 2*(a*b^6
- 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (45*(b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*d^8 + a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 56*(
b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^6 + 15*(b^7 - 6*a*b^5*c +
32*a^3*b*c^3)*d^4 + 12*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^
2)*e^2*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2 + 2*(5*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^9 + 8*(b^6*c - 8*a*b^4*c^2
+ 16*a^2*b^2*c^3)*d^7 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^5 +
4*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3 + (a^2*b^5 - 8*a^3*
b^3*c + 16*a^4*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d

```
^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
 12*((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^10*x^10 + 10*(b^5*c
^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^9*x^9 + (2*b^6*c - 16*a*b^4*
c^2 + 32*a^2*b^2*c^3 + 45*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
d^2)*e^8*x^8 + 8*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3 +
 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d)*e^7*x^7 + (b^7 - 6*a
*b^5*c + 32*a^3*b*c^3 + 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4
)*d^4 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^2)*e^6*x^6 +
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^10 + 2*(126*(b^5*c^2 - 8
*a*b^3*c^3 + 16*a^2*b*c^4)*d^5 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2
*b^2*c^3)*d^3 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d)*e^5*x^5 + 2
*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^8 + (2*a*b^6 - 16*a^2*b
^4*c + 32*a^3*b^2*c^2 + 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4
)*d^6 + 140*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^4 + 15*(b^7
 - 6*a*b^5*c + 32*a^3*b*c^3)*d^2)*e^4*x^4 + (b^7 - 6*a*b^5*c + 32*
a^3*b*c^3)*d^6 + 4*(30*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^7
 + 28*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^5 + 5*(b^7 - 6*a*b
^5*c + 32*a^3*b*c^3)*d^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^
2)*d)*e^3*x^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (4
5*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^8 + a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^
6 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^4 + 12*(a*b^6 - 8*a^2*b
^4*c + 16*a^3*b^2*c^2)*d^2)*e^2*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16
*a^4*b*c^2)*d^2 + 2*(5*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^9
 + 8*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^7 + 3*(b^7 - 6*a*b^
5*c + 32*a^3*b*c^3)*d^5 + 4*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2
)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e*x)*log(e*x +
d))*sqrt(b^2 - 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^
4)*e^11*x^10 + 10*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d*e^
10*x^9 + (2*a^4*b^5*c - 16*a^5*b^3*c^2 + 32*a^6*b*c^3 + 45*(a^4*b
^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^2)*e^9*x^8 + 8*(15*(a^4*b^
4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^3 + 2*(a^4*b^5*c - 8*a^5*b^
3*c^2 + 16*a^6*b*c^3)*d)*e^8*x^7 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^
7*c^3 + 210*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^4 + 56*(
a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^2)*e^7*x^6 + 2*(126*(
a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^5 + 56*(a^4*b^5*c - 8
*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^3 + 3*(a^4*b^6 - 6*a^5*b^4*c + 32*
a^7*c^3)*d)*e^6*x^5 + (2*a^5*b^5 - 16*a^6*b^3*c + 32*a^7*b*c^2 +
210*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^6 + 140*(a^4*b^5
*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^4 + 15*(a^4*b^6 - 6*a^5*b^4*
c + 32*a^7*c^3)*d^2)*e^5*x^4 + 4*(30*(a^4*b^4*c^2 - 8*a^5*b^2*c^3
 + 16*a^6*c^4)*d^7 + 28*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3
)*d^5 + 5*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^3 + 2*(a^5*b^5 -
 8*a^6*b^3*c + 16*a^7*b*c^2)*d)*e^4*x^3 + (a^6*b^4 - 8*a^7*b^2*c
+ 16*a^8*c^2 + 45*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^8
+ 56*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^6 + 15*(a^4*b^6
 - 6*a^5*b^4*c + 32*a^7*c^3)*d^4 + 12*(a^5*b^5 - 8*a^6*b^3*c + 16
*a^7*b*c^2)*d^2)*e^3*x^2 + 2*(5*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16
*a^6*c^4)*d^9 + 8*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^7
+ 3*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^5 + 4*(a^5*b^5 - 8*a^6
*b^3*c + 16*a^7*b*c^2)*d^3 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)
*d)*e^2*x + ((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^10 + 2*
(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^8 + (a^4*b^6 - 6*a^5
*b^4*c + 32*a^7*c^3)*d^6 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^
```

2)*d^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*d^2)*e)*sqrt(b^2 - 4*a*c)), 1/4*(6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*e^8*x^8 + 48*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d*e^7*x^7 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3 + 56*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^2)*e^6*x^6 + 6*(56*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^3 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d)*e^5*x^5 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^8 + (6*a*b^6 - 36*a^2*b^4*c + 14*a^3*b^2*c^2 + 100*a^4*c^3 + 420*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^4 + 45*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^2)*e^4*x^4 + 2*a^3*b^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^6 + 4*(84*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^5 + 15*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d)*e^3*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d^4 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2 + 168*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^6 + 45*(4*a*b^5*c - 29*a^2*b^3*c

^2 + 46*a^3*b*c^3)*d^4 + 12*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c
^2 + 50*a^4*c^3)*d^2)*e^2*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a
^4*b*c^2)*d^2 + 2*(24*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^
7 + 9*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^5 + 4*(3*a*b^
6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d^3 + (9*a^2*b^5 -
68*a^3*b^3*c + 122*a^4*b*c^2)*d)*e*x - 3*((b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*e^10*x^10 + 10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b
*c^4)*d*e^9*x^9 + (2*b^6*c - 16*a*b^4*c^2 + 32*a^2*b^2*c^3 + 45*(
b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2)*e^8*x^8 + 8*(15*(b^5*c
^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3 + 2*(b^6*c - 8*a*b^4*c^2 + 1
6*a^2*b^2*c^3)*d)*e^7*x^7 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3 + 210
*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 + 56*(b^6*c - 8*a*b^4
*c^2 + 16*a^2*b^2*c^3)*d^2)*e^6*x^6 + (b^5*c^2 - 8*a*b^3*c^3 + 16
*a^2*b*c^4)*d^10 + 2*(126*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
d^5 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^3 + 3*(b^7 - 6*
a*b^5*c + 32*a^3*b*c^3)*d)*e^5*x^5 + 2*(b^6*c - 8*a*b^4*c^2 + 16*
a^2*b^2*c^3)*d^8 + (2*a*b^6 - 16*a^2*b^4*c + 32*a^3*b^2*c^2 + 210
*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^6 + 140*(b^6*c - 8*a*b^
4*c^2 + 16*a^2*b^2*c^3)*d^4 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)
*d^2)*e^4*x^4 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^6 + 4*(30*(b^5
*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^7 + 28*(b^6*c - 8*a*b^4*c^2
+ 16*a^2*b^2*c^3)*d^5 + 5*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^3 +
2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d)*e^3*x^3 + 2*(a*b^6 -
8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (45*(b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*d^8 + a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 56*(b^
6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^6 + 15*(b^7 - 6*a*b^5*c + 3
2*a^3*b*c^3)*d^4 + 12*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^2)
*e^2*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2 + 2*(5*(b^5
*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^9 + 8*(b^6*c - 8*a*b^4*c^2 +
16*a^2*b^2*c^3)*d^7 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^5 + 4
*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3 + (a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 1
2*((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^10*x^10 + 10*(b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^9*x^9 + (2*b^6*c - 16*a*b^4*c^
2 + 32*a^2*b^2*c^3 + 45*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^
2)*e^8*x^8 + 8*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3 + 2
*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d)*e^7*x^7 + (b^7 - 6*a*b
^5*c + 32*a^3*b*c^3 + 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
d^4 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^2)*e^6*x^6 + (b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^10 + 2*(126*(b^5*c^2 - 8*a
*b^3*c^3 + 16*a^2*b*c^4)*d^5 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b
^2*c^3)*d^3 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d)*e^5*x^5 + 2*(
b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^8 + (2*a*b^6 - 16*a^2*b^4
*c + 32*a^3*b^2*c^2 + 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
d^6 + 140*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^4 + 15*(b^7 -
6*a*b^5*c + 32*a^3*b*c^3)*d^2)*e^4*x^4 + (b^7 - 6*a*b^5*c + 32*a^
3*b*c^3)*d^6 + 4*(30*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^7 +
28*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^5 + 5*(b^7 - 6*a*b^5
*c + 32*a^3*b*c^3)*d^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)
*d)*e^3*x^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (45*
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^8 + a^2*b^5 - 8*a^3*b^3*
c + 16*a^4*b*c^2 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^6
+ 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^4 + 12*(a*b^6 - 8*a^2*b^4

```
*c + 16*a^3*b^2*c^2)*d^2)*e^2*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a
^4*b*c^2)*d^2 + 2*(5*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^9 +
 8*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^7 + 3*(b^7 - 6*a*b^5*
c + 32*a^3*b*c^3)*d^5 + 4*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e*x)*log(e*x + d)
)*sqrt(-b^2 + 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4
)*e^11*x^10 + 10*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d*e^1
0*x^9 + (2*a^4*b^5*c - 16*a^5*b^3*c^2 + 32*a^6*b*c^3 + 45*(a^4*b^
4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^2)*e^9*x^8 + 8*(15*(a^4*b^4
*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^3 + 2*(a^4*b^5*c - 8*a^5*b^3
*c^2 + 16*a^6*b*c^3)*d)*e^8*x^7 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7
*c^3 + 210*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^4 + 56*(a
^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^2)*e^7*x^6 + 2*(126*(a
^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^5 + 56*(a^4*b^5*c - 8*
a^5*b^3*c^2 + 16*a^6*b*c^3)*d^3 + 3*(a^4*b^6 - 6*a^5*b^4*c + 32*a
^7*c^3)*d)*e^6*x^5 + (2*a^5*b^5 - 16*a^6*b^3*c + 32*a^7*b*c^2 + 2
10*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^6 + 140*(a^4*b^5*
c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^4 + 15*(a^4*b^6 - 6*a^5*b^4*c
 + 32*a^7*c^3)*d^2)*e^5*x^4 + 4*(30*(a^4*b^4*c^2 - 8*a^5*b^2*c^3
+ 16*a^6*c^4)*d^7 + 28*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)
*d^5 + 5*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^3 + 2*(a^5*b^5 -
8*a^6*b^3*c + 16*a^7*b*c^2)*d)*e^4*x^3 + (a^6*b^4 - 8*a^7*b^2*c +
 16*a^8*c^2 + 45*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^8 +
 56*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^6 + 15*(a^4*b^6
- 6*a^5*b^4*c + 32*a^7*c^3)*d^4 + 12*(a^5*b^5 - 8*a^6*b^3*c + 16*
a^7*b*c^2)*d^2)*e^3*x^2 + 2*(5*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*
a^6*c^4)*d^9 + 8*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^7 +
 3*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^5 + 4*(a^5*b^5 - 8*a^6*
b^3*c + 16*a^7*b*c^2)*d^3 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*
d)*e^2*x + ((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^10 + 2*(
a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^8 + (a^4*b^6 - 6*a^5*
b^4*c + 32*a^7*c^3)*d^6 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2
)*d^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*d^2)*e)*sqrt(-b^2 +
4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.294791, size = 509, normalized size = 1.57

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^{2}}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{2\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,be^{(-1)}\ln\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)}{4\,a^{4}}-\frac{e^{(-1)}}{2\,(xe+d)^{2}a^{3}}+\frac{\left(5\,b^{5}c^{2}-36\,ab^{3}c^{3}+58\,a^{2}bc^{4}+\frac{2\left(5\,b^{6}ce-38\,ab^{4}c^{2}e+71\,a^{2}b^{2}c^{3}e-14\,a^{3}c^{4}e\right)e^{(-1)}}{(xe+d)^{2}}+\frac{\left(5\,b^{7}e^{2}-34\,ab^{5}ce^{2}+41\,a^{2}b^{3}c^{2}e^{2}+42\,a^{3}bc^{3}e^{2}\right)e^{(-2)}}{(xe+d)^{4}}+\frac{6\left(ab^{6}e^{3}-\cdots\right.}{\cdots}}{4\left(b^{2}-4\,ac\right)^{2}a^{4}\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*x + d)^3),x, algorithm="giac
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b +
 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/((a^4*b^4 - 8*a^5*b^
2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 3/4*b*e^(-1)*ln(c + b/(x*
e + d)^2 + a/(x*e + d)^4)/a^4 - 1/2*e^(-1)/((x*e + d)^2*a^3) + 1/
4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*c^4 + 2*(5*b^6*c*e - 38*a*
b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)*e^(-1)/(x*e + d)^2 +
 (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*b*c^3*
e^2)*e^(-2)/(x*e + d)^4 + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3
*b^2*c^2*e^3 - 6*a^4*c^3*e^3)*e^(-3)/(x*e + d)^6)*e^(-1)/((b^2 -
4*a*c)^2*a^4*(c + b/(x*e + d)^2 + a/(x*e + d)^4)^2)
```

3.638 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=202

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.938251, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 77.2259, size = 214, normalized size = 1.06

$$\frac{f^4(d+ex)}{ce}-\frac{\sqrt{2}f^4\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}e\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}f^4\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}e\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] f**4*(d + e*x)/(c*e) - sqrt(2)*f**4*(-2*a*c + b**2 + b*sqrt(-4*a*
c + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c +
b**2)))/(2*c**(3/2)*e*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c +
 b**2)) + sqrt(2)*f**4*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*at
an(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c*
*(3/2)*e*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.253376, size = 222, normalized size = 1.1

$$\frac{f^4\left(-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+2\sqrt{c}(d+ex)\right)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
+ Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

Maple [C] time = 0.004, size = 164, normalized size = 0.8

$$\frac{f^4x}{c}+\frac{f^4}{2\,ce}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-\_R^2be^2-2\,\_R\,bde-bd^2-a\right)\ln\left(x-\_R\right)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,\_R\,cd^2e+2\,cd^3+be\_R+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] f^4*x/c+1/2*f^4/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*
c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R
=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d
```

```
^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{f^4x}{c} - \frac{f^4 \int \frac{be^2x^2+2\,bdex+bd^2+a}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima
```

```
[Out] f^4*x/c - f^4*integrate((b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2
*(2*c*d^3 + b*d)*e*x + a), x)/c
```

Fricas [A] time = 0.313256, size = 1817, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas
```

```
[Out] 1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3
- 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)
*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*
c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^
2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3
*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4
*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))
) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*
e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x -
 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a
*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8
 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4
))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/
2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*
c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 -
 a^2*c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 +
 sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))
```

```
*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3
 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt(-
((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^
2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c
^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f
^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 + sqrt((b^4
- 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 -
 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)
*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))))/c
```

Sympy [A] time = 12.7821, size = 219, normalized size = 1.08

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\left(48a^2bc^2e^2f^8-28ab^3ce^2f^8+4b^5e^2f^8\right)+a^3f^{16},\left(t\mapsto t\log\left(x+\frac{32\ldots}{\ldots}\right.\right.\right.$$

$$+\frac{f^4x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**
4*c**3*e**4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2
*f**8 + 4*b**5*e**2*f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32
*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*
f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b**4*e*f**8 + a**2*c*d*f**12 -
 a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f**12)))) + f**4*x/c
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx+df)^4}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.639 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal. Leaf size=87

$$\frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3 \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqr
t[b^2 - 4*a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/
(4*c*e)
```

Rubi [A] time = 0.280523, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3 \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqr
t[b^2 - 4*a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/
(4*c*e)
```

Rubi in Sympy [A] time = 32.9442, size = 75, normalized size = 0.86

$$\frac{bf^3 \operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2ce\sqrt{-4ac+b^2}}+\frac{f^3 \log\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}{4ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] b*f**3*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*c*e*s
qrt(-4*a*c + b**2)) + f**3*log(a + b*(d + e*x)**2 + c*(d + e*x)**
4)/(4*c*e)
```

Mathematica [A] time = 0.0735401, size = 80, normalized size = 0.92

$$\frac{f^3\left(\log\left(a+b(d+ex)^2+c(d+ex)^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e
)
```

Maple [C] time = 0.003, size = 154, normalized size = 1.8

$$\frac{f^3}{2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^3e^3+3\_R^2de^2+3\_Rd^2e+d^3\right)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6\_Rcd^2e+2cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^
3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=Roo
tOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx+df)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima
```

```
[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A] time = 0.282743, size = 1, normalized size = 0.01

$$\left[\frac{bf^3\log\left(\frac{2\,(b^2c-4\,ac^2)\,e^2x^2+4\,(b^2c-4\,ac^2)\,dex+b^3-4\,abc+2\,(b^2c-4\,ac^2)\,d^2+(2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)\,e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)\,ex+}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{4\sqrt{b^2-4\,ac}ce}\right.$$

$$-\frac{2\,bf^3\arctan\left(-\frac{(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}f^3\log\left(ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\,(\right.}{4\sqrt{-b^2+4\,ac}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas
```

```
[Out] [1/4*(b*f^3*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2
)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^
4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2
*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + sqrt(b^2 - 4*a*c)*f^3*log
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^
2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqrt(b^2 - 4*a*c)*c*e), -1/4*(2*
b*f^3*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 +
 4*a*c)/(b^2 - 4*a*c)) - sqrt(-b^2 + 4*a*c)*f^3*log(c*e^4*x^4 + 4
*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
 + b*d)*e*x + a))/(sqrt(-b^2 + 4*a*c)*c*e)]
```

Sympy [A] time = 8.32427, size = 332, normalized size = 3.82

$$\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)$$
$$+\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e
))*log(2*d*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4
*c*e*(4*a*c - b**2)) + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f*
*3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e)) + b
*d**2*f**3)/(b*e**2*f**3)) + (b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(
4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*
f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e)) +
 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c -
b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx + df)^3}{(ex + d)^4 c + (ex + d)^2 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.640 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*
a*c]*e)) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt
[b^2 - 4*a*c]*e)
```

Rubi [A] time = 0.447407, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*
a*c]*e)) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt
[b^2 - 4*a*c]*e)
```

Rubi in Sympy [A] time = 42.6939, size = 158, normalized size = 0.93

$$-\frac{\sqrt{2}f^2\sqrt{b-\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}e\sqrt{-4ac+b^2}}+\frac{\sqrt{2}f^2\sqrt{b+\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}e\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(2)*f**2*sqrt(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*
(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*e*sqrt(-4*a*c
 + b**2)) + sqrt(2)*f**2*sqrt(b + sqrt(-4*a*c + b**2))*atan(sqrt(
2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*e*
sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.166787, size = 178, normalized size = 1.05

$$\frac{f^2\left(\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]]))/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] time = 0.003, size = 143, normalized size = 0.8

$$\frac{f^2}{2\,e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^2e^2+2\,\_R\,de+d^2\right)\ln\left(x-\_R\right)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,\_R\,cd^2e+2\,cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^
2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+
4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*
d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(efx+df)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A]   time = 0.324789, size = 1079, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f
^6 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^
4))*e^3*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a
*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))) - 1/2*sqrt(1/2)*sqrt(-
(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^
2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt(1/2)*(b^2*
c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4
 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b
^2*c - 4*a*c^2)*e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*
a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2
)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^
8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)
*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2)
)) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^
2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x
+ d*f^6 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^
3)*e^4))*e^3*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
- 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2)))
```

Sympy [A]   time = 6.98454, size = 124, normalized size = 0.73

$$\mathrm{RootSum}\left(t^4\left(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4\right) + t^2\left(-16abce^2f^4 + 4b^3e^2f^4\right) + af^8, \left(t \mapsto t\log\left(x + \frac{64t^3ac^2e^3 - 16t^3b^2}{e}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**
4*c*e**4) + _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f*
*8, Lambda(_t, _t*log(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c
*e**3 - 2*_t*b*e*f**4 + d*f**6)/(e*f**6))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx+df)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.641 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=44

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2
- 4*a*c]*e))
```

Rubi [A] time = 0.134701, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2
- 4*a*c]*e))
```

Rubi in Sympy [A] time = 18.7569, size = 41, normalized size = 0.93

$$-\frac{f\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -f*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*sqrt(-4*a
*c + b**2))
```

Mathematica [A] time = 0.0271992, size = 47, normalized size = 1.07

$$\frac{f\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 +
 4*a*c]*e)
```

Maple [C] time = 0.002, size = 130, normalized size = 3.

$$\frac{f}{2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)}\frac{(e\_R+d)\ln(x-\_R)}{2ce^3\_R^3+6cde^2\_R^2+6\_Rcd^2e+2cd^3+be\_R+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6
*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{efx+df}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Fricas [A] time = 0.319564, size = 1, normalized size = 0.02

$$\left[\frac{f\log\left(-\frac{2\left(b^2c-4\,ac^2\right)e^2x^2+4\left(b^2c-4\,ac^2\right)dex+b^3-4\,abc+2\left(b^2c-4\,ac^2\right)d^2-\left(2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+\right.}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{2\sqrt{b^2-4\,ace}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="fricas")
```

```
[Out] [1/2*f*log(-(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*
e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - (2*c^2*e^4*x^4 +
8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c
*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqrt(b^2 - 4*a*c)*e), f*arctan
(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c))/(sqrt(-b^2 + 4*a*c)*e)]
```

Sympy [A] time = 5.23891, size = 189, normalized size = 4.3

$$-\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4acf\sqrt{-\frac{1}{4ac-b^2}}+b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}+\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4acf\sqrt{-\frac{1}{4ac-b^2}}-b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1
/(4*a*c - b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**
2*f)/(2*c*e**2*f))/(2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e
+ x**2 + (4*a*c*f*sqrt(-1/(4*a*c - b**2)) - b**2*f*sqrt(-1/(4*a*c
 - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))/(2*e)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{efx+df}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a),x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.642 $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=103

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^
2 - 4*a*c]*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 +
c*(d + e*x)^4]/(4*a*e*f)
```

Rubi [A] time = 0.283488, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^
2 - 4*a*c]*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 +
c*(d + e*x)^4]/(4*a*e*f)
```

Rubi in Sympy [A] time = 39.0735, size = 87, normalized size = 0.84

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2aef\sqrt{-4ac+b^2}}+\frac{\log\left((d+ex)^2\right)}{2aef}-\frac{\log\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}{4aef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] b*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*a*e*f*sqrt
(-4*a*c + b**2)) + log((d + e*x)**2)/(2*a*e*f) - log(a + b*(d + e
*x)**2 + c*(d + e*x)**4)/(4*a*e*f)
```

Mathematica [A] time = 0.126468, size = 131, normalized size = 1.27

$$\frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2c(\right.}{4aef\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b
 - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*
a*c]*e*f)
```

Maple [C] time = 0.007, size = 190, normalized size = 1.8

$$\frac{1}{2\,aef}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+\left(6\,cd^2e^2+be^2\right)\_Z^2+\left(4\,cd^3e+2\,bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-ce^3\_R^3-3\,cde^2\_R^2+e\left(-3\,cd^2-b\right)\_R-cd^3-\right.}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,\_R\,cd^2e+2\,cd^3+b\left.\right.}+\frac{\ln(ex+d)}{aef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f/a/e*sum((-c*e^3*_R^3-3*c*d*e^2*_R^2+e*(-3*c*d^2-b)*_R-c*d^3
-b*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*
d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^
2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+ln(e*x+d)/a/e/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{ce^3x^3+3\,cde^2x^2+cd^3+\left(3\,cd^2+b\right)ex+bd}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\,dx}{af}+\frac{\log(ex+d)}{aef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)),x, algorithm="maxi
```

```
[Out] -integrate((c*e^3*x^3 + 3*c*d*e^2*x^2 + c*d^3 + (3*c*d^2 + b)*e*x
 + b*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^
2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a*f) + log(e*x + d)/(
a*e*f)
```

Fricas [A] time = 0.294892, size = 1, normalized size = 0.01

$$\Bigg[\frac{b\log\Big(\frac{2\,(b^2c-4\,ac^2)\,e^2x^2+4\,(b^2c-4\,ac^2)\,dex+b^3-4\,abc+2\,(b^2c-4\,ac^2)\,d^2+(2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)\,e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)\,ex+b^2-}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}}{4\,\sqrt{b^2-}}$$

$$-\frac{2\,b\arctan\Big(-\frac{(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\Big)+\sqrt{-b^2+4\,ac}\,\big(\log\big(ce^4x^4+4\,cde^3x^3+cd^4+\big(6\,cd^2+b\big)\,e^2x^2+bd^2+2\,\big(2\,cd}{4\,\sqrt{-b^2+4\,ac}aef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)),x, algorithm="fric
```

```
[Out] [1/4*(b*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*
e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 +
8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c
*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - sqrt(b^2 - 4*a*c)*(log(c*e^4*
x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a) - 4*log(e*x + d)))/(sqrt(b^2 - 4*a*c)*a*e
*f), -1/4*(2*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*e^4*x
^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2
*c*d^3 + b*d)*e*x + a) - 4*log(e*x + d)))/(sqrt(-b^2 + 4*a*c)*a*e
*f)]
```

Sympy [A] time = 20.4612, size = 348, normalized size = 3.38

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\frac{\log\left(\frac{d}{e}+x\right)}{aef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*l
og(2*d*x/e + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e
*f*(4*a*c - b**2)) - 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c
+ b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) - 2*a*c + b**2 +
b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c -
b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-8*a**2*c*e*f*(b*sqrt
(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) + 2*a*b**
2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*
f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)),x, algorithm="giac
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)),
x)
```

3.643 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rubi [A] time = 0.694051, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rubi in Sympy [A] time = 64.9485, size = 202, normalized size = 0.99

$$\frac{\sqrt{2}\sqrt{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2aef^2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2aef^2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{1}{aef^2\,(d+ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d
 + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*e*f**2*sqrt(b + sqrt(
-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*sqrt(c)*(b + sqrt(
-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a
*c + b**2)))/(2*a*e*f**2*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*
c + b**2)) - 1/(a*e*f**2*(d + e*x))
```

Mathematica [A] time = 0.643669, size = 209, normalized size = 1.02

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2aef^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b +
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
))/(2*a*e*f^2)
```

Maple [C] time = 0.007, size = 174, normalized size = 0.9

$$-\frac{1}{aef^2(ex+d)}+\frac{1}{2\,aef^2}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(-\_R^2ce^2-2\,\_R\,cde-cd^2-b\right)\ln(x-\_R)}{2\,ce^3\_R^3+6\,cde^2\_R^2+6\,\_R\,cd^2e+2\,cd^3+be\_R\,-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] -1/a/e/f^2/(e*x+d)+1/2/f^2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-
b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*
ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*
_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{ae^2f^2x + adef^2} - \frac{\int \frac{ce^2x^2+2\,cdex+cd^2+b}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\,dx}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^2),x, algorithm="ma
```

```
[Out] -1/(a*e^2*f^2*x + a*d*e*f^2) - integrate((c*e^2*x^2 + 2*c*d*e*x +
 c*d^2 + b)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^
2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a*f^2)
```

Fricas [A] time = 0.289381, size = 1994, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^2),x, algorithm="fr
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*
c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*
(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b
^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b
*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/(
(a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f
^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2
- 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 -
a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8))
- (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*
c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt
(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8))
- b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 -
 a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4
*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^
6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^
2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 -
4*a^4*c)*e^2*f^4))) + sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((
```

```
a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6
*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e
^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - s
qrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a
*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b
^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - 2)/(a*e^2*f^2*x +
 a*d*e*f^2)
```

Sympy [A] time = 16.9901, size = 258, normalized size = 1.26

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2e^4f^8 - 128a^4b^2ce^4f^8 + 16a^3b^4e^4f^8\right) + t^2\left(48a^2bc^2e^2f^4 - 28ab^3ce^2f^4 + 4b^5e^2f^4\right) + c^3, \left(t \mapsto t\log\left(x\right.\right.\right.$$
$$- \frac{1}{adef^2 + ae^2f^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**
8 + 16*a**3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 2
8*a*b**3*c*e**2*f**4 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*lo
g(x + (-64*_t**3*a**5*c**2*e**3*f**6 + 48*_t**3*a**4*b**2*c*e**3*
f**6 - 8*_t**3*a**3*b**4*e**3*f**6 - 10*_t*a**2*b*c**2*e*f**2 + 1
0*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c**3*d - b**2*c**2*d)
/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*f**2*x)
```

GIAC/XCAS [A] time = 1.01916, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^2),x, algorithm="gi
```

```
[Out] Done
```

3.644 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=133

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

```
[Out] -1/(2*a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d +
 e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b
*Log[d + e*x])/(a^2*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*
x)^4])/(4*a^2*e*f^3)
```

Rubi [A] time = 0.431615, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d +
 e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b
*Log[d + e*x])/(a^2*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*
x)^4])/(4*a^2*e*f^3)
```

Rubi in Sympy [A] time = 56.6593, size = 124, normalized size = 0.93

$$-\frac{1}{2aef^3\,(d+ex)^2}-\frac{b\log\left((d+ex)^2\right)}{2a^2ef^3}+\frac{b\log\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)}{4a^2ef^3}-\frac{(-2ac+b^2)\,\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^2ef^3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -1/(2*a*e*f**3*(d + e*x)**2) - b*log((d + e*x)**2)/(2*a**2*e*f**3
) + b*log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(4*a**2*e*f**3) -
(-2*a*c + b**2)*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))
```

```
/(2*a**2*e*f**3*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.245937, size = 157, normalized size = 1.18

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{(d+ex)^2}-4b\log(d+ex)}{4a^2ef^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b
^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b
^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[
b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

Maple [C] time = 0.007, size = 222, normalized size = 1.7

$$\frac{1}{2f^3a^2e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)}\frac{\left(\_R^3bce^3+3\_R^2bcde^2+e\left(3bcd^2-ac+b^2\right)\_R\right.}{2ce^3\_R^3+6cde^2\_R^2+6\_Rcd^2e+}$$
$$-\frac{1}{2aef^3(ex+d)^2}-\frac{b\ln(ex+d)}{f^3a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^3/a^2/e*sum((_R^3*b*c*e^3+3*_R^2*b*c*d*e^2+e*(3*b*c*d^2-a*c
+b^2)*_R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c
*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^
3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^
2+a))-1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)/a^2/e/f^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\left(ae^3f^3x^2+2ade^2f^3x+ad^2ef^3\right)}+\frac{\int\frac{bce^3x^3+3bcde^2x^2+bcd^3+\left(3bcd^2+b^2-ac\right)ex+\left(b^2-ac\right)d}{ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a}\,dx}{a^2f^3}-\frac{b\log(ex+d)}{a^2ef^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^3),x, algorithm="ma
```

```
[Out] -1/2/(a*e^3*f^3*x^2 + 2*a*d*e^2*f^3*x + a*d^2*e*f^3) + integrate(
(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - a*c
)*e*x + (b^2 - a*c)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*f^
3) - b*log(e*x + d)/(a^2*e*f^3)
```

Fricas [A] time = 0.399025, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^3),x, algorithm="fr
```

```
[Out] [-1/4*(((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*
a*c)*d^2)*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*
d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4
+ 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b
*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*
c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - sqrt(b^2 - 4*a*c)*((b*e^2*x
^2 + 2*b*d*e*x + b*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (
6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - 4*(b*
e^2*x^2 + 2*b*d*e*x + b*d^2)*log(e*x + d) - 2*a))/((a^2*e^3*f^3*x
^2 + 2*a^2*d*e^2*f^3*x + a^2*d^2*e*f^3)*sqrt(b^2 - 4*a*c)), 1/4*(
2*((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*
d^2)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*((b*e^2*x^2 + 2*b*d*e*
x + b*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*
e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - 4*(b*e^2*x^2 + 2*b
*d*e*x + b*d^2)*log(e*x + d) - 2*a))/((a^2*e^3*f^3*x^2 + 2*a^2*d*
e^2*f^3*x + a^2*d^2*e*f^3)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 58.4927, size = 532, normalized size = 4.

$$\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)}\right) + 2a^2b^2ef^3\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)}\right)}{2ac^2e^2 - b^2ce^2}\right.$$

$$+\left(\frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)}\right) + 2a^2b^2ef^3\left(\frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)}\right) +}{2ac^2e^2 - b^2ce^2}\right.$$

$$-\frac{1}{2ad^2ef^3 + 4ade^2f^3x + 2ae^3f^3x^2} - \frac{b\log\left(\frac{d}{e}+x\right)}{a^2ef^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e
*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*f**3*(b/
(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f*
*3*(4*a*c - b**2))) + 2*a**2*b**2*e*f**3*(b/(4*a**2*e*f**3) - sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) +
 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b
**2*c*e**2)) + (b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a*
*3*c*e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 2*a**2*b**2*e*f**3*(b/(4*a**
2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*
a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*
a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e*f**3 + 4*a*d*e**2*f**
3*x + 2*a*e**3*f**3*x**2) - b*log(d/e + x)/(a**2*e*f**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c + (ex+d)^2b + a\right)(efx+df)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^3),x, algorithm="gi
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^3)
, x)
```

3.645 $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

```
[Out] -1/(3*a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(
b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*e*f^4) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```

Rubi [A] time = 1.1997, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(3*a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(
b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*e*f^4) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```

Rubi in Sympy [A] time = 115.569, size = 241, normalized size = 1.02

$$-\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}-\frac{\sqrt{2}\sqrt{c}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a^2ef^4\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a^2ef^4\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -1/(3*a*e*f**4*(d + e*x)**3) + b/(a**2*e*f**4*(d + e*x)) - sqrt(2
)*sqrt(c)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sq
rt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a**2*e*f**4*sqr
t(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*sqrt(c)
*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d
+ e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(2*a**2*e*f**4*sqrt(b - sqr
t(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.372991, size = 238, normalized size = 1.01

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2ef^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 -
 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2*e*f
^4)
```

Maple [C] time = 0.007, size = 197, normalized size = 0.8

$$\frac{1}{2 f^4 a^2 e} \sum_{\_R = RootOf\left(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a\right)} \frac{\left(\_R^2bce^2 + 2\_Rbcde + bcd^2 - ac + b^2\right)\ln\left(x - \_\right.}{2ce^3\_R^3 + 6cde^2\_R^2 + 6\_Rcd^2e + 2cd^3 + be\_R +} - \frac{1}{3aef^4(ex+d)^3} + \frac{b}{f^4a^2e(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^4/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*
_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_
R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(
4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/3/a/e/f^4/(e*x+d)^3+b/a^2
/e/f^4/(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3be^2x^2 + 6bdex + 3bd^2 - a}{3\left(a^2e^4f^4x^3 + 3a^2de^3f^4x^2 + 3a^2d^2e^2f^4x + a^2d^3ef^4\right)} + \frac{\int \frac{bce^2x^2 + 2bcdex + bcd^2 + b^2 - ac}{ce^4x^4 + 4cde^3x^3 + cd^4 + (6cd^2 + b)e^2x^2 + bd^2 + 2(2cd^3 + bd)ex + a}\,dx}{a^2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^4),x, algorithm="ma
```

```
[Out] 1/3*(3*b*e^2*x^2 + 6*b*d*e*x + 3*b*d^2 - a)/(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4) + integr
ate((b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - a*c)/(c*e^4*x^4
+ 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a), x)/(a^2*f^4)
```

Fricas [A] time = 0.302659, size = 2986, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^4),x, algorithm="fr
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^
4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4
```

```
)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*
f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*
f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 -
3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12
*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 -
8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*s
qrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^1
6)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8
))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*
d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c
^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 +
 a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)
*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4
*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*
b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
(a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)
/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 +
 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(
((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) -
 b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*lo
g(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*
c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c
^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + (b^8 - 8*a*b^6*
c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(((a^
5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5
 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) + 3*s
qrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f
^4*x + a^2*d^3*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*
b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*
e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5
 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e
^4*f^16)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 +
4*a^4*c^4)*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 -
 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2
- 4*a^6*c)*e^2*f^8))) - 2*a)/(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x
^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)
```

Sympy [A] time = 56.729, size = 411, normalized size = 1.74

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3} +\text{RootSum}\left(t^4\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)+t^2\left(-80a^3bc^3e^2f^8+100a^2b^3c^2e^2f^8-36ab^5ce^2f^8+4b^7e^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 +
 9*a**2*d**2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*
f**4*x**3) + RootSum(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b
**2*c*e**4*f**16 + 16*a**5*b**4*e**4*f**16) + _t**2*(-80*a**3*b*c
**3*e**2*f**8 + 100*a**2*b**3*c**2*e**2*f**8 - 36*a*b**5*c*e**2*f
**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t*log(x + (-96*_t**3*
a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**12 - 8*_t**
3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**2*
c**3*e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4
 - 2*_t*b**8*e*f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d
)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^4),x, algorithm="gi
```

```
[Out] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)*(e*f*x + d*f)^4)
, x)
```

3.646 $\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$

Optimal. Leaf size=279

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a
*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqr
t[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.21415, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a
*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqr
t[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 82.4703, size = 253, normalized size = 0.91

$$\frac{f^4\left(2a+b\left(d+ex\right)^2\right)\left(d+ex\right)}{2e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}+\frac{\sqrt{2}f^4\left(4ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}e\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}f^4\left(4ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}e\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] f**4*(2*a + b*(d + e*x)**2)*(d + e*x)/(2*e*(-4*a*c + b**2)*(a + b
*(d + e*x)**2 + c*(d + e*x)**4)) + sqrt(2)*f**4*(4*a*c + b**2 + b
*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqr
t(-4*a*c + b**2)))/(4*sqrt(c)*e*sqrt(b + sqrt(-4*a*c + b**2))*(-4
*a*c + b**2)**(3/2)) - sqrt(2)*f**4*(4*a*c + b**2 - b*sqrt(-4*a*c
 + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b
**2)))/(4*sqrt(c)*e*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)
**(3/2))
```

Mathematica [A] time = 0.884126, size = 266, normalized size = 0.95

$$\frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

Maple [C] time = 0.008, size = 695, normalized size = 2.5

$$-\frac{f^4be^2x^3}{(2ce^4x^4+8cde^3x^3+12cd^2e^2x^2+8cd^3ex+2be^2x^2+2cd^4+4bdex+2bd^2+2a)(4ac-b^2)}$$
$$-\frac{3f^4dbex^2}{(2ce^4x^4+8cde^3x^3+12cd^2e^2x^2+8cd^3ex+2be^2x^2+2cd^4+4bdex+2bd^2+2a)(4ac-b^2)}$$
$$-\frac{3f^4xbd^2}{(2ce^4x^4+8cde^3x^3+12cd^2e^2x^2+8cd^3ex+2be^2x^2+2cd^4+4bdex+2bd^2+2a)(4ac-b^2)}$$
$$-\frac{f^4ax}{(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)(4ac-b^2)}$$
$$-\frac{f^4d^3b}{(2ce^4x^4+8cde^3x^3+12cd^2e^2x^2+8cd^3ex+2be^2x^2+2cd^4+4bdex+2bd^2+2a)e(4ac-b^2)}$$
$$-\frac{f^4da}{(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)e(4ac-b^2)}$$
$$+\frac{f^4}{4e}\sum_{\_R=RootOf(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a)}\frac{(-\_R^2be^2-2\_Rbde-bd^2+2a)\ln(x-\_}{(4ac-b^2)(2ce^3\_R^3+6cde^2\_R^2+6\_Rcd^2e+2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*e^2/(4*a*c-b^2)*x^3-3/2*f^4/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)*d*b*e/(4*a*c-b^2)*x^2-3/2*f^4/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)/(4*a*c-b^2)*x*b*d^2-f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*a/(4*a*c-b
^2)*x-1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e*d^3/(4*a*c-b^2)*b-f^4/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)/e*d/(4*a*c-b^2)*a+1/4*f^4/e*sum((-_R^2*b*e^2-2
*_R*b*d*e-b*d^2+2*a)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_
R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d
*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b
*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}f^4\int$$
$$-\frac{be^2x^2+2bdex+bd^2-2a}{(b^2c-4ac^2)e^4x^4+4(b^2c-4ac^2)de^3x^3+(b^2c-4ac^2)d^4+(b^3-4abc+6(b^2c-4ac^2)d^2)e^2x^2+ab^2-4a^2c+(b^3-4abc}$$
$$+\frac{be^3f^4x^3+3bde^2f^4x^2+(3bd^2+2a)ef^4x+(bd^3+2ad)f^4}{2((b^2c-4ac^2)e^5x^4+4(b^2c-4ac^2)de^4x^3+(b^3-4abc+6(b^2c-4ac^2)d^2)e^3x^2+2(2(b^2c-4ac^2)d^3+(b^3-4abc)d)e^2x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxi
```

```
[Out] -1/2*f^4*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c
 - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*
a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 +
a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^
3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^3*f^4*x^3 + 3*b*d*e^2*
f^4*x^2 + (3*b*d^2 + 2*a)*e*f^4*x + (b*d^3 + 2*a*d)*f^4)/((b^2*c
- 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b
*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^
3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4
*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Fricas [A] time = 0.329669, size = 3480, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fric
```

```
[Out] 1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^
4*x + 2*(b*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^
4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*
c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*
a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*
c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^
12*x + (3*b^2 + 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*
a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
- 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c
- 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*
c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 -
 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 +
sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((
3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^
4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^
```

```
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f
^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)))
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e
^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2
*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a
*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3
 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
- 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a
^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^
12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 - 2*sqrt(f^1
6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-
((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b
^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*
d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^
2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*
d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48
*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (
3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)
*e*f^8 - 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3
*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))))/((b^2*c - 4*a*c^2)*e^5*x^
4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*
c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx+df)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac
```

```
[Out] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,
x)
```

3.647 $\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=103

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi [A] time = 0.287812, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi in Sympy [A] time = 29.3546, size = 88, normalized size = 0.85

$$-\frac{bf^3\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{f^3\left(2a+b\left(d+ex\right)^2\right)}{2e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -b*f**3*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(-4*
a*c + b**2)**(3/2)) + f**3*(2*a + b*(d + e*x)**2)/(2*e*(-4*a*c +
b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 0.226007, size = 103, normalized size = 1.

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
c*(d + e*x)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(3/2)))/(2*e)
```

Maple [C] time = 0.008, size = 500, normalized size = 4.9

$$-\frac{bef^3x^2}{(2\,ce^4x^4+8\,cde^3x^3+12\,cd^2e^2x^2+8\,cd^3ex+2\,be^2x^2+2\,cd^4+4\,bdex+2\,bd^2+2\,a)(4\,ac-b^2)}$$
$$-\frac{f^3bdx}{(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a)(4\,ac-b^2)}$$
$$-\frac{f^3bd^2}{(2\,ce^4x^4+8\,cde^3x^3+12\,cd^2e^2x^2+8\,cd^3ex+2\,be^2x^2+2\,cd^4+4\,bdex+2\,bd^2+2\,a)\,e\,(4\,ac-b^2)}$$
$$-\frac{f^3a}{(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a)\,e\,(4\,ac-b^2)}$$
$$+\frac{f^3b}{2\,e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{(-e\_R-d)\ln(x-\_R)}{(4\,ac-b^2)\left(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,\_R\,cd^2e+2\,cd^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*e/(4*a*c-b^2)*x^2-f^3/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)*b*d/(4*a*c-b^2)*x-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/
(4*a*c-b^2)*b*d^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*a+1/2*
f^3*b/e*sum((-_R*e-d)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*
_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+
```

```
b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-bf^3\int -\frac{ex+d}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc} + \frac{be^2f^3x^2+2\,bdef^3x+\left(bd^2+2\,a\right)f^3}{2\,((b^2c-4\,ac^2)e^5x^4+4\,(b^2c-4\,ac^2)de^4x^3+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^3x^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)e^2x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxi
```

```
[Out] -b*f^3*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c
 - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c +
6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x
) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3*x + (b*d^2 + 2*a)*f^3)/((b^2
*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Fricas [A] time = 0.286937, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fric
```

```
[Out] [-1/2*((b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)
*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2
 + a*b)*f^3)*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^
2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x
^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 +
2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - (b*e^2*f^3*x^2 + 2*b*d*e
*f^3*x + (b*d^2 + 2*a)*f^3)*sqrt(b^2 - 4*a*c))/(((b^2*c - 4*a*c^2
)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b
^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
```

```
- 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*d^2)*e)*sqrt(b^2 - 4*a*c)), 1/2*(2*(b*c*e^4*f^3*x
^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b
*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2 + a*b)*f^3)*arctan(-
(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) + (b*e^2*f^3*x^2 + 2*b*d*e*f^3*x + (b*d^2 + 2*a)*f^3)*sq
rt(-b^2 + 4*a*c))/(((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^
2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2
+ 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c
 - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(
-b^2 + 4*a*c))]
```

Sympy [A] time = 167.661, size = 554, normalized size = 5.38

$$\frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3cf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e} - \frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^3cf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^5f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e} - \frac{2af^3+bd^2f^3+2bdef^3x+be^2f^3x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\left(8ac^2e^5-2b^2ce^5\right)+x^3\left(32ac^2de^4-8b^2cde^4\right)+x^2\left(8abce^3+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*
b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/
(4*a*c - b**2)**3) - b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*
f**3 + 2*b*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) - b*f**3*sqrt(-1
/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*b*c**2*f**3*sqr
t(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)*
*3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d*
*2*f**3)/(2*b*c*e**2*f**3))/(2*e) - (2*a*f**3 + b*d**2*f**3 + 2*b
*d*e*f**3*x + b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*
c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x*
*4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b
**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3
*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**
3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx + df)^3}{\left((ex + d)^4 c + (ex + d)^2 b + a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac
```

```
[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,
x)
```

3.648 $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}f^2\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*
(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c]
)*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
 - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 0.967378, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}f^2\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```


```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*
(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c]
)*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
 - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 72.5918, size = 236, normalized size = 0.9

$$-\frac{\sqrt{2}\sqrt{c}f^2\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{e\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}f^2\left(b-\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{e\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{f^2\left(b+2c\left(d+ex\right)^2\right)\left(d+ex\right)}{2e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*f**2*(b + sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sq
rt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(e*sqrt(b + sqrt(-
4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(c)*f**2*(b
- sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b -
sqrt(-4*a*c + b**2)))/(e*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c +
b**2)**(3/2)) - f**2*(b + 2*c*(d + e*x)**2)*(d + e*x)/(2*e*(-4*a*
c + b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 1.72752, size = 250, normalized size = 0.95

$$-\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
+ Sqrt[b^2 - 4*a*c]])))/(2*e)
```

Maple [C] time = 0.008, size = 693, normalized size = 2.6

$$\frac{ce^2f^2x^3}{(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)(4ac-b^2)} + 3\frac{cdef^2x^2}{(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)(4ac-b^2)} + 3\frac{f^2xcd^2}{(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)(4ac-b^2)} + \frac{bf^2x}{(2ce^4x^4+8cde^3x^3+12cd^2e^2x^2+8cd^3ex+2be^2x^2+2cd^4+4bdex+2bd^2+2a)(4ac-b^2)} + \frac{f^2d^3c}{(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a)e(4ac-b^2)} + \frac{df^2b}{(2ce^4x^4+8cde^3x^3+12cd^2e^2x^2+8cd^3ex+2be^2x^2+2cd^4+4bdex+2bd^2+2a)e(4ac-b^2)} + \frac{f^2}{4e}\sum_{\_R=RootOf(ce^4\_Z^4+4cde^3\_Z^3+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+cd^4+bd^2+a)} \frac{(2\_R^2ce^2+4\_Rcde+2cd^2-b)\ln(x-\_}{(4ac-b^2)(2ce^3\_R^3+6cde^2\_R^2+6\_Rcd^2e+2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)*e^2*c/(4*a*c-b^2)*x^3+3*f^2/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)*d*e*c/(4*a*c-b^2)*x^2+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(
4*a*c-b^2)*x*c*d^2+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b/(4*a*c-b^2)*x
+f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c+1/2*f^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b+1/4*f^2/e*sum((2*_R^2*c*e^2+4*_R*
c*d*e+2*c*d^2-b)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2
+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}f^2\int -\frac{2ce^2x^2+4cdex+2cd^2-b}{(b^2c-4ac^2)e^4x^4+4(b^2c-4ac^2)de^3x^3+(b^2c-4ac^2)d^4+(b^3-4abc+6(b^2c-4ac^2)d^2)e^2x^2+ab^2-4a^2c+(b^3-4abc} - \frac{2ce^3f^2x^3+6cde^2f^2x^2+(6cd^2+b)ef^2x+(2cd^3+bd)f^2}{2((b^2c-4ac^2)e^5x^4+4(b^2c-4ac^2)de^4x^3+(b^3-4abc+6(b^2c-4ac^2)d^2)e^3x^2+2(2(b^2c-4ac^2)d^3+(b^3-4abc)d)e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxi
```

```
[Out] 1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*
c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4
*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 +
 a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^3*f^2*x^3 + 6*c*d*e
^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3 + b*d)*f^2)/((b^2*c
 - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*
b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 -
4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Fricas [A] time = 0.308677, size = 3510, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fric
```

```
[Out] -1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^
2*x + 2*(2*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^
4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*
c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e
*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^
3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a
*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*
f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f
^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*
e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*
log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2
*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a
^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12
```

```
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3
+ 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c -
4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^
3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + (
(b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2
*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2
*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2
*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*e*f^4 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4
)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c
+ 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c
+ 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^
3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 -
12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 -
12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c +
4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a*b^8 - 8*a^2*b^6*c + 128*
a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4
 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/
((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2
)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))))/(
(b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*
c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [A] time = 157.283, size = 646, normalized size = 2.46

$$\frac{bdf^2 + 2cd^3f^2 + 6cde^2f^2x^2 + 2ce^3f^2x^3 + x\left(bef^2 + 6\right.}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3 + 48\right.}$$

$$+\mathrm{RootSum}\left(t^4\left(1048576a^7c^6e^4 - 1572864a^6b^2c^5e^4 + 983040a^5b^4c^4e^4 - 327680a^4b^6c^3e^4 + 61440a^3b^8c^2e^4 - 6144a^2b^{10}ce^4 + 2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d*f**2 + 2*c*d**3*f**2 + 6*c*d*e**2*f**2*x**2 + 2*c*e**3*f**2*
x**3 + x*(b*e*f**2 + 6*c*d**2*e*f**2))/(8*a**2*c*e - 2*a*b**2*e +
 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4
*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**
4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 -
 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c
**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t
**4*(1048576*a**7*c**6*e**4 - 1572864*a**6*b**2*c**5*e**4 + 98304
0*a**5*b**4*c**4*e**4 - 327680*a**4*b**6*c**3*e**4 + 61440*a**3*b
**8*c**2*e**4 - 6144*a**2*b**10*c*e**4 + 256*a*b**12*e**4) + _t**
2*(-12288*a**4*b*c**4*e**2*f**4 + 8192*a**3*b**3*c**3*e**2*f**4 -
 1536*a**2*b**5*c**2*e**2*f**4 + 16*b**9*e**2*f**4) + 16*a**2*c**
3*f**8 + 24*a*b**2*c**2*f**8 + 9*b**4*c*f**8, Lambda(_t, _t*log(x
 + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a**4*b**2*c**3*e**3 +
 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 128*_t*a**2*
b*c**2*e*f**4 - 16*_t*a*b**3*c*e*f**4 - 4*_t*b**5*e*f**4 + 4*a*c*
*2*d*f**6 + 3*b**2*c*d*f**6)/(4*a*c**2*e*f**6 + 3*b**2*c*e*f**6))
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx+df)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,
x)
```

3.649 $\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=98

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi [A] time = 0.259565, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)
```

Rubi in Sympy [A] time = 25.3718, size = 87, normalized size = 0.89

$$\frac{2cf\,\mathrm{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{f\left(b+2c\left(d+ex\right)^2\right)}{2e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] 2*c*f*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(-4*a*
c + b**2)**(3/2)) - f*(b + 2*c*(d + e*x)**2)/(2*e*(-4*a*c + b**2)
*(a + b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 0.22294, size = 99, normalized size = 1.01

$$-\frac{f\left(\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\right)}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) +
(4*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2
+ 4*a*c]))/(2*(b^2 - 4*a*c)*e)
```

Maple [C] time = 0.007, size = 484, normalized size = 4.9

$$\frac{cefx^2}{(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a)(4\,ac-b^2)}+2\,\frac{cdfx}{(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a)(4\,ac-b^2)}+\frac{cd^2f}{(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+be^2x^2+cd^4+2\,bdex+bd^2+a)\,e\,(4\,ac-b^2)}+\frac{bf}{(2\,ce^4x^4+8\,cde^3x^3+12\,cd^2e^2x^2+8\,cd^3ex+2\,be^2x^2+2\,cd^4+4\,bdex+2\,bd^2+2\,a)\,e\,(4\,ac-b^2)}+\frac{cf}{e}\sum_{\_R=RootOf\left(ce^4\_Z^4+4\,cde^3\_Z^3+(6\,cd^2e^2+be^2)\_Z^2+(4\,cd^3e+2\,bde)\_Z+cd^4+bd^2+a\right)}\frac{(e\_R+d)\ln(x-\_R)}{(4\,ac-b^2)\left(2\,ce^3\_R^3+6\,cde^2\_R^2+6\,\_R\,cd^2e+2\,cd^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)*e*c/(4*a*c-b^2)*x^2+2*f/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*c*d/(4*a*c-b^2)*x+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c
*d^2+1/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b+f*c/e*sum((_R*e
+d)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3
+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^
2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,cf\int -\frac{ex+d}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc}$$
$$-\frac{2\,ce^2fx^2+4\,cdefx+\left(2\,cd^2+b\right)f}{2\,((b^2c-4\,ac^2)e^5x^4+4\,(b^2c-4\,ac^2)de^4x^3+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^3x^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="maxima
```

```
[Out] 2*c*f*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c
- 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6
*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*
c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
 - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x + (2*c*d^2 + b)*f)/((b^2*c -
4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c
 + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3
+ (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Fricas [A] time = 0.288469, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="fricas
```

```
[Out] [-1/2*(2*(c^2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e
^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c
)*f)*log(-(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*
x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - (2*c^2*e^4*x^4 + 8*
c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d
^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^
2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (2*c*e^2*f*x^2 + 4*c*d*e*f*x +
(2*c*d^2 + b)*f)*sqrt(b^2 - 4*a*c))/(((b^2*c - 4*a*c^2)*e^5*x^4 +
 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*
c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*
d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2)*e)*sqrt(b^2 - 4*a*c)), -1/2*(4*(c^2*e^4*f*x^4 + 4*c^2*d
*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*
```

```
e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*arctan(-(2*c*e^2*x^2 + 4*c*d
*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*e^2*
f*x^2 + 4*c*d*e*f*x + (2*c*d^2 + b)*f)*sqrt(-b^2 + 4*a*c))/(((b^2
*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 115.044, size = 525, normalized size = 5.36

$$-\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e}+\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e}+\frac{bf+2cd^2f+4cdefx+2ce^2fx^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\left(8ac^2e^5-2b^2ce^5\right)+x^3\left(32ac^2de^4-8b^2cde^4\right)+x^2\left(8abce^3+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c*
*3*f*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c
- b**2)**3) - b**4*c*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**
2*d**2*f)/(2*c**2*e**2*f))/e + c*f*sqrt(-1/(4*a*c - b**2)**3)*log
(2*d*x/e + x**2 + (16*a**2*c**3*f*sqrt(-1/(4*a*c - b**2)**3) - 8*
a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*f*sqrt(-1/(4*a*
c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e + (b*f
+ 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a*b
**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2
*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**
2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2
*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 +
 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{efx+df}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2,x, algorithm="giac")

[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^2, x)
```

3.650 $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=174

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d
 + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) +
Log[d + e*x]/(a^2*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(
4*a^2*e*f)
```

Rubi [A] time = 0.629708, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d
 + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) +
Log[d + e*x]/(a^2*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(
4*a^2*e*f)
```

Rubi in Sympy [A] time = 73.2488, size = 153, normalized size = 0.88

$$\frac{-2ac+b^2+bc\left(d+ex\right)^2}{2aef\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}+\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^2ef\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\log\left(\left(d+ex\right)^2\right)}{2a^2ef}-\frac{\log\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}{4a^2ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(2*a*e*f*(-4*a*c + b**2)*(a +
b*(d + e*x)**2 + c*(d + e*x)**4)) + b*(-6*a*c + b**2)*atanh((b +
2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*a**2*e*f*(-4*a*c + b**2
)**(3/2)) + log((d + e*x)**2)/(2*a**2*e*f) - log(a + b*(d + e*x)*
*2 + c*(d + e*x)**4)/(4*a**2*e*f)
```

Mathematica [A] time = 0.715288, size = 238, normalized size = 1.37

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log(}{(b^2-4ac)^{3/2}}}{4a^2ef}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e
*x)^2*(b + c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c +
b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2
 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b
*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2*e*
f)
```

Maple [C] time = 0.018, size = 714, normalized size = 4.1

$$-\frac{bce\,x^{2}}{2 fa\left(ce^{4}x^{4}+4 cde^{3}x^{3}+6 cd^{2}e^{2}x^{2}+4 cd^{3}ex+be^{2}x^{2}+cd^{4}+2 bdex+bd^{2}+a\right)\left(4 ac-b^{2}\right)}-\frac{bcdx}{fa\left(ce^{4}x^{4}+4 cde^{3}x^{3}+6 cd^{2}e^{2}x^{2}+4 cd^{3}ex+be^{2}x^{2}+cd^{4}+2 bdex+bd^{2}+a\right)\left(4 ac-b^{2}\right)}-\frac{bcd^{2}}{2 fa\left(ce^{4}x^{4}+4 cde^{3}x^{3}+6 cd^{2}e^{2}x^{2}+4 cd^{3}ex+be^{2}x^{2}+cd^{4}+2 bdex+bd^{2}+a\right)e\left(4 ac-b^{2}\right)}+\frac{c}{f\left(ce^{4}x^{4}+4 cde^{3}x^{3}+6 cd^{2}e^{2}x^{2}+4 cd^{3}ex+be^{2}x^{2}+cd^{4}+2 bdex+bd^{2}+a\right)e\left(4 ac-b^{2}\right)}-\frac{b^{2}}{2 fa\left(ce^{4}x^{4}+4 cde^{3}x^{3}+6 cd^{2}e^{2}x^{2}+4 cd^{3}ex+be^{2}x^{2}+cd^{4}+2 bdex+bd^{2}+a\right)e\left(4 ac-b^{2}\right)}-\frac{1}{2 a^{2}fe}\sum_{\_R=\mathit{RootOf}\left(ce^{4}\_Z^{4}+4 cde^{3}\_Z^{3}+\left(6 cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4 cd^{3}e+2 bde\right)\_Z+cd^{4}+bd^{2}+a\right)}\frac{\left(ce^{3}\left(4 ac-b^{2}\right)\_R^{3}+3 cde^{2}\left(4 ac-b^{2}\right)\_R^{2}+e\right.}{\left(4 ac-b^{2}\right)\left(2 ce\right.}+\frac{\ln\left(ex+d\right)}{a^{2}fe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*e*c/(4*a*c-b^2)*x^2-1/f/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)*b*c*d/(4*a*c-b^2)*x-1/2/f/a/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)/e/(4*a*c-b^2)*b*c*d^2+1/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2
)*c-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/f/a^2/e*
sum((c*e^3*(4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^
2*d^2-3*b^2*c*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d
-b^3*d)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c
*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*
c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+ln(e
*x+d)/a^2/e/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bce^{2}x^{2}+2 bcdex+bcd^{2}+b^{2}-2 ac}{2\left(\left(ab^{2}c-4 a^{2}c^{2}\right)e^{5}fx^{4}+4\left(ab^{2}c-4 a^{2}c^{2}\right)de^{4}fx^{3}+\left(ab^{3}-4 a^{2}bc+6\left(ab^{2}c-4 a^{2}c^{2}\right)d^{2}\right)e^{3}fx^{2}+2\left(2\left(ab^{2}c-4 a^{2}c^{2}\right)d^{3}+\left(ab^{3}\right.\right.\right.}-\frac{\int\frac{\left(b^{2}c-4 ac^{2}\right)e^{3}x^{3}+3\left(b^{2}c-4 ac^{2}\right)de^{2}x^{2}+\left(b^{2}c-4 ac^{2}\right)d^{3}+\left(b^{3}-5 abc+3\left(b^{2}c-4 ac^{2}\right)d^{2}\right)ex+\left(b^{3}-5 abc\right)d}{\left(b^{2}c-4 ac^{2}\right)e^{4}x^{4}+4\left(b^{2}c-4 ac^{2}\right)de^{3}x^{3}+\left(b^{2}c-4 ac^{2}\right)d^{4}+\left(b^{3}-4 abc+6\left(b^{2}c-4 ac^{2}\right)d^{2}\right)e^{2}x^{2}+ab^{2}-4 a^{2}c+\left(b^{3}-4 abc\right)d^{2}+2\left(2\left(b^{2}c-4 ac^{2}\right)d^{3}+\left(b^{3}-4 abc\right)d\right)ex}\,dx}{a^{2}f}+\frac{\log\left(ex+d\right)}{a^{2}ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)),x, algorithm="ma
```

```
[Out] 1/2*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c
 - 4*a^2*c^2)*e^5*f*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*f*x^3 + (
a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*f*x^2 + 2*(2
*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*f*x + ((a
*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)
*d^2)*e*f) - integrate(((b^2*c - 4*a*c^2)*e^3*x^3 + 3*(b^2*c - 4*
a*c^2)*d*e^2*x^2 + (b^2*c - 4*a*c^2)*d^3 + (b^3 - 5*a*b*c + 3*(b^
2*c - 4*a*c^2)*d^2)*e*x + (b^3 - 5*a*b*c)*d)/((b^2*c - 4*a*c^2)*e
^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*
a*b*c)*d)*e*x), x)/(a^2*f) + log(e*x + d)/(a^2*e*f)
```

Fricas [A] time = 0.640099, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)),x, algorithm="fr
```

```
[Out] [1/4*(((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*
x^3 + (b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a
*b*c^2)*d^2)*e^2*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2
+ 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*log((2
*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*
a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^
3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^
2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d
^3 + b*d)*e*x + a)) + (2*a*b*c*e^2*x^2 + 4*a*b*c*d*e*x + 2*a*b*c*
d^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c -
 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x)*log(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2
 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^2*c - 4*a*c^2)*e^4*x^4 + 4*
(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*
b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e
*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/(((a^2*b^2*c - 4*a^3*c^2)*e^
5*f*x^4 + 4*(a^2*b^2*c - 4*a^3*c^2)*d*e^4*f*x^3 + (a^2*b^3 - 4*a^
3*b*c + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^2*
c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)*d)*e^2*f*x + (a^3*b^2
- 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*d^4 + (a^2*b^3 - 4*a^3*b*c)*d
```

```
^2)*e*f)*sqrt(b^2 - 4*a*c)), -1/4*(2*((b^3*c - 6*a*b*c^2)*e^4*x^4
 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b*c^2)*d^4 + (b
^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 - 6*a
^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (
b^4 - 6*a*b^2*c)*d)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*a*b*c*e^2*x^2 + 4*
a*b*c*d*e*x + 2*a*b*c*d^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2
)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 -
 4*a*b*c)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^
2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^2*c -
 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*
c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3
+ (b^3 - 4*a*b*c)*d)*e*x)*log(e*x + d))*sqrt(-b^2 + 4*a*c))/(((a^
2*b^2*c - 4*a^3*c^2)*e^5*f*x^4 + 4*(a^2*b^2*c - 4*a^3*c^2)*d*e^4*
f*x^3 + (a^2*b^3 - 4*a^3*b*c + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^3
*f*x^2 + 2*(2*(a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)
*d)*e^2*f*x + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*d^4 +
(a^2*b^3 - 4*a^3*b*c)*d^2)*e*f)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)),x, algorithm="gi

[Out] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f))
, x)
```

3.651 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=360

$$-\frac{3b^2-10ac}{2a^2ef^2(b^2-4ac)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2ef^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2ef^2(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^2(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 -
2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3
*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c]*(3*b^3 - 16*a*b
*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rubi [A] time = 3.29624, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$

$$-\frac{3b^2-10ac}{2a^2ef^2(b^2-4ac)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2ef^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2ef^2(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^2(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 -
2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3
```

```
*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c]*(3*b^3 - 16*a*b
*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rubi in Sympy [A] time = 178.924, size = 326, normalized size = 0.91

$$\frac{-2ac+b^2+bc\,(d+ex)^2}{2aef^2\,(d+ex)\,(-4ac+b^2)\left(a+b\,(d+ex)^2+c\,(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3-\left(-10ac+3b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a^2ef^2\sqrt{b+\sqrt{-4ac+b^2}}\,(-4ac+b^2)^{\frac{3}{2}}} - \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3+\left(-10ac+3b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a^2ef^2\sqrt{b-\sqrt{-4ac+b^2}}\,(-4ac+b^2)^{\frac{3}{2}}} - \frac{-10ac+3b^2}{2a^2ef^2\,(d+ex)\,(-4ac+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(2*a*e*f**2*(d + e*x)*(-4*a*c
+ b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) + sqrt(2)*sqrt(c)*
(-16*a*b*c + 3*b**3 - (-10*a*c + 3*b**2)*sqrt(-4*a*c + b**2))*ata
n(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a**
2*e*f**2*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) -
sqrt(2)*sqrt(c)*(-16*a*b*c + 3*b**3 + (-10*a*c + 3*b**2)*sqrt(-4*
a*c + b**2))*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c
+ b**2)))/(4*a**2*e*f**2*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c +
b**2)**(3/2)) - (-10*a*c + 3*b**2)/(2*a**2*e*f**2*(d + e*x)*(-4*a
*c + b**2))
```

Mathematica [A] time = 2.97798, size = 342, normalized size = 0.95

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b \cdots\right.}{(b^2-4a \cdots}}{4a^2ef^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 -
 2*a*c^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d
+ e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2
 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
+ e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b
^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

Maple [C] time = 0.017, size = 1346, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*e^2*c^2/(4*a*c-b^2)*x^3+1/2/f^2/a
^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)*e^2*c/(4*a*c-b^2)*x^3*b^2-3/f^2/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)*d*e*c^2/(4*a*c-b^2)*x^2+3/2/f^2/a^2/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*d*e*c/(4*a*c-b^2)*x^2*b^2-3/f^2/a/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)/(4*a*c-b^2)*x*c^2*d^2+3/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a
*c-b^2)*x*b^2*c*d^2-3/2/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c/(4*a*c-b
^2)*x+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3-1/f^2/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c^2+1/2/f^2/a^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^2*c-3/2/f^2/a/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)*d/e/(4*a*c-b^2)*b*c+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d
/e/(4*a*c-b^2)*b^3-1/4/f^2/a^2/e*sum((e^2*c*(10*a*c-3*b^2)*_R^2+2
*c*d*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)
/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R
*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e
^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/f^2/a^2/e
/(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\,b^2c-10\,ac^2\right)e^4x^4+4\left(3\,b^2c-10\,ac^2\right)de^3x^3+\left(3\,b^2c-10\,ac^2\right)d^4+\left(3\,b^3-\right.}{2\left((a^2b^2c-4\,a^3c^2)e^6f^2x^5+5\,(a^2b^2c-4\,a^3c^2)de^5f^2x^4+(a^2b^3-4\,a^3bc+10\,(a^2b^2c-4\,a^3c^2)d^2)e^4f^2x^3+(10\,(a^2b^2c-4\,a^3c\right.}$$
$$-\frac{\int\frac{\left(3\,b^2c-10\,ac^2\right)e^2x^2+2\left(3\,b^2c-10\,ac^2\right)dex+3\,b^3-13\,abc+\left(3\,b^2c-10\,ac^2\right)d^2}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc)d^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)ex}\,dx}{2\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^2),x, algorithm='
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*e^4*x^4 + 4*(3*b^2*c - 10*a*c^2)*d*e^3
*x^3 + (3*b^2*c - 10*a*c^2)*d^4 + (3*b^3 - 11*a*b*c + 6*(3*b^2*c
- 10*a*c^2)*d^2)*e^2*x^2 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)
*d^2 + 2*(2*(3*b^2*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x)
/((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)
*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2
)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3
 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c
- 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2
*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 -
4*a^4*c)*d)*e*f^2) - 1/2*integrate(((3*b^2*c - 10*a*c^2)*e^2*x^2
+ 2*(3*b^2*c - 10*a*c^2)*d*e*x + 3*b^3 - 13*a*b*c + (3*b^2*c - 10
*a*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e
^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*
c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*
(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^2*f^2)
```

Fricas [A] time = 0.394464, size = 6102, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^2),x, algorithm='
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e
^3*x^3 + 2*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*
b^2*c - 10*a*c^2)*d^2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 -
11*a*b*c)*d^2 + 4*(2*(3*b^2*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c
)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^
2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a
^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c
^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a
```

^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^
2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*
c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b
^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*
b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10
*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) +
 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(18
9*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x
 - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^
6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^
2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*
sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
+ 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 -
 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*s
qrt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*
f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 -
 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^
8*c^3)*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^
5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*
c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c
- 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*
b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^
3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 -
 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(-((a^5*b^6 -
 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^
4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^
4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/
((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))
*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3
*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 25
00*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a
^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)
*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3
*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*
b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)
*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*
c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*
b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e
^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 -
 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a
^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^
2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b
^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt

((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 62
5*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13
*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3
*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e
^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c
 + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a
^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^11 - 486*a*b^9*c + 3
330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^
5*b*c^5)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 3
85*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^
2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 +
 (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3
*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3
*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*
d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*s
qrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f
^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c
^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 +
420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8
*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2
*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a
^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^
6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 -
 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4
*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^11 - 486*a*b^
9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 -
5200*a^5*b*c^5)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^
5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))))/((a^2*b^2*c - 4*a^3*c^2
)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^
3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10
*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2
*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^
2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5
+ (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^2),x, algorithm='
```

```
[Out] Exception raised: TypeError
```

3.652 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=228

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log(d+ex)}{a^3ef^3}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2
*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)
*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*
a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(a^3*e*f^3) + (b*Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rubi [A] time = 0.761414, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log(d+ex)}{a^3ef^3}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2
*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)
*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*
a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(a^3*e*f^3) + (b*Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rubi in Sympy [A] time = 104.26, size = 209, normalized size = 0.92

$$\frac{-2ac+b^2+bc(d+ex)^2}{2aef^3(d+ex)^2(-4ac+b^2)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{-3ac+b^2}{a^2ef^3(d+ex)^2(-4ac+b^2)}-\frac{b\log\left((d+ex)^2\right)}{a^3ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{a^3ef^3(-4ac+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(2*a*e*f**3*(d + e*x)**2*(-4*a
*c + b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) - (-3*a*c + b**
2)/(a**2*e*f**3*(d + e*x)**2*(-4*a*c + b**2)) - b*log((d + e*x)**
2)/(a**3*e*f**3) + b*log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(2*
a**3*e*f**3) - (6*a**2*c**2 - 6*a*b**2*c + b**4)*atanh((b + 2*c*(
d + e*x)**2)/sqrt(-4*a*c + b**2))/(a**3*e*f**3*(-4*a*c + b**2)**(
3/2))
```

Mathematica [A] time = 0.857838, size = 287, normalized size = 1.26

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+\right.}{(b^2-4ac)^{3/2}}}{2a^3ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c
^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - 4*b*Log[d + e*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqr
t[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c
 - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2
))/(2*a^3*e*f^3)
```

Maple [C] time = 0.015, size = 1047, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/f^3/a^2
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2*b^2-2/f^3/a/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)*c*d/(4*a*c-b^2)*x*b^2-1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b
^2)*c^2*d^2+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2*
c*d^2-3/2/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c+1/2/f^3/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^3+1/f^3/a^3/e*sum((b*e
^3*c*(4*a*c-b^2)*_R^3+3*b*d*e^2*c*(4*a*c-b^2)*_R^2+e*(12*a*b*c^2*
d^2-3*b^3*c*d^2-3*a^2*c^2+5*a*b^2*c-b^4)*_R+4*a*b*c^2*d^3-b^3*c*d
^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^
2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e
^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*
e)*_Z+c*d^4+b*d^2+a))-1/2/f^3/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e
/f^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^3),x, algorithm='
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*e^4*x^4 + 8*(b^2*c - 3*a*c^2)*d*e^3*x^3
 + 2*(b^2*c - 3*a*c^2)*d^4 + (2*b^3 - 7*a*b*c + 12*(b^2*c - 3*a*c
^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + 2*(4
*(b^2*c - 3*a*c^2)*d^3 + (2*b^3 - 7*a*b*c)*d)*e*x)/((a^2*b^2*c -
4*a^3*c^2)*e^7*f^3*x^6 + 6*(a^2*b^2*c - 4*a^3*c^2)*d*e^6*f^3*x^5
+ (a^2*b^3 - 4*a^3*b*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^5*f^3*
x^4 + 4*(5*(a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)*d)
*e^4*f^3*x^3 + (a^3*b^2 - 4*a^4*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^
4 + 6*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^3*f^3*x^2 + 2*(3*(a^2*b^2*c -
4*a^3*c^2)*d^5 + 2*(a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c
)*d)*e^2*f^3*x + ((a^2*b^2*c - 4*a^3*c^2)*d^6 + (a^2*b^3 - 4*a^3*
b*c)*d^4 + (a^3*b^2 - 4*a^4*c)*d^2)*e*f^3) + 2*integrate(((b^3*c
- 4*a*b*c^2)*e^3*x^3 + 3*(b^3*c - 4*a*b*c^2)*d*e^2*x^2 + (b^3*c -
 4*a*b*c^2)*d^3 + (b^4 - 5*a*b^2*c + 3*a^2*c^2 + 3*(b^3*c - 4*a*b
```

```
*c^2)*d^2)*e*x + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*d)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d
^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 -
4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e*x), x)/(a^3*f^3) - 2*b*log(e*x + d)/(a^3*e*f^3)
```

Fricas [A] time = 1.59886, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^3),x, algorithm='
```

```
[Out] [-1/2*(((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6
*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c
^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a
*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15
*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 +
2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c +
 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*log
((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 -
 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2
*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^
4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*
c*d^3 + b*d)*e*x + a)) + (2*(a*b^2*c - 3*a^2*c^2)*e^4*x^4 + 8*(a*
b^2*c - 3*a^2*c^2)*d*e^3*x^3 + 2*(a*b^2*c - 3*a^2*c^2)*d^4 + (2*a
*b^3 - 7*a^2*b*c + 12*(a*b^2*c - 3*a^2*c^2)*d^2)*e^2*x^2 + a^2*b^
2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*d^2 + 2*(4*(a*b^2*c - 3*a^2*c
^2)*d^3 + (2*a*b^3 - 7*a^2*b*c)*d)*e*x - ((b^3*c - 4*a*b*c^2)*e^6
*x^6 + 6*(b^3*c - 4*a*b*c^2)*d*e^5*x^5 + (b^4 - 4*a*b^2*c + 15*(b
^3*c - 4*a*b*c^2)*d^2)*e^4*x^4 + (b^3*c - 4*a*b*c^2)*d^6 + 4*(5*(
b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 4*a*b^2*c)*d)*e^3*x^3 + (b^4 - 4*
a*b^2*c)*d^4 + (15*(b^3*c - 4*a*b*c^2)*d^4 + a*b^3 - 4*a^2*b*c +
6*(b^4 - 4*a*b^2*c)*d^2)*e^2*x^2 + (a*b^3 - 4*a^2*b*c)*d^2 + 2*(3
*(b^3*c - 4*a*b*c^2)*d^5 + 2*(b^4 - 4*a*b^2*c)*d^3 + (a*b^3 - 4*a
^2*b*c)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
+ b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3*c - 4
*a*b*c^2)*e^6*x^6 + 6*(b^3*c - 4*a*b*c^2)*d*e^5*x^5 + (b^4 - 4*a*
b^2*c + 15*(b^3*c - 4*a*b*c^2)*d^2)*e^4*x^4 + (b^3*c - 4*a*b*c^2)
*d^6 + 4*(5*(b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 4*a*b^2*c)*d)*e^3*x^
3 + (b^4 - 4*a*b^2*c)*d^4 + (15*(b^3*c - 4*a*b*c^2)*d^4 + a*b^3 -
 4*a^2*b*c + 6*(b^4 - 4*a*b^2*c)*d^2)*e^2*x^2 + (a*b^3 - 4*a^2*b*
c)*d^2 + 2*(3*(b^3*c - 4*a*b*c^2)*d^5 + 2*(b^4 - 4*a*b^2*c)*d^3 +
 (a*b^3 - 4*a^2*b*c)*d)*e*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/(((
a^3*b^2*c - 4*a^4*c^2)*e^7*f^3*x^6 + 6*(a^3*b^2*c - 4*a^4*c^2)*d*
```

```
e^6*f^3*x^5 + (a^3*b^3 - 4*a^4*b*c + 15*(a^3*b^2*c - 4*a^4*c^2)*d
^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4
*a^4*b*c)*d)*e^4*f^3*x^3 + (a^4*b^2 - 4*a^5*c + 15*(a^3*b^2*c - 4
*a^4*c^2)*d^4 + 6*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^3*f^3*x^2 + 2*(3*(
a^3*b^2*c - 4*a^4*c^2)*d^5 + 2*(a^3*b^3 - 4*a^4*b*c)*d^3 + (a^4*b
^2 - 4*a^5*c)*d)*e^2*f^3*x + ((a^3*b^2*c - 4*a^4*c^2)*d^6 + (a^3*
b^3 - 4*a^4*b*c)*d^4 + (a^4*b^2 - 4*a^5*c)*d^2)*e*f^3)*sqrt(b^2 -
 4*a*c)), 1/2*(2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(
b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6
*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 +
(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2
 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 +
(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^
3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b
^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6
*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d
)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) - (2*(a*b^2*c - 3*a^2*c^2)*e^4*x^4 + 8*(a
*b^2*c - 3*a^2*c^2)*d*e^3*x^3 + 2*(a*b^2*c - 3*a^2*c^2)*d^4 + (2*
a*b^3 - 7*a^2*b*c + 12*(a*b^2*c - 3*a^2*c^2)*d^2)*e^2*x^2 + a^2*b
^2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*d^2 + 2*(4*(a*b^2*c - 3*a^2*
c^2)*d^3 + (2*a*b^3 - 7*a^2*b*c)*d)*e*x - ((b^3*c - 4*a*b*c^2)*e^
6*x^6 + 6*(b^3*c - 4*a*b*c^2)*d*e^5*x^5 + (b^4 - 4*a*b^2*c + 15*(
b^3*c - 4*a*b*c^2)*d^2)*e^4*x^4 + (b^3*c - 4*a*b*c^2)*d^6 + 4*(5*
(b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 4*a*b^2*c)*d)*e^3*x^3 + (b^4 - 4
*a*b^2*c)*d^4 + (15*(b^3*c - 4*a*b*c^2)*d^4 + a*b^3 - 4*a^2*b*c +
 6*(b^4 - 4*a*b^2*c)*d^2)*e^2*x^2 + (a*b^3 - 4*a^2*b*c)*d^2 + 2*(
3*(b^3*c - 4*a*b*c^2)*d^5 + 2*(b^4 - 4*a*b^2*c)*d^3 + (a*b^3 - 4*
a^2*b*c)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3*c -
4*a*b*c^2)*e^6*x^6 + 6*(b^3*c - 4*a*b*c^2)*d*e^5*x^5 + (b^4 - 4*a
*b^2*c + 15*(b^3*c - 4*a*b*c^2)*d^2)*e^4*x^4 + (b^3*c - 4*a*b*c^2
)*d^6 + 4*(5*(b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 4*a*b^2*c)*d)*e^3*x
^3 + (b^4 - 4*a*b^2*c)*d^4 + (15*(b^3*c - 4*a*b*c^2)*d^4 + a*b^3
- 4*a^2*b*c + 6*(b^4 - 4*a*b^2*c)*d^2)*e^2*x^2 + (a*b^3 - 4*a^2*b
*c)*d^2 + 2*(3*(b^3*c - 4*a*b*c^2)*d^5 + 2*(b^4 - 4*a*b^2*c)*d^3
+ (a*b^3 - 4*a^2*b*c)*d)*e*x)*log(e*x + d))*sqrt(-b^2 + 4*a*c))/(
((a^3*b^2*c - 4*a^4*c^2)*e^7*f^3*x^6 + 6*(a^3*b^2*c - 4*a^4*c^2)*
d*e^6*f^3*x^5 + (a^3*b^3 - 4*a^4*b*c + 15*(a^3*b^2*c - 4*a^4*c^2)
*d^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 -
 4*a^4*b*c)*d)*e^4*f^3*x^3 + (a^4*b^2 - 4*a^5*c + 15*(a^3*b^2*c -
 4*a^4*c^2)*d^4 + 6*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^3*f^3*x^2 + 2*(3
*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 2*(a^3*b^3 - 4*a^4*b*c)*d^3 + (a^4
*b^2 - 4*a^5*c)*d)*e^2*f^3*x + ((a^3*b^2*c - 4*a^4*c^2)*d^6 + (a^
3*b^3 - 4*a^4*b*c)*d^4 + (a^4*b^2 - 4*a^5*c)*d^2)*e*f^3)*sqrt(-b^
2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^3),x, algorithm='
```

```
[Out] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^
3), x)
```

3.653 $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal. Leaf size=423

$$\frac{b(5b^2-19ac)}{2a^3ef^4(b^2-4ac)(d+ex)}-\frac{5b^2-14ac}{6a^2ef^4(b^2-4ac)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3ef^4(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3ef^4(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^4(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}$$

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5
*b^2 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*
a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c +
28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) - (Sqrt[c
]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e*f^4)
```

Rubi [A] time = 6.70817, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$

$$\frac{b(5b^2-19ac)}{2a^3ef^4(b^2-4ac)(d+ex)}-\frac{5b^2-14ac}{6a^2ef^4(b^2-4ac)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3ef^4(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3ef^4(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^4(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5
*b^2 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*
a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c +
28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) - (Sqrt[c
]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e*f^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.63944, size = 387, normalized size = 0.91

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2\right)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+\frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-28\right.}{}}{12a^3ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*
b^2*c + 2*a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/(
(b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]
*Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*
c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x
))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c - 28*
a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
```

```
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

Maple [C] time = 0.015, size = 1569, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 3/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c^2*e^2/(4*a*c-b^2)*x^3-1/2/
f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b^3*c*e^2/(4*a*c-b^2)*x^3+9/2/f^4/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*c^2*e/(4*a*c-b^2)*x^2-3/2/f^4/a^3/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)*d*b^3*c*e/(4*a*c-b^2)*x^2+9/2/f^4/a^2/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c^2*d^2-3/2/f^4/a^3/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3*c*d^2-1/f^4/a/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)/(4*a*c-b^2)*x*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*x*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^4+3/2
/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b*c^2-1/2/f^4/a
^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-1/f^4/a/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)*d/e/(4*a*c-b^2)*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/
(4*a*c-b^2)*b^4+1/4/f^4/a^3/e*sum((b*c*e^2*(19*a*c-5*b^2)*_R^2+2*
b*c*d*e*(19*a*c-5*b^2)*_R+19*a*b*c^2*d^2-5*b^3*c*d^2-14*a^2*c^2+2
4*a*b^2*c-5*b^4)/(4*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2
+a))-1/3/f^4/a^2/e/(e*x+d)^3+2/f^4/a^3*b/e/(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^4),x, algorithm='
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 18*(5*b^3*c - 19*a*b*c^2)
*d*e^5*x^5 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19
*a*b*c^2)*d^2)*e^4*x^4 + 3*(5*b^3*c - 19*a*b*c^2)*d^6 + 4*(15*(5*
b^3*c - 19*a*b*c^2)*d^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e
^3*x^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^4 + (45*(5*b^3*c -
19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(15*b^4 - 62*a*b^2*c
+ 14*a^2*c^2)*d^2)*e^2*x^2 - 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*
a^2*b*c)*d^2 + 2*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62*a
*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x)/((a^3*b
^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f
^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e
^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*
b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*
c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b
^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2
 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(
a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x +
 ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*
b^2 - 4*a^5*c)*d^3)*e*f^4) + 1/2*integrate(((5*b^3*c - 19*a*b*c^2
)*e^2*x^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + 2*(5*b^3*c - 19*a*b
*c^2)*d*e*x + (5*b^3*c - 19*a*b*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*
x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^
3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b
*c)*d)*e*x), x)/(a^3*f^4)
```

Fricas [A] time = 0.532661, size = 8038, normalized size = 19.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^4),x, algorithm='
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2
)*d*e^5*x^5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c -
 19*a*b*c^2)*d^2)*e^4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*
(5*b^3*c - 19*a*b*c^2)*d^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d
)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^4 + 2*(45*(5*b
^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(15*b^4 - 62*a
*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 20*(a*
b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^
4 - 62*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x
- 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c
 - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*
```

c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^
3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 3
5*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4
*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*
b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c -
4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^
5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4
*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 -
12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 766
86*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 1
2*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9
- 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b
*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^
2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 -
50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*
b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d
+ 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 -
2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b
^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((
a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^1
6)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*
b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2
*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*
a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c
+ 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24
108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 13
86*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))) + 3*sqrt(1
/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c
^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*
c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b
^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2
*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 +
(21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 +
3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2
)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*
e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*
d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^
4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250
*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4
*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b
^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*lo
g((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3
*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 +
43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqr
t(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*
b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt((625
*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 -

12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - (125
*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 +
160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 109
76*a^7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^
2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a
^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^
2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^
2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*
c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))) + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*
f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*
e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4
*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4
*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*
b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^
2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*
(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x
+ ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4
*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^
9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c +
 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 2410
8*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386
*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8
*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 +
9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b
^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*
a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4
672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 825
0*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + (125*b^14 - 242
5*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*
b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)
*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*
c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 -
 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401
*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*
a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b
^2*c^2 - 64*a^10*c^3)*e^2*f^8))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^
4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a
^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5
+ 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5
*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 +
10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^
4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)
*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4
*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*
c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5
*c)*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b

```
^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^
5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 -
64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2
 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c +
48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 12325
*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8
)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 9
4*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3
*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c +
 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 2410
8*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + (125*b^14 - 2425*a*b^12*c +
 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 17
2990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt
(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b
^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((
a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^1
6)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3
- 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*
a^10*c^3)*e^2*f^8))))/((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a
^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(
a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4
*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a
^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*
d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3
- 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3
*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^
2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3
*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^4),x, algorithm='
```

```
[Out] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^2*(e*f*x + d*f)^
4), x)
```

3.654 $\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$

Optimal. Leaf size=353

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}f^4\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c +
12*b*c*(d + e*x)^2))/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]
)*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sq
rt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 1.89515, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}f^4\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c +
```

```
12*b*c*(d + e*x)^2))/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]
)*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sq
rt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 131.81, size = 328, normalized size = 0.93

$$-\frac{3\sqrt{2}\sqrt{c}f^4\left(ac+\frac{3b^2}{4}+\frac{b\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2e\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{3\sqrt{2}\sqrt{c}f^4\left(ac+\frac{3b^2}{4}-\frac{b\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2e\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{f^4\left(2a+b\left(d+ex\right)^2\right)\left(d+ex\right)}{4e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)^2}-\frac{f^4\left(d+ex\right)\left(-4ac+7b^2+12bc\left(d+ex\right)^2\right)}{8e\left(-4ac+b^2\right)^2\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*sqrt(2)*sqrt(c)*f**4*(a*c + 3*b**2/4 + b*sqrt(-4*a*c + b**2)/2
)*atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(
2*e*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + 3*sqr
t(2)*sqrt(c)*f**4*(a*c + 3*b**2/4 - b*sqrt(-4*a*c + b**2)/2)*atan
(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(2*e*sq
rt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + f**4*(2*a +
 b*(d + e*x)**2)*(d + e*x)/(4*e*(-4*a*c + b**2)*(a + b*(d + e*x)*
*2 + c*(d + e*x)**4)**2) - f**4*(d + e*x)*(-4*a*c + 7*b**2 + 12*b
*c*(d + e*x)**2)/(8*e*(-4*a*c + b**2)**2*(a + b*(d + e*x)**2 + c*
(d + e*x)**4))
```

Mathematica [A] time = 6.21997, size = 351, normalized size = 0.99

$$f^4\left(-\frac{-2a(d+ex)-b(d+ex)^3}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{4ac(d+ex)-7b^2(d+ex)-12bc(d+ex)^3}{8e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}-4ac-3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^4*(-(-2*a*(d + e*x) - b*(d + e*x)^3)/(4*(b^2 - 4*a*c)*e*(a + b*
(d + e*x)^2 + c*(d + e*x)^4)^2) + (-7*b^2*(d + e*x) + 4*a*c*(d +
e*x) - 12*b*c*(d + e*x)^3)/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) - (3*Sqrt[c]*(-3*b^2 - 4*a*c + 2*b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqr
t[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e))
```

Maple [C] time = 0.015, size = 3432, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -63/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^5*b*d^2+3/16*f^4/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c
*d^2+4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*
_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*
_Z+c*d^4+b*d^2+a))-3/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/e/(16*a^2*c
```

^2-8*a*b^2*c+b^4)*a*b^2-63/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^2*b+5*f^4/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2*d^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c^2-95/4*f^4/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*d^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c-21/2*f^4
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^6-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
*a^2*c-6*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^2*a*b*c-3/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*x*a*b^2-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c
^2-8*a*b^2*c+b^4)*b^3-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^3*b^3-15/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*d^2+5/2*f^4/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*c^2*d^4-95/8*f^4/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c*d^4-3/2*f^4/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*d^7/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2+1/2*f^4/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*d^5/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*c^2-3
/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^7-15/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^2*b^3-21/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*x*b*c^2*d^6-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/e/(
16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*
d/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*c+1/2*f^4/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*a-19/8*f^4/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b^2-6*f^4/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b*c*d^2-2*f^
4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b*c
-105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^4*b+5/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^4*a-95/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*

```
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*
e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2-105/2*f^4/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*c^2*d^4*b+5*f^4/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*c^2*d^2-
95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3*b^2*c*d^2-2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*
d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3*a*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxi
```

```
[Out] -3/8*f^4*integrate((4*b*c*e^2*x^2 + 8*b*c*d*e*x + 4*b*c*d^2 - b^2
 - 4*a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*
x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) - 1/8*(12*b*c^2*e^7*f^4*x^7 + 84*b*c^2*d*e^6*f^4*x^6 +
 (252*b*c^2*d^2 + 19*b^2*c - 4*a*c^2)*e^5*f^4*x^5 + 5*(84*b*c^2*d
^3 + (19*b^2*c - 4*a*c^2)*d)*e^4*f^4*x^4 + (420*b*c^2*d^4 + 5*b^3
 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*e^3*f^4*x^3 + (252*b*c
^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^
2*f^4*x^2 + (84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2
+ 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + (12*b*c^2*d^7 +
(19*b^2*c - 4*a*c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*
a^2*c)*d)*f^4)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d
^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3
*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8
*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Fricas [A] time = 0.509447, size = 9140, normalized size = 25.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fric
```

```
[Out] -1/16*(24*b*c^2*e^7*f^4*x^7 + 168*b*c^2*d*e^6*f^4*x^6 + 2*(252*b*
c^2*d^2 + 19*b^2*c - 4*a*c^2)*e^5*f^4*x^5 + 10*(84*b*c^2*d^3 + (1
9*b^2*c - 4*a*c^2)*d)*e^4*f^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16
*a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*e^3*f^4*x^3 + 2*(252*b*c^2*
d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*f
^4*x^2 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 +
 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + 2*(12*b*c^2*d^7 +
 (19*b^2*c - 4*a*c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4
*a^2*c)*d)*f^4 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)
*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5
 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2
*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6
 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3
*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-
((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c +
 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*
c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*
a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^1
0 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b
^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*
a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^
12 + 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4
*c^4)*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*
b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)
*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*
b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*sqrt(-((b^5
 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c + 160*
a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*
sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b
^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 2
```

```
0*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^
4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^
8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e
^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2
)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d
^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*
x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)
*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a
^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*
c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^
2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 +
1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2
*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2
*c^3)*d*f^12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3
 - 256*a^4*c^4)*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 +
 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*
a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2
- 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*s
qrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^
8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((
a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*
a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)
*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + (
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6
- 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b
```

*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b
^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5
*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 16
0*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5
)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*
b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c
+ 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^
2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a
^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 + (a*b^13 - 8*a^2*b^11*c - 80*a^3
*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^
5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^
4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^
4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4
*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4
))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c
^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*
b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)
*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c
^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*
f^8 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3
+ 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*
b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 10
24*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2
- 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27
*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 4
0*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6
*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 + (a*b^13 - 8*a^2*b^11*
c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*
a^6*b^3*c^5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*
c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a
^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 -
 (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 128
0*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c
 + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^
7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640
*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))))/((b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 +

```
 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 +
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^
2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx + df)^4}{\left((ex + d)^4 c + (ex + d)^2 b + a\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac
```

```
[Out] integrate((e*f*x + d*f)^4/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,
x)
```

3.655 $\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 -
4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcT
anh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2
)*e)
```

Rubi [A] time = 0.402498, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 -
4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcT
anh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2
)*e)
```

Rubi in Sympy [A] time = 39.0261, size = 144, normalized size = 0.91

$$\frac{3bcf^3\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\left(-4ac+b^2\right)^{\frac{5}{2}}}-\frac{3bf^3\left(b+2c\left(d+ex\right)^2\right)}{4e\left(-4ac+b^2\right)^2\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}+\frac{f^3\left(2a+b\left(d+ex\right)^2\right)}{4e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*f**3*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(
-4*a*c + b**2)**(5/2)) - 3*b*f**3*(b + 2*c*(d + e*x)**2)/(4*e*(-4
*a*c + b**2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) + f**3*(2*
a + b*(d + e*x)**2)/(4*e*(-4*a*c + b**2)*(a + b*(d + e*x)**2 + c*
(d + e*x)**4)**2)
```

Mathematica [A] time = 0.355179, size = 149, normalized size = 0.94

$$\frac{f^3\left(\frac{(b^2-4ac)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2(b+c(d+ex)^2)\right)^2}-\frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*
x)^4) + ((b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b
 + c*(d + e*x)^2))^2 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[
-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.016, size = 2181, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -3/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^6-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^4*b*c^2*d/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^5-45/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^2*e^3/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2-9/4*f^3/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*b^2*c*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-30*f^3/(c*e^4*x^4+4*c*d
```

```
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*c^2*d^3*e^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-9*f^3/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2*c*d*e^2*b^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-45
/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2*c^2*d^4-27/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*e/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^2*c*d^2-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*a*c-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^3
*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*
d^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-9*f^3/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*d^3*b^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c-5*f^3/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*d*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*c-f^3/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2*d*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*f^3/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^6-9/4*f^3/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c*d^4-5/2*f
^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b*c*d^
2-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b
^3*d^2-2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4
)*a^2*c-1/4*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+
b^4)*a*b^2+3/2*f^3*b*c/e*sum((-_R*e-d)/(16*a^2*c^2-8*a*b^2*c+b^4)
/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln
(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxi
```

```
[Out] -3*b*c*f^3*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)
/(b^4 - 8*a*b^2*c + 16*a^2*c^2) - 1/4*(6*b*c^2*e^6*f^3*x^6 + 36*b
*c^2*d*e^5*f^3*x^5 + 9*(10*b*c^2*d^2 + b^2*c)*e^4*f^3*x^4 + 12*(1
```

```
0*b*c^2*d^3 + 3*b^2*c*d)*e^3*f^3*x^3 + 2*(45*b*c^2*d^4 + 27*b^2*c
*d^2 + b^3 + 5*a*b*c)*e^2*f^3*x^2 + 4*(9*b*c^2*d^5 + 9*b^2*c*d^3
+ (b^3 + 5*a*b*c)*d)*e*f^3*x + (6*b*c^2*d^6 + 9*b^2*c*d^4 + a*b^2
 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2)*f^3)/((b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*
e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)
*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d
^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^
3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^
2)*e)
```

Fricas [A] time = 0.43656, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fric
```

```
[Out] [1/4*(6*(b*c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*
d^2 + b^2*c^2)*e^6*f^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f
^3*x^5 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*
f^3*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*
d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(
b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*
d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 +
 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*
c)*f^3)*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*
e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 +
8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c
*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - (6*b*c^2*e^6*f^3*x^6 + 36*b*c
^2*d*e^5*f^3*x^5 + 9*(10*b*c^2*d^2 + b^2*c)*e^4*f^3*x^4 + 12*(10*
b*c^2*d^3 + 3*b^2*c*d)*e^3*f^3*x^3 + 2*(45*b*c^2*d^4 + 27*b^2*c*d
^2 + b^3 + 5*a*b*c)*e^2*f^3*x^2 + 4*(9*b*c^2*d^5 + 9*b^2*c*d^3 +
(b^3 + 5*a*b*c)*d)*e*f^3*x + (6*b*c^2*d^6 + 9*b^2*c*d^4 + a*b^2 +
 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2)*f^3)*sqrt(b^2 - 4*a*c))/(((b^4*
```

```
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*
b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)
*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c
^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*d^2)*e)*sqrt(b^2 - 4*a*c)), -1/4*(12*(b*c^3*e^8*f
^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*f
^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*d
^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^
3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(
14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d
^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b
^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 + 2*b^2*c^2*d^6 + 2*a
*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*arctan(-(2*c
*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) + (6*b*c^2*e^6*f^3*x^6 + 36*b*c^2*d*e^5*f^3*x^5 + 9*(10*b*c^
2*d^2 + b^2*c)*e^4*f^3*x^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)*e^3*f^
3*x^3 + 2*(45*b*c^2*d^4 + 27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*f^3*x
^2 + 4*(9*b*c^2*d^5 + 9*b^2*c*d^3 + (b^3 + 5*a*b*c)*d)*e*f^3*x +
(6*b*c^2*d^6 + 9*b^2*c*d^4 + a*b^2 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*
d^2)*f^3)*sqrt(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 +
2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 +
 (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^
4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*
(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d
^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt
(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(efx + df)^3}{\left((ex + d)^4 c + (ex + d)^2 b + a\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac
```

```
[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,
x)
```

3.656 $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} \, dx$

Optimal. Leaf size=375

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}f^2\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}f^2\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*
(d + e*x)^2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c)
 + c*(b^2 + 20*a*c)*(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 -
 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x
))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 -
52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x)
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] time = 2.08738, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}f^2\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}f^2\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*
(d + e*x)^2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c)
```

```
 + c*(b^2 + 20*a*c)*(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 -
 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x
))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 -
52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x)
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rubi in Sympy [A] time = 123.568, size = 345, normalized size = 0.92

$$-\frac{f^2\left(b+2c(d+ex)^2\right)(d+ex)}{4e\left(-4ac+b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{\sqrt{2}\sqrt{c}f^2\left(b\left(-52ac+b^2\right)-\sqrt{-4ac+b^2}\left(20ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{16ae\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{\sqrt{2}\sqrt{c}f^2\left(b\left(-52ac+b^2\right)+\sqrt{-4ac+b^2}\left(20ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{16ae\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{f^2(d+ex)\left(b\left(8ac+b^2\right)+c(d+ex)^2\left(20ac+b^2\right)\right)}{8ae\left(-4ac+b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -f**2*(b + 2*c*(d + e*x)**2)*(d + e*x)/(4*e*(-4*a*c + b**2)*(a +
b*(d + e*x)**2 + c*(d + e*x)**4)**2) - sqrt(2)*sqrt(c)*f**2*(b*(-
52*a*c + b**2) - sqrt(-4*a*c + b**2)*(20*a*c + b**2))*atan(sqrt(2
)*sqrt(c)*(d + e*x)/sqrt(b + sqrt(-4*a*c + b**2)))/(16*a*e*sqrt(b
 + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + sqrt(2)*sqrt(c)
*f**2*(b*(-52*a*c + b**2) + sqrt(-4*a*c + b**2)*(20*a*c + b**2))*
atan(sqrt(2)*sqrt(c)*(d + e*x)/sqrt(b - sqrt(-4*a*c + b**2)))/(16
*a*e*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + f**2
*(d + e*x)*(b*(8*a*c + b**2) + c*(d + e*x)**2*(20*a*c + b**2))/(8
*a*e*(-4*a*c + b**2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4))
```

Mathematica [A] time = 6.24641, size = 409, normalized size = 1.09

$$f^2\left(-\frac{b(d+ex)+2c(d+ex)^3}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{8abc(d+ex)+20ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{8ae\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^2*(-(b*(d + e*x) + 2*c*(d + e*x)^3)/(4*(b^2 - 4*a*c)*e*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)^2) + (b^3*(d + e*x) + 8*a*b*c*(d + e*
x) + b^2*c*(d + e*x)^3 + 20*a*c^2*(d + e*x)^3)/(8*a*(-b^2 + 4*a*c
)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^3 - 52*a
*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[
2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c
]*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]
]*e))
```

Maple [C] time = 0.015, size = 4751, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 35*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
3*b*c^2*d^2+1/16*f^2/a/e*sum((e^2*c*(20*a*c+b^2)*_R^2+2*c*d*e*(20
*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c*d^2-16*a*b*c+b^3)/(16*a^2*c^2-8*a
```

```
*b^2*c+b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*
b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^
2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+1/8*f^2/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^4+7/2
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*
c^2+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a*b^2*c+
b^4)*a*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a
*b^2*c+b^4)/a*b^4+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*a*x*c^2*d^2+3/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^
2*c^2-8*a*b^2*c+b^4)/a*x*b^4*d^2+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*d^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^3*c+21/8*f^2/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^2*c^2+5/4
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/
a*x^4*b^3+7/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^5/(16*a^2*c^2-
8*a*b^2*c+b^4)/a*x^6*b^2+21/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^
4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5*b^2*d^2+35/8*f^2/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^2+35/8
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^
3*b^2*c^2*d^4+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*
a*b^2*c+b^4)/a*x^3*b^3*c*d^2+2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*a*x*b*c-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/
e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*d^7/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3+35/2*f^2/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3*d^6+5/2*f^2/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*c^3*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+35/2*f^2/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2*c^3*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+105/
2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^5*d^2+5/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/e/(16*a^2*c^2-8*a*b
^2*c+b^4)*b^2*c+7/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^5*b+175/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d
```

```
^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+175/2*f^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*c^3*d^4+5/8*f^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c+105/2
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2*c^3+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*x*b*c^2*d^4+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*x*b^2*c*d^2+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c+35*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d
^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+1/4*f^2/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*d^5/e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*b^3*c+1/8*f^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*c^2*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7*b^2+2
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b*
c+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^
4)/a*x^5*b^3+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*
a*b^2*c+b^4)*a*x^2*c^2+5/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c
^2-8*a*b^2*c+b^4)/a*x*b^3*c*d^4+3/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*
d*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^4+7/8*f^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^2*c^2*d^6+35/2*f^2/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+1
/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^7/e/(16*a^2*c^2-8*a*b^2*c+b^4)/
a*b^2*c^2+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*a*x^3*c^2-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxi
```

```
[Out] 1/8*f^2*integrate(((b^2*c + 20*a*c^2)*e^2*x^2 + 2*(b^2*c + 20*a*c
^2)*d*e*x + b^3 - 16*a*b*c + (b^2*c + 20*a*c^2)*d^2)/(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d
^3 + b*d)*e*x + a), x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*(
(b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 7*(b^2*c^2 + 20*a*c^3)*d*e^6*f
^2*x^6 + (2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5
*f^2*x^5 + 5*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)
*d)*e^4*f^2*x^4 + (35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c
+ 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*f^2*x^3 + (21*(b^
2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*
a*b^2*c + 36*a^2*c^2)*d)*e^2*f^2*x^2 + (7*(b^2*c^2 + 20*a*c^3)*d^
6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c + 3*(b^4 + 5
*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*x + ((b^2*c^2 + 20*a*c^3)*d^7 +
 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3
- (a*b^3 - 16*a^2*b*c)*d)*f^2)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^
8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2
 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16
*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*
b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)
*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*
c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2)*d^2)*e)
```

Fricas [A] time = 0.582921, size = 10581, normalized size = 28.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fric
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 14*(b^2*c^2 + 20*a*c^3
)*d*e^6*f^2*x^6 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^
3)*d^2)*e^5*f^2*x^5 + 10*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c +
 14*a*b*c^2)*d)*e^4*f^2*x^4 + 2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^
4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*f^2
*x^3 + 2*(21*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d
^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*e^2*f^2*x^2 + 2*(7*(b^2*
```

```
c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^
2*b*c + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*x + 2*((b^2*c
^2 + 20*a*c^3)*d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^2*
c + 36*a^2*c^2)*d^3 - (a*b^3 - 16*a^2*b*c)*d)*f^2 + sqrt(1/2)*((a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8
*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2
 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2
)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 +
3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 -
6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*
x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c
+ 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c
 - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*
b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16
*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3
*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4
 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 -
 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 -
 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*
c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a
^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280
*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8
*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c
^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 1500
0*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*
a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4
- 25600*a^5*b*c^5)*e*f^4 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^
10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 -
 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625
*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a
^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-
((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3
*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a
^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*
f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3
 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*
a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4
- 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4
*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 +
70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8
*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*
a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*
a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 +
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2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b
*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4
(2(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8
*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*
c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 +
(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c
 + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c
^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6
*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 16
0*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c
^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*
a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*
c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x
 + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c
^5)*d*f^6 - 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 -
 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 -
(a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 +
 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*
a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 -
20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*
c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^
2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8
*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024
*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2
 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + sq
rt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*
a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5
+ (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 +
15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^
2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c
- 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a
^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sq
rt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 -
(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
80*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c
^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4

*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 102

```
4*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 -
 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2
*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*
e^2))))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*
a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5
+ (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 +
15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^
2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c
- 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a
^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.284021, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac

[Out] Done
```

3.657 $\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a
*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(
b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi [A] time = 0.382989, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$-\frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a
*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(
b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rubi in Sympy [A] time = 34.3245, size = 139, normalized size = 0.91

$$-\frac{6c^2f\,\mathrm{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{e\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{3cf\left(b+2c\left(d+ex\right)^2\right)}{2e\left(-4ac+b^2\right)^2\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)}-\frac{f\left(b+2c\left(d+ex\right)^2\right)}{4e\left(-4ac+b^2\right)\left(a+b\left(d+ex\right)^2+c\left(d+ex\right)^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -6*c**2*f*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(e*(-
4*a*c + b**2)**(5/2)) + 3*c*f*(b + 2*c*(d + e*x)**2)/(2*e*(-4*a*c
 + b**2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) - f*(b + 2*c*(
d + e*x)**2)/(4*e*(-4*a*c + b**2)*(a + b*(d + e*x)**2 + c*(d + e*
x)**4)**2)
```

Mathematica [A] time = 0.318006, size = 148, normalized size = 0.97

$$\frac{f\left(\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2
+ c*(d + e*x)^4) + (24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2
 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] time = 0.014, size = 2132, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^6+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^4*d*c^3/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^5+45*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^3/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^4*d^2+9/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^4*b+60*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
```

```
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2
*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b+45*f/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*c^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*d^4+27*f/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*d^2+5*f/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a+f/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*c*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2+18*f/(c
*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*c^3*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x+18*f/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b+1
0*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a
+2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^
2+3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d
^6+9/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^
2*d^4*b+5*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)
*a*c^2*d^2+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4
)*b^2*c*d^2+5/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*
c+b^4)*a*b*c-1/4*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2
*c+b^4)*b^3+3*f*c^2/e*sum((_R*e+d)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*
_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_
R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(
4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="maxima")
```

```
[Out] 6*c^2*f*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
(6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b
^4 - 8*a*b^2*c + 16*a^2*c^2) + 1/4*(12*c^3*e^6*f*x^6 + 72*c^3*d*e
^5*f*x^5 + 18*(10*c^3*d^2 + b*c^2)*e^4*f*x^4 + 24*(10*c^3*d^3 + 3
*b*c^2*d)*e^3*f*x^3 + 4*(45*c^3*d^4 + 27*b*c^2*d^2 + b^2*c + 5*a*
```

```
c^2)*e^2*f*x^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2*c + 5*a*c^2)*d
)*e*f*x + (12*c^3*d^6 + 18*b*c^2*d^4 - b^3 + 10*a*b*c + 4*(b^2*c
+ 5*a*c^2)*d^2)*f)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8
+ 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3
+ 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*
b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4
+ 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 +
 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4
 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Fricas [A] time = 0.444369, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="fricas
```

```
[Out] [1/4*(12*(c^4*e^8*f*x^8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c
^3)*e^6*f*x^6 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^
4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 +
 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6
+ 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 +
 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3
)*e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a
*c^3)*d^4 + a^2*c^2)*f)*log(-(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^
2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 -
(2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c
)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*
sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
+ b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (12*c^3*e^6*
f*x^6 + 72*c^3*d*e^5*f*x^5 + 18*(10*c^3*d^2 + b*c^2)*e^4*f*x^4 +
24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*f*x^3 + 4*(45*c^3*d^4 + 27*b*c^2*
d^2 + b^2*c + 5*a*c^2)*e^2*f*x^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (
b^2*c + 5*a*c^2)*d)*e*f*x + (12*c^3*d^6 + 18*b*c^2*d^4 - b^3 + 10
*a*b*c + 4*(b^2*c + 5*a*c^2)*d^2)*f)*sqrt(b^2 - 4*a*c))/(((b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 +
 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^
```

```
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 +
 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6
- 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*
a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e
^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c +
16*a^3*b*c^2)*d^2)*e)*sqrt(b^2 - 4*a*c)), 1/4*(24*(c^4*e^8*f*x^8
+ 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^
4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c
^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2
 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2
 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*
d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*f*x + (c^4*d^8 + 2*b
*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f)*
arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c
)/(b^2 - 4*a*c)) + (12*c^3*e^6*f*x^6 + 72*c^3*d*e^5*f*x^5 + 18*(1
0*c^3*d^2 + b*c^2)*e^4*f*x^4 + 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*f*
x^3 + 4*(45*c^3*d^4 + 27*b*c^2*d^2 + b^2*c + 5*a*c^2)*e^2*f*x^2 +
 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2*c + 5*a*c^2)*d)*e*f*x + (12*c^
3*d^6 + 18*b*c^2*d^4 - b^3 + 10*a*b*c + 4*(b^2*c + 5*a*c^2)*d^2)*
f)*sqrt(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9
*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*
x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)
*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-b^2 +
 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{efx + df}{\left((ex+d)^4c + (ex+d)^2b + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a)^3, x)
```

3.658 $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=270

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}+\frac{\log(d+ex)}{a^3ef}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 +
 2*b*c*(b^2 - 7*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*
c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^
2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - Log[a + b*(d + e
*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rubi [A] time = 1.0043, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}+\frac{\log(d+ex)}{a^3ef}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 +
 2*b*c*(b^2 - 7*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*
c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^
2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - Log[a + b*(d + e
*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rubi in Sympy [A] time = 113.605, size = 246, normalized size = 0.91

$$\frac{-2ac+b^2+bc(d+ex)^2}{4aef(-4ac+b^2)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{16a^2c^2-15ab^2c+2b^4+2bc(d+ex)^2(-7ac+b^2)}{4a^2ef(-4ac+b^2)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b(30a^2c^2-10ab^2c+b^4)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^3ef(-4ac+b^2)^{\frac{5}{2}}}+\frac{\log\left((d+ex)^2\right)}{2a^3ef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(4*a*e*f*(-4*a*c + b**2)*(a +
b*(d + e*x)**2 + c*(d + e*x)**4)**2) + (16*a**2*c**2 - 15*a*b**2*
c + 2*b**4 + 2*b*c*(d + e*x)**2*(-7*a*c + b**2))/(4*a**2*e*f*(-4*
a*c + b**2)**2*(a + b*(d + e*x)**2 + c*(d + e*x)**4)) + b*(30*a**
2*c**2 - 10*a*b**2*c + b**4)*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4
*a*c + b**2))/(2*a**3*e*f*(-4*a*c + b**2)**(5/2)) + log((d + e*x)
**2)/(2*a**3*e*f) - log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(4*a
**3*e*f)
```

Mathematica [A] time = 6.20379, size = 436, normalized size = 1.61

$$\frac{\log(d+ex)}{a^3ef}+\frac{16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^4+2b^3c(d+ex)^2}{4a^2ef(4ac-b^2)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\left(-16a^2c^2\sqrt{b^2-4ac}-30a^2bc^2+10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac}-b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^3ef(b^2-4ac)^{5/2}}+\frac{\left(-16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac}+b^5\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^3ef(b^2-4ac)^{5/2}}+\frac{2ac-b^2-bc(d+ex)^2}{4aef(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-b^2 + 2*a*c - b*c*(d + e*x)^2)/(4*a*(-b^2 + 4*a*c)*e*f*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2
 + 2*b^3*c*(d + e*x)^2 - 14*a*b*c^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4
*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + Log[d + e*x]/(
a^3*e*f) + ((-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*
a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
```

```
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^3*(b^2 - 4*a
*c)^(5/2)*e*f) + ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2
 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4
*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^3*(b^2
- 4*a*c)^(5/2)*e*f)
```

Maple [C] time = 0.034, size = 4606, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^
3-21/4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c*
b^2+4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3
*d^4+6/f*a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*
c^2+3/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)
*b^4+4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4+6/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^2*b^4*c*d^2+15/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^3*d^2-70/f/a/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2*c^3*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b+10/f/a^2/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^3-
21/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*
b*c^3-21/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^3*d*e^4/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^5+3/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^3*c^2*d*e^4/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^5-105/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e
^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*d^2+3/f/a^2/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c^2-29/f/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2-1/f/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
```

d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2
/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^
2+16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3+24/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2*c^3*d^2-1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*
d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^2*b*c^2+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^2*b^5+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*
a^2*c^2-8*a*b^2*c+b^4)*x*b^5+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e
/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2-1/2/f/a^3/e*sum((c*e^3*(16*a^
2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R
^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*
a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b
*c^2*d-9*a*b^3*c*d+b^5*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^3*c*e^
3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=Roo
tOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+c*d^4+b*d^2+a))-29/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*
d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2+4/f/a^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*c*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-105/2/f/a/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+
15/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2*b^3*c^2*d^4-87/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^2*b^2*c^2*d^2+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^
3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-6/f/a/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-7/2/f/a/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^3*d^6+1/2/f/a^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c^2*d^6-29/4/f/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c^2*d
^4+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*
b^4*c*d^4-3/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+
b^4)*b^3*c*d^2-7/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5*b/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^6+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^
5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-29/4/f/a/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2

```
+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2+1/f/a^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4-3/f/a/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c+ln
(e*x+d)/a^3/e/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)),x, algorithm="ma
```

```
[Out] 1/4*(2*(b^3*c^2 - 7*a*b*c^3)*e^6*x^6 + 12*(b^3*c^2 - 7*a*b*c^3)*d
*e^5*x^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3 + 30*(b^3*c^2 - 7
*a*b*c^3)*d^2)*e^4*x^4 + 2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 4*(10*(b^3
*c^2 - 7*a*b*c^3)*d^3 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d)*
e^3*x^3 + 3*a*b^4 - 21*a^2*b^2*c + 24*a^3*c^2 + (4*b^4*c - 29*a*b
^2*c^2 + 16*a^2*c^3)*d^4 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2 + 15*(b
^3*c^2 - 7*a*b*c^3)*d^4 + 3*(4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)
*d^2)*e^2*x^2 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 4*(3*(b^3*c
^2 - 7*a*b*c^3)*d^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^3 +
 (b^5 - 6*a*b^3*c - a^2*b*c^2)*d)*e*x)/((a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4)*e^9*f*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16
*a^4*c^4)*d*e^8*f*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c
^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*f*x^6
 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*
b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*f*x^5 + (a^2*b^6 - 6
*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^
4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e
^5*f*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 +
 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6
*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*f*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c
+ 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^
6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^
6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*f*x^2 + 4*(2*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*f*x + ((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a
^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*
b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c +
16*a^5*b*c^2)*d^2)*e*f) - integrate(((b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*e^3*x^3 + 3*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^2*x^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 9*a*b^3*c + 23*a
^2*b*c^2 + 3*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e*x + (b^5 -
 9*a*b^3*c + 23*a^2*b*c^2)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/
```

```
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f) + log(e*x + d)/(a^3*e*f)
```

Fricas [A] time = 2.10442, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)),x, algorithm="fr
```

```
[Out] [1/4*(((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4
*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4
)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^
3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2
*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c
^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4
 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^
2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*
b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^
7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7
- 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*
a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*
b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(
b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2
*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10
*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2
*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)
*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*log((2*(b^
2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b*
c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c
*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 +
 b*d)*e*x + a)) + (2*(a*b^3*c^2 - 7*a^2*b*c^3)*e^6*x^6 + 12*(a*b^
3*c^2 - 7*a^2*b*c^3)*d*e^5*x^5 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16
*a^3*c^3 + 30*(a*b^3*c^2 - 7*a^2*b*c^3)*d^2)*e^4*x^4 + 2*(a*b^3*c
^2 - 7*a^2*b*c^3)*d^6 + 4*(10*(a*b^3*c^2 - 7*a^2*b*c^3)*d^3 + (4*
a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d)*e^3*x^3 + 3*a^2*b^4 - 2
1*a^3*b^2*c + 24*a^4*c^2 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c
^3)*d^4 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2 + 15*(a*b^3*c^2 - 7*
a^2*b*c^3)*d^4 + 3*(4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d^2)
*e^2*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*d^2 + 4*(3*(a*b^3*
c^2 - 7*a^2*b*c^3)*d^5 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3
)*d^3 + (a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*d)*e*x - ((b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*e^8*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d*e^7*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^6*x^6 + 4*(14*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2
```

```
*b*c^3)*d)*e^5*x^5 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4
+ 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(14*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^3*x^3 + a^2*b^4 -
 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 +
2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
+ 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 8*a^
2*b^3*c + 16*a^3*b*c^2)*d^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 -
6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e
^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c^2 - 8*a*b
^2*c^3 + 16*a^2*c^4)*e^8*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d*e^7*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^6*x^6 + 4*(14*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d)*e^5*x^5 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + 2
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(14*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^3*x^3 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3
*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)*d^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a
*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d
)*e*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/(((a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*e^9*f*x^8 + 8*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*d*e^8*f*x^7 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b
*c^3 + 14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^7*f*x
^6 + 4*(14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + 3*(a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^6*f*x^5 + (a^3*b^6 -
 6*a^4*b^4*c + 32*a^6*c^3 + 70*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*
a^5*c^4)*d^4 + 30*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)
*e^5*f*x^4 + 4*(14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5
 + 10*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + (a^3*b^6 -
 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^4*f*x^3 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2 + 14*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*
d^6 + 15*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 3*(a^3*
b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 3*(a^3*b^5*c - 8*a^4*b^3*c^
2 + 16*a^5*b*c^3)*d^5 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3
+ (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^2*f*x + (a^5*b^4 -
8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + (a^
3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 2*(a^4*b^5 - 8*a^5*b^3*c
+ 16*a^6*b*c^2)*d^2)*e*f)*sqrt(b^2 - 4*a*c)), -1/4*(2*((b^5*c^2 -
 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3
```

```
 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2
*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 +
4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10
*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3
 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3
*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6
*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a
^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^
3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(
b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 1
0*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*
a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^
3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(
b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c +
 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*
b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a
^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (
b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 1
0*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*arctan(-(2*c*e^2*x^2 + 4*c*
d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*(a*b^
3*c^2 - 7*a^2*b*c^3)*e^6*x^6 + 12*(a*b^3*c^2 - 7*a^2*b*c^3)*d*e^5
*x^5 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3 + 30*(a*b^3*c^2 -
 7*a^2*b*c^3)*d^2)*e^4*x^4 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*d^6 + 4*
(10*(a*b^3*c^2 - 7*a^2*b*c^3)*d^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 +
 16*a^3*c^3)*d)*e^3*x^3 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 +
 (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d^4 + 2*(a*b^5 - 6*a^2
*b^3*c - a^3*b*c^2 + 15*(a*b^3*c^2 - 7*a^2*b*c^3)*d^4 + 3*(4*a*b^
4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 6*a^
2*b^3*c - a^3*b*c^2)*d^2 + 4*(3*(a*b^3*c^2 - 7*a^2*b*c^3)*d^5 + (
4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*d^3 + (a*b^5 - 6*a^2*b^3
*c - a^3*b*c^2)*d)*e*x - ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^
8*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^7*x^7 + 2*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^2)*e^6*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^5*x^5 + (b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + (b^6 - 6*a*b^4*c + 32*a^3*c
^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + 2*(b^5*c - 8*a*b^3*c^2 +
 16*a^2*b*c^3)*d^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d
^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*d)*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(14*(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*
a^3*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^
2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 +
4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^
3 + b*d)*e*x + a) + 4*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^8*x
^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^7*x^7 + 2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^2)*e^6*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d
^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^5*x^5 + (b^4*c^2
```

```
 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
+ 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5
+ 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d)*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3
*c^3)*d^2)*e^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 +
 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))*sqrt(
-b^2 + 4*a*c))/(((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^9*f
*x^8 + 8*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^8*f*x^7 +
 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 14*(a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^4*c^2 -
 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 +
 16*a^5*b*c^3)*d)*e^6*f*x^5 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3
 + 70*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 30*(a^3*b^
5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 10*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^3 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3
)*d)*e^4*f*x^3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 14*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 15*(a^3*b^5*c - 8*a
^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 3*(a^3*b^6 - 6*a^4*b^4*c + 32*a^
6*c^3)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*d^7 + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + (
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + (a^4*b^5 - 8*a^5*b^3*c
+ 16*a^6*b*c^2)*d)*e^2*f*x + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
+ (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 2*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*
a^6*c^3)*d^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e*f)
*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

<u>GIAC/XCAS</u> [A] time = 0.338432, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)),x, algorithm="gi
```

```
[Out] Done
```

3.659 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=499

$$-\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3ef^2(b^2-4ac)^2(d+ex)}+\frac{36a^2c^2+bc(5b^2-32ac)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2(b^2-4ac)^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)(b^2-5ac)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3ef^2(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3ef^2(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^2(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*
(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e
*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 -
35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^
2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 -
47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b
^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rubi [A] time = 2.73162, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3ef^2(b^2-4ac)^2(d+ex)}+\frac{36a^2c^2+bc(5b^2-32ac)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2(b^2-4ac)^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)(b^2-5ac)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3ef^2(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3ef^2(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^2(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*
(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e
*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 -
35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^
2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 -
47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b
^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.325, size = 575, normalized size = 1.15

$$-\frac{1}{a^3ef^2(d+ex)}+\frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{4a^2ef^2\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}-5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-84a^2bc^2(d+ex)-52a^2c^3(d+ex)^3+52ab^3c(d+ex)+47ab^2c^2(d+ex)^3-7b^5(d+ex)-7b^4c(d+ex)^3}{8a^3ef^2\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) +
 b^2*c*(d + e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e
*f^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) +
 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x
)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d + e*x)^3)/(8*a^3*(
-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*S
qrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a
*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
e*f^2) - (3*Sqrt[c]*(-5*b^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*
Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Maple [C] time = 0.033, size = 7019, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^2),x, algorithm='
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 24*(5*b
^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + (30*b^5*c - 227*a
*b^3*c^2 + 392*a^2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*
c^4)*d^2)*e^6*x^6 + 6*(28*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)
*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d)*e^5*x^5 + 3*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + (15*b^6 - 91*a*b^4*
c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37*a*b^2*c^3
+ 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)
```

```
*d^2)*e^4*x^4 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 +
4*(42*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a
^2*b^2*c^2 + 324*a^3*c^3)*d)*e^3*x^3 + 8*a^2*b^4 - 64*a^3*b^2*c +
 128*a^4*c^2 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^
3)*d^4 + (84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^6 + 25*a*b
^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c - 227*a*b^3*c^2
 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 +
 324*a^3*c^3)*d^2)*e^2*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*
b*c^2)*d^2 + 2*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 +
3*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91
*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^
2*b^3*c + 364*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*f^2*x^
7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 -
6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*
a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)
*e^6*f^2*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^
6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 + 2*(a^4*b^5 - 8*a^5
*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*
(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a
^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^
6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*
x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^
4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*
a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(
a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*
c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^
5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*d)*e*f^2) - 3/8*integrate((5*b^5 - 42*a*b^
3*c + 92*a^2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*e^2*x^
2 + 2*(5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*d*e*x + (5*b^4*c - 37
*a*b^2*c^2 + 60*a^2*c^3)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f^2)
```

Fricas [A] time = 1.15901, size = 14199, normalized size = 28.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^2),x, algorithm='
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*
b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 22
7*a*b^3*c^2 + 392*a^2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a
^2*c^4)*d^2)*e^6*x^6 + 12*(28*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*
c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d)*e^5*x^5
+ 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + 2*(15*b^6 - 91*
a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37*a*b^
2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*
b*c^3)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^6 + 8*(42*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30
*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*
c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d)*e^3*x^3 + 16*a^2*b^4 - 128*a
^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2
+ 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4
)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25
*a^2*b^2*c^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*
b^3*c + 364*a^3*b*c^2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60
*a^2*c^4)*d^7 + 3*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5
+ 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^3 + (2
5*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x - 3*sqrt(1/2)*((a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3
+ 16*a^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)
*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*
b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5
*f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)
*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c
^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^
7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*
b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2
)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 -
 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^
6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*sqr
t(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3
+ 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a
^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*
c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^
5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2
 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)
))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3
 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^1
```

0*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^
7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4
 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2
835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b
^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 +
68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 -
323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 1
2250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a
^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20
*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2
*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (125*b^17 - 3775*a*b^15*c + 493
60*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 446
3140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 13
24800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b
^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5
+ (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 1225
0*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*
b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^
15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^
4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9
*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*
e^2*f^4))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c
^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(
a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(
6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
d^4 + 42(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2
x^5 + (126(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^
4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c +
 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6
 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c
^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d
^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a
^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^
3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5
 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8*
a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a
^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)*d)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*
b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5
 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3
+ 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 122
50*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4
*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a
^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c
^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^

```
9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)
*e^2*f^4))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*
b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*
c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c
^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*
d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*
c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*
c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8
)*e^3*f^6*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 3
51310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 506
25*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*
a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (12
5*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3
 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^
6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^1
1 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^
4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*
b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e
^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 3513
10*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*
a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^1
7*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b
^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^
11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))) + 3*sqrt(1/2)*((a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a
^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f
^2*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 -
 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2
+ 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*
c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4
+ 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a
^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4
*f^2*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 +
 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 -
 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^
6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9
*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 3
2*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^
2*f^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*
d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*sqrt(-(25*b^
11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a
^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9
*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*
e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351
310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^
```

17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*
b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a
^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 7
7825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 28350
00*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*
a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^
4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152
*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^1
1*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^
14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250*a*b^
10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^
4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 10
24*a^19*c^5)*e^4*f^8)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^
13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*
b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8
*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 -
15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^
10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^1
1*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*
c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 -
312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c
+ 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*
a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2
- 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))
) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*
f^2*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*
x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c
^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*
c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*
a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (1
26*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c +
32*a^6*c^3)*d)*e^5*f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b
c^2 + 42(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*
b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a
^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a
^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 -
8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 +
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b
^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7
*c^2)*d)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 -
15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b
^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^
11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10
*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4

```
- 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c
 + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024
*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2
 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4)
)*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 -
 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x
- 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957
349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d - 27/2*
sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 149
60*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342
528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6
*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3
*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^
6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*
c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) + (125*b^17 -
3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 162353
4*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 56846
72*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a
*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4
 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 -
 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sq
rt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^
6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/
((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3
 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*
a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^
4 - 1024*a^12*c^5)*e^2*f^4))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16
*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3
+ 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*f^2*x^7
+ 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*
a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e
^6*f^2*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5
+ 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6
- 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b
^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^
4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a
^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*
b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*
c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^
2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^
5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^
4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^
2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5
+ 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)*d)*e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^2),x, algorithm='
```

```
[Out] Exception raised: TypeError
```

3.660 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=343

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3ef^3\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4ef^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d
+ e*x)^2) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*
f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 -
 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^
2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*Arc
Tanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*
c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rubi [A] time = 1.22462, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3ef^3\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4ef^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d
+ e*x)^2) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*
```

```
f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 -
 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^
2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*Arc
Tanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*
c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rubi in Sympy [A] time = 176.663, size = 333, normalized size = 0.97

$$\frac{-2ac+b^2+bc(d+ex)^2}{4aef^3(d+ex)^2(-4ac+b^2)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{20a^2c^2-20ab^2c+3b^4+3bc(d+ex)^2\left(-6ac+b^2\right)}{4a^2ef^3(d+ex)^2(-4ac+b^2)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{3\left(-5ac+b^2\right)\left(-2ac+b^2\right)}{2a^3ef^3(d+ex)^2(-4ac+b^2)^2} - \frac{3b\log\left((d+ex)^2\right)}{2a^4ef^3} + \frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3} - \frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\operatorname{atanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{-4ac+b^2}}\right)}{2a^4ef^3(-4ac+b^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] (-2*a*c + b**2 + b*c*(d + e*x)**2)/(4*a*e*f**3*(d + e*x)**2*(-4*a
*c + b**2)*(a + b*(d + e*x)**2 + c*(d + e*x)**4)**2) + (20*a**2*c
**2 - 20*a*b**2*c + 3*b**4 + 3*b*c*(d + e*x)**2*(-6*a*c + b**2))/
(4*a**2*e*f**3*(d + e*x)**2*(-4*a*c + b**2)**2*(a + b*(d + e*x)**
2 + c*(d + e*x)**4)) - 3*(-5*a*c + b**2)*(-2*a*c + b**2)/(2*a**3*
e*f**3*(d + e*x)**2*(-4*a*c + b**2)**2) - 3*b*log((d + e*x)**2)/(
2*a**4*e*f**3) + 3*b*log(a + b*(d + e*x)**2 + c*(d + e*x)**4)/(4*
a**4*e*f**3) - 3*(-20*a**3*c**3 + 30*a**2*b**2*c**2 - 10*a*b**4*c
 + b**6)*atanh((b + 2*c*(d + e*x)**2)/sqrt(-4*a*c + b**2))/(2*a**
4*e*f**3*(-4*a*c + b**2)**(5/2))
```

Mathematica [A] time = 6.24747, size = 509, normalized size = 1.48

$$-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{1}{2a^3ef^3(d+ex)^2}+\frac{-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2}{4a^2ef^3(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)^2}$$
$$+\frac{-46a^2bc^2-28a^2c^3(d+ex)^2+29ab^3c+26ab^2c^2(d+ex)^2-4b^5-4b^4c(d+ex)^2}{4a^3ef^3(4ac-b^2)^2(a+b(d+ex)^2+c(d+ex)^4)}$$
$$+\frac{3\left(-20a^3c^3+30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+b^6\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d\right.}{4a^4ef^3(b^2-4ac)^{5/2}}$$
$$+\frac{3\left(20a^3c^3-30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}+10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}-b^6\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+e\right.}{4a^4ef^3(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/(2*a^3*e*f^3*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2
 - 2*a*c^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*f^3*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 -
 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d +
 e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*f^3*(a + b*(d + e*x)^2 + c*(d
+ e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*(b^6 - 10*a*b^4*
c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3
*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sq
rt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*
f^3) + (3*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*
Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sq
rt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4
*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)
```

Maple [C] time = 0.027, size = 5737, normalized size = 16.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^3),x, algorithm='
```

```
[Out] -1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*e^8*x^8 + 48*(b^4*c^
2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d*e^7*x^7 + 3*(4*b^5*c - 29*a*b^3*c
^2 + 46*a^2*b*c^3 + 56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^2)*
e^6*x^6 + 6*(56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^3 + 3*(4*b
^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d)*e^5*x^5 + 6*(b^4*c^2 - 7*a
*b^2*c^3 + 10*a^2*c^4)*d^8 + (6*b^6 - 36*a*b^4*c + 14*a^2*b^2*c^2
 + 100*a^3*c^3 + 420*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^4 + 4
5*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^2)*e^4*x^4 + 3*(4*b^5
*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 4*(84*(b^4*c^2 - 7*a*b^2*
c^3 + 10*a^2*c^4)*d^5 + 15*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3
)*d^3 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d)*e^
3*x^3 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 2*(3*b^6 - 18*a*b
^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^4 + (168*(b^4*c^2 - 7*a*b^2*
c^3 + 10*a^2*c^4)*d^6 + 9*a*b^5 - 68*a^2*b^3*c + 122*a^3*b*c^2 +
45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^4 + 12*(3*b^6 - 18*a
*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^2)*e^2*x^2 + (9*a*b^5 - 68
*a^2*b^3*c + 122*a^3*b*c^2)*d^2 + 2*(24*(b^4*c^2 - 7*a*b^2*c^3 +
10*a^2*c^4)*d^7 + 9*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^5 +
 4*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^3 + (9*a*b
^5 - 68*a^2*b^3*c + 122*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*e^11*f^3*x^10 + 10*(a^3*b^4*c^2 - 8*a^4*b^2
*c^3 + 16*a^5*c^4)*d*e^10*f^3*x^9 + (2*a^3*b^5*c - 16*a^4*b^3*c^2
 + 32*a^5*b*c^3 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^2)*e^9*f^3*x^8 + 8*(15*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4
)*d^3 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^8*f^3*x
^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 210*(a^3*b^4*c^2 - 8*a
^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d^2)*e^7*f^3*x^6 + 2*(126*(a^3*b^4*c^2 - 8*a^4*b^2*c^
3 + 16*a^5*c^4)*d^5 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^3 + 3*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^6*f^3*x^5 +
(2*a^4*b^5 - 16*a^5*b^3*c + 32*a^6*b*c^2 + 210*(a^3*b^4*c^2 - 8*a
^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 140*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^4 + 15*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e
^5*f^3*x^4 + 4*(30*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7
 + 28*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*
a^6*b*c^2)*d)*e^4*f^3*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 +
 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 56*(a^3*b^5*
c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 15*(a^3*b^6 - 6*a^4*b^4*c
 + 32*a^6*c^3)*d^4 + 12*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^
2)*e^3*f^3*x^2 + 2*(5*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*
d^9 + 8*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + 3*(a^3*b
^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 4*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e^2*f^
3*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^10 + 2*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^8 + (a^3*b^6 - 6*a^4*b^4*c
 + 32*a^6*c^3)*d^6 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^4
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d^2)*e*f^3) + 3*integrate
(((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^3*x^3 + 3*(b^5*c - 8*a*b
```

```
^3*c^2 + 16*a^2*b*c^3)*d*e^2*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*d^3 + (b^6 - 9*a*b^4*c + 23*a^2*b^2*c^2 - 10*a^3*c^3 + 3*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e*x + (b^6 - 9*a*b^4*c +
 23*a^2*b^2*c^2 - 10*a^3*c^3)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d
^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a),
x)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*f^3) - 3*b*log(e*x + d)/
(a^4*e*f^3)
```

Fricas [A] time = 6.10038, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^3),x, algorithm='
```

```
[Out] [-1/4*(3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*
e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*
c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^
3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^
5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20
*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a
^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c
^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^
2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 3
0*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a
^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*
c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b
^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^
3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2
+ 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a
^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*
(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 +
28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5
*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)
*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d
)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c
^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c
^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 +
56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 1
5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4
)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)
*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5
*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^
3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*
c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 -
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40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*
a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^
5*c^3)*d)*e*x)*log((2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*
c^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + (2*c^2*e^4
*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2
+ 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c)*sqrt(b^2 -
4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x
^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (6*(a*b^4*c^2 - 7*a^2*
b^2*c^3 + 10*a^3*c^4)*e^8*x^8 + 48*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 1
0*a^3*c^4)*d*e^7*x^7 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c
^3 + 56*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^2)*e^6*x^6 + 6
(56(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^3 + 3*(4*a*b^5*c
- 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d)*e^5*x^5 + 6*(a*b^4*c^2 - 7*a^
2*b^2*c^3 + 10*a^3*c^4)*d^8 + (6*a*b^6 - 36*a^2*b^4*c + 14*a^3*b^
2*c^2 + 100*a^4*c^3 + 420*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4
)*d^4 + 45*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^2)*e^4*x
^4 + 2*a^3*b^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 3*(4*a*b^5*c - 29*a^
2*b^3*c^2 + 46*a^3*b*c^3)*d^6 + 4*(84*(a*b^4*c^2 - 7*a^2*b^2*c^3
+ 10*a^3*c^4)*d^5 + 15*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3
)*d^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d
)*e^3*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^
3)*d^4 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2 + 168*(a*b^4*c
^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^6 + 45*(4*a*b^5*c - 29*a^2*b^3
*c^2 + 46*a^3*b*c^3)*d^4 + 12*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2
*c^2 + 50*a^4*c^3)*d^2)*e^2*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122
*a^4*b*c^2)*d^2 + 2*(24*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*
d^7 + 9*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^5 + 4*(3*a*
b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d^3 + (9*a^2*b^5
- 68*a^3*b^3*c + 122*a^4*b*c^2)*d)*e*x - 3*((b^5*c^2 - 8*a*b^3*c
^3 + 16*a^2*b*c^4)*e^10*x^10 + 10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*d*e^9*x^9 + (2*b^6*c - 16*a*b^4*c^2 + 32*a^2*b^2*c^3 + 45
*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2)*e^8*x^8 + 8*(15*(b^5
*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3 + 2*(b^6*c - 8*a*b^4*c^2 +
16*a^2*b^2*c^3)*d)*e^7*x^7 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3 + 2
10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 + 56*(b^6*c - 8*a*b
^4*c^2 + 16*a^2*b^2*c^3)*d^2)*e^6*x^6 + (b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*d^10 + 2*(126*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4
)*d^5 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^3 + 3*(b^7 -
6*a*b^5*c + 32*a^3*b*c^3)*d)*e^5*x^5 + 2*(b^6*c - 8*a*b^4*c^2 + 1
6*a^2*b^2*c^3)*d^8 + (2*a*b^6 - 16*a^2*b^4*c + 32*a^3*b^2*c^2 + 2
10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^6 + 140*(b^6*c - 8*a*
b^4*c^2 + 16*a^2*b^2*c^3)*d^4 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^
3)*d^2)*e^4*x^4 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^6 + 4*(30*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^7 + 28*(b^6*c - 8*a*b^4*c^
2 + 16*a^2*b^2*c^3)*d^5 + 5*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^3
+ 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d)*e^3*x^3 + 2*(a*b^6
- 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (45*(b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*d^8 + a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 56*(
b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^6 + 15*(b^7 - 6*a*b^5*c +
32*a^3*b*c^3)*d^4 + 12*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^
2)*e^2*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2 + 2*(5*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^9 + 8*(b^6*c - 8*a*b^4*c^2
+ 16*a^2*b^2*c^3)*d^7 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^5 +
4*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3 + (a^2*b^5 - 8*a^3*

b^3*c + 16*a^4*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d
^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
 12*((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^10*x^10 + 10*(b^5*c
^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^9*x^9 + (2*b^6*c - 16*a*b^4*
c^2 + 32*a^2*b^2*c^3 + 45*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
d^2)*e^8*x^8 + 8*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3 +
 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d)*e^7*x^7 + (b^7 - 6*a
*b^5*c + 32*a^3*b*c^3 + 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4
)*d^4 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^2)*e^6*x^6 +
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^10 + 2*(126*(b^5*c^2 - 8
*a*b^3*c^3 + 16*a^2*b*c^4)*d^5 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2
*b^2*c^3)*d^3 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d)*e^5*x^5 + 2
*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^8 + (2*a*b^6 - 16*a^2*b
^4*c + 32*a^3*b^2*c^2 + 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4
)*d^6 + 140*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^4 + 15*(b^7
- 6*a*b^5*c + 32*a^3*b*c^3)*d^2)*e^4*x^4 + (b^7 - 6*a*b^5*c + 32*
a^3*b*c^3)*d^6 + 4*(30*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^7
 + 28*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^5 + 5*(b^7 - 6*a*b
^5*c + 32*a^3*b*c^3)*d^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^
2)*d)*e^3*x^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (4
5*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^8 + a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^
6 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^4 + 12*(a*b^6 - 8*a^2*b
^4*c + 16*a^3*b^2*c^2)*d^2)*e^2*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16
*a^4*b*c^2)*d^2 + 2*(5*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^9
 + 8*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^7 + 3*(b^7 - 6*a*b^
5*c + 32*a^3*b*c^3)*d^5 + 4*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2
)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e*x)*log(e*x +
d))*sqrt(b^2 - 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^
4)*e^11*f^3*x^10 + 10*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*
d*e^10*f^3*x^9 + (2*a^4*b^5*c - 16*a^5*b^3*c^2 + 32*a^6*b*c^3 + 4
5*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^2)*e^9*f^3*x^8 + 8
(15(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^3 + 2*(a^4*b^5*
c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d)*e^8*f^3*x^7 + (a^4*b^6 - 6*a
^5*b^4*c + 32*a^7*c^3 + 210*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6
*c^4)*d^4 + 56*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^2)*e^
7*f^3*x^6 + 2*(126*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^5
 + 56*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^3 + 3*(a^4*b^6
 - 6*a^5*b^4*c + 32*a^7*c^3)*d)*e^6*f^3*x^5 + (2*a^5*b^5 - 16*a^6
*b^3*c + 32*a^7*b*c^2 + 210*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6
*c^4)*d^6 + 140*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^4 +
15*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^2)*e^5*f^3*x^4 + 4*(30*
(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^7 + 28*(a^4*b^5*c -
8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^5 + 5*(a^4*b^6 - 6*a^5*b^4*c + 32
*a^7*c^3)*d^3 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d)*e^4*f
^3*x^3 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2 + 45*(a^4*b^4*c^2 -
8*a^5*b^2*c^3 + 16*a^6*c^4)*d^8 + 56*(a^4*b^5*c - 8*a^5*b^3*c^2 +
 16*a^6*b*c^3)*d^6 + 15*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^4
+ 12*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d^2)*e^3*f^3*x^2 + 2*
(5*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^9 + 8*(a^4*b^5*c
- 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^7 + 3*(a^4*b^6 - 6*a^5*b^4*c +
32*a^7*c^3)*d^5 + 4*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d^3 +
(a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*d)*e^2*f^3*x + ((a^4*b^4*c^2
 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^10 + 2*(a^4*b^5*c - 8*a^5*b^3*c^

2 + 16*a^6*b*c^3)*d^8 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^6
+ 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d^4 + (a^6*b^4 - 8*a^7
*b^2*c + 16*a^8*c^2)*d^2)*e*f^3)*sqrt(b^2 - 4*a*c)), 1/4*(6*((b^6
*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10
*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9
 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(
b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x^
8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*
d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)
*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 4
0*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3
*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*
c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20
*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 2
0*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b
^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^
2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*
b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2
*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c
^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 4
0*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10*
a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 -
 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10
*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^
6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^
7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*
(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*
(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 + a^2*
b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10
*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b
^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*
b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2
+ (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*
(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8
*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^7 + 3*(
b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d
^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3
 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)
*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) - (6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*e
^8*x^8 + 48*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d*e^7*x^7 +
3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3 + 56*(a*b^4*c^2 - 7*
a^2*b^2*c^3 + 10*a^3*c^4)*d^2)*e^6*x^6 + 6*(56*(a*b^4*c^2 - 7*a^2
*b^2*c^3 + 10*a^3*c^4)*d^3 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a
^3*b*c^3)*d)*e^5*x^5 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)
*d^8 + (6*a*b^6 - 36*a^2*b^4*c + 14*a^3*b^2*c^2 + 100*a^4*c^3 + 4
20*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^4 + 45*(4*a*b^5*c -
 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^2)*e^4*x^4 + 2*a^3*b^4 - 16*a^4
*b^2*c + 32*a^5*c^2 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^
3)*d^6 + 4*(84*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^5 + 15*
(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^3 + 2*(3*a*b^6 - 18
*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d)*e^3*x^3 + 2*(3*a*b^6
- 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d^4 + (9*a^2*b^5 - 6

8*a^3*b^3*c + 122*a^4*b*c^2 + 168*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10
*a^3*c^4)*d^6 + 45*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^
4 + 12*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*d^2)
*e^2*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2)*d^2 + 2*(24
*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*d^7 + 9*(4*a*b^5*c - 29
*a^2*b^3*c^2 + 46*a^3*b*c^3)*d^5 + 4*(3*a*b^6 - 18*a^2*b^4*c + 7*
a^3*b^2*c^2 + 50*a^4*c^3)*d^3 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a
^4*b*c^2)*d)*e*x - 3*((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^10
x^10 + 10(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^9*x^9 + (2*
b^6*c - 16*a*b^4*c^2 + 32*a^2*b^2*c^3 + 45*(b^5*c^2 - 8*a*b^3*c^3
 + 16*a^2*b*c^4)*d^2)*e^8*x^8 + 8*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16
*a^2*b*c^4)*d^3 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d)*e^7
*x^7 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3 + 210*(b^5*c^2 - 8*a*b^3*c
^3 + 16*a^2*b*c^4)*d^4 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3
)*d^2)*e^6*x^6 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^10 + 2*
(126*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^5 + 56*(b^6*c - 8*a
*b^4*c^2 + 16*a^2*b^2*c^3)*d^3 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^
3)*d)*e^5*x^5 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^8 + (2
*a*b^6 - 16*a^2*b^4*c + 32*a^3*b^2*c^2 + 210*(b^5*c^2 - 8*a*b^3*c
^3 + 16*a^2*b*c^4)*d^6 + 140*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^
3)*d^4 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^2)*e^4*x^4 + (b^7
- 6*a*b^5*c + 32*a^3*b*c^3)*d^6 + 4*(30*(b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*d^7 + 28*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^5
 + 5*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^3 + 2*(a*b^6 - 8*a^2*b^4*
c + 16*a^3*b^2*c^2)*d)*e^3*x^3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*
b^2*c^2)*d^4 + (45*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^8 + a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 56*(b^6*c - 8*a*b^4*c^2 + 1
6*a^2*b^2*c^3)*d^6 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^4 + 12
*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^2)*e^2*x^2 + (a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2 + 2*(5*(b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*d^9 + 8*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^7
+ 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^5 + 4*(a*b^6 - 8*a^2*b^4*c
 + 16*a^3*b^2*c^2)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d
)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*
x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4)*e^10*x^10 + 10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*d*e^9*x^9 + (2*b^6*c - 16*a*b^4*c^2 + 32*a^2*b^2*c^3 +
45*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2)*e^8*x^8 + 8*(15*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3 + 2*(b^6*c - 8*a*b^4*c^2
 + 16*a^2*b^2*c^3)*d)*e^7*x^7 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3 +
 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 + 56*(b^6*c - 8*a
*b^4*c^2 + 16*a^2*b^2*c^3)*d^2)*e^6*x^6 + (b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*d^10 + 2*(126*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c
^4)*d^5 + 56*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^3 + 3*(b^7
- 6*a*b^5*c + 32*a^3*b*c^3)*d)*e^5*x^5 + 2*(b^6*c - 8*a*b^4*c^2 +
 16*a^2*b^2*c^3)*d^8 + (2*a*b^6 - 16*a^2*b^4*c + 32*a^3*b^2*c^2 +
 210*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^6 + 140*(b^6*c - 8*
a*b^4*c^2 + 16*a^2*b^2*c^3)*d^4 + 15*(b^7 - 6*a*b^5*c + 32*a^3*b*
c^3)*d^2)*e^4*x^4 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^6 + 4*(30*
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^7 + 28*(b^6*c - 8*a*b^4*
c^2 + 16*a^2*b^2*c^3)*d^5 + 5*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^
3 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d)*e^3*x^3 + 2*(a*b^
6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (45*(b^5*c^2 - 8*a*b^3*c^
3 + 16*a^2*b*c^4)*d^8 + a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 56

```
*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^6 + 15*(b^7 - 6*a*b^5*c
 + 32*a^3*b*c^3)*d^4 + 12*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
d^2)*e^2*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2 + 2*(5*
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^9 + 8*(b^6*c - 8*a*b^4*c
^2 + 16*a^2*b^2*c^3)*d^7 + 3*(b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^5
 + 4*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3 + (a^2*b^5 - 8*a^
3*b^3*c + 16*a^4*b*c^2)*d)*e*x)*log(e*x + d))*sqrt(-b^2 + 4*a*c))
/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*e^11*f^3*x^10 + 10*
(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d*e^10*f^3*x^9 + (2*a^
4*b^5*c - 16*a^5*b^3*c^2 + 32*a^6*b*c^3 + 45*(a^4*b^4*c^2 - 8*a^5
*b^2*c^3 + 16*a^6*c^4)*d^2)*e^9*f^3*x^8 + 8*(15*(a^4*b^4*c^2 - 8*
a^5*b^2*c^3 + 16*a^6*c^4)*d^3 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16
*a^6*b*c^3)*d)*e^8*f^3*x^7 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3
+ 210*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^4 + 56*(a^4*b^
5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^2)*e^7*f^3*x^6 + 2*(126*(a^
4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^5 + 56*(a^4*b^5*c - 8*a
^5*b^3*c^2 + 16*a^6*b*c^3)*d^3 + 3*(a^4*b^6 - 6*a^5*b^4*c + 32*a^
7*c^3)*d)*e^6*f^3*x^5 + (2*a^5*b^5 - 16*a^6*b^3*c + 32*a^7*b*c^2
+ 210*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*d^6 + 140*(a^4*b
^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^4 + 15*(a^4*b^6 - 6*a^5*b^
4*c + 32*a^7*c^3)*d^2)*e^5*f^3*x^4 + 4*(30*(a^4*b^4*c^2 - 8*a^5*b
^2*c^3 + 16*a^6*c^4)*d^7 + 28*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6
*b*c^3)*d^5 + 5*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^3 + 2*(a^5
*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d)*e^4*f^3*x^3 + (a^6*b^4 - 8*
a^7*b^2*c + 16*a^8*c^2 + 45*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6
*c^4)*d^8 + 56*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^6 + 1
5*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^4 + 12*(a^5*b^5 - 8*a^6*
b^3*c + 16*a^7*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(5*(a^4*b^4*c^2 - 8*a^
5*b^2*c^3 + 16*a^6*c^4)*d^9 + 8*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a
^6*b*c^3)*d^7 + 3*(a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^5 + 4*(a
^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d^3 + (a^6*b^4 - 8*a^7*b^2*c
 + 16*a^8*c^2)*d)*e^2*f^3*x + ((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*
a^6*c^4)*d^10 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*d^8
+ (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*d^6 + 2*(a^5*b^5 - 8*a^6*b
^3*c + 16*a^7*b*c^2)*d^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*d
^2)*e*f^3)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.335031, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(((e*x + d)^4*c + (e*x + d)^2*b + a)^3*(e*f*x + d*f)^3),x, algorithm='
```

```
[Out] Done
```

3.661 $\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal. Leaf size=340

$$\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

```
[Out] -((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/
3, 1/2, 1/2, 4/3, (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2
*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^2*Sqrt[a + b*(d + e*
x)^3 + c*(d + e*x)^6])) + ((d + e*x)^2*Sqrt[1 + (2*c*(d + e*x)^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])
/(2*e^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])
```

Rubi [A] time = 1.5102, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```


```
[Out] -((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/
3, 1/2, 1/2, 4/3, (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2
*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^2*Sqrt[a + b*(d + e*
x)^3 + c*(d + e*x)^6])) + ((d + e*x)^2*Sqrt[1 + (2*c*(d + e*x)^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])
/(2*e^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])
```

Rubi in Sympy [A] time = 85.5295, size = 299, normalized size = 0.88

$$-\frac{d(d+ex)\sqrt{a+b(d+ex)^3+c(d+ex)^6}\operatorname{appellf}_1\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}},-\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}\right)}{ae^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}+1}}+\frac{(d+ex)^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}\operatorname{appellf}_1\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}},-\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}\right)}{2ae^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] -d*(d + e*x)*sqrt(a + b*(d + e*x)**3 + c*(d + e*x)**6)*appellf1(1
/3, 1/2, 1/2, 4/3, -2*c*(d + e*x)**3/(b - sqrt(-4*a*c + b**2)), -
2*c*(d + e*x)**3/(b + sqrt(-4*a*c + b**2)))/(a*e**2*sqrt(2*c*(d +
 e*x)**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*(d + e*x)**3/(b
+ sqrt(-4*a*c + b**2)) + 1)) + (d + e*x)**2*sqrt(a + b*(d + e*x)*
*3 + c*(d + e*x)**6)*appellf1(2/3, 1/2, 1/2, 5/3, -2*c*(d + e*x)*
*3/(b - sqrt(-4*a*c + b**2)), -2*c*(d + e*x)**3/(b + sqrt(-4*a*c
+ b**2)))/(2*a*e**2*sqrt(2*c*(d + e*x)**3/(b - sqrt(-4*a*c + b**2
)) + 1)*sqrt(2*c*(d + e*x)**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [A] time = 20.6174, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+b(ex+d)^3+c(ex+d)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

```
[Out] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x}{\sqrt{ce^6x^6+6\,cde^5x^5+15\,cd^2e^4x^4+cd^6+(20\,cd^3+b)e^3x^3+3\,(5\,cd^4+bd)e^2x^2+bd^3+3\,(2\,cd^5+bd^2)ex+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*
d^6 + (20*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3
+ 3*(2*c*d^5 + b*d^2)*e*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5x^5+ce^6x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e
**3*x**3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**
3*e**3*x**3 + 15*c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.662 $\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal. Leaf size=398

$$\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3}$$

```
[Out] (d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/
3, 1/2, 1/2, 4/3, (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2
*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^3*Sqrt[a + b*(d + e*
x)^3 + c*(d + e*x)^6]) - (d*(d + e*x)^2*Sqrt[1 + (2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]
)/(e^3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ArcTanh[(b + 2*
c*(d + e*x)^3)/(2*Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]
)]/(3*Sqrt[c]*e^3)
```

Rubi [A] time = 1.6095, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] (d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/
3, 1/2, 1/2, 4/3, (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2
*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^3*Sqrt[a + b*(d + e*
x)^3 + c*(d + e*x)^6]) - (d*(d + e*x)^2*Sqrt[1 + (2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]
)/(e^3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ArcTanh[(b + 2*
c*(d + e*x)^3)/(2*Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]
)]/(3*Sqrt[c]*e^3)
```

Rubi in Sympy [A] time = 101.353, size = 354, normalized size = 0.89

$$\frac{\operatorname{atanh}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3} + \frac{d^2\,(d+ex)\,\sqrt{a+b\,(d+ex)^3+c\,(d+ex)^6}\operatorname{appellf_1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}},-\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}\right)}{ae^3\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}+1}} - \frac{d\,(d+ex)^2\,\sqrt{a+b\,(d+ex)^3+c\,(d+ex)^6}\operatorname{appellf_1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}},-\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}\right)}{ae^3\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2c(d+ex)^3}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] atanh((b + 2*c*(d + e*x)**3)/(2*sqrt(c)*sqrt(a + b*(d + e*x)**3 +
 c*(d + e*x)**6)))/(3*sqrt(c)*e**3) + d**2*(d + e*x)*sqrt(a + b*(
d + e*x)**3 + c*(d + e*x)**6)*appellf1(1/3, 1/2, 1/2, 4/3, -2*c*(
d + e*x)**3/(b - sqrt(-4*a*c + b**2)), -2*c*(d + e*x)**3/(b + sqr
t(-4*a*c + b**2)))/(a*e**3*sqrt(2*c*(d + e*x)**3/(b - sqrt(-4*a*c
 + b**2)) + 1)*sqrt(2*c*(d + e*x)**3/(b + sqrt(-4*a*c + b**2)) +
1)) - d*(d + e*x)**2*sqrt(a + b*(d + e*x)**3 + c*(d + e*x)**6)*ap
pellf1(2/3, 1/2, 1/2, 5/3, -2*c*(d + e*x)**3/(b - sqrt(-4*a*c + b
**2)), -2*c*(d + e*x)**3/(b + sqrt(-4*a*c + b**2)))/(a*e**3*sqrt(
2*c*(d + e*x)**3/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*(d + e*x
)**3/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [A] time = 2.21127, size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a+b\,(ex+d)^3+c\,(ex+d)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

```
[Out] int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{ce^6x^6+6\,cde^5x^5+15\,cd^2e^4x^4+cd^6+(20\,cd^3+b)e^3x^3+3\,(5\,cd^4+bd)e^2x^2+bd^3+3\,(2\,cd^5+bd^2)ex+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 +
c*d^6 + (20*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^
3 + 3*(2*c*d^5 + b*d^2)*e*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bd^3 + 3bd^2ex + 3bde^2x^2 + be^3x^3 + cd^6 + 6cd^5ex + 15cd^4e^2x^2 + 20cd^3e^3x^3 + 15cd^2e^4x^4 + 6cde^5x^5 + ce^6x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 +
b*e**3*x**3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*
d**3*e**3*x**3 + 15*c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x
**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(ex+d)^6c + (ex+d)^3b + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.663 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx$

Optimal. Leaf size=34

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rubi [A] time = 0.0797298, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3x+2)^{21}}{63}+\frac{(3x+2)^7}{21}+\frac{\int^{(3x+2)^7} x\,dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

```
[Out] (3*x + 2)**21/63 + (3*x + 2)**7/21 + Integral(x, (x, (3*x + 2)**7
))/21
```

Mathematica [A] time = 0.0139212, size = 34, normalized size = 1.

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Maple [B] time = 0.003, size = 105, normalized size = 3.1

$$\frac{1162261467\,x^{21}}{7}+2324522934\,x^{20}+15496819560\,x^{19}+65431015920\,x^{18}+196293047760\,x^{17}+444930908256\,x^{16}+790988281344\,x^{15}+\frac{15819767221203\,x^{14}}{14}+1318314865122\,x^{13}+1269491970942\,x^{12}+1015602174288\,x^{11}+677082445416\,x^{10}+376174427616\,x^{9}+173635132896\,x^{8}+66158154783\,x^{7}+20588764518\,x^{6}+5149786572\,x^{5}+1010576952\,x^{4}+149902032\,x^{3}+15808800\,x^{2}+1056832\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x)
```

```
[Out] 1162261467/7*x^21+2324522934*x^20+15496819560*x^19+65431015920*x^
18+196293047760*x^17+444930908256*x^16+790988281344*x^15+15819767
221203/14*x^14+1318314865122*x^13+1269491970942*x^12+101560217428
8*x^11+677082445416*x^10+376174427616*x^9+173635132896*x^8+661581
54783*x^7+20588764518*x^6+5149786572*x^5+1010576952*x^4+149902032
*x^3+15808800*x^2+1056832*x
```

Maxima [A] time = 0.782857, size = 140, normalized size = 4.12

$$\frac{1162261467}{7}\,x^{21}+2324522934\,x^{20}+15496819560\,x^{19}+65431015920\,x^{18}+196293047760\,x^{17}+444930908256\,x^{16}+790988281344\,x^{15}+\frac{15819767221203}{14}\,x^{14}+1318314865122\,x^{13}+1269491970942\,x^{12}+1015602174288\,x^{11}+677082445416\,x^{10}+376174427616\,x^{9}+173635132896\,x^{8}+66158154783\,x^{7}+20588764518\,x^{6}+5149786572\,x^{5}+1010576952\,x^{4}+149902032\,x^{3}+15808800\,x^{2}+1056832\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((3*x + 2)^14 + (3*x + 2)^7 + 1)*(3*x + 2)^6,x, algorithm="maxima")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015
920*x^18 + 196293047760*x^17 + 444930908256*x^16 + 790988281344*x
^15 + 15819767221203/14*x^14 + 1318314865122*x^13 + 1269491970942
```

```
*x^12 + 1015602174288*x^11 + 677082445416*x^10 + 376174427616*x^9
 + 173635132896*x^8 + 66158154783*x^7 + 20588764518*x^6 + 5149786
572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15808800*x^2 + 1056832
*x
```

Fricas [A] time = 0.244678, size = 1, normalized size = 0.03

$$\frac{1162261467}{7}x^{21} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203}{14}x^{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^{9} + 173635132896x^{8} + 66158154783x^{7} + 20588764518x^{6} + 5149786572x^{5} + 1010576952x^{4} + 149902032x^{3} + 15808800x^{2} + 1056832x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((3*x + 2)^14 + (3*x + 2)^7 + 1)*(3*x + 2)^6,x, algorithm="fricas")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015
920*x^18 + 196293047760*x^17 + 444930908256*x^16 + 790988281344*x
^15 + 15819767221203/14*x^14 + 1318314865122*x^13 + 1269491970942
*x^12 + 1015602174288*x^11 + 677082445416*x^10 + 376174427616*x^9
 + 173635132896*x^8 + 66158154783*x^7 + 20588764518*x^6 + 5149786
572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15808800*x^2 + 1056832
*x
```

Sympy [A] time = 0.176918, size = 107, normalized size = 3.15

$$\frac{1162261467x^{21}}{7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203x^{14}}{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^{9} + 173635132896x^{8} + 66158154783x^{7} + 20588764518x^{6} + 5149786572x^{5} + 1010576952x^{4} + 149902032x^{3} + 15808800x^{2} + 1056832x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

```
[Out] 1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431
015920*x**18 + 196293047760*x**17 + 444930908256*x**16 + 79098828
```

```
1344*x**15 + 15819767221203*x**14/14 + 1318314865122*x**13 + 1269
491970942*x**12 + 1015602174288*x**11 + 677082445416*x**10 + 3761
74427616*x**9 + 173635132896*x**8 + 66158154783*x**7 + 2058876451
8*x**6 + 5149786572*x**5 + 1010576952*x**4 + 149902032*x**3 + 158
08800*x**2 + 1056832*x
```

GIAC/XCAS [A] time = 0.27301, size = 38, normalized size = 1.12

$$\frac{1}{63}\,(3\,x+2)^{21}+\frac{1}{42}\,(3\,x+2)^{14}+\frac{1}{21}\,(3\,x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((3*x + 2)^14 + (3*x + 2)^7 + 1)*(3*x + 2)^6,x, algorithm="giac")
```

```
[Out] 1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

3.664 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28
/42 + (2 + 3*x)^35/105
```

Rubi [A] time = 0.190905, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28
/42 + (2 + 3*x)^35/105
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3x+2)^{35}}{105}+\frac{(3x+2)^{28}}{42}+\frac{(3x+2)^{21}}{21}+\frac{(3x+2)^{7}}{21}+\frac{2\int^{(3x+2)^7} x\,dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

```
[Out] (3*x + 2)**35/105 + (3*x + 2)**28/42 + (3*x + 2)**21/21 + (3*x +
2)**7/21 + 2*Integral(x, (x, (3*x + 2)**7))/21
```

Mathematica [B] time = 0.0124685, size = 188, normalized size = 3.36

$$\frac{16677181699666569x^{35}}{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256x^{30}}{5} + 67899784121041365504x^{29} + \frac{2625458326972530284475x^{28}}{14} + 437576396725285446564x^{27} + 875152864622814086340x^{26} + \frac{7584660010542711771792x^{25}}{5} + 2298383223254096766840x^{24} + 3064515076512846852480x^{23} + 3614565944605222108800x^{22} + \frac{26506949038858918036881x^{21}}{7} + 3534290697929473864098x^{20} + 2945285062308448290360x^{19} + 2194577166014752240080x^{18} + 1463104032160519033200x^{17} + 872775774067455498528x^{16} + 465517091041681015296x^{15} + 221699757548270194389x^{14} + 94069263918929616324x^{13} + 35454069480572048124x^{12} + 11821487501620716192x^{11} + \frac{17344958593049772048x^{10}}{5} + 889942562270387136x^{9} + 197897276851452864x^{8} + 37727143432895007x^{7} + 6077684727888102x^{6} + \frac{4057390785756924x^{5}}{5} + 87406679578680x^{4} + 7299544818384x^{3} + 443569828128x^{2} + 17451466816x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] 17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 8740667957
8680*x^4 + (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 3772
7143432895007*x^7 + 197897276851452864*x^8 + 889942562270387136*x
^9 + (17344958593049772048*x^10)/5 + 11821487501620716192*x^11 +
35454069480572048124*x^12 + 94069263918929616324*x^13 + 221699757
548270194389*x^14 + 465517091041681015296*x^15 + 8727757740674554
98528*x^16 + 1463104032160519033200*x^17 + 2194577166014752240080
*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^20
 + (26506949038858918036881*x^21)/7 + 3614565944605222108800*x^22
 + 3064515076512846852480*x^23 + 2298383223254096766840*x^24 + (7
584660010542711771792*x^25)/5 + 875152864622814086340*x^26 + 4375
76396725285446564*x^27 + (2625458326972530284475*x^28)/14 + 67899
784121041365504*x^29 + (101849676181562048256*x^30)/5 + 492821013
7817518464*x^31 + 924039400840784712*x^32 + 126005372841925188*x^
33 + 11118121133111046*x^34 + (16677181699666569*x^35)/35
```

Maple [B] time = 0.005, size = 175, normalized size = 3.1

$$17451466816x + 353429069792947386409 8x^{20} + 2194577166014752240080x^{18} + 2945285062308448290360x^{19} + \frac{26506949038858918036881x^{21}}{7} + \frac{26254583269725302844 75x^{28}}{14} + 2298383223254096766840x^{24} + 875152864622814086340x^{26} + 443569828128x^{2} + 87406679578680x^{4} + \frac{4057390785756924x^{5}}{5} + 7299544818384x^{3} + \frac{16677181699666569x^{35}}{35} + 4928210137817518464x^{31} + \frac{101849676181562048256x^{30}}{5} + 67899784121041365504x^{29} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 1463104032160519033200x^{17} + 872775774067455498528x^{16} + 197897276851452864x^{8} + 6077684727888102x^{6} + 221699757548270194389x^{14} + 465517091041681015296x^{15} + 889942562270387136x^{9} + \frac{17344958593049772048x^{10}}{5} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 94069263918929616324x^{13} + 437576396725285446564x^{27} + 3614565944605222108800x^{22} + 3064515076512846852480x^{23} + \frac{7584660010542711771792x^{25}}{5} + 37727143432895007x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x)
```

```
[Out] 17451466816*x+353429069792947386409 8*x^20+2194577166014752240080*
x^18+2945285062308448290360*x^19+26506949038858918036881/7*x^21+2
6254583269725302844 75/14*x^28+2298383223254096766840*x^24+8751528
64622814086340*x^26+443569828128*x^2+87406679578680*x^4+405739078
5756924/5*x^5+7299544818384*x^3+16677181699666569/35*x^35+4928210
137817518464*x^31+101849676181562048256/5*x^30+678997841210413655
04*x^29+11118121133111046*x^34+126005372841925188*x^33+9240394008
40784712*x^32+1463104032160519033200*x^17+872775774067455498528*x
^16+197897276851452864*x^8+6077684727888102*x^6+22169975754827019
4389*x^14+465517091041681015296*x^15+889942562270387136*x^9+17344
958593049772048/5*x^10+11821487501620716192*x^11+3545406948057204
8124*x^12+94069263918929616324*x^13+437576396725285446564*x^27+36
14565944605222108800*x^22+3064515076512846852480*x^23+75846600105
42711771792/5*x^25+37727143432895007*x^7
```

Maxima [A] time = 0.782346, size = 235, normalized size = 4.2

$$\frac{16677181699666569}{35}\,x^{35} + 11118121133111046\,x^{34} + 126005372841925188\,x^{33} + 924039400840784712\,x^{32} + 4928210137817518464\,x^{31} + \frac{10184967618156204825 6}{5}\,x^{30} + 6789978412104136550 4\,x^{29} + \frac{2625458326972530284475}{14}\,x^{28} + 437576396725285446564\,x^{27} + 875152864622814086340\,x^{26} + \frac{7584660010542711771792}{5}\,x^{25} + 2298383223254096766840\,x^{24} + 3064515076512846852480\,x^{23} + 3614565944605222108800\,x^{22} + \frac{26506949038858918036881}{7}\,x^{21} + 3534290697929473864098\,x^{20} + 2945285062308448290360\,x^{19} + 2194577166014752240080\,x^{18} + 1463104032160519033200\,x^{17} + 872775774067455498528\,x^{16} + 465517091041681015296\,x^{15} + 221699757548270194389\,x^{14} + 94069263918929616324\,x^{13} + 35454069480572048124\,x^{12} + 11821487501620716192\,x^{11} + \frac{17344958593049772048}{5}\,x^{10} + 889942562270387136\,x^{9} + 197897276851452864\,x^{8} + 37727143432895007\,x^{7} + 6077684727888102\,x^{6} + \frac{4057390785756924}{5}\,x^{5} + 87406679578680\,x^{4} + 7299544818384\,x^{3} + 443569828128\,x^{2} + 17451466816\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((3*x + 2)^14 + (3*x + 2)^7 + 1)^2*(3*x + 2)^6,x, algorithm="maxima")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841
925188*x^33 + 924039400840784712*x^32 + 4928210137817518464*x^31
+ 101849676181562048256/5*x^30 + 67899784121041365504*x^29 + 2625
458326972530284475/14*x^28 + 437576396725285446564*x^27 + 8751528
64622814086340*x^26 + 7584660010542711771792/5*x^25 + 22983832232
54096766840*x^24 + 3064515076512846852480*x^23 + 3614565944605222
108800*x^22 + 26506949038858918036881/7*x^21 + 353429069792947386
4098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*
x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 +
 465517091041681015296*x^15 + 221699757548270194389*x^14 + 940692
63918929616324*x^13 + 35454069480572048124*x^12 + 118214875016207
16192*x^11 + 17344958593049772048/5*x^10 + 889942562270387136*x^9
 + 197897276851452864*x^8 + 37727143432895007*x^7 + 6077684727888
102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 + 729954481
8384*x^3 + 443569828128*x^2 + 17451466816*x
```

Fricas [A] time = 0.225785, size = 1, normalized size = 0.02

$$\frac{16677181699666569}{35}x^{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256}{5}x^{30} + 67899784121041365504x^{29} + \frac{2625458326972530284475}{14}x^{28} + 437576396725285446564x^{27} + 875152864622814086340x^{26} + \frac{7584660010542711771792}{5}x^{25} + 2298383223254096766840x^{24} + 3064515076512846852480x^{23} + 3614565944605222108800x^{22} + \frac{26506949038858918036881}{7}x^{21} + 3534290697929473864098x^{20} + 2945285062308448290360x^{19} + 2194577166014752240080x^{18} + 1463104032160519033200x^{17} + 872775774067455498528x^{16} + 465517091041681015296x^{15} + 221699757548270194389x^{14} + 94069263918929616324x^{13} + 35454069480572048124x^{12} + 11821487501620716192x^{11} + \frac{17344958593049772048}{5}x^{10} + 889942562270387136x^{9} + 197897276851452864x^{8} + 37727143432895007x^{7} + 6077684727888102x^{6} + \frac{4057390785756924}{5}x^{5} + 87406679578680x^{4} + 7299544818384x^{3} + 443569828128x^{2} + 17451466816x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((3*x + 2)^14 + (3*x + 2)^7 + 1)^2*(3*x + 2)^6,x, algorithm="fricas")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841
925188*x^33 + 924039400840784712*x^32 + 4928210137817518464*x^31
+ 101849676181562048256/5*x^30 + 67899784121041365504*x^29 + 2625
458326972530284475/14*x^28 + 437576396725285446564*x^27 + 8751528
64622814086340*x^26 + 7584660010542711771792/5*x^25 + 22983832232
54096766840*x^24 + 3064515076512846852480*x^23 + 3614565944605222
108800*x^22 + 26506949038858918036881/7*x^21 + 353429069792947386
4098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*
x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 +
 465517091041681015296*x^15 + 221699757548270194389*x^14 + 940692
63918929616324*x^13 + 35454069480572048124*x^12 + 118214875016207
16192*x^11 + 17344958593049772048/5*x^10 + 889942562270387136*x^9
 + 197897276851452864*x^8 + 37727143432895007*x^7 + 6077684727888
102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 + 729954481
8384*x^3 + 443569828128*x^2 + 17451466816*x
```

Sympy [A] time = 0.307388, size = 187, normalized size = 3.34

$$\frac{16677181699666569x^{35}}{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256x^{30}}{5} + 67899784121041365504x^{29} + \frac{2625458326972530284475x^{28}}{14} + 437576396725285446564x^{27} + 875152864622814086340x^{26} + \frac{7584660010542711771792x^{25}}{5} + 2298383223254096766840x^{24} + 3064515076512846852480x^{23} + 3614565944605222108800x^{22} + \frac{26506949038858918036881x^{21}}{7} + 3534290697929473864098x^{20} + 2945285062308448290360x^{19} + 2194577166014752240080x^{18} + 1463104032160519033200x^{17} + 872775774067455498528x^{16} + 465517091041681015296x^{15} + 221699757548270194389x^{14} + 94069263918929616324x^{13} + 35454069480572048124x^{12} + 11821487501620716192x^{11} + \frac{17344958593049772048x^{10}}{5} + 889942562270387136x^{9} + 197897276851452864x^{8} + 37727143432895007x^{7} + 6077684727888102x^{6} + \frac{4057390785756924x^{5}}{5} + 87406679578680x^{4} + 7299544818384x^{3} + 443569828128x^{2} + 17451466816x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

```
[Out] 16677181699666569*x**35/35 + 11118121133111046*x**34 + 1260053728
41925188*x**33 + 924039400840784712*x**32 + 4928210137817518464*x
**31 + 101849676181562048256*x**30/5 + 67899784121041365504*x**29
 + 2625458326972530284475*x**28/14 + 437576396725285446564*x**27
+ 875152864622814086340*x**26 + 7584660010542711771792*x**25/5 +
2298383223254096766840*x**24 + 3064515076512846852480*x**23 + 361
4565944605222108800*x**22 + 26506949038858918036881*x**21/7 + 353
4290697929473864098*x**20 + 2945285062308448290360*x**19 + 219457
7166014752240080*x**18 + 1463104032160519033200*x**17 + 872775774
067455498528*x**16 + 465517091041681015296*x**15 + 22169975754827
0194389*x**14 + 94069263918929616324*x**13 + 35454069480572048124
*x**12 + 11821487501620716192*x**11 + 17344958593049772048*x**10/
5 + 889942562270387136*x**9 + 197897276851452864*x**8 + 377271434
32895007*x**7 + 6077684727888102*x**6 + 4057390785756924*x**5/5 +
 87406679578680*x**4 + 7299544818384*x**3 + 443569828128*x**2 + 1
7451466816*x
```

GIAC/XCAS [A] time = 0.269389, size = 62, normalized size = 1.11

$$\frac{1}{105}\,(3\,x+2)^{35}+\frac{1}{42}\,(3\,x+2)^{28}+\frac{1}{21}\,(3\,x+2)^{21}+\frac{1}{21}\,(3\,x+2)^{14}+\frac{1}{21}\,(3\,x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((3*x + 2)^14 + (3*x + 2)^7 + 1)^2*(3*x + 2)^6,x, algorithm="giac")
```

```
[Out] 1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21
*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```